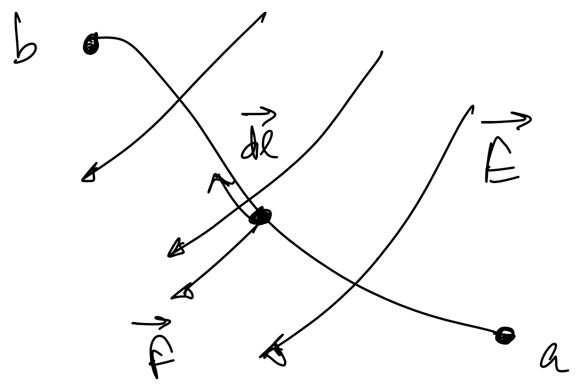
Moving charge in an electric field



$\vec{E} = -\nabla\phi$ → potential

$\vec{F} = Z\vec{E}$

charge

$\delta W = -\vec{F}\cdot\vec{dl}$ → $\delta W = \vec{f}\cdot\vec{dl}$

force we are applying to move the object

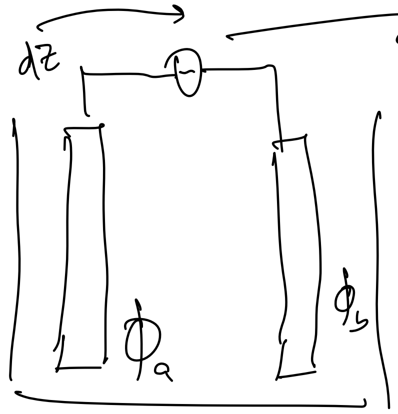
"work done $\underline{\underline{\text{on}}}$ the charge against the action of the field"

$$\delta W = Z\nabla\phi\, \vec{dl}$$

$$= Z\left(\frac{\partial\phi}{\partial x}dx + \frac{\partial\phi}{\partial y}dy + \frac{\partial\phi}{\partial z}dz\right)$$

$$W = \int_a^b Z\nabla\phi\cdot\vec{dl} = Z(\phi_b - \phi_a) \Rightarrow \boxed{W = \Delta\phi Z}$$

battery

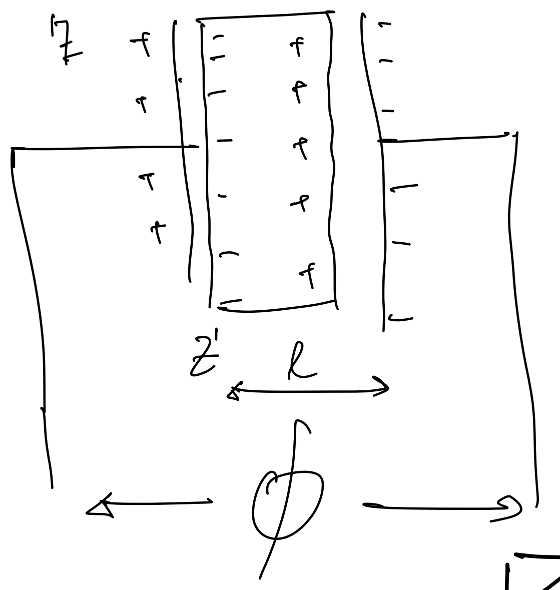


→ high resistance (trickle of charge)

$$\delta W = \Delta\phi\, dZ$$

$\mathcal{E}$ electromotive force

* work to polarize a material (eg a ferroelectric material)



$\vec{E} \to$ electric field

$\vec{P} = \varepsilon_0 \chi_e \vec{E}$ → material property.

$\vec{P}$ ↳ induced polarization in the material (per unit volume)

$$\boxed{\begin{array}{l} \delta W = \vec{E}\, d\vec{\mathcal{P}} \\ \text{where } \vec{\mathcal{P}} = V\vec{P} \end{array}}$$

V ↳ volume of the material.

$\vec{\mathcal{P}}$ → dipole moment

* work to magnetize a material

$$\delta W = \mu_0 H \, dM$$

M → total magnetic moment

μ_0: permeability of free space

H: applied magnetic field

SUMMARY:

form of work	δW	state variables	
lengthen a rod	$F\,dl$	F	l
hydrostatic pressure	$-p\,dV$	p	V
general strain energy	$\sum_i \sum_j V_0 \sigma_{ij}\, d\varepsilon_{ij}$	σ_{ij}	$V_0 \varepsilon_{ij}$
moving charge through a potential	$\phi\, dZ_i$	ϕ	Z_i
polarization	$\vec{E}\, d\vec{P}$	$\vec{E}$	$\vec{P}$
magnetization	$\mu_0 \vec{H}\, d\vec{M}$	$\vec{H}$	$\vec{M}$

GENERAL $\quad \delta W = Y \cdot dX \longrightarrow$ reversible work expressed in terms of state variables of the system

Y: intensive (force)

X: extensive (displacement)

Y & X are **CONJUGATE VARIABLES**

Exact & Inexact differentials:

$d \longrightarrow$ exact

$\delta \longrightarrow$ inexact

$\vec{F} \cdot d\vec{r} \longrightarrow$ exact?

exact = differential of a function

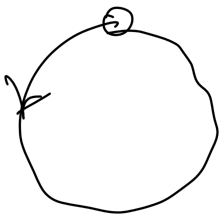
$$\vec{F} = \vec{\nabla} \phi(r)$$

$$F_x = \left(\frac{\partial \phi}{\partial x}\right)_{y,z} \qquad F_y = \left(\frac{\partial \phi}{\partial y}\right)_{x,z} \qquad F_x = \left(\frac{\partial \phi}{\partial z}\right)_{x,y}$$

Consequence

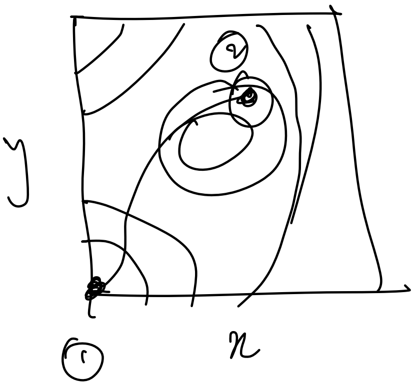
$$\int_{r_1}^{r_2} \vec{F} \cdot d\vec{r} = \int_{r_1}^{r_2} \vec{\nabla}\phi(r) \cdot d\vec{r} = \phi(\vec{r}_2) - \phi(\vec{r}_1)$$

$\vec{F} \cdot d\vec{r}$ = exact $\longrightarrow$ F is "conservative"



$\oint \vec{F} \cdot d\vec{r} = 0 \longrightarrow$ no net energy transfer

EXAMPLE:



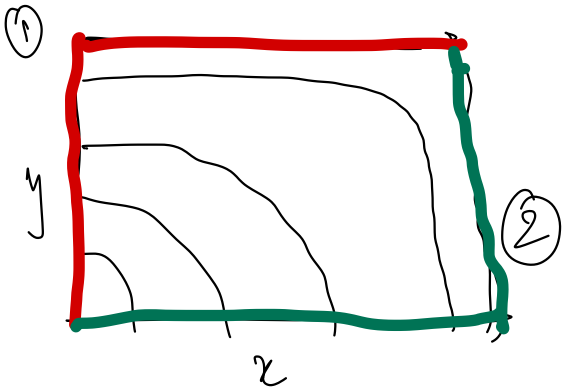
Calculate energy transfer

$$E_{\text{potential in field}} = mgh(x,y)$$

$$W = \int_1^2 \vec{F}\cdot d\vec{r} = \int_1^2 -\vec{\nabla}E \cdot d\vec{r}$$

$$\boxed{W = -mg(h_2 - h_1)}$$

What makes a differential inexact?



I Missing forces

$$dW = \int f_x\, dx$$

$$W(\text{path } 1) = \underbrace{0}_{dx = 0} + \underbrace{0}_{f_x = 0}$$

$$W(\text{path } 2) \neq 0$$

$$\delta W = \int f_x\, dx \longrightarrow$$ not exact

not complete / inexact

II Dissipative forces (eg friction)

$$\vec{F} = -\vec{\nabla}E - f\hat{e}_r$$

$\hookrightarrow$ unit vector along the path

$$\delta W = \int \vec{F}\cdot d\vec{r} = -\int_1^2 \vec{\nabla} E \cdot d\vec{r} - \int_1^2 f\,\hat{e}_r \cdot d\vec{r}$$

$$= -mg(h_2 - h_1) - f\,L$$ → length of the path

The force is not conservative:

$$\oint \delta W = -fL \neq 0$$

In all these "inexact" differentials we are missing additional contributions ⟹ "HEAT"

$$\underbrace{dU}_{\text{exact differential}} = \underbrace{\delta W}_{\text{inexact differential}} + \underbrace{\delta q}_{\text{inexact differential.}}$$

⟹ THERMO HAS 3 LAWS THAT INTRODUCE NEW STATE FUNCTIONS

* 0^{th} law ⟶ T (temperature)
* 1^{st} law ⟶ U (internal energy)
* 2^{nd} law ⟶ S (entropy)

FIRST LAW OF THERMODYNAMICS

- introduces internal energy state function
- equivalence of work and heat

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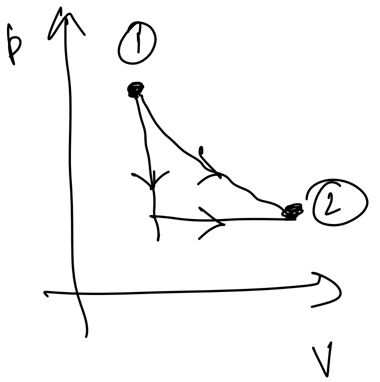
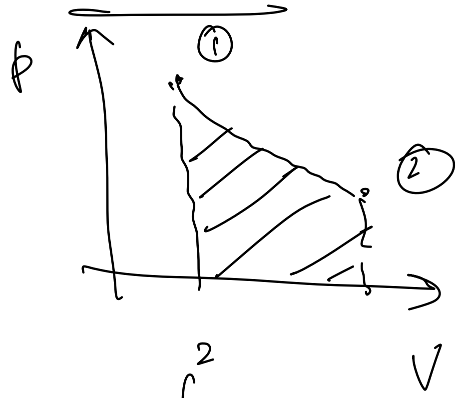
* work depends on path : (eg) ideal gas → reversible changes of state

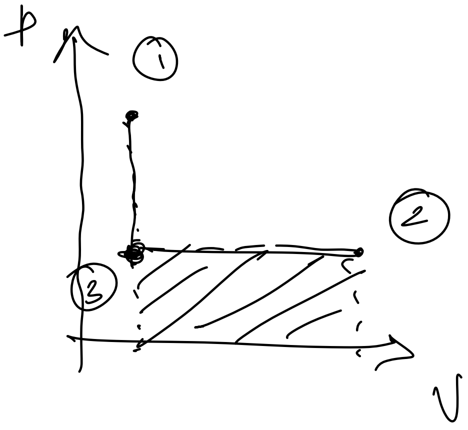


isothermal path



$$W = \int_1^2 -p\,dV$$

$$= \int_1^2 -\frac{RT}{V}\,dV = -RT\log\left(\frac{V_2}{V_1}\right)$$



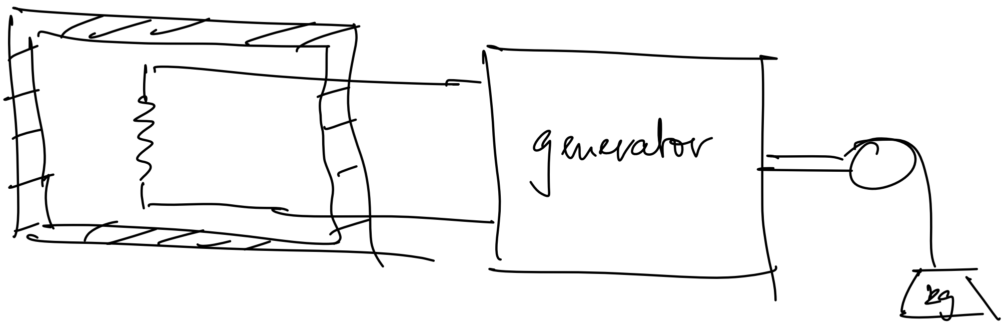
$$W = \underbrace{\int_{(1)}^{(3)} -p\,dV}_{0} + \underbrace{\int_{(3)}^{(2)} -p\,dV}_{-(V_2 - V_3)p}$$

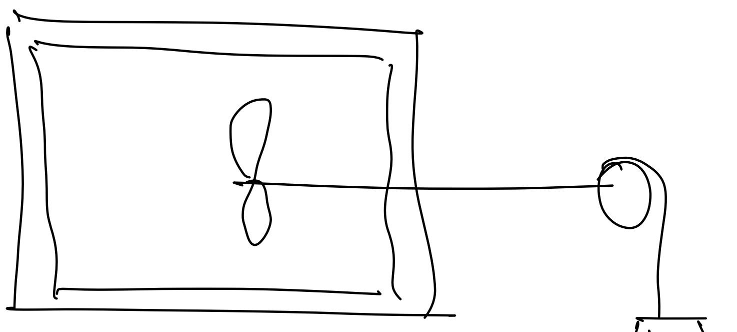
JOULE'S STATEMENT OF FIRST LAW:

If an adiabatic system is caused to change from a prescribed initial to a prescribed final state, the work expended is the same for all paths connecting the two states.

$$W^{adiabatic}_{1\to 2} = U_2 - U_1 = \Delta U$$

* Work performed adiabatically is independent of the process





same amount of work connects the same two states $(p_i, T_i) \mapsto (p_f, T_f)$

1^{st} law: * system caused to change from an initial state to a final state adiabatically

$\Rightarrow$ Work done on the system is the same for all adiabatic paths connecting the two states

$\Rightarrow$ IMPLIES STATE FUNCTION

INTERNAL ENERGY:

$$W_{i \to f} (\text{adiabatic}) = U_f - U_i = \Delta U$$

* general non-adiabatic transformation

$$\Delta U \neq W$$

$$\Delta U = W + Q \longrightarrow \text{valid for any process}$$

$$\boxed{dU = \delta W + \delta Q} \longrightarrow \text{differential form}$$

exact differential

inexact differential

reversible:

$$\boxed{dU = \sum_i Y_i \, dX_i + \delta Q}$$

SECOND LAW OF THERMODYNAMICS

Kelvin-Planck Statements $\longleftrightarrow$ Clausius Statements $\longleftrightarrow$ Carotheodory Statement

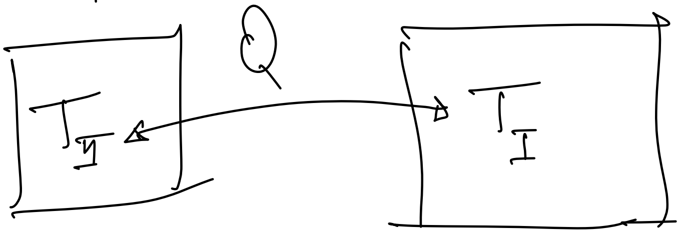
All equivalent $\Longrightarrow$ proofs can be found in any standard thermo book

$$dU = \delta Q + \sum_i Y_i \, dX_i$$

??
what do we do about the "heat" term?

expressed in terms of state variables of the system

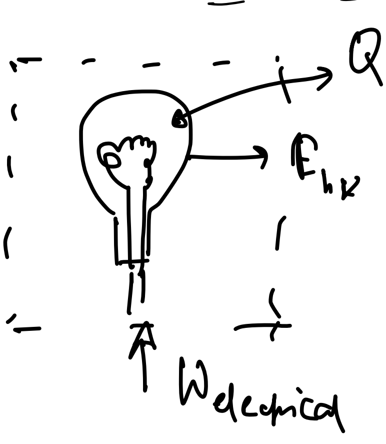
NOTICE: The I^{st} law does not make any statements about the direction of the process.



$T_{II} > T_I$

Based on the I^{st} law of thermo heat can flow from $II \rightarrow I$ & $I \rightarrow II$

(eg)

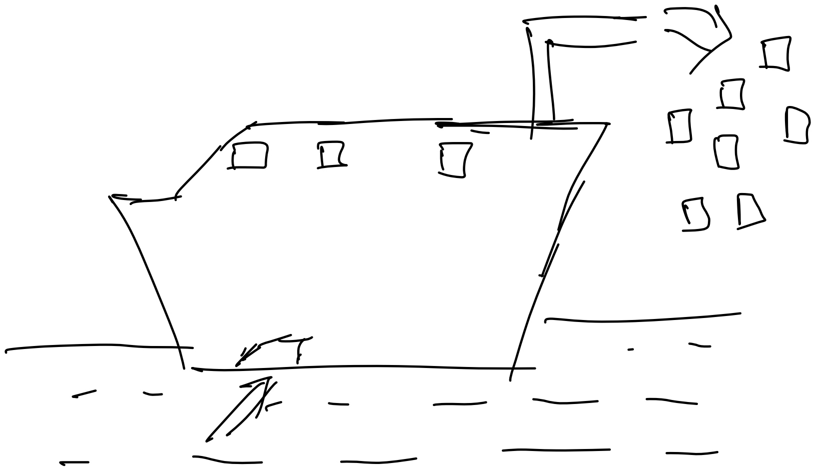


No reason (from the 1^{st} law) that we can't run this process in reverse.

★ POWERING THE BOATS IN LAC LEMAN WITH LAKE WATER:

Simplon | La Suisse : CGN boats $\longrightarrow$ 850 people !!

Ice $\rightarrow$ h_{ice}



"warm water" $\longrightarrow$ h_{in}

$\simeq$ 1500 horsepower (actually 1400) $\longrightarrow$ LOTS OF ENERGY: cars are usually $\simeq$ 100-200 hp

$\simeq 1120 \ kJ/s$

$$W = -1120 \ kJ/s = (h_{ice} - h_{in})\,\dot{m}$$

$$h_{ice} - h_{in} \simeq -334 \ J/g$$

$$\Rightarrow \boxed{\dot{m} \simeq 3.35 \ kg/s}$$