Solution – Series 2

Exercise 1: Water jet impacting a curved plate

We consider a two-dimensional water jet impacting a fixed curved plate. The inlet velocity $\boldsymbol{V}_0$ of the fluid is assumed to be uniform over the section A_0. This jet separates into two streams at the impingement on the plate and the outlet velocities $\boldsymbol{V}_1$ and $\boldsymbol{V}_2$ far from the point of impact are assumed uniform over the sections A_1 and A_2, respectively. $\boldsymbol{n}$ refers to the outward normal vectors at the control volume boundaries. The flow is considered to be steady, incompressible, inviscid and to remain in an horizontal plane. The surrounding air is considered to be at atmospheric pressure p_{atm}.

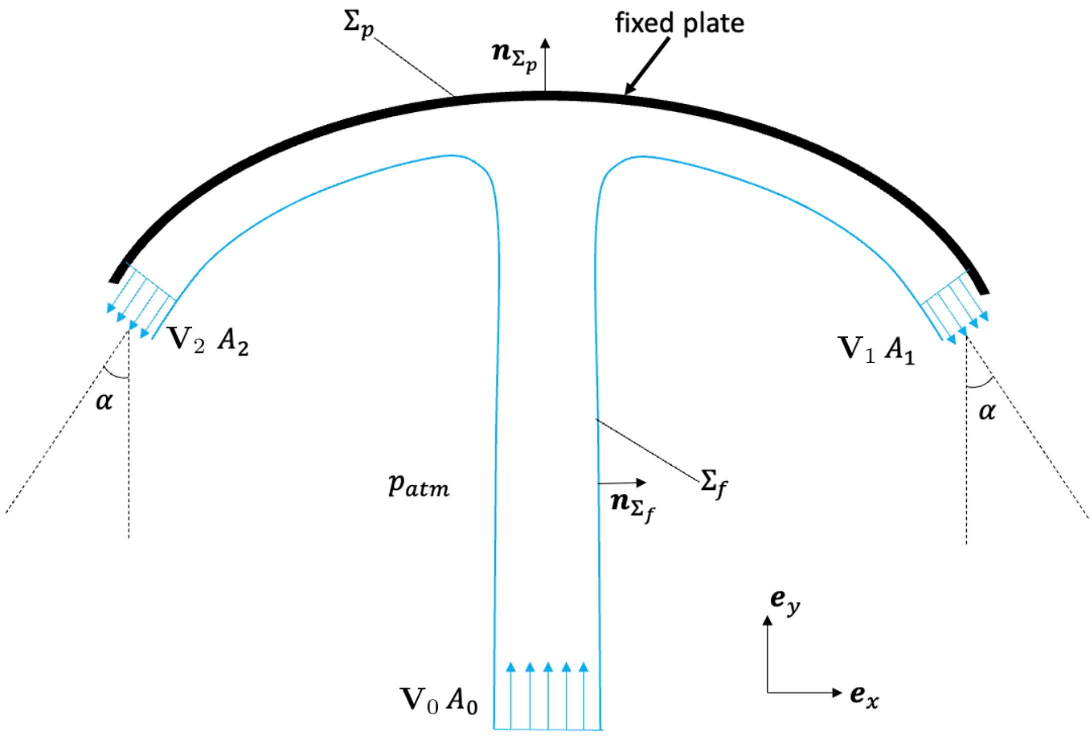


Figure 1: Water jet impacting a fixed curved plate

1. Using the Bernoulli equation, determine the relationship between the velocity magnitudes V_0, V_1 and V_2 ($V_i = \|\boldsymbol{V}_i\|$ $\mathrm{i} = 0, 1, 2$).
2. Using the conservation of mass, find the relationship between the flow rates Q_0, Q_1 and Q_2.
3. Using the conservation of momentum, determine the force $\boldsymbol{F}_p = F_{\parallel}\boldsymbol{e}_x + F_{\perp}\boldsymbol{e}_y$ exerted by the jet on the plate. For which angle α is the force maximized?
4. In the case of a curved plate moving with a constant velocity $\boldsymbol{U} = U\boldsymbol{e}_y$ (equivalent to a Pelton turbine bucket), what do you expect the expression of the force to become?

Exercise 2: Gravity and pressure effects in hydroelectric plants

The Bieudron Hydroelectric Power Station is a hydroelectric power plant located in Riddes in the Canton of Valais in Switzerland (altitude above sea level $Z_{\bar{B}} = 500$ m, latitude $\varphi \approx 46.2°N$). The power plant is fed with water from the Grande Dixence Dam's reservoir (altitude above sea level $Z_B = 2380$ m, latitude $\varphi \approx 46.2°N$). The facility houses 3 Pelton turbines, each fed by 5 nozzles having a jet exit diameter of $d_s = 184$ mm and velocity $V_I = 191.5\ m/s$. The overall efficiency, η, of each turbine is 92.2%. Given the hydropower plant large head (height difference between the reservoir and power station), we expect the change in gravitational acceleration and atmospheric pressure along the head height to have an impact on the plant's power production. Point I corresponds to the inlet of the turbine while point $\bar{I}$ corresponds to the outlet of the turbine.

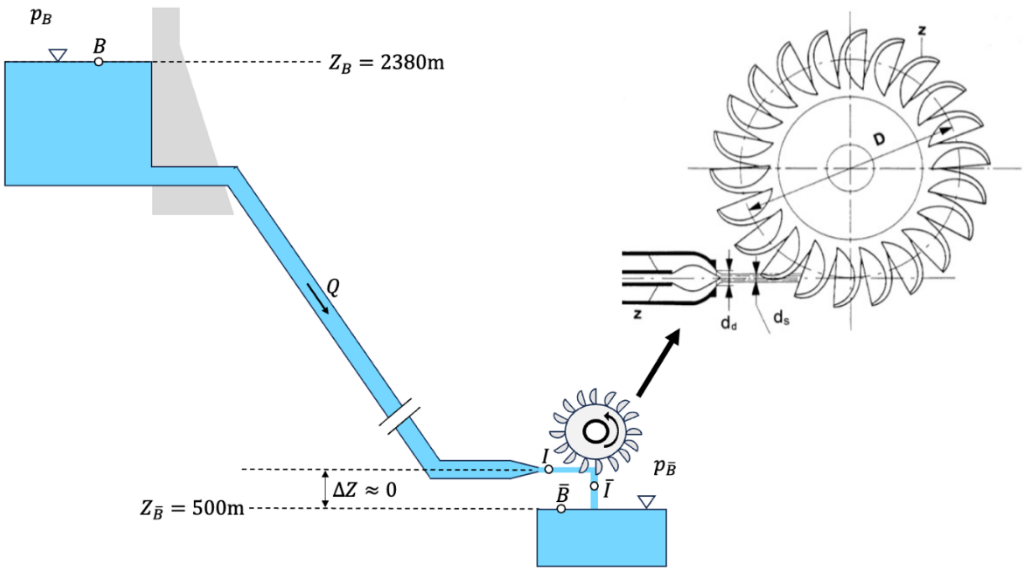


Figure 2: Schematic of a Pelton turbine with one nozzle.

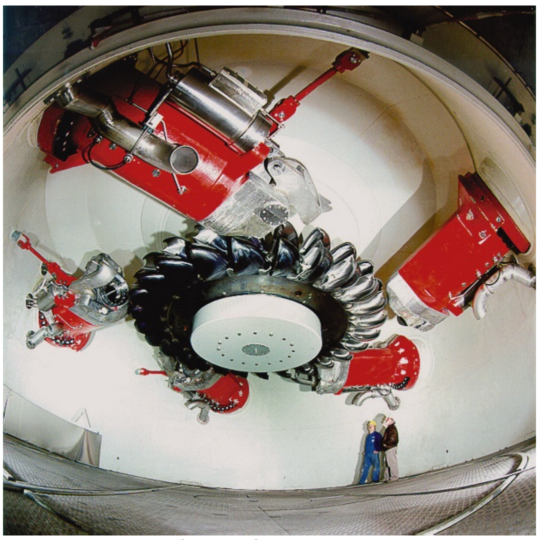


Figure 3: Photography of one of the three Bieudron Pelton turbine.

1. Calculate the total flow rate, Q, of the Bieudron power station.

2. Compute the head losses h_L between the free surface of the reservoir and the inlet of the turbine. Are they negligible in comparison with the elevation head? We here consider that the atmospheric pressure at the power station and at the reservoir are the same.

3. Compute the specific energy (energy per unit mass) available for the turbine $E = E_I - E_{\bar{I}}$ (The inlet specific energy E_I can be written gh_I), neglecting the head losses and the kinetic energy at the outlet of the turbine and using for the earth gravitational acceleration g:

   a) the value evaluated at the power plant altitude and latitude.
   b) the value evaluated at the reservoir altitude and latitude.
   c) the value integrated over the head height (averaged value).

We here consider that the atmospheric pressure at the power station and at the reservoir are the same.

4. Calculate the related facility power productions (water density $\rho = 998\ kg/m^3$).

5. We assume that the power plant operates at full capacity over a year (24h per day over 365 days). Compute the total electric energy (in kilowatt hours, kWh) produced at Bieudron for the three specific energies identified at the previous question.

6. We now want to assess what effect the change in atmospheric pressure between the power station and the reservoir has on the revenue estimation. Calculate the facility revenue estimation by taking into account the atmospheric pressure difference between the power station and the reservoir and compare your result to the case where this difference is neglected. In both cases, use the averaged value of the gravitational acceleration and neglect the head losses as well as the kinetic energy at the outlet of the turbine. The atmospheric pressure at different altitudes Z can be calculated with the following formula:

$$p(Z) = p_{ref}\left(1 - \frac{L_b Z}{T_{ref}}\right)^{\frac{g_0 M}{R L_b}}$$

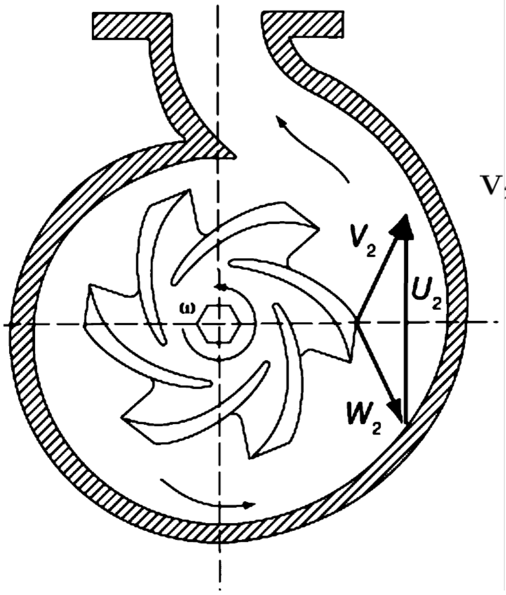
where $p_{ref} = 101325$ Pa, $T_{ref} = 288.15$ K, $L_b = 0.0065$ K/m is the temperature lapse rate, $g_0 = 9.81\ m/s^2$, $M = 0.028$ kg/mol is the molar mass of air and $R = 8.314$ J/(mol.K) is the universal gas constant.

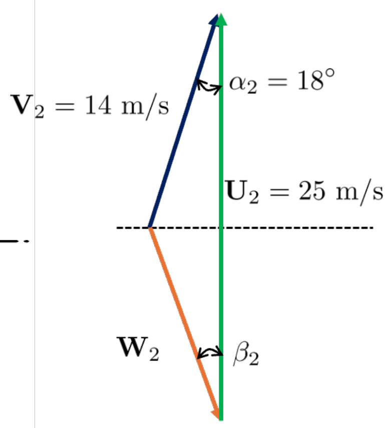
Exercise 3: Similarity laws for pumps

A model of centrifugal pump has a scale ratio of 1:15. When tested at 3600 RPM, the model delivers 0.1 m^3/s of water at a head of 40 m with an efficiency of 80%. Assuming the prototype has an efficiency of 88%, what will be its rotational speed, flowrate and power consumption at a head of 50 m. The water density is $\rho = 1000\ kg/m^3$.
What is the power consumption of the pump model?

Exercise 4: Centrifugal pump

Water at 20 °C leaves a pump impeller with an absolute velocity of 14 m/s at the angle α_2 =18°. The blade speed at the exit is 25 m/s, and the shaft speed is 3450 rpm. The absolute velocity is axial at the inlet. The flow rate is 18.0 L/s. The pump and the exit velocity diagram is shown in Figure below. Density is 998 kg/m^3.





1. Find the magnitude of the relative velocity and its flow angle β_2
2. Find the required power
3. Find the outlet blade radius and the blade height assuming that the open area at the periphery is 93 % of the total area