ME-351 THERMODYNAMICS AND ENERGETICS II

SPRING 2025

QUESTION 1 : EXACT AND INEXACT DIFFERENTIALS

Consider the following differentials:

$$df = xy(3x + 2y)dx + x^2(x + 2y)dy$$

$$dg = xy(3x + 2y^2)dx + x^2(x + 2y)dy$$

1. Integrate the two differentials from $x = 0, y = 0$ to $x = 1, y = 1$. Compute the integral along two paths, the first is along $y = x$, and the second along $y = x^2$
2. Which of the two differentials is an exact differential?
3. How would you identify the exact differential without performing an integral along different paths?

QUESTION 2 : IDEAL GAS

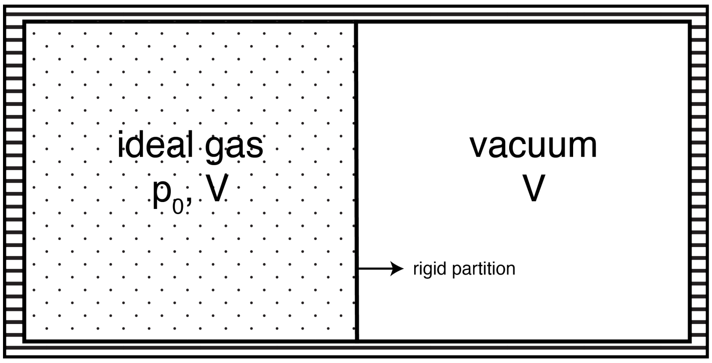

Figure 1

A container of total volume $2V$ contains two chambers of equal size separated by a rigid partition. One of the chambers is filled with one mole of ideal gas with an initial pressure, p_0, and volume V. The other half of the container is empty (i.e. contains vacuum). A schematic picture of the container is shown in fig. 2. The rigid partition is suddenly removed and the gas expands to fill the entire container

1. Assume the walls of the container are adiabatic:
   (a) What is the work performed by the gas during expansion?
   (b) Estimate the change in internal energy (ΔU) due to the removal of the wall.
   (c) What is the final temperature (T_1) of the gas?
2. Instead of adiabatic walls, what if the container was made of diathermic walls? Estimate the work, change in internal energy and final temperature of the gas.

QUESTION 3 : CALLEN 1.8-6

For a particular gas it is found that if the volume is kept constant at the value V_0 and the pressure is changed from p_0 to an arbitrary pressure p, the heat transfer (q) into the system is:

$$q = A(p - p_0)$$

where A is a positive constant. In addition it is known that the adiabats of the system are of the form:

$$pV^\gamma = \text{constant}$$

where $\gamma > 0$. Calculate an expression for the internal energy of the gas at any arbitrary point (p, V). Express the internal energy, $U(p, V)$, in terms of p_0, V_0, A, γ, and $U_0 = U(p_0, V_0)$

QUESTION 4

A thin-walled metal container of volume V contains a gas at high pressure. The container is connected to a capillary tube with a frictionless piston (fig. 2). The valve connecting the gas container to the capillary tube is opened such that the gas leaks slowly into the tube. What is the work done by the gas after it reaches its final equilibrium volume of V_f?

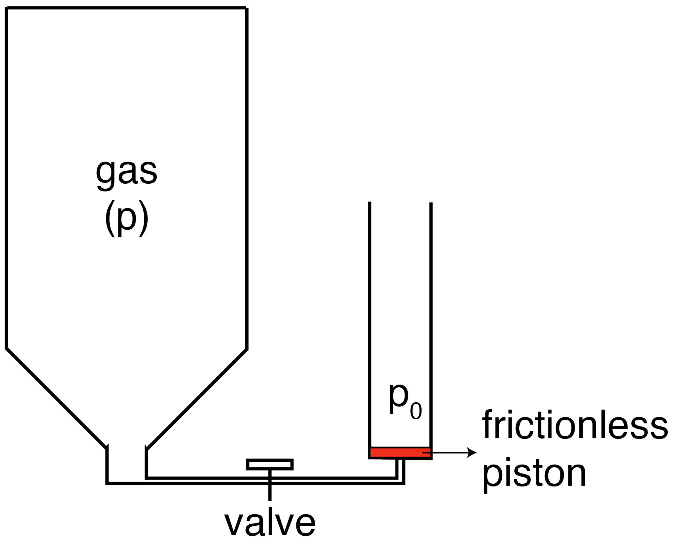

Figure 2

QUESTION 5

Compute the work done upon the quasi-static isothermal expansion of 1 mol of a van der Waals gas from an initial volume V_i to V_f. The equation of state of a van der Waals gas is:

$$\left(p + \frac{a}{V^2}\right)(V - b) = RT \tag{1}$$

where a , b and R are constants.

QUESTION 6

The pressure on 100g of nickel is increased quasi-statically and isothermally from 0 to 500 atm. Assuming the isothermal compressibility ($\kappa = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T$) remains constant at a value of 6.75×10^{-12}Pa^{-1}, compute the work. The density of nickel at 0 atm is 8.9×10^3 kg/m^3.

QUESTION 7

The equation of state of an ideal elastic wire is:

$$f = CT\left(\frac{l}{l_0} - \frac{l_0^2}{l^2}\right) \tag{2}$$

where C is a constant, l_0 is the length of the wire at zero tension (dependent on only temperature), f is the force applied and l is the length of the wire. Compute the work necessary to compress the wire from an initial length of l_0 to $l_0/2$ quasi-statically and isothermally.