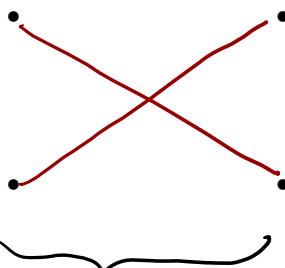


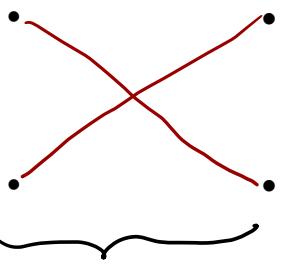
Problem 1

Match the definitions with lines (2 pt)

Minor loss



Due to viscous effect



$$K_L \frac{V^2}{2g}$$

Major loss

Due to various pipe components

$$f \frac{l}{D} \frac{V^2}{2g}$$

1 pt

1 pt

Problem 2

Complete the velocity equation between relative (W), rotational (U) and absolute (V) velocities for turbomachinery (2 pt)

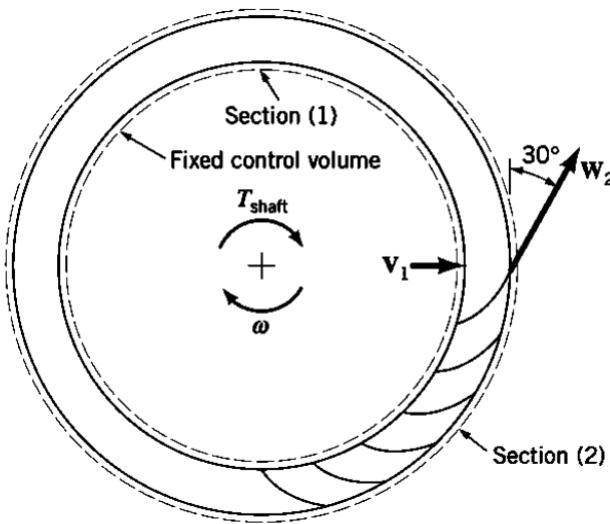
$$\boxed{V} = \boxed{W} + \boxed{U}$$

OR

$$V = U + w$$

Problem 3

An air system has a bladed rotor of 30 cm outside **diameter** and 25 cm inside **diameter**. The height of each rotor blade is constant at 2.5 cm from blade inlet to outlet. The **flowrate** is steady, on a time-average basis, at 0.1 m³/s and the absolute velocity of the air at blade inlet, \mathbf{V}_1 , is radial. The blade discharge angle is 30° from the tangential direction as shown in the figure. The rotor rotates at a constant speed of 1725 rpm. Air density is $\rho = 1.2 \text{ kg/m}^3$, the only torque we consider is the driving shaft torque.



3.1 Find the rotational velocity at r_1 and r_2 (2 pt).

$$1725 \text{ rpm} = 1725 \text{ [rev/min]} \times \frac{2\pi \text{ [rad/rev]}}{60 \text{ [s/min]}} = 180.6 \text{ rad/s}$$

$$U_1 = 0.25 \times \frac{1}{2} \times 180.6 = 22.58 \text{ m/s}$$

$$U_2 = 0.3 \times \frac{1}{2} \times 180.6 = 27.09 \text{ m/s}$$

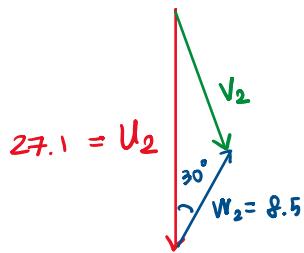
3.2 Compute the relative velocity magnitude at r_2 (3 pt).

$$Q = V A = V_{r2} A = \overline{W_{r2}} \cdot 2\pi r_2 \cdot h = \overline{W \sin \theta} \cdot 2\pi r_2 \cdot h$$

$$= \overline{W \sin \theta}$$

$$W = \frac{Q}{\sin 30^\circ \cdot 2\pi r_2 \cdot h} = \frac{0.1}{0.5 \times 2 \times 3.14 \times 0.15 \times 0.025} = 8.5 \text{ m/s}$$

3.3 Draw the velocity triangle at r_2 (3 pt).



3.4 Is this system turbine or pump (1pt), explain why (2 pt).

the system is pump

the tangential velocity of absolute velocity $V_{\theta 2}$ is in the same direction of the rotational velocity U_2 . (Note that because the inlet tangential velocity is zero, only the direction of tangential velocity is important in this situation)

3.5 Compute the shaft power of the system (3 pt).

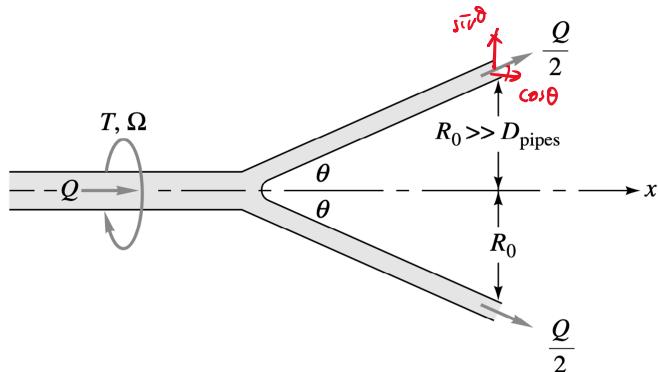
$$V_{\theta 2} = V_2 - U_2 = 27.1 - 8.5 \cos 30^\circ = 19.7 \text{ m/s}$$

$$T_{\text{shaft}} = \dot{m} r_2 V_{\theta 2} = \rho Q r_2 V_{\theta 2} = 1.2 \times 0.1 \times 0.15 \times 19.7 = 0.35 \text{ N.m}$$

$$\dot{W}_{\text{shaft}} = T_{\text{shaft}} \cdot \omega = 63.2 \text{ W}$$

Problem 4

The "Y" joint splits the pipe flow into equal amounts $Q/2$, which exit, as shown, at a distance R_0 from the axis. Neglect gravity and friction. Find an expression for the torque T_{shaft} about the x-axis required to keep the system rotating at angular velocity Ω .



The following are the answers from ChatGPT and Claude.

4.1 Which answer do you think is correct? (2pt)

ChatGPT
 Claude
 None of them

4.2 What is your answer and why? (5pt)

$$\vec{\tau} = \dot{m} (\vec{\tau} \times \vec{v})$$

$$\vec{v} = \vec{w} + \vec{u} = \left[\frac{r}{A} \sin\theta \ 0 \ \frac{r}{A} \cos\theta \right] + [0, R_0\Omega, 0]$$

$$= \left[\frac{Q}{A} \sin\theta, R_0\Omega, \frac{Q}{A} \cos\theta \right]$$

$$\vec{\tau} = \dot{m} \begin{vmatrix} i & j & k \\ R_0 & 0 & x \\ \frac{Q}{A} \sin\theta & R_0\Omega & \frac{Q}{A} \cos\theta \end{vmatrix}, \vec{T}_{\text{axial}} = \vec{T}_{\text{shaft}} = \dot{m} R_0^2 \Omega$$

$$= \underline{\underline{3QR_0^2\Omega}}$$

ChatGPT

To find the required torque T to maintain the system's rotation at angular velocity Ω , we apply the principle of angular momentum.

Step 1: Define the Given Parameters

- The inlet mass flow rate: $\dot{m} = \rho Q$
- The exit mass flow rates: $\frac{\rho Q}{2}$ for each branch
- The velocity of fluid at inlet: V_{in}
- The velocity of fluid at exits: V_{out}

Since the flow is incompressible and the pipe maintains constant cross-sectional area:

$$V_{\text{in}} A_{\text{in}} = Q$$

$$V_{\text{out}} A_{\text{out}} = \frac{Q}{2}$$

which implies $V_{\text{in}} = V_{\text{out}}$.

Step 2: Compute the Angular Momentum

The moment of momentum about the x -axis is given by:

$$H_x = \sum (r_i \times m_i V_i)$$

where r is the perpendicular distance from the x -axis.

- At the inlet, the fluid moves along the axis (x -direction), so the angular momentum contribution is **zero**.
- At the outlet, the velocity components perpendicular to the x -axis contribute to angular momentum.

For each branch, the velocity component perpendicular to the x -axis is:

$$V_{\perp} = V_{\text{out}} \sin \theta$$

The angular momentum for one branch:

$$H_{x,\text{one}} = \left(\frac{\rho Q}{2} \right) R_0 V_{\text{out}} \sin \theta$$

Since there are two branches:

$$H_x = 2 \times \left(\frac{\rho Q}{2} R_0 V_{\text{out}} \sin \theta \right) = \rho Q R_0 V_{\text{out}} \sin \theta$$

Step 3: Compute the Required Torque

The torque needed to maintain constant angular velocity is obtained from the rate of change of angular momentum:

$$T = \frac{dH_x}{dt}$$

Since the flow is steady,

$$T = \rho Q R_0 V_{\text{out}} \sin \theta$$

Final Expression:

$$T = \rho Q R_0 V_{\text{out}} \sin \theta$$

where $V_{\text{out}} = Q/A$.

Claude

Let's solve this problem step by step:

1. First, let's understand the scenario:

- A pipe with total flow Q splits into two equal flows of $Q/2$
- The split occurs at an angle θ in a wye joint
- The flow exits at a distance R_0 from the x-axis
- The system is rotating at an angular velocity Ω
- We need to find the torque T about the x-axis

2. To find the torque, we'll use the principle of conservation of angular momentum.

3. The momentum of each flow segment:

- Mass flow rate for each branch = $Q/2$
- Velocity components are perpendicular to the pipe axis
- Moment arm is $R_0 \sin(\theta)$

4. The torque will be the sum of the moments of each flow: $T = 2 * (Q/2) * R_0 * \sin(\theta)$

5. Simplifying: $T = Q * R_0 * \sin(\theta)$

Therefore, the expression for the torque T about the x-axis required to keep the system rotating at angular velocity Ω is:

$$T = Q * R_0 * \sin(\theta)$$

Cheat Sheet

$$T_{\text{shaft}} = -\dot{m}_1 (r_1 V_{\theta 1}) + \dot{m}_2 (r_2 V_{\theta 2})$$

$$\dot{W}_{\text{shaft}} = T_{\text{shaft}} \omega$$

$$\sum (\mathbf{r} \times \mathbf{F}) = \int_{cs} (\mathbf{r} \times \mathbf{V}) \rho \mathbf{V} \cdot \hat{\mathbf{n}} dA$$