Fluid Dynamics

- Mechanical Energy Equation per unit weight between two streamline points:

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1 \pm h_s-\sum h_L=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2$$

- Viscous Shear Stress (Newton's Law of Viscosity):

$$\tau=\mu \frac{du}{dy}$$

- Entrance Length:

$$\begin{aligned} \frac{\ell_e}{D} &= 0.06Re && : \text{laminar flow} \\ \frac{\ell_e}{D} &= 4.4(Re)^{1/6} && : \text{turbulent flow} \end{aligned}$$

- Hagen–Poiseuille Equation (Fully Developed Laminar Flow):

$$Q=\frac{\pi D^4 \Delta p}{128 \mu \ell}$$

- Darcy Friction Factor for Laminar Flow:

$$f=\frac{64}{\mathrm{Re}}$$

- Losses

$$\begin{aligned} h_L &= h_{L,\text{ major}}+h_{L,\text{ minor}} \\ &= f \frac{l}{D} \frac{V^2}{2g}+K_L \frac{V^2}{2g} \end{aligned}$$

- Hydraulic Diameter for Non-circular Ducts:

$$D_h=\frac{4A}{P}$$

- Lift, Drag and Pressure Coefficients:

$$C_L=\frac{L}{\frac{1}{2}\rho U^2 A}, \quad C_D=\frac{D}{\frac{1}{2}\rho U^2 A}, \quad C_p=\frac{p-p_{\infty}}{\frac{1}{2}\rho U^2}$$

- Stagnation Pressure:

$$p_0=p+\frac{1}{2}\rho v^2$$

- Total Pressure

$$p_T=p+\frac{1}{2}\rho V^2+\gamma z= \text{ constant along a streamline}$$

- Continuity Equation (Mass Conservation):

$$\frac{\partial}{\partial t}\int_{\mathrm{cv}} \rho d\forall+\int_{\mathrm{cs}} \rho \mathbf{V} \cdot \hat{\mathbf{n}} dA=0$$

- Linear Momentum Equation (Integral Form):

$$\frac{\partial}{\partial t}\int_{\mathrm{cv}} \mathbf{V}\rho d\forall+\int_{\mathrm{cs}} \mathbf{V}\rho \mathbf{V} \cdot \hat{\mathbf{n}} dA=\sum \mathbf{F}_{\text{contents of the control volume}}$$

- Energy Equation (Integral Form):

$$\dot{Q}_{\substack{\text{net}\\\text{in}}}+\dot{W}_{\substack{\text{shaft}\\\text{net in}}}=\frac{\partial}{\partial t}\int_{\mathrm{cv}} e\rho d\forall+\int_{\mathrm{cs}}\left(\check{u}+\frac{p}{\rho}+\frac{V^2}{2}+gz\right)\rho \mathbf{V}\cdot\hat{\mathbf{n}} dA$$

Propellers & Vortex

- Induced Velocity for Rotor Helicopter (Hovering):

$$v_i=\sqrt{\frac{\mathcal{T}}{2\rho A}}$$

- Thrust and Power Helicopter:

$$\mathcal{T}=\dot{m}V_3=2\rho A v_i^2, \quad P_{\text{rotor}}=\mathcal{T} v_i$$

- Figure of Merit for Helicopter:

$$FM=\frac{\text{Ideal Rotor Power}}{\text{Actual Rotor Power}}$$

- Free Vortex Velocity:

$$u_\theta=\frac{\text{const.}}{r}=\frac{\Gamma}{2\pi r}$$

- Free Vortex Pressure

$$\frac{\partial p}{\partial r}=\rho \frac{u_\theta^2}{r}$$

Euler equation, incompressible Pumps and Turbines

- Shaft Work per Unit Mass:

$$w_s=U_2 V_{\theta 2}-U_1 V_{\theta 1}$$

- Shaft Power:

$$\dot{W}_{\text{shaft}}=-\dot{m}_1\left(U_1 V_{\theta 1}\right)+\dot{m}_2\left(U_2 V_{\theta 2}\right)$$

- Euler Turbomachine Equation (Torque form):

$$T_{\text{shaft}}=-\dot{m}_1\left(r_1 V_{\theta 1}\right)+\dot{m}_2\left(r_2 V_{\theta 2}\right)$$

- Total Head:

$$H=\frac{U_2 V_{\theta 2}-U_1 V_{\theta 1}}{g}$$

- Power gained by/from fluid:

$$P = \gamma Q H$$

- Ideal head rise pump:

$$h_i = \frac{U_2 V_{\theta 2} - U_1 V_{\theta 1}}{g}$$

- Actual head rise pump:

$$h_a = \frac{p_2 - p_1}{\gamma} + z_2 - z_1 + \frac{V_2^2 - V_1^2}{2g}$$

- Available Net Positive Suction Head ($\mathrm{NPSH_A}$):

$$\mathrm{NPSH_A} = \frac{p_{\text{atm}} - p_v}{\gamma} - z_s \pm \sum h_L$$

- Flow, Head and Power Coefficients:

$$C_Q = \frac{Q}{\omega D^3}, \quad C_H = \frac{g h_a}{\omega^2 D^2}, \quad C_{\mathscr{P}} = \frac{\dot{W}_{\text{shaft}}}{\rho \omega^3 D^5}$$

- Specific Speed (Dimensionless):

$$N_s = \frac{\omega \sqrt{Q}}{(g h_a)^{3/4}}$$

- Power Specific Speed (Hydraulic Turbines):

$$N_s' = \frac{\omega \sqrt{\dot{W}_{\text{shaft}} / \rho}}{(\mathrm{g} h_a)^{5/4}}$$

- (static) Enthalpy and Stagnation enthalpy:

$$\check{h} = \check{u} + \frac{p}{\rho}, \quad \check{h}_0 = \check{h} + \frac{V^2}{2} + gz$$

- Degree of Reaction:

$$R = \frac{\check{h}_{\text{rotor}}}{\check{h}_{\text{stage}}}$$

- Tygun's empirical formula for the number of buckets (Z) in the wheel:

$$Z = \frac{D_{\text{wheel}}}{2 D_{\text{nozzle}}} + 15$$

- Head available at the turbine inlet relative to the surface of the tailrace:

$$H_E = h_g - h_{LP}$$

- Turbine Efficiency:

$$\eta_{\text{turbine}} = \frac{\dot{W}_{\text{shaft}}}{\gamma Q H_E}$$

(when mechanical and volumetric losses are neglected)

$$\eta_{\text{turbine}} = \eta_h = \frac{|w_{\text{shaft}}|}{g H_E} = \frac{|-U_1 V_{\theta 1} + U_2 V_{\theta 2}|}{g H_E}$$

- Pump Efficiency:

$$\eta_{\text{pump}} = \frac{\gamma Q h_a}{\dot{W}_{\text{shaft}}}$$

- Thoma Cavitation Factor:

$$\sigma = \frac{\mathrm{NPSH_A}}{H_E}$$

- Cavitation number (with velocity definition):

$$\sigma = \frac{p_{ref} - p_v(T)}{\frac{1}{2} \rho V_{ref}^2}$$

Compressible turbomachines

- Ideal gas law and Isentropic compression

$$\rho = \frac{p}{\mathcal{R} T}, \quad \frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\mathsf{k}-1}{\mathsf{k}}}, \ \mathsf{k} = c_p / c_v$$

- Pressure ratio, PR (stagnation pressure)

$$\mathrm{PR} = \frac{p_2}{p_1}$$

- Propulsive efficiency

$$\eta_p = \frac{V\left[\left(\dot{m}_{\text{air}} + \dot{m}_f\right) V_j - \dot{m}_{\text{air}} V\right]}{\frac{1}{2}\left[\left(\dot{m}_{\text{air}} + \dot{m}_f\right) V_j^2 - \dot{m}_{\text{air}} V^2\right]} \simeq \frac{2}{1 + \frac{V_j}{V}}$$

- Bypass ratio

$$bpr = \frac{\dot{m}_b}{\dot{m}_c}$$

- Navier–Stokes equations

$$\rho\left(\frac{\partial \mathbf{V}}{\partial t}+\mathbf{V}\cdot\nabla\mathbf{V}\right)=-\nabla p+\rho\mathbf{g}+\mu\nabla^2\mathbf{V}$$

- Expanded in cylindrical coordinates:

r-direction

$$\rho\left(\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta}-\frac{v_\theta^2}{r}+v_z\frac{\partial v_r}{\partial z}\right)=-\frac{\partial p}{\partial r}+\rho g_r+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_r}{\partial r}\right)-\frac{v_r}{r^2}+\frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2}-\frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}+\frac{\partial^2 v_r}{\partial z^2}\right]$$

θ-direction

$$\rho\left(\frac{\partial v_\theta}{\partial t}+v_r\frac{\partial v_\theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta}+\frac{v_r v_\theta}{r}+v_z\frac{\partial v_\theta}{\partial z}\right)=-\frac{1}{r}\frac{\partial p}{\partial \theta}+\rho g_\theta+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_\theta}{\partial r}\right)-\frac{v_\theta}{r^2}+\frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2}+\frac{2}{r^2}\frac{\partial v_r}{\partial \theta}+\frac{\partial^2 v_\theta}{\partial z^2}\right]$$

z-direction

$$\rho\left(\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta}+v_z\frac{\partial v_z}{\partial z}\right)=-\frac{\partial p}{\partial z}+\rho g_z+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2}+\frac{\partial^2 v_z}{\partial z^2}\right]$$

- Incompressible Continuity Equation (Differential Form):

$$\nabla\cdot\mathbf{V}=0,\quad \frac{1}{r}\frac{\partial(rv_r)}{\partial r}+\frac{1}{r}\frac{\partial v_\theta}{\partial \theta}+\frac{\partial v_z}{\partial z}=0$$