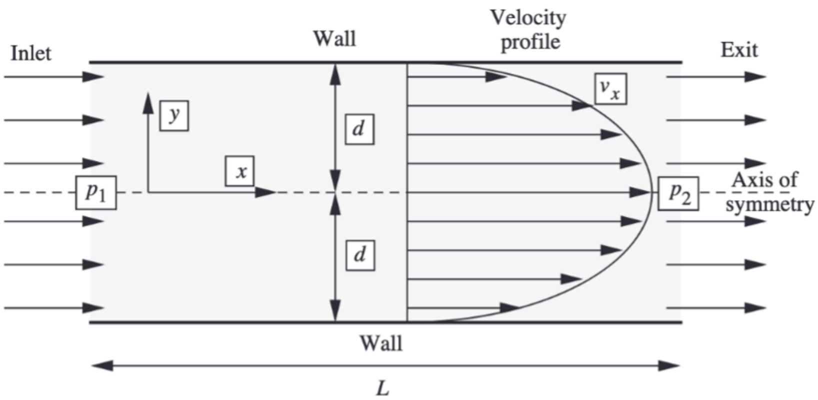
Solutions– Serie 1 – Fluid Flows

Exercise 1

Flow between two parallel plates.

Consider the 2D steady flow of a Newtonian fluid of velocity $\boldsymbol{v} = v_x\boldsymbol{e}_x + v_y\boldsymbol{e}_y$, density ρ and dynamic viscosity μ between two plates, located at $y = -d$ and $y = +d$ (see the figure below). The flow is driven by a constant pressure difference $\Delta p = p_1 - p_2$. We make the following assumptions:

- The flow is steady, laminar, and incompressible.
- The flow is unidirectional (x-direction).
- We neglect the effect of gravity.
- The velocity is zero on the walls (no slip condition) and symmetric about the $y = 0$ axis.



1. Using the Navier-Stokes continuity equation and the hypotheses listed above, show that $\boldsymbol{v} = v_x(y)\boldsymbol{e}_x$.
2. Use the Navier-Stokes momentum equations and the hypotheses listed above to express the pressure gradient $\nabla p = \frac{\partial p}{\partial x}\boldsymbol{e}_x + \frac{\partial p}{\partial y}\boldsymbol{e}_y$.
3. Why can you integrate the x-momentum equation?
4. Derive and qualitatively draw the shear rate profile $\frac{\partial v_x}{\partial y}$.
5. Derive the velocity profile v_x.
6. Deduce the expression of the flow rate Q (per unit length).
7. Express the pressure field $p(x)$ along the flow direction between $x = 0$ $(p(x = 0) = p_1)$ and $x = L$ $(p(x = L) = p_2)$ and find the expression of the pressure gradient ∇p as a function of Δp and L.
8. Deduce the relationship between the pressure difference Δp and the flow rate Q.

9. The force (per unit length) exerted by the flow on each wall of length L is given by $\boldsymbol{F} = \int_0^L \boldsymbol{\sigma}|_{y=\pm d}\boldsymbol{n}dx = F_x\boldsymbol{e}_x + F_y\boldsymbol{e}_y$, where $\boldsymbol{n}$ is the inward normal vector to the wall and $\boldsymbol{\sigma}$ is the stress tensor. Give the expression of the horizontal force F_x exerted by the flow on each wall.

   Hint: The stress tensor for a Newtonian fluid is given by $\boldsymbol{\sigma} = -p\boldsymbol{I} + 2\mu\boldsymbol{D}$, and the strain rate tensor is defined as $\boldsymbol{D} = \frac{1}{2}(\nabla\boldsymbol{v} + \nabla\boldsymbol{v}^T)$. In 2D cartesian coordinates, its components are:

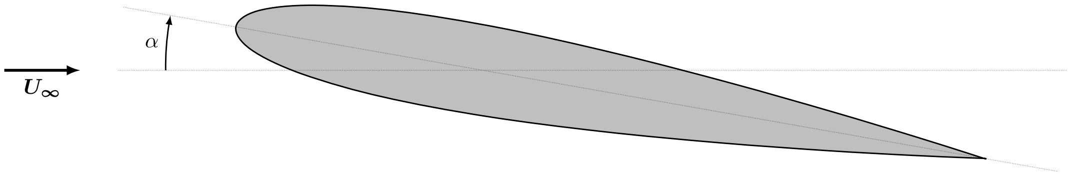
$$D_{xx} = \frac{\partial v_x}{\partial x}, D_{xy} = D_{yx} = \frac{1}{2}\left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right), D_{yy} = \frac{\partial v_y}{\partial y} \tag{1}$$

Exercise 2

Flow around a hydrofoil.

Fully completing this exercise requires having JavaFoil installed on your computer.

We consider a symmetrical NACA 0009 hydrofoil with a 1m chord length in a flow of water at an upstream velocity is $U = 10\text{ms}^{-1}$. (Kinematic viscosity: $\nu_{water} = 10^{-6}\ \text{m}^2\text{s}^{-1}$).



Hint: The NACA (predecessor of NASA) four-digit wing sections were created to establish an extensive database of airfoil shapes. Using a four-digit system where the first digit denotes camber percentage, the second digit indicates camber location, and the last two digits signify thickness percentage. These numerical parameters can be incorporated into equations to produce precise airfoil shapes.
Visit https://en.wikipedia.org/wiki/NACA_airfoil#Four-digit_series for more details.

1. Find the Reynolds number related to this flow.
2. Give the expression of the pressure coefficient C_p.
3. What is the value of C_p at the stagnation point? Justify.
4. In JavaFoil's velocity tab, you can either compute the local velocity ratio or the local pressure coefficient distribution along the hydrofoil surface. The velocity field distribution is computed using a panel method and considering potential flow. The local velocity and the local pressure are related by the Bernoulli equation. Do the following using the JavaFoil solver:

a. Compute the pressure coefficient distribution over the NACA 0009 hydrofoil for different values of the incidence angle, α, ranging between $0°$ and $10°$. What can you conclude from this graph?

b. Why is the pressure coefficient, as computed in JavaFoil, independent of the Reynolds number?

c. Cite a few limitations inherent to this computing technique.

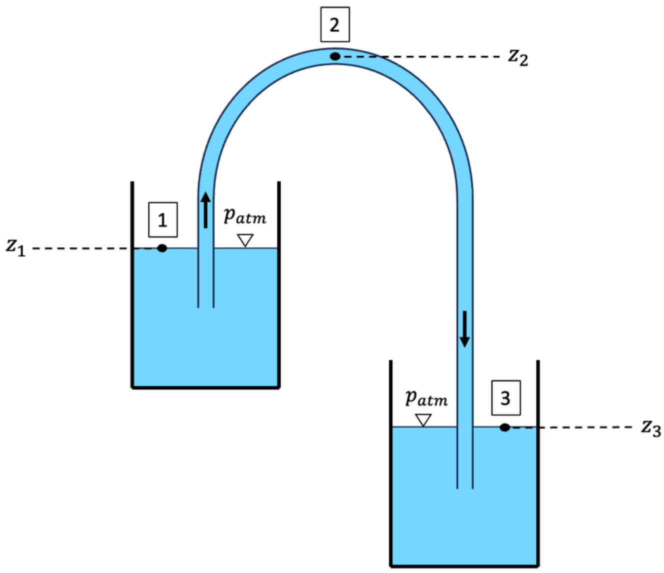
d. Modify the NACA 0009 hydrofoil by adding 2% camber at 50% of its chord length. What is the name of this hydrofoil. By comparing the pressure coefficient distribution of this new hydrofoil with the one of the symmetrical hydrofoil, explain how a cambered hydrofoil can generate lift at $\alpha = 0°$.

5. Draw the lift and drag vectors on the hydrofoil and indicate where they are acting. Justify your choice.
6. Give the expression of the lift and drag coefficients (C_l and C_d).
   With JavaFoil's polar tab, compute the $C_l - \alpha$ curves for the symmetrical and the cambered hydrofoil. Compare the computed $C_l - \alpha$ curves to the one derived from the Thin Airfoil Theory.

Exercise 3

Siphon

Consider the siphon shown below used to convey water ($\rho = 1000\ kg/m^3$, $\mu = 10^{-3} Pa.s$) from an upper to a lower reservoir. The height difference between the free surfaces of the two reservoirs is $z_1 - z_3 = 3m$. The pipe is rigid and has an inner diameter of $D = 25mm$ and a total length of $L = 6m$. The highest point of the pipe is located at an elevation z_2 which is $z_2 - z_1 = 2m$ above the upper tank free surface. The length of pipe from its inlet to its highest point is $L_{in:2} = 2.5m$. The friction factor of the pipe is $f = 0.028$. The pipe inlet loss coefficient is $K_{L,in} = 0.7$ and the pipe outlet loss coefficient is $K_{L,out} = 1$. We neglect the losses due to the pipe curvature and we consider that the free surface levels of the two reservoirs stay constant so that the flow is steady. The atmospheric pressure is $p_{atm} = 1.0 \cdot 10^5\ Pa$.



1. Calculate the total head losses h_L between points 1 and 3.
2. Why is the mean flow velocity the same at each pipe cross-section?
3. Find the flow rate Q, and the flow regime using the following equations for the individual head losses:

Major losses (regular) in the pipe

$$h_{L,major} = f\frac{L}{D}\frac{V^2}{2g}$$

Minor losses (singular) at pipe inlet (2)

$$h_{L,minor,in} = K_{L,in}\frac{V^2}{2g}$$

Minor losses (singular) pipe outlet

$$h_{L,minor,out} = K_{L,out}\frac{V^2}{2g}$$

Where g is the gravitational acceleration and $V = Q/A$ is the mean flow velocity at a given cross-section $A = \frac{\pi D^2}{4}$ of the pipe.

4. Find the pressure p_2 at point 2. What could happen if the height of z_2 is increased beyond a certain threshold (by keeping the difference $z_1 - z_3$ constant and increasing the overall tube length)?

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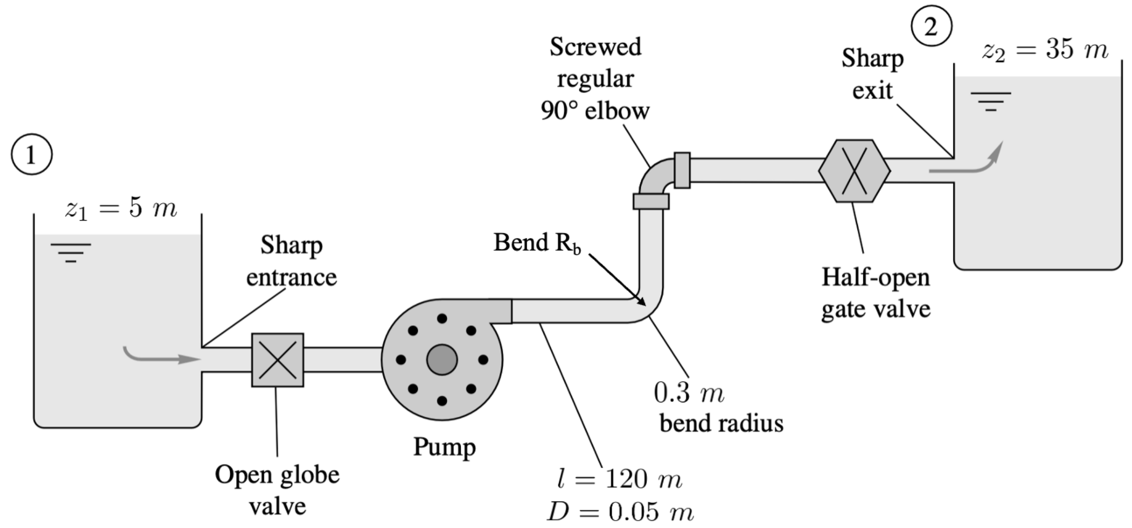
Exercise 4

Combine losses

Water, $\rho = 1000 \frac{kg}{m^3}$ and $\nu = 1.02 \times 10^{-6} \frac{m^2}{s}$, is pumped between two reservoirs at Q = 0.005 m^3/s through L=120 m of D=0.05 m diameter pipe and several minor (singular) losses (given in the table), as shown in the figure. The roughness ratio is ε/D = 0.001. Compute the pump power required for a pump of efficiency 0.8.



Minor losses	K_L
Sharp entrance	0.5
Open globe valve	6.3
0.3 m Bend	0.15
Regular 90degree elbow	1.5
Half-closed gate valve	2.7
Sharp exit	1

The friction factor, f, can be estimated as

$$\frac{1}{\sqrt{f}} = -1.8 \log\left[\left(\frac{\varepsilon/D}{3.7}\right)^{1.11} + \frac{6.9}{\mathrm{Re}}\right]$$

1. Calculate the average velocity
2. Calculate the Reynolds number
3. Write a steady-flow energy equation between sections 1 and 2 and make an expression for the pump head, h_p (keep h_L)
4. Calculate the head losses ($h_L = h_{L,major} + h_{L,minor}$)
5. Calculate the required pump power