ME-351 THERMODYNAMICS AND ENERGETICS II

SPRING 2025

QUESTION 1 : MORE FUN WITH IDEAL GASES

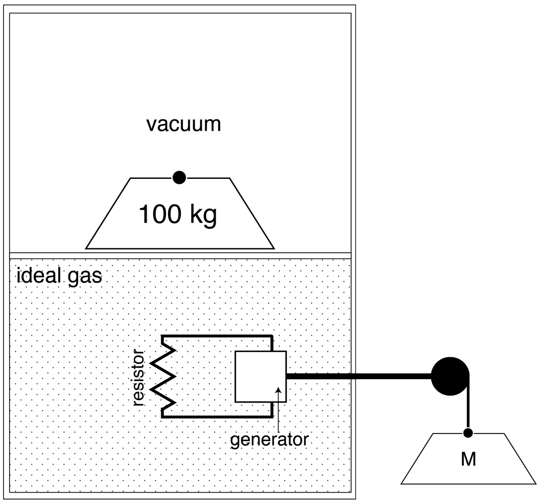

Figure 1

Figure 1 shows one mole of an ideal gas in the bottom compartment of a diathermal container that is held in an environment at 298K. The ideal gas is separated from the top of the container by a frictionless piston (with surface area 1 m^2). A constant weight of 100 kg is placed on the piston. A circuit with a resistor and a generator passes through the compartment with the gas. A frictionless pulley system converts the mechanical work of dropping a weight of mass M =10 kg by a height of 10m into an electric current that passes through the resistor. The specific heats of an ideal gas are given by $C_V = \frac{5}{2}R$ and $C_p = \frac{7}{2}R$. You can assume that the environment is very large and stays at constant temperature and pressure.

1. Calculate the entropy change of the gas after it equilibrates with the environment
2. Calculate the entropy change of the universe after the gas equilibrates with the enviroment.

QUESTION 2 : CAN WE BUILD A BETTER ENGINE THAN CARNOT?

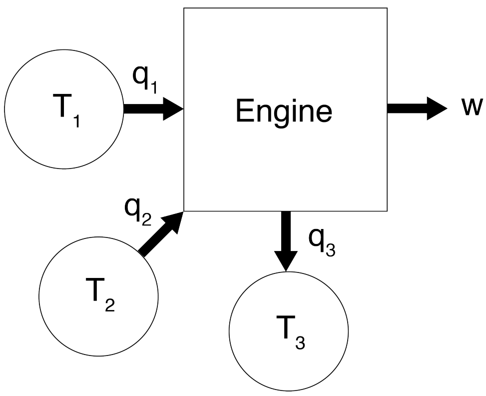

Figure 2

You are tasked with assessing the following claim made in a new patent application. The patent applicant claims to have invented an engine that is more efficient than the Carnot limit by introducing two stages of heat input to the engine. The process diagram for the cyclic operation of the engine is shown in fig. 2. In the first stage, the engine takes in q_1 amount of heat from the heat reservoir at a temperature T_1. This is followed by another stage where the engine accepts additional heat, q_2, from a heat reservoir at a temperature T_2. In the final stage, the engine returns to its original state by rejecting heat q_3 into a cold reservoir at temperature T_3 and performing W amount of work. The temperatures of the reservoirs are such that $T_1 > T_2 > T_3$.

1. Express the work generated by the engine in terms of the heat entering / leaving the system
2. Apply the second law of thermodynamics to the cyclic process performed by the engine.
3. What is the maximum amount of work that can be extracted from the system?
4. Can the efficiency of this engine be higher than the Carnot limit?

QUESTION 3 : REVERSIBLE VS IRREVERSIBLE WORK

Show that irreversible work will always be greater than the reversible work done by the system during a process that changes the state of the system:

$$w_{\text{reversible}} < w_{\text{irreversible}}$$

HINT: Consider a change of state of the system between two states 1 and 2 that are very close to each other. Start by writing down the differential form of the first law.

QUESTION 4 : ADIABATIC WORK ON AN IDEAL GAS

An ideal gas has the following equations of state:

$$pV = nRT$$

$$U = nC_V T$$

where $p, V, T, n, U,$ and C_v are the pressure, volume, temperature, number of moles, internal energy and specific heat capacity at constant volume of the ideal gas. R is the ideal gas constant. The specific heat capacity of an ideal gas is constant.

1. During the reversible adiabatic compression (or expansion) of the ideal gas, show that pV^γ is constant. where $\gamma = \frac{C_V+R}{C_V}$. *HINT: Start from the differential form of the first law*
2. Show that the work required to adiabatically change the state of an ideal gas from (p_1, V_1) to (p_2, V_2) is $w = \frac{p_2V_2 - p_1V_1}{1-\gamma}$

QUESTION 5 : EFFICIENCY OF THE JOULE CYCLE

Figure 3 shows the Joule cycle of an ideal gas on a p-V diagram. All processes are quasi-static. Show that the efficiency of an engine performing this cycle is:

$$\eta = 1 - \left(\frac{p_1}{p_2}\right)^{\frac{\gamma-1}{\gamma}}$$

where $\gamma = \frac{C_p}{C_V} = \frac{C_V+R}{C_V}$

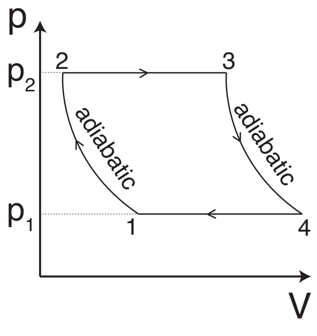

Figure 3

QUESTION 6 : IDEAL GASES

Consider a gas in state 1 (p_1, V_1, T_1) that is composed of one mole of an ideal gas for which $pV = nRT$ and $U = A + BT$. A and B are constants. The system is taken through a cyclic process in four steps. The first step is a very slow compression to state 2 (p_2, V_2) at a constant temperature of T_1. The second step is a quick compression to state 3 (p_3, V_3, T_3). The third step is a very slow expansion to state 4 (p_4, V_4) at a constant temperature T_3. The final step involves a quick compression back to state 1. The second and final steps are performed so quickly that there is not heat exchange.

1. Sketch the four steps in the cyclic process on a $p - V$ diagram. Clearly indicate which of the steps are reversible and irreversible.
2. For each process, calculate the work done, heat exchanged and internal energy change.
3. Express the total work done during the cyclic process in terms of T_1, T_3, and the heat exchange during the first and third steps