ME-351 THERMODYNAMICS AND ENERGETICS II

SPRING 2025

QUESTION 1 : LEGENDRE TRANSFORMS AND CHARACTERISTIC POTENTIALS

An electrically polarizable material is placed between the plates of a capacitor under ambient temperature and pressure as shown in fig. 1. Applying a voltage across the capacitor plates results in an electric field of magnitude $\vec{E}$ across the material. Let the induced polarization in the material be denoted as $\vec{\mathcal{P}}$

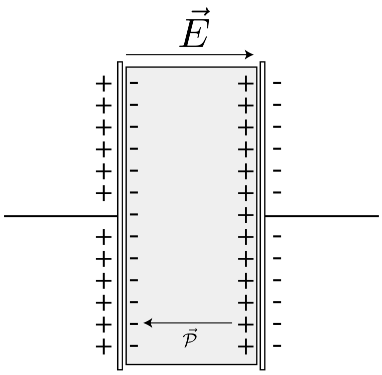

Figure 1

1. List all the relevant state variables of the *polarizable material*. Be sure to indicate the pairs of variables that are *conjugate* to each other.
2. What are the state variables of the polarizable material that are controlled experimentally?
3. Write down an expression for dU (where U is the internal energy of the polarizable material) in terms of the state variables of the material.
4. Write down the expression for the characteristic potential of the material under these boundary conditions
5. Calculate the equations of state for the material based on the characteristic potential.

QUESTION 2 : MASSIEU FUNCTIONS

During class, we derived the Legendre transforms of the internal energy $U(S, V)$. Similar Legendre transforms can also be derived for the entropy $S(U, V)$. Start from the fundamental relation in thermodynamics:

$$dS = \frac{dU}{T} + \frac{p}{T}dV$$

1. Identify the natural variables of S and the variables that are conjugate to the natural variables.
2. Define the three Legendre transforms of the entropy $S(U, V)$. *HINT : The three Legendre transforms of $U(S, V)$, transformed the internal energy into three different characteristic potentials: $H(S, p)$, $F(T, V)$ and $G(T, p)$*
3. Write the differential form of each Legendre transform

The Legendre transformations in the entropy representation are called Massieu functions - named after François Massieu who developed them in 1869.

QUESTION 3

A rod having initial length L_0 is elongated to a length L_1 by applying a force f under isothermal conditions. The temperature of the rod is T_0. The state variables of the rod are related by the following equation:

$$L = L_0 + \alpha(T - T_0) + \beta f$$

where L, T, and f are the length, temperature and force applied on the rod. α, β, T_0, L_0 are constants.

1. What are the thermodynamic state variables of the rod that are controlled experimentally?
2. Write down an expression for the characteristic potential of the rod and derive the equations of state.
3. Show that :

$$\left(\frac{\partial S}{\partial f}\right)_T = \left(\frac{\partial L}{\partial T}\right)_f$$

4. The rod is elongated reversibly from a length L_0 to L_1.
   (a) What is the work done on the rod during elongation?
   (b) Use the Maxwell relation from above to calculate the heat exchanged between the rod and the environment
5. The rod is elongated by applying a constant force of magnitude $f = \frac{L_1 - L_0}{\beta}$
   (a) Is this process reversible or irreversible?
   (b) What is the work done on the rod by the environment?
   (c) Calculate the heat exchanged between the rod and the environment.

QUESTION 4

Consider a solid that is placed in an environment where we control the temperature (T), pressure (p), and magnetic field (H) across it.

1. How many independent response functions can be defined for this system? Write down expressions for each response function. Give them different Greek letter names. Ensure that each response function is independent of system size.
   (*HINT : Notice that response functions that we have encountered so far have the form $\partial X/\partial Y$, where X is an extensive variable and* Y *is an intensive variable*)
2. The solid at constant T and p and zero applied magnetic field is suddenly placed in a magnetic field of magnitude H_1. Derive an expression for the change in internal energy of the solid after it has reached equilibrium at the same temperature and pressure, but in a magnetic field of H_1. Write the change in internal energy in terms of measurable state variables and response functions.