

Name :

Grade

/25

Problem 1

Match the definitions with lines (2 pt)

Minor loss

Due to viscos effect

Major loss

Due to various pipe components

$$K_L \frac{V^2}{2g}$$

$$f \frac{l}{D} \frac{V^2}{2g}$$

1 pt

1 pt

Problem 2

Complete the velocity equation between relative (W), rotational (U) and absolute (V) velocities for turbomachinery (2 pt)

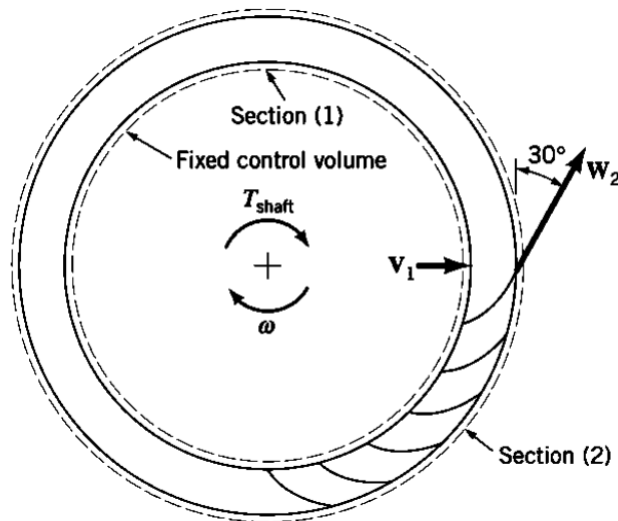
$$\boxed{V} = \boxed{W} + \boxed{U}$$

OR

$$V = U + W$$

Problem 3

An air system has a bladed rotor of 30 cm outside **diameter** and 25 cm inside **diameter**. The height of each rotor blade is constant at 2.5 cm from blade inlet to outlet. The **flowrate** is steady, on a time-average basis, at $0.1 \text{ m}^3/\text{s}$ and the absolute velocity of the air at blade inlet, \mathbf{V}_1 , is radial. The blade discharge angle is 30° from the tangential direction as shown in the figure. The rotor rotates at a constant speed of 1725 rpm. Air density is $\rho = 1.2 \text{ kg/m}^3$, the only torque we consider is the driving shaft torque.



3.1 Find the rotational velocity at r_1 and r_2 (2 pt).

$$1725 \text{ rpm} = 1725 [\text{rev/min}] \times \frac{2\pi [\text{rad/rev}]}{60 [\text{s/min}]} = 180.6 \text{ rad/s}$$

$$u_1 = 0.25 \times \frac{1}{2} \times 180.6 = 22.58 \text{ m/s}$$

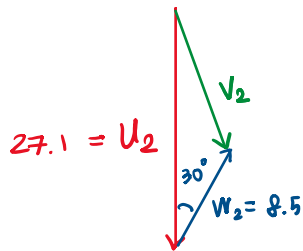
$$u_2 = 0.3 \times \frac{1}{2} \times 180.6 = 27.09 \text{ m/s}$$

3.2 Compute the relative velocity magnitude at r_2 (3 pt).

$$Q = VA = V_{r2} A = \underbrace{W_{r2}}_{= W \sin \theta} \cdot 2\pi r_2 \cdot h = W \sin \theta \cdot 2\pi r_2 \cdot h$$

$$W = \frac{Q}{\sin 30^\circ \cdot 2\pi r_2 \cdot h} = \frac{0.1}{0.5 \times 2 \times 3.14 \times 0.15 \times 0.025} = 8.5 \text{ m/s}$$

3.3 Draw the velocity triangle at r_2 (3 pt).



3.4 Is this system turbine or pump (1pt), explain why (2 pt).

the system is pump

the tangential velocity of absolute velocity $V_{\theta 2}$ is in the same direction of the rotational velocity U_2 . (Note that because the inlet tangential velocity is zero, only the direction of tangential velocity is important in this situation)

3.5 Compute the shaft power of the system (3 pt).

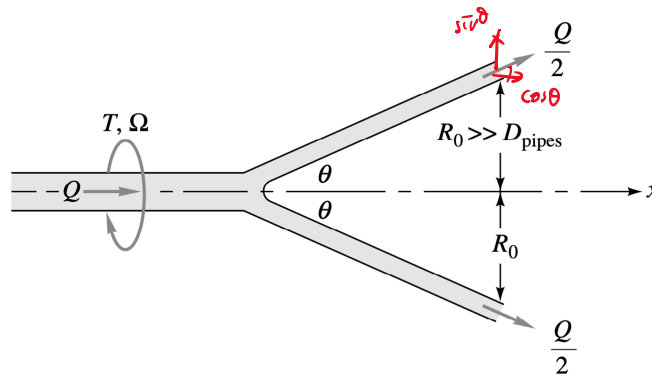
$$V_{\theta 2} = U_2 - W_{\theta 2} = 27.1 - 8.5 \cos 30^\circ = 19.7 \text{ m/s}$$

$$T_{\text{shaft}} = \dot{m} r_2 V_{\theta 2} = \rho Q r_2 V_{\theta 2} = 1.2 \times 0.1 \times 0.15 \times 19.7 = 0.35 \text{ N}\cdot\text{m}$$

$$\dot{W}_{\text{shaft}} = T_{\text{shaft}} \cdot \omega = 63.2 \text{ W}$$

Problem 4

The “Y” joint splits the pipe flow into equal amounts $Q/2$, which exit, as shown, at a distance R_0 from the axis. Neglect gravity and friction. Find an expression for the torque T_{shaft} about the x-axis required to keep the system rotating at angular velocity Ω .



The following are the answers from ChatGPT and Claude.

4.1 Which answer do you think is correct? (2pt)

☐ ChatGPT

☐ Claude

☒ None of them

4.2 What is your answer and why? (5pt)

$$\begin{aligned}
 \vec{T} &= \dot{m} (\vec{r} \times \vec{v}) \\
 \vec{v} &= \vec{W} + \vec{U} = \begin{bmatrix} \frac{Q}{A} \sin \theta & 0 & \frac{Q}{A} \cos \theta \end{bmatrix} + \begin{bmatrix} 0 & R_0 \Omega & 0 \end{bmatrix} \\
 &= \begin{bmatrix} \frac{Q}{A} \sin \theta & R_0 \Omega & \frac{Q}{A} \cos \theta \end{bmatrix} \\
 \vec{T} &= \dot{m} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ R_0 & 0 & x \\ \frac{Q}{A} \sin \theta & R_0 \Omega & \frac{Q}{A} \cos \theta \end{vmatrix}, \quad \vec{T}_{\text{axial}} = T_{\text{shaft}} = \dot{m} R_0^2 \Omega \\
 &= \underline{\underline{P Q R_0^2 \Omega}}
 \end{aligned}$$

ChatGPT

To find the required torque T to maintain the system's rotation at angular velocity Ω , we apply the principle of angular momentum.

Step 1: Define the Given Parameters

- The inlet mass flow rate: $\dot{m} = \rho Q$
- The exit mass flow rates: $\frac{\rho Q}{2}$ for each branch
- The velocity of fluid at inlet: V_{in}
- The velocity of fluid at exits: V_{out}

Since the flow is incompressible and the pipe maintains constant cross-sectional area:

$$V_{\text{in}} A_{\text{in}} = Q$$

$$V_{\text{out}} A_{\text{out}} = \frac{Q}{2}$$

which implies $V_{\text{in}} = V_{\text{out}}$.

Step 2: Compute the Angular Momentum

The moment of momentum about the x -axis is given by:

$$H_x = \sum (r_i \times m_i V_i)$$

where r is the perpendicular distance from the x -axis.

- At the inlet, the fluid moves along the axis (x -direction), so the angular momentum contribution is **zero**.
- At the outlet, the velocity components perpendicular to the x -axis contribute to angular momentum.

For each branch, the velocity component perpendicular to the x -axis is:

$$V_{\perp} = V_{\text{out}} \sin \theta$$

The angular momentum for one branch:

$$H_{x,\text{one}} = \left(\frac{\rho Q}{2} \right) R_0 V_{\text{out}} \sin \theta$$

Since there are two branches:

$$H_x = 2 \times \left(\frac{\rho Q}{2} R_0 V_{\text{out}} \sin \theta \right) = \rho Q R_0 V_{\text{out}} \sin \theta$$

Step 3: Compute the Required Torque

The torque needed to maintain constant angular velocity is obtained from the rate of change of angular momentum:

$$T = \frac{dH_x}{dt}$$

Since the flow is steady,

$$T = \rho Q R_0 V_{\text{out}} \sin \theta$$

Final Expression:

$$T = \rho Q R_0 V_{\text{out}} \sin \theta$$

where $V_{\text{out}} = Q/A$.

Claude

Let's solve this problem step by step:

1. First, let's understand the scenario:

- A pipe with total flow Q splits into two equal flows of $Q/2$
- The split occurs at an angle θ in a wye joint
- The flow exits at a distance R_0 from the x-axis
- The system is rotating at an angular velocity Ω
- We need to find the torque T about the x-axis

2. To find the torque, we'll use the principle of conservation of angular momentum.

3. The momentum of each flow segment:

- Mass flow rate for each branch = $Q/2$
- Velocity components are perpendicular to the pipe axis
- Moment arm is $R_0 \sin(\theta)$

4. The torque will be the sum of the moments of each flow: $T = 2 * (Q/2) * R_0 * \sin(\theta)$

5. Simplifying: $T = Q * R_0 * \sin(\theta)$

Therefore, the expression for the torque T about the x-axis required to keep the system rotating at angular velocity Ω is:

$$T = Q * R_0 * \sin(\theta)$$

Cheat Sheet

$$T_{\text{shaft}} = -\dot{m}_1 (r_1 V_{\theta 1}) + \dot{m}_2 (r_2 V_{\theta 2})$$

$$\dot{W}_{\text{shaft}} = T_{\text{shaft}} \omega$$

$$\sum (\mathbf{r} \times \mathbf{F}) = \int_{cs} (\mathbf{r} \times \mathbf{V}) \rho \mathbf{V} \cdot \hat{\mathbf{n}} dA$$