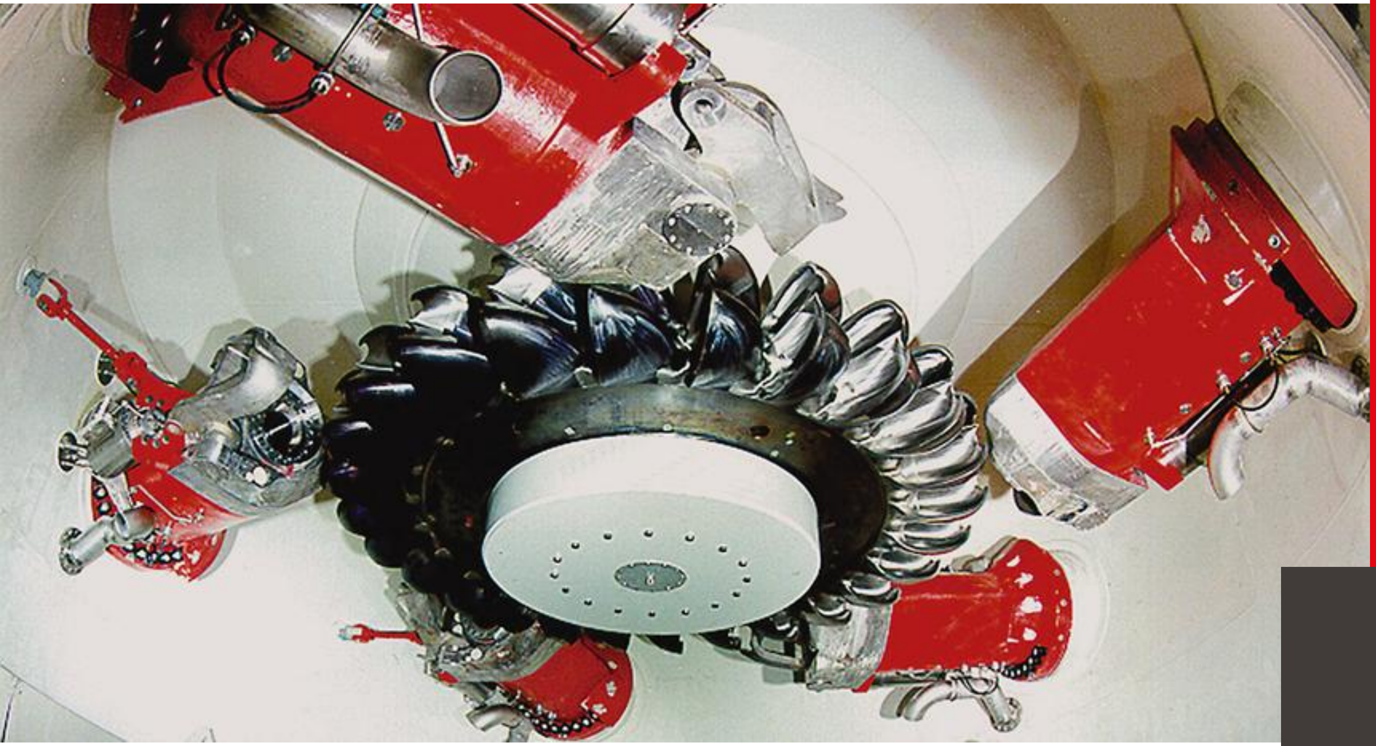
Chapter 8: Hydraulic Turbines



ME-342 Introduction to turbomachinery

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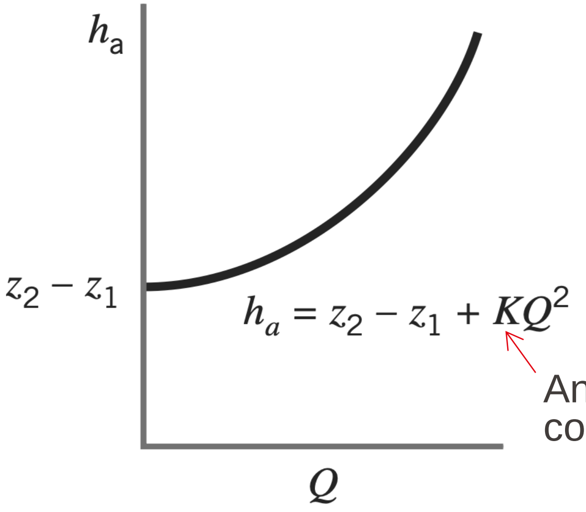
Last week

- **K vs. K_L**

Loss coefficient for Minor loss

System equation



Exercise

Pump water with 0.014 m³/s the required NPSH is 4.5 m specified pump manufacturer. The water temperature is 30 ºC, 101.3 kPa. The loss occurs mainly due to the filter at inlet with K_L = 20. Friction loss is neglected. Pipe is with diameter of 10 cm. Determine the max height z the pump can be located without the cavitation.

$$\mathrm{NPSH_A} = \frac{p_{atm}}{\gamma} - \Delta z - \sum h_L - \frac{p_v}{\gamma}$$

Cavitation occurs when $NPSH_R = NPSH_A$

$\sum h_L = K_L \frac{V^2}{2g}$

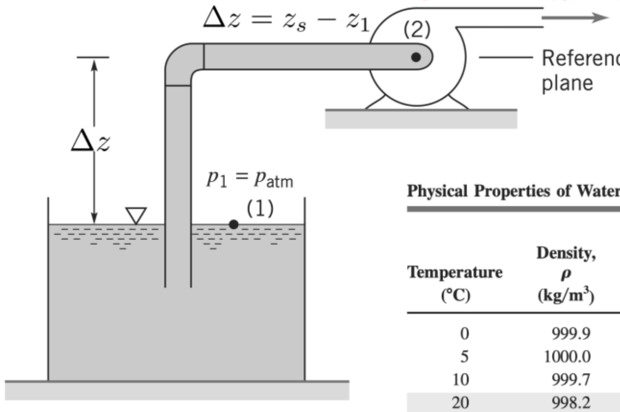
$NPSH_A = NPSH_R = 4.5\ m = \frac{101.3 \times 10^3\ Pa}{9.765 \times 10^3\ N/m^3} - \Delta z - 20 \cdot \frac{V^2}{2g} - \frac{4.243 \times 10^3}{9.765 \times 10^3}$ ($20 \cdot \frac{V^2}{2g} = 3.23$)

$V = \frac{Q}{A} = \frac{0.014}{\pi \cdot 0.05^2} = 1.78\ m/s$

$= 10.37 - \Delta z - 3.23 - 0.43$

$\Delta z = 10.37 - 4.5 - 3.23 - 0.43$

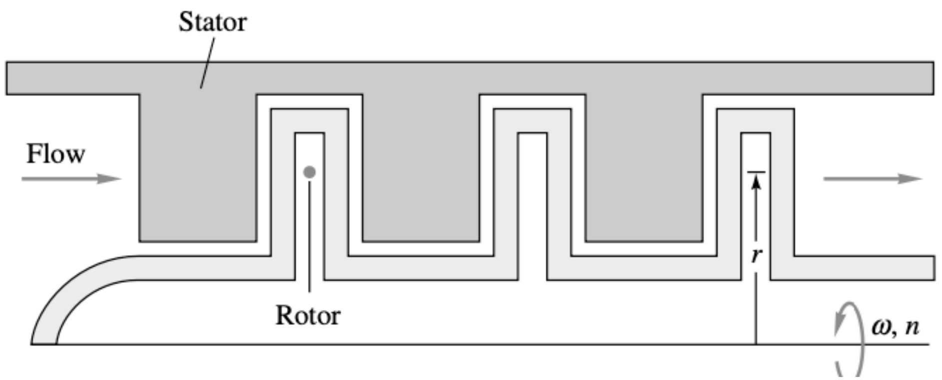
$= 2.21\ m$

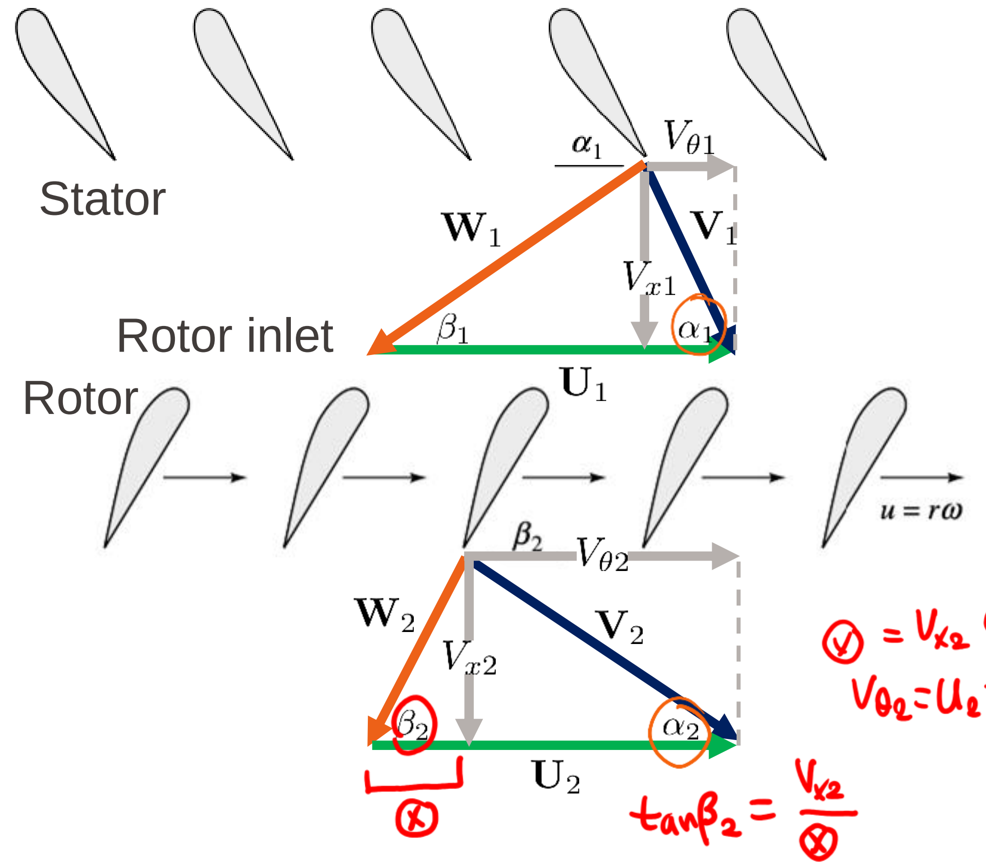


Physical Properties of Water (SI Units)[a]

Temperature (°C)	Density, ρ (kg/m³)	Specific Weight[b], γ (kN/m³)	Dynamic Viscosity, μ (N·s/m²)	Kinematic Viscosity, ν (m²/s)	Surface Tension[c], σ (N/m)	Vapor Pressure, p_v [N/m²(abs)]
0	999.9	9.806	1.787 E − 3	1.787 E − 6	7.56 E − 2	6.105 E + 2
5	1000.0	9.807	1.519 E − 3	1.519 E − 6	7.49 E − 2	8.722 E + 2
10	999.7	9.804	1.307 E − 3	1.307 E − 6	7.42 E − 2	1.228 E + 3
20	998.2	9.789	1.002 E − 3	1.004 E − 6	7.28 E − 2	2.338 E + 3
30	995.7	9.765	7.975 E − 4	8.009 E − 7	7.12 E − 2	4.243 E + 3
40	992.2	9.731	6.529 E − 4	6.580 E − 7	6.96 E − 2	7.376 E + 3

Last week - Axial pump





Since the stator is fixed, ideally the absolute velocity V_1 is parallel to the trailing edge of the blade

$$V_{x1} = V_{x2} = V_x = \frac{Q}{A} = \text{const}$$

$u_1 = u_2 = u$

The ideal head expressed stator angle α_1 and rotor angle β_2

$$h_i = \frac{U_2 V_{\theta 2} - U_1 V_{\theta 1}}{g}$$

$\tan\alpha_1 = \frac{V_{x1}}{V_{\theta 1}}$, $\tan\alpha_2 = \frac{V_{x2}}{V_{\theta 2}}$

$V_{\theta 1} = V_{x1}\cot\alpha_1$, $V_{\theta 2} = V_{x2}\cot\alpha_2 = u_2 - V_{x2}\cot\beta_2$

$\tan\beta_2 = \frac{V_{x2}}{\otimes}$, $\otimes = V_{x2}\cot\beta_2$

$V_{\theta 2} = u_2 - V_{x2}\cot\beta_2$

$$h_i = \frac{1}{g}\left(u_2(u_2 - V_{x2}\cot\beta_2) - u_1 V_{x1}\cot\alpha_1\right)$$

$$= \frac{1}{g}\left[u_2^2 - u_2 V_{x2}\cot\beta_2 - u_1 V_{x1}\cot\alpha_1\right]$$

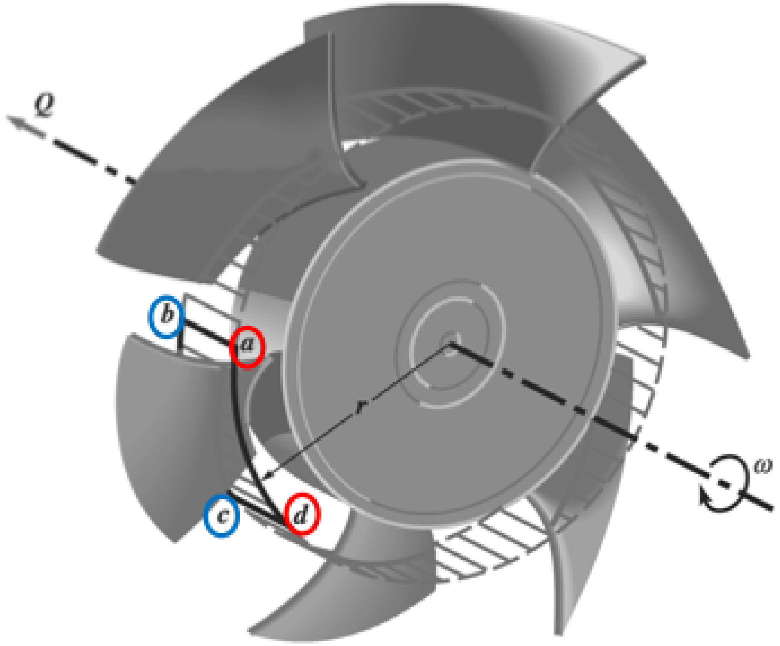
$$= \frac{1}{g}\left(u^2 - u V_x (\cot\beta_2 + \cot\alpha_1)\right)]$$

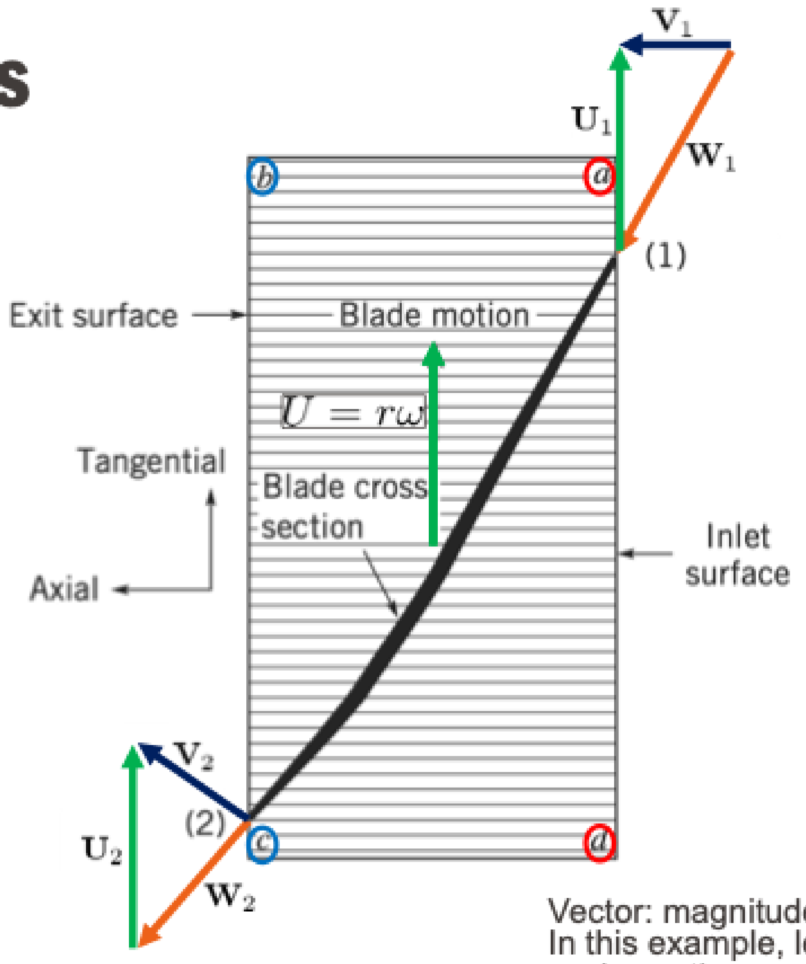
Strictly speaking, this applies only to a single streamtube of radius *r*, but it is a good approximation for very short blades if *r* denotes the average radius.

Recall - Chapter 5

Basic Energy Considerations

- **Velocity diagram (fan)**





Vector: magnitude and angle
In this example, let's assume we know the magnitude of **W**

Assumptions

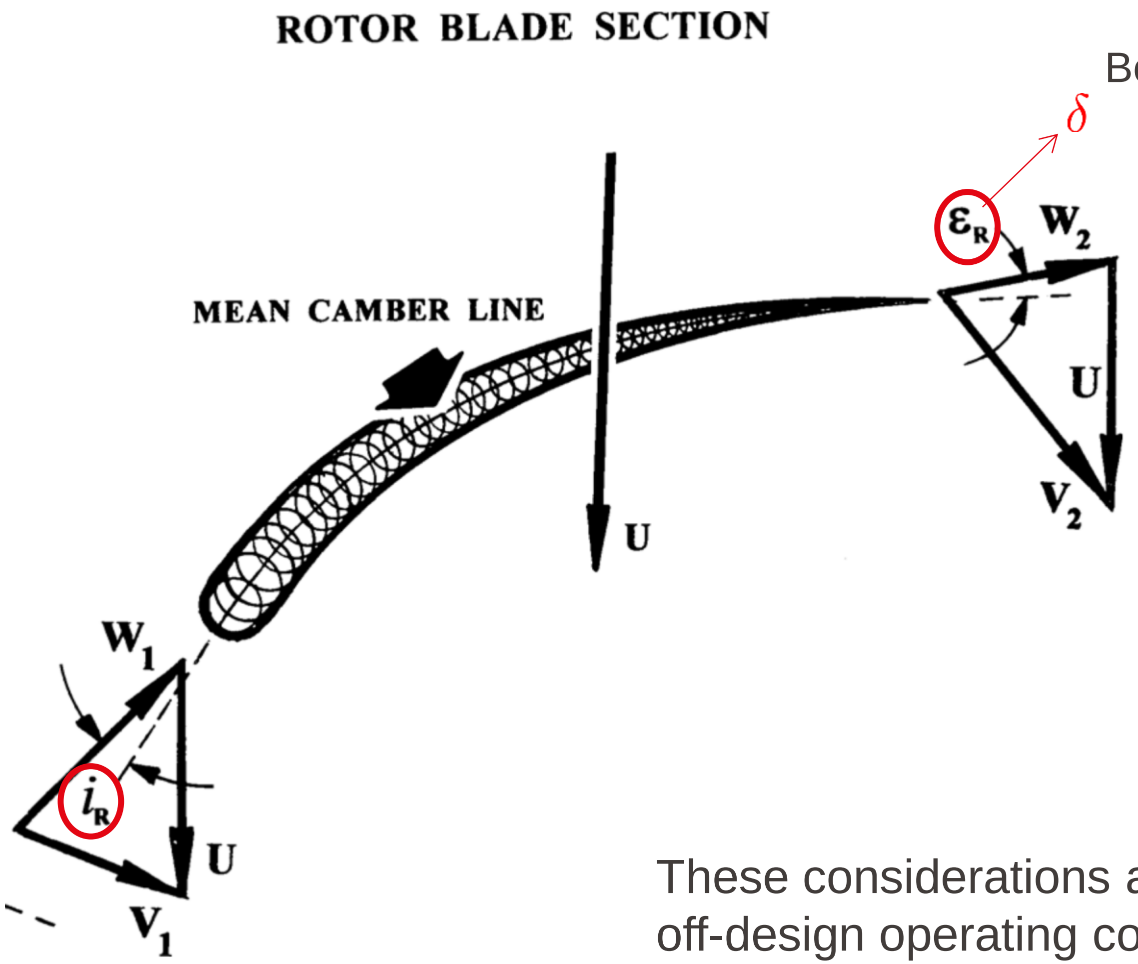
- Fluid is **perfectly guided by the blade** (relative velocity is parallel to the leading and trailing edge)*
- Flow leaves at $r = r_{\text{in}} = r_{\text{out}}$

*sometimes leading edge (inlet) **V** is prescribed

Incidence and deviation angles

Assumptions

- Fluid **is perfectly guided by the blade** (relative velocity is parallel to the leading and trailing edge)
- Flow leaves at $r = r_{\text{in}} = r_{\text{out}}$



Both ε and δ are used..

These considerations are important at design and off-design operating condition

Incidence and deviation angles

The **incidence** is the difference between the inlet flow angle (α) and the blade **inlet** angle (α'):

$$i = \alpha_1 - \alpha_1'$$

The **deviation** is the difference between the exit flow angle and the blade **exit** angle:

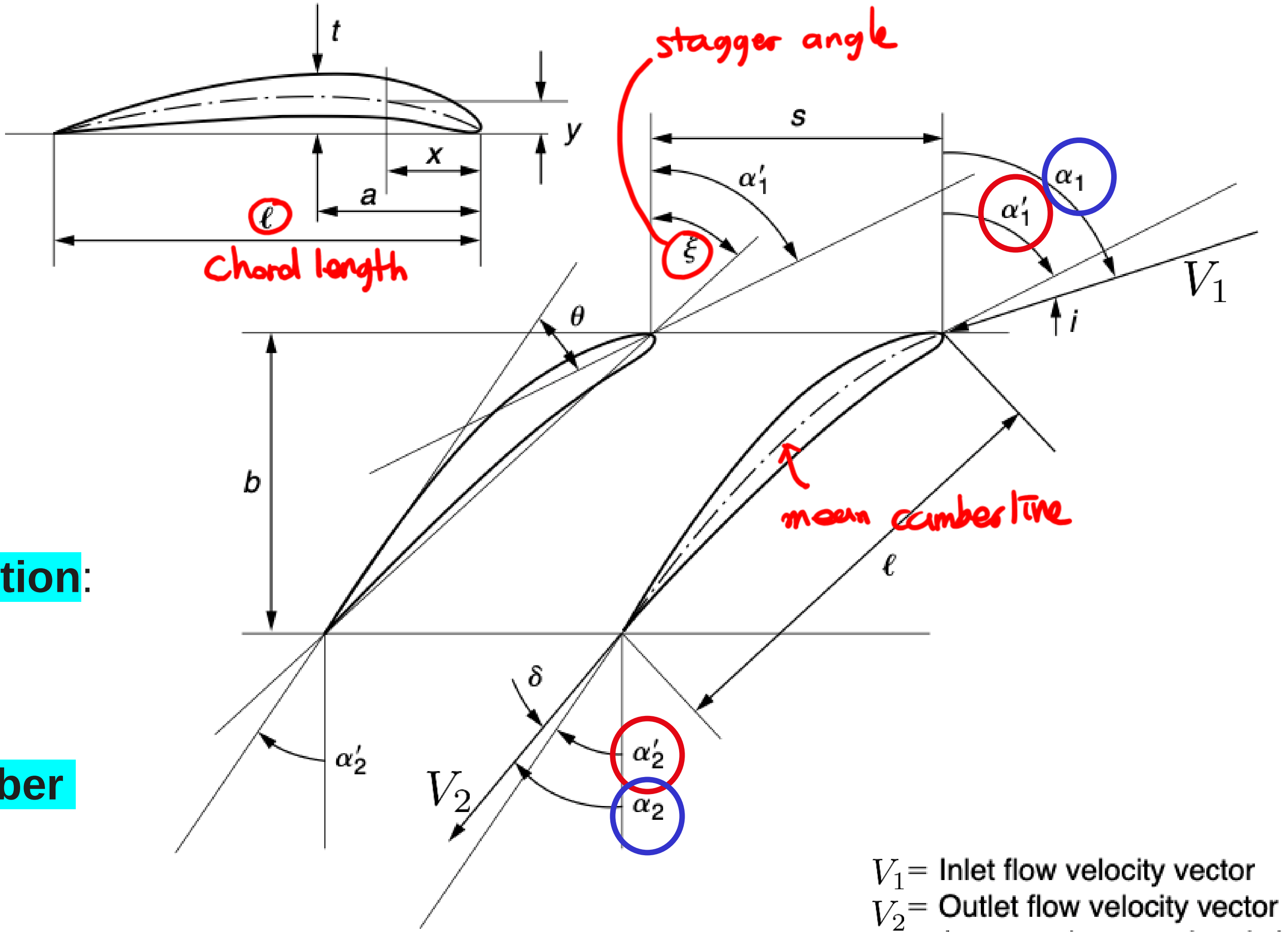
$$\delta = \alpha_2 - \alpha_2'$$

The change in angle of the flow is called **deflection**:

$$\varepsilon = \alpha_1 - \alpha_2$$

The change in angle of the blade is called **camber angle**:

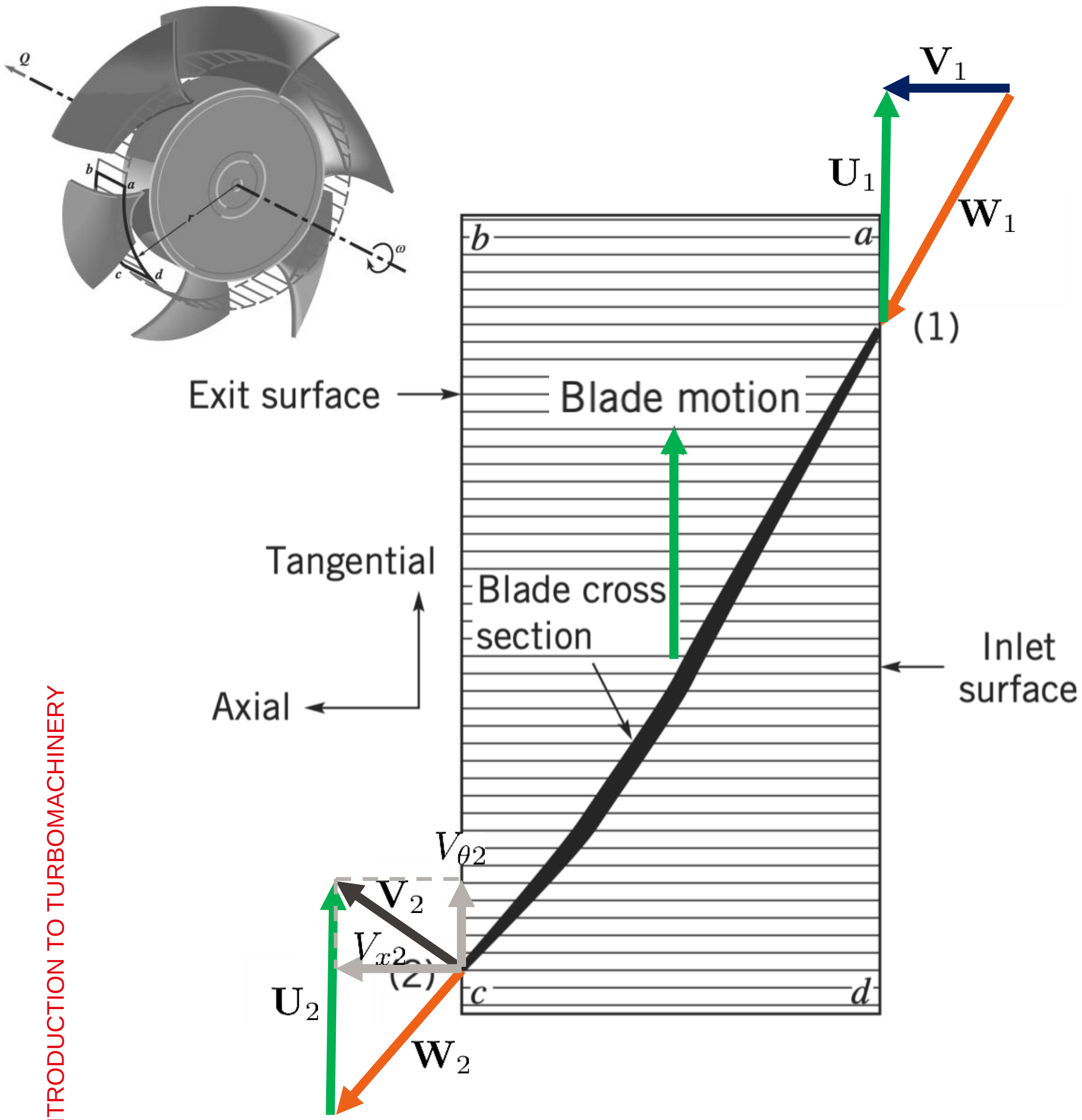
$$\theta = \alpha_1' - \alpha_2'$$

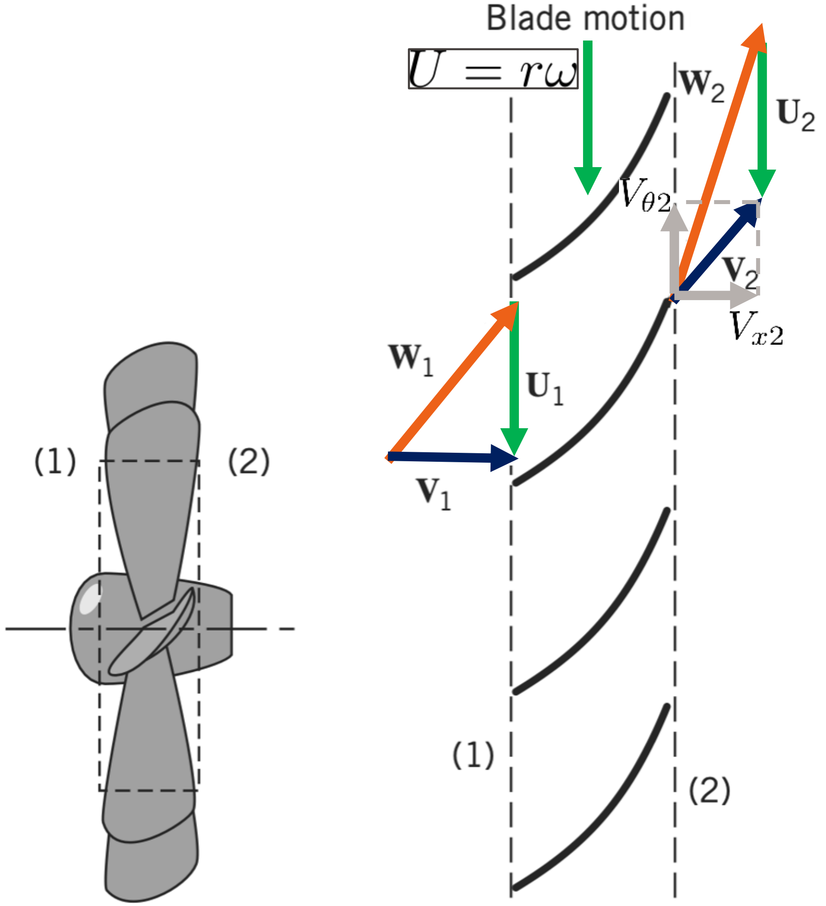


V_1 = Inlet flow velocity vector
V_2 = Outlet flow velocity vector
(averaged across the pitch

EPFL

Actung! Pump vs. turbine

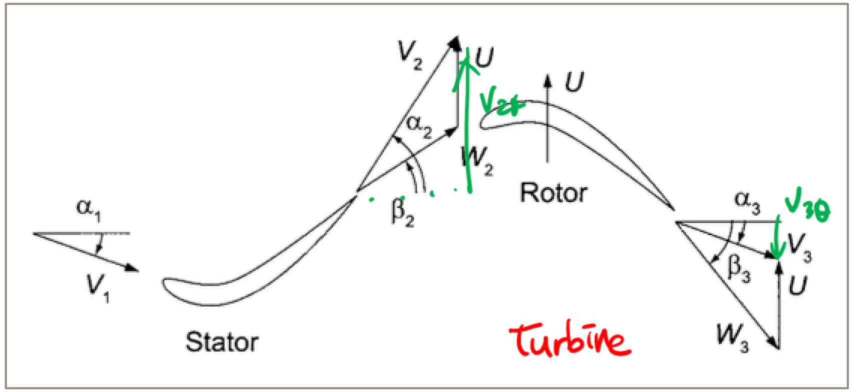


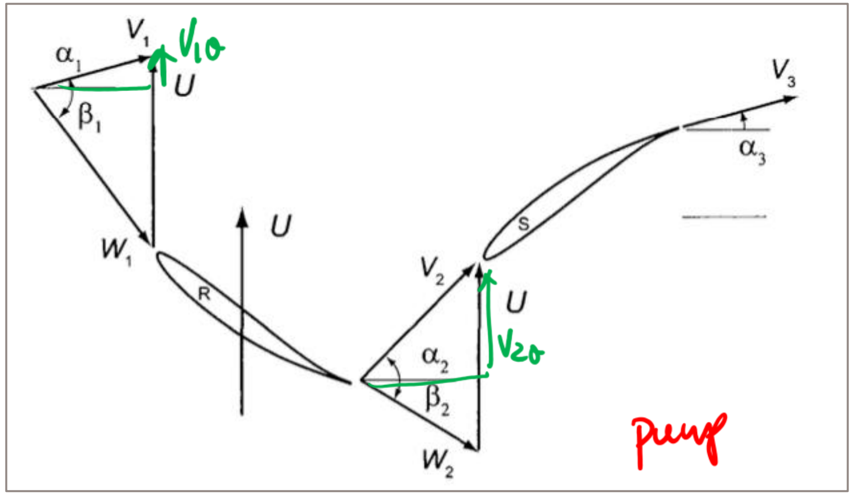


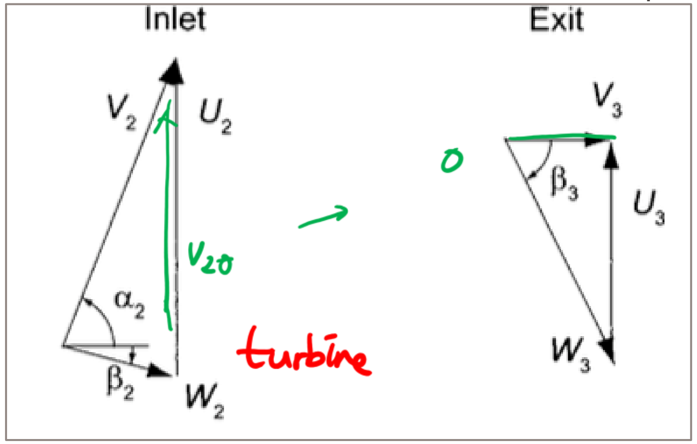
$$w_{\text{shaft}} = \frac{\dot{W}_{\text{shaft}}}{\rho Q} = U_2 V_{\theta 2} - U_1 V_{\theta 1}$$

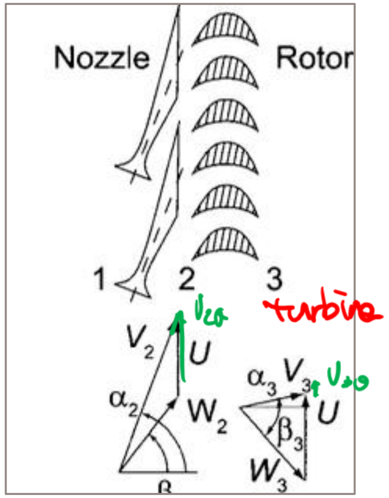
- The **direction argument** is valid only when inlet $V_{\theta 1} = 0$
- For nonzero $V_{\theta 1} \neq 0$, compare the magnitudes of $U_1 V_{\theta 1}$ and $U_2 V_{\theta 2}$
  - For $U_1 = U_2$, compare only V_θ

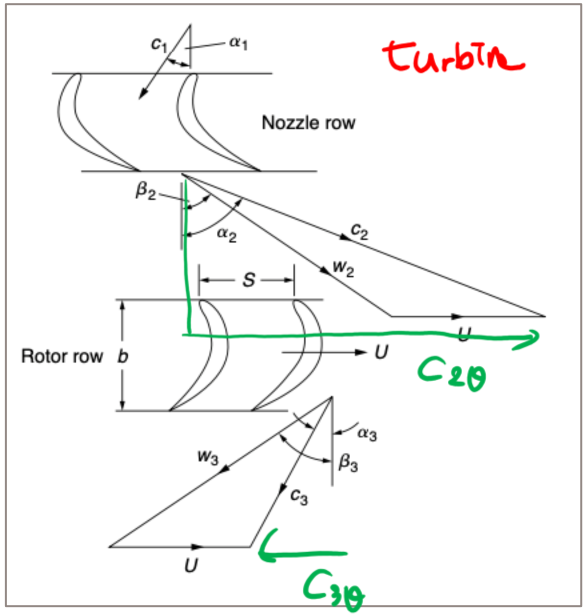
Pump or turbine? – when inlet $V_{\theta 1}$ is non-zero

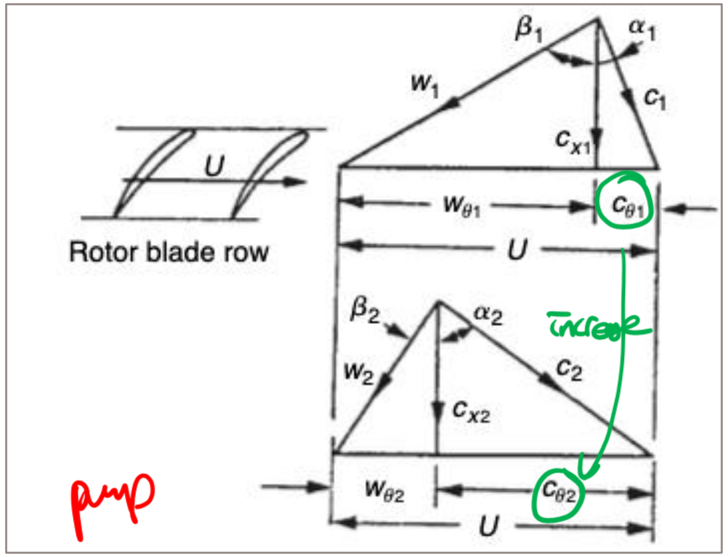


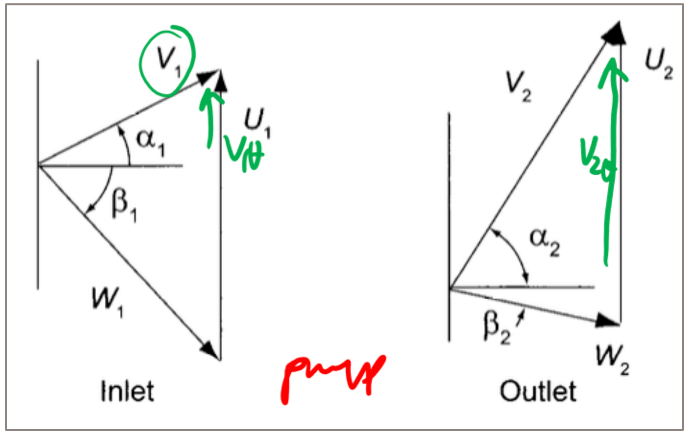












Commonly used notations

Relative	W	w
Rotation	U	u
Absolute	V	c

Hydraulic Turbines

Turbines

Impulse or **reaction** : No matter the working fluid, turbines can be broadly classified into two types based on the mechanism of the fluid interaction

Impulse turbine: the force on the blades is produced solely by turning the fluid, without appreciable pressure drop in the blade passage, with all of the pressure drop occurring in a fixed nozzle.

Reaction turbine: some of the fluid-vane force is from fluid turning and some of the force is a reaction to acceleration of the fluid relative to the vane. In reaction blading, a pressure drop occurs in both a fixed nozzle and the moving vane.

Turbine blading is characterized by the **degree of reaction (R) (or simply reaction)**, which is the ratio of the drop in static pressure (or enthalpy) across the moving blade to the overall drop in static pressure (or enthalpy) across the fixed nozzle plus the moving blade. Impulse turbines have $R = 0$ while reaction turbines typically have $0.1 < R < 0.7$.

Degree of reaction or Reaction (R)

Change in static enthalpy across the **rotor** divided by the static enthalpy change across the entire **stage**

(static) Enthalpy: internal energy and flow work

$$\check{h} = \check{u} + \frac{p}{\rho} \quad \text{[J/kg]}$$

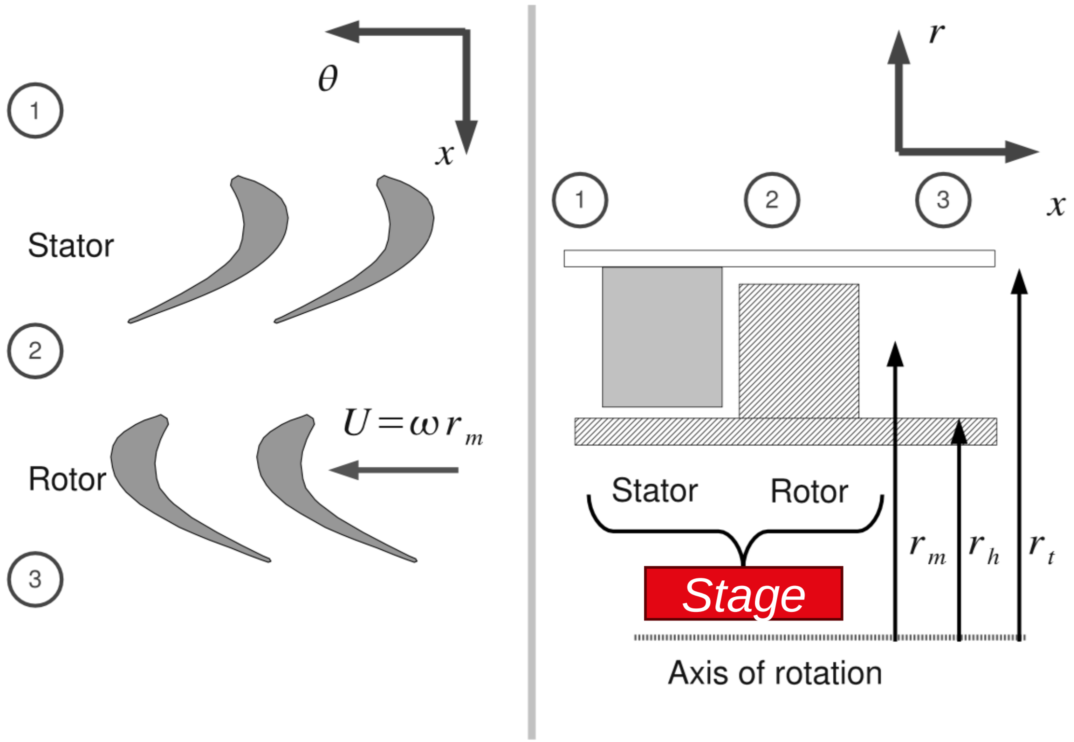
Internal energy

$\mathrm{Pa} \cdot \ \mathrm{m}^3/\mathrm{kg} \equiv \ \mathrm{J/kg}$

Stagnation enthalpy: sum of enthalpy, kinetic energy and potential energy

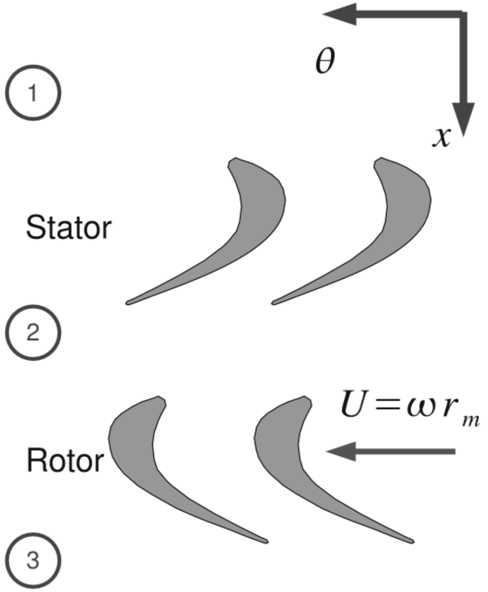
$$\check{h}_0 = \check{h} + \frac{V^2}{2} + gz$$

Achtung! $\check{h}$ is enthalpy, not head h or h_a



Degree of reaction or Reaction (R)

This concept is much used in axial flow machines as a measure of the relative proportions of energy transfer obtained by static and dynamic pressure change



(static) Enthalpy: internal energy and flow work

$$\check{h} = \check{u} + \frac{p}{\rho} \quad \text{[J/kg]}$$

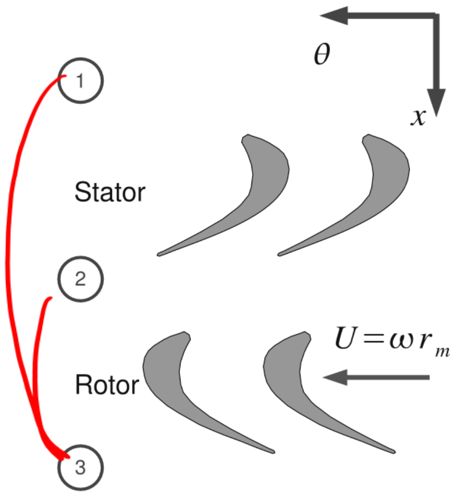
$$R = \frac{\text{energy change due to, or resulting from, static pressure change in the } \textbf{rotor}}{\text{total energy change for a } \textbf{stage}}$$

$$R = \frac{\text{static enthalpy change in } \textbf{rotor}}{\text{static enthalpy change in } \textbf{stage}}$$

$$= \frac{\check{h}_2 - \check{h}_3}{\check{h}_1 - \check{h}_3}$$

If no internal energy is changed, incompressible, $R \simeq \dfrac{p_2 - p_3}{p_1 - p_3}$

Degree of reaction or Reaction (R)



$$\frac{p_1}{\rho} + \frac{V_1^2}{2} + g z_1 = \frac{p_3}{\rho} + \frac{V_3^2}{2} + g z_3 + w_f$$

$$\frac{p_2}{\rho} + \frac{V_2^2}{2} + g z_2 = \frac{p_3}{\rho} + \frac{V_3^2}{2} + g z_3 + w_f$$

equal, Stator does not work!

$$\frac{p_1 - p_3}{\rho} = \frac{V_3^2}{2} - \frac{V_1^2}{2} + w_f$$

$$\frac{p_2 - p_3}{\rho} = \frac{V_3^2}{2} - \frac{V_2^2}{2} + w_f$$

$$R \simeq \frac{p_2 - p_3}{p_1 - p_3}$$

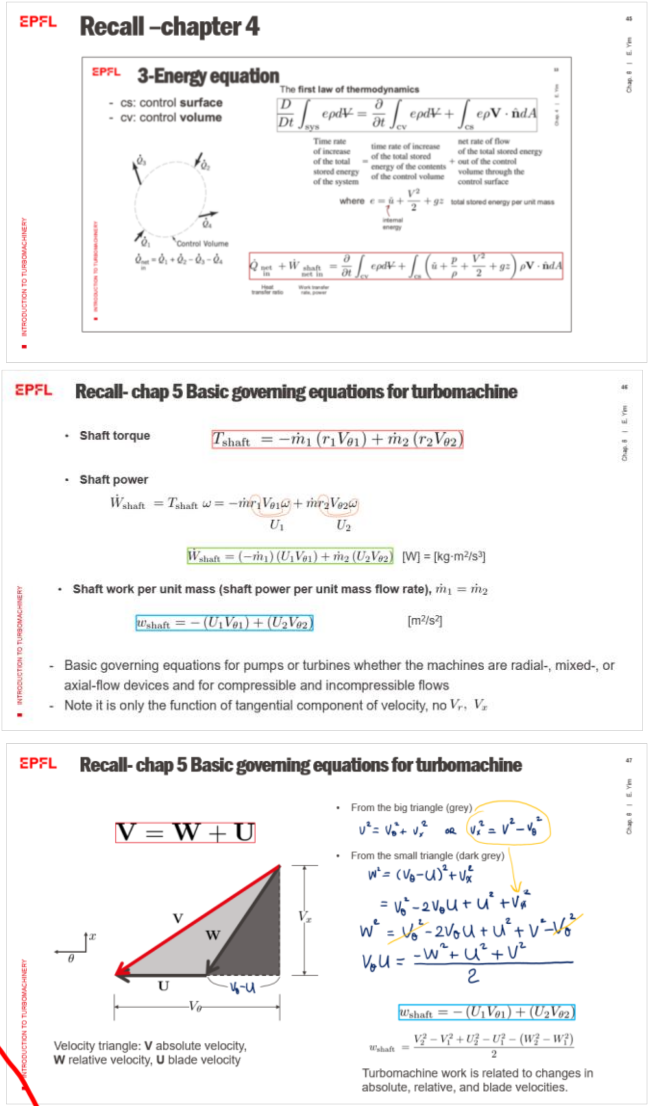
$$w_f = -w_{\text{shaft}}$$

$$R = \frac{\frac{1}{2}\left(V_3^2 - V_2^2\right) - w_{\text{shaft}}}{\frac{1}{2}\left(V_3^2 - V_1^2\right) - w_{\text{shaft}}} = \frac{\frac{1}{2}\left(V_3^2 - V_2^2\right) - \frac{1}{2}\left(V_3^2 - V_2^2 + U_3^2 - U_2^2 - \left(W_3^2 - W_2^2\right)\right)}{\frac{1}{2}\left(V_3^2 - V_1^2\right) - \frac{1}{2}\left(V_3^2 - V_2^2 + U_3^2 - U_2^2 - \left(W_3^2 - W_2^2\right)\right)}$$

$$w_{\text{shaft}} = \frac{V_3^2 - V_2^2 + U_3^2 - U_2^2 - \left(W_3^2 - W_2^2\right)}{2}$$

$$R = \frac{U_2^2 - U_3^2 + W_3^2 - W_2^2}{V_2^2 - V_1^2 + U_2^2 - U_3^2 + W_3^2 - W_2^2}$$

- If $U_2=U_3$ (axial machine), $W_3=W_2$ → R=0
- If $V_1=V_2$ → R=1



Types of hydraulic turbines

- Impulse turbines: Pelton turbines, R~0
- Reaction turbines: R~[0.1, 0.7]

  Francis turbines (radial and axial), Kaplan turbines (axial)
  Propeller turbines (similar to Kaplan turbines with fixed pitch)

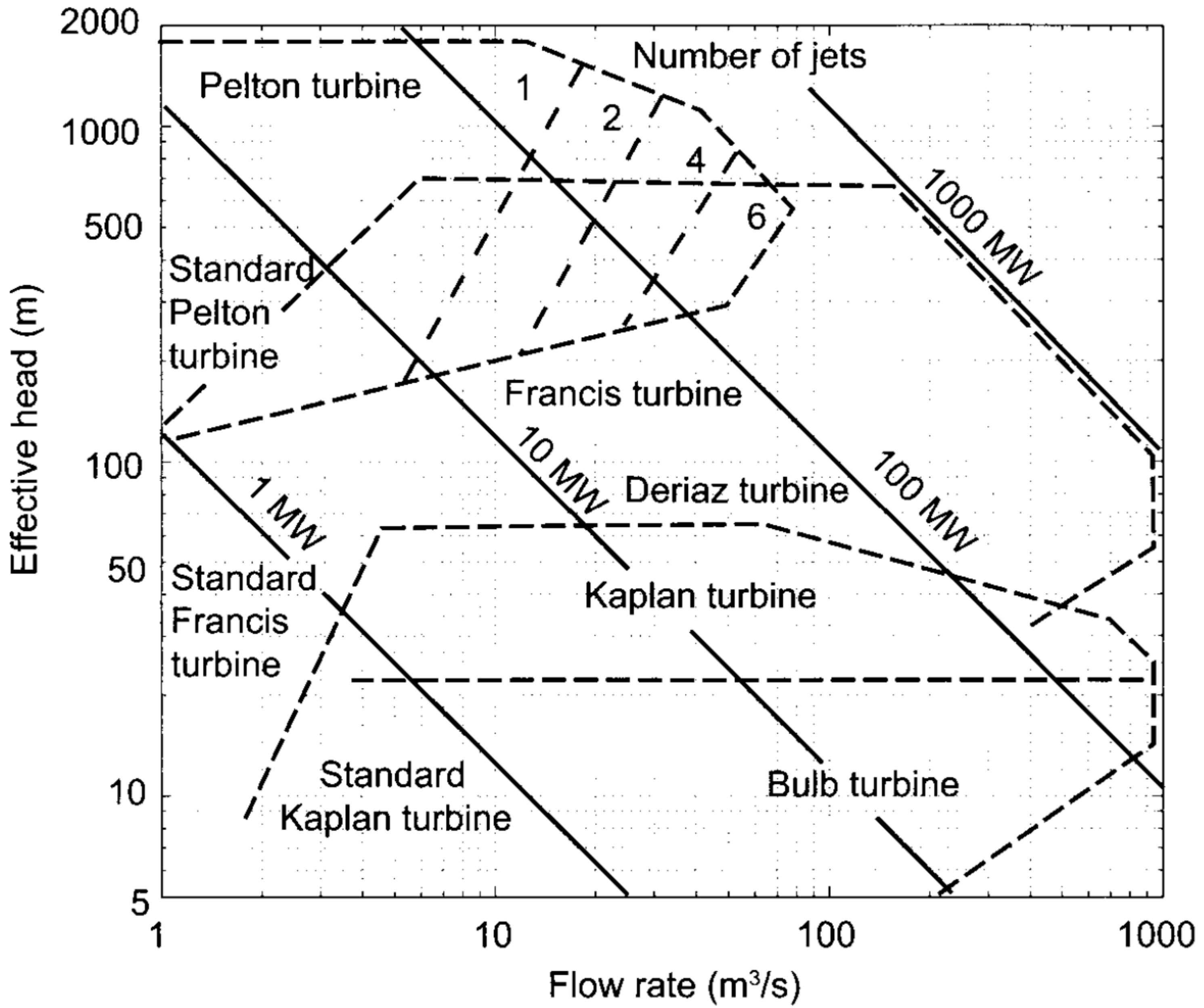
Pelton

Francis

Kaplan & Bulb

Fixed pitch propeller

Types of hydraulic turbines



Power specific speed

For hydraulic turbines, the rotor diameter D is eliminated between the flow coefficient and the power coefficient to obtain the power-specific speed

Power specific speed (Hydraulic turbines)

$$N_s' = \frac{\omega\sqrt{\dot{W}_{\text{shaft}}/\rho}}{(gh_a)^{5/4}}$$

Specific speed

$$N_s = \frac{\omega\sqrt{Q}}{(gh_a)^{3/4}}$$

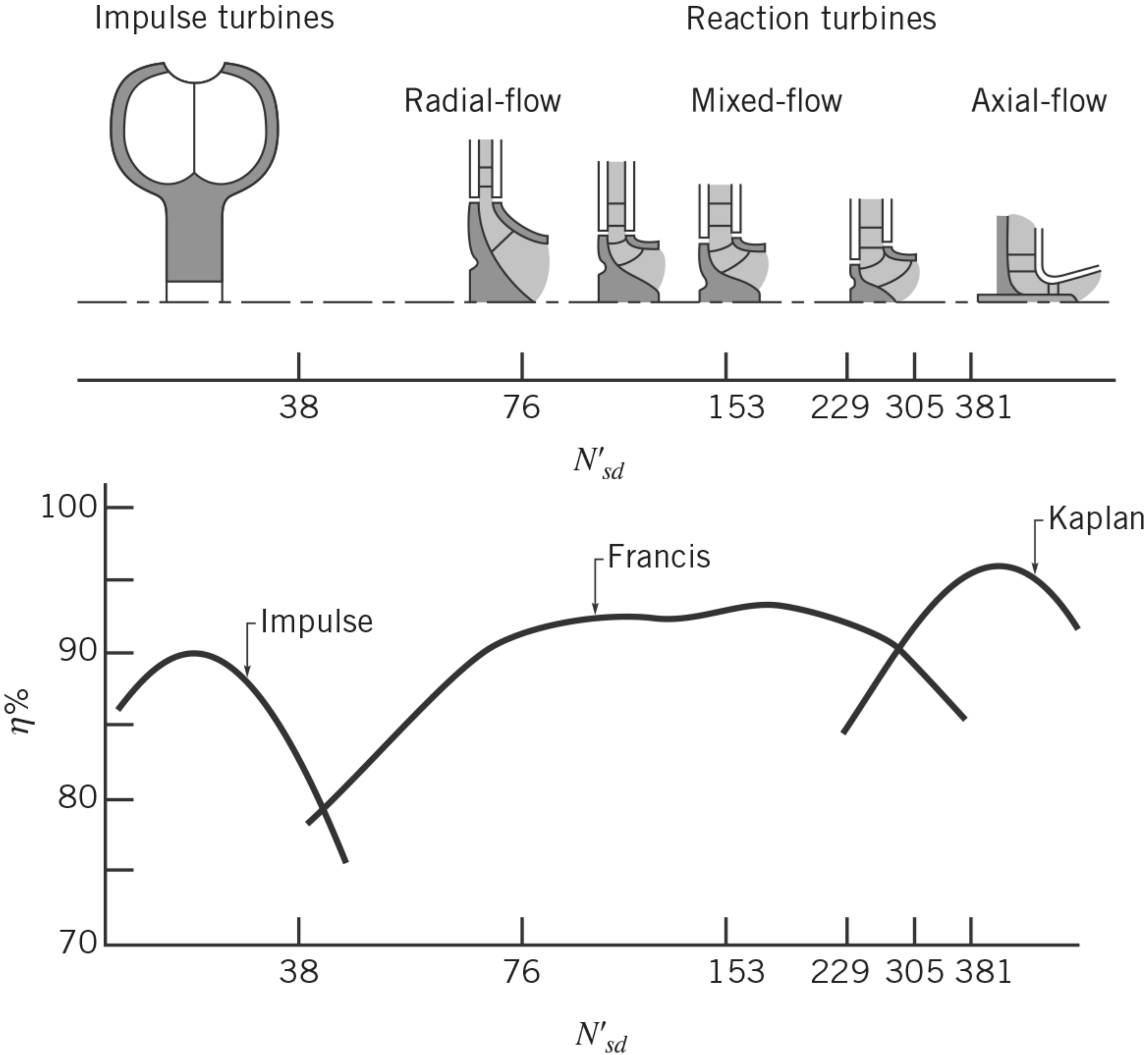
Commonly used, but not dimensionless, definition of power specific speed $N_{sd}' = \frac{\omega(\text{rpm})\sqrt{\dot{W}_{\text{shaft}}(\text{W})}}{[h_a(\text{m})]^{5/4}}$

In hydraulic machine, the actual head (in pump) h_a is commonly called '**effective or net head**'.
The elevation head (physical difference between upper reservoir's surface and the one of the lower one) is called '**gross head**', h_g.

$$h_a = h_g - h_L$$

- Power gained by/extracted from the fluids, $P_f = \gamma Q h_a$
- Efficiency pump $\eta = \frac{P_f}{\dot{W}_{\text{shaft}}}$ ⟶ $\dot{W}_{\text{shaft}} = P_f/\eta = \gamma Q h_a/\eta$
- Efficiency turbine $\eta = \frac{\dot{W}_{\text{shaft}}}{P_f}$ ⟶ $\dot{W}_{\text{shaft}} = \eta P_f = \eta\gamma Q h_a$

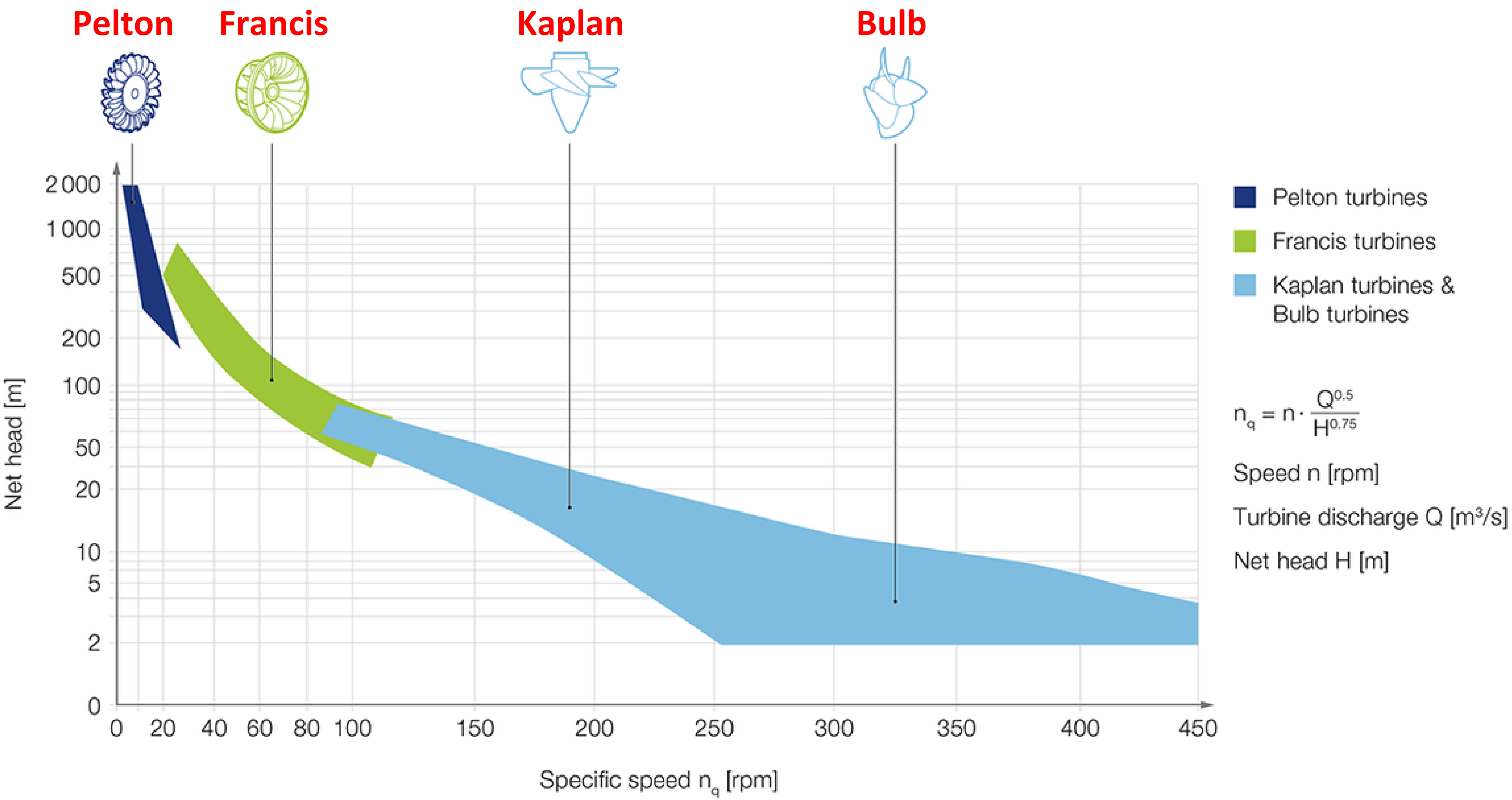
Turbines



Type		N_s'	η %
Pelton wheel			
	Single jet	0.02 – 0.18	88 – 90
	Twin jet	0.09 – 0.26	89 – 92
	Three jet	0.10 – 0.30	89 – 92
	Four jet	0.12 – 0.36	86
Francis			
	Low-speed	0.39 – 0.65	90 – 92
	Medium-speed	0.65 – 1.2	93
	High-speed	1.2 – 1.9	93 – 96
	Extreme-speed	1.9 – 2.3	89 – 91
Kaplan turbine		1.55 – 5.17	87 – 94
Bulb turbine		3 – 8	

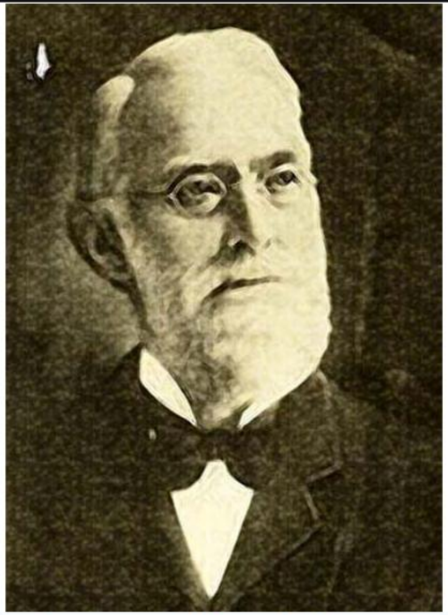
Turbines

- Classification of turbine types as a function of the head and unit specific speed

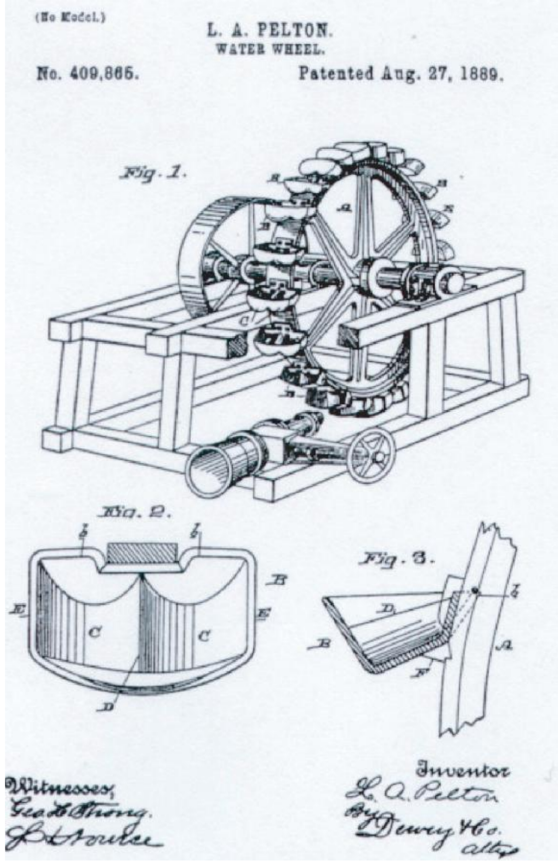


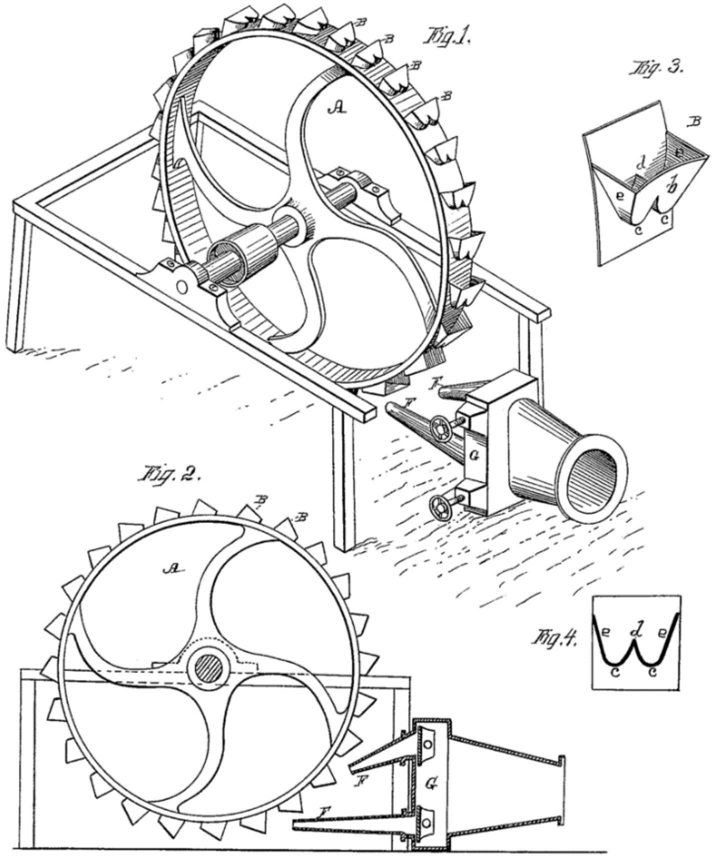
Impulse turbines – Pelton turbine

- **Impulse-type Turbines**
  - Among several types of impulse turbines, the so-called Pelton turbine is the most used
  - Patented by L. A. Pelton in 1889
  - The rotor is made of several buckets and the motion is obtained by high-speed jet(s)



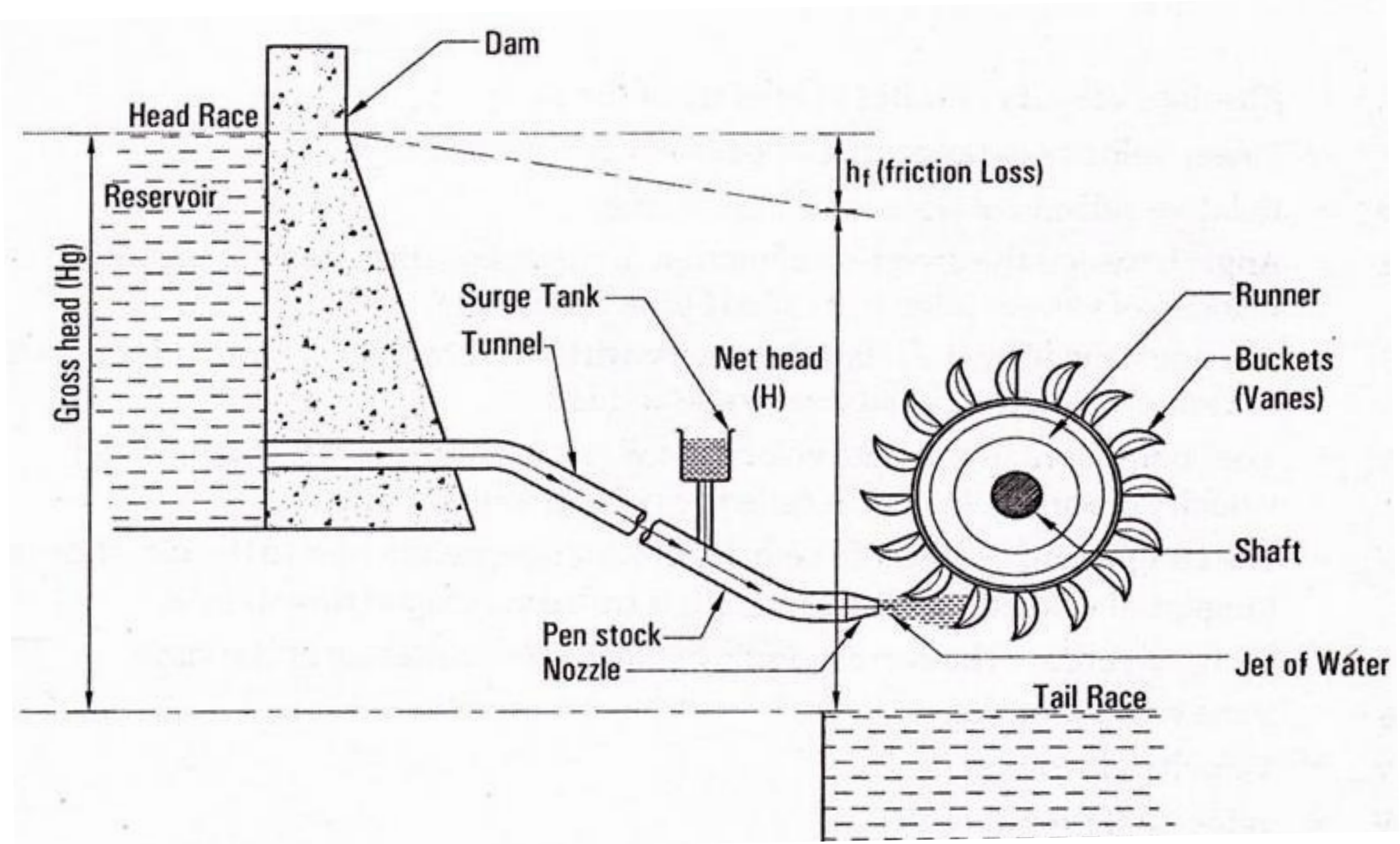
Lester Allan Pelton
(Sept. 5, 1829 – Mar. 14, 1908)
Pelton made his living as a carpenter and a millwright. He created the most efficient form of impulse water turbine.





Pelton turbines

- Pelton wheel is not typical turbomachines (no axial flow, no radial flow), tangential flow
    - Pelton turbines are the most used turbines in **Switzerland** in hydropower generation

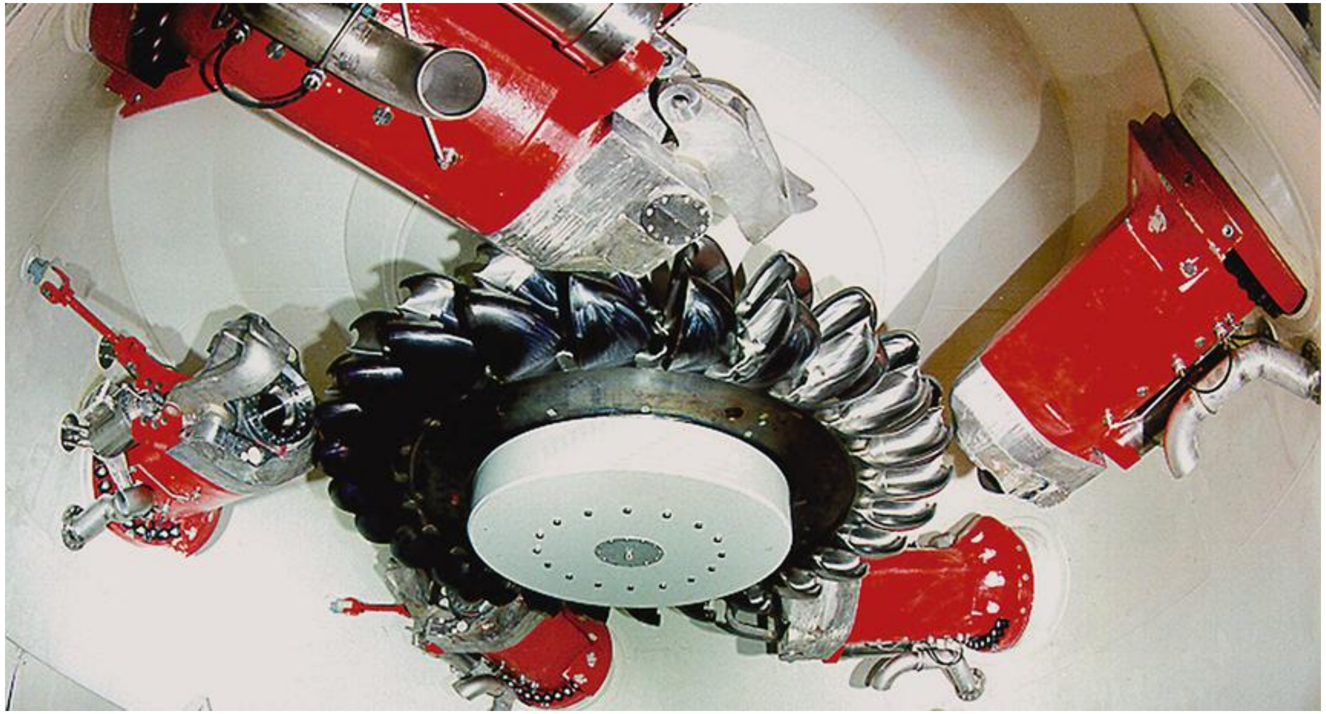


Pelton turbines

- The water enters and leaves the control volume surrounding the wheel as free jets (at **atmospheric** pressure)
- A person riding on the bucket would note that the speed of the water does not change as it slides across the buckets (assuming viscous effects are negligible)

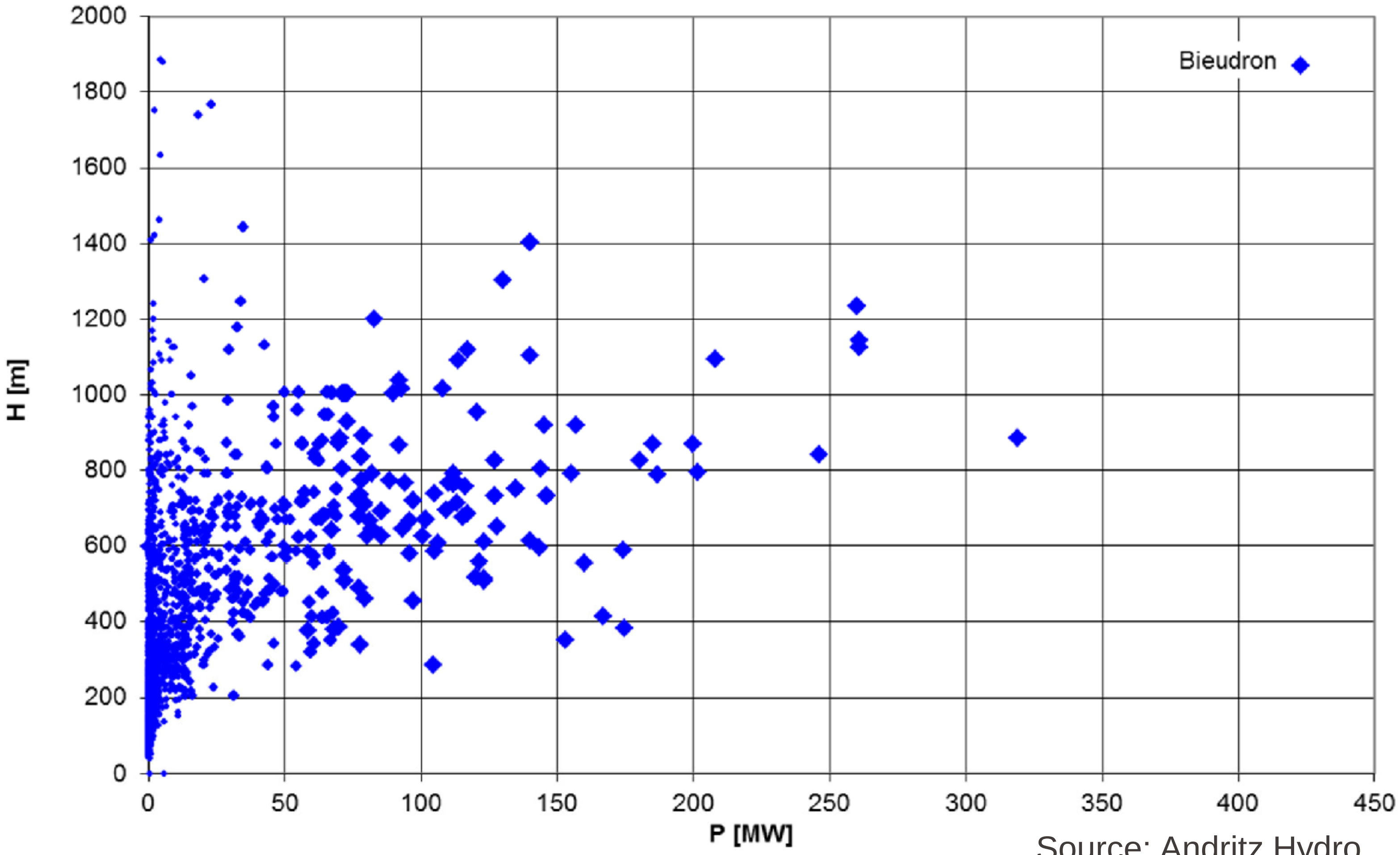
  → the magnitude of the relative velocity does not change, but its direction does.

- The change in direction of the velocity of the fluid jet causes a torque on the rotor, resulting in a power output from the turbine



Pelton turbines

- Suitable for high heads (100’s meter range):

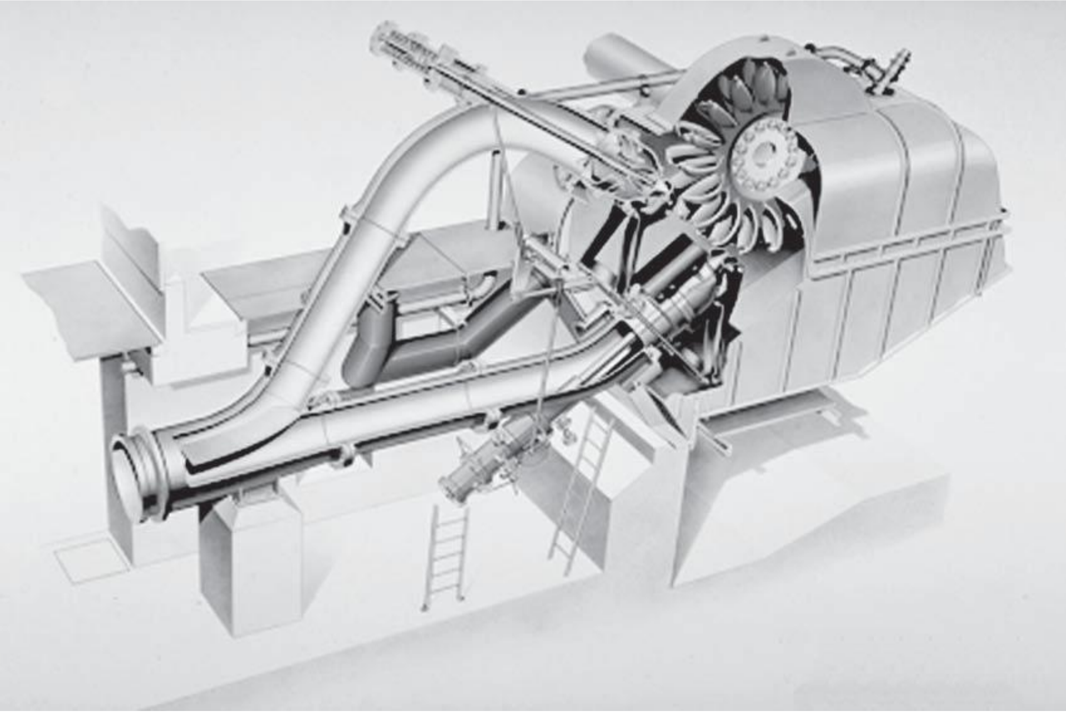


Source: Andritz Hydro

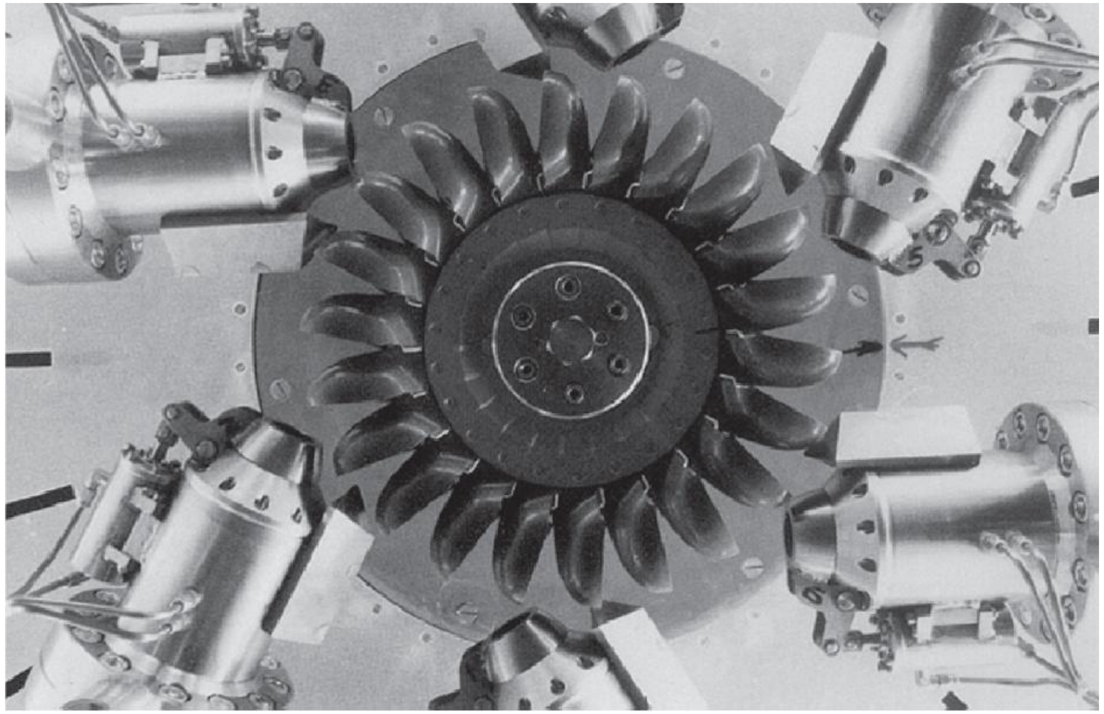
Pelton turbines

- Types of Pelton Turbines:

Horizontal shaft: 1, 2 or 3 Jets

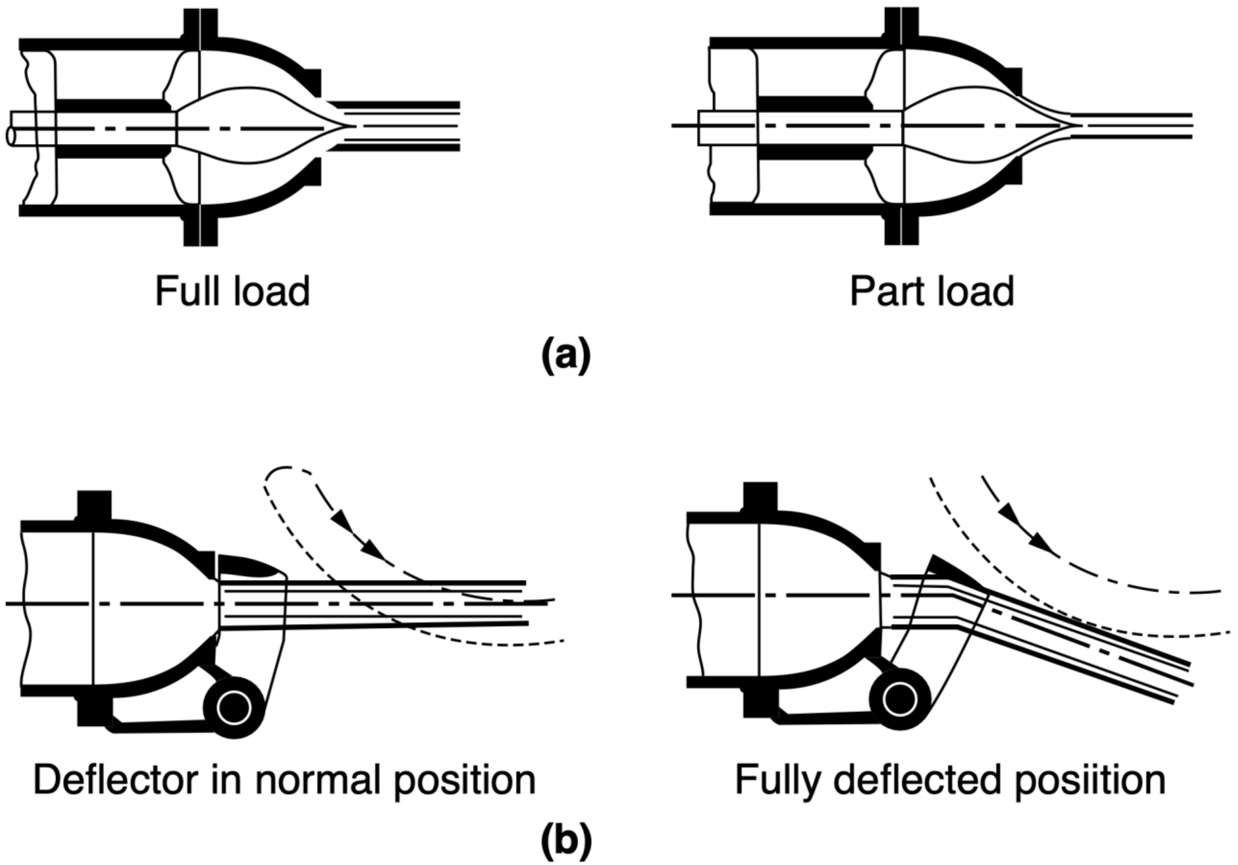


Vertical shaft: 1 to 6 Jets



Pelton turbines

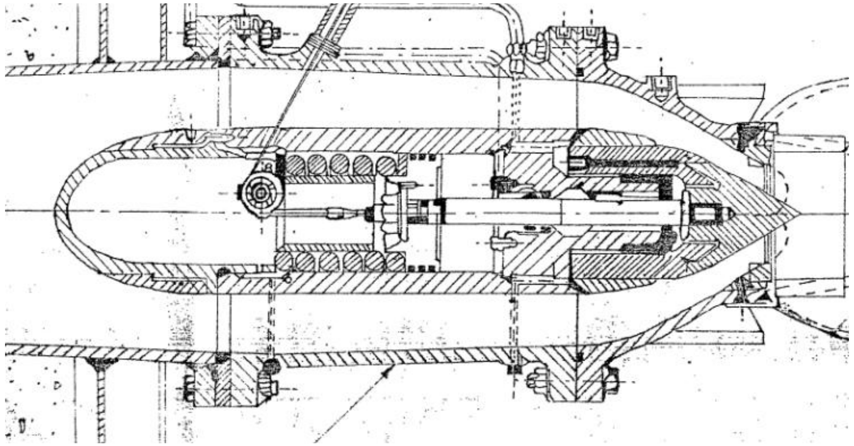
- **The nozzle:**
  - Equipped with a needle for flowrate control (e.g. valve)
  - Actioned by a servomotor, which may be internal or external
  - Equipped with a deflector for emergency stops



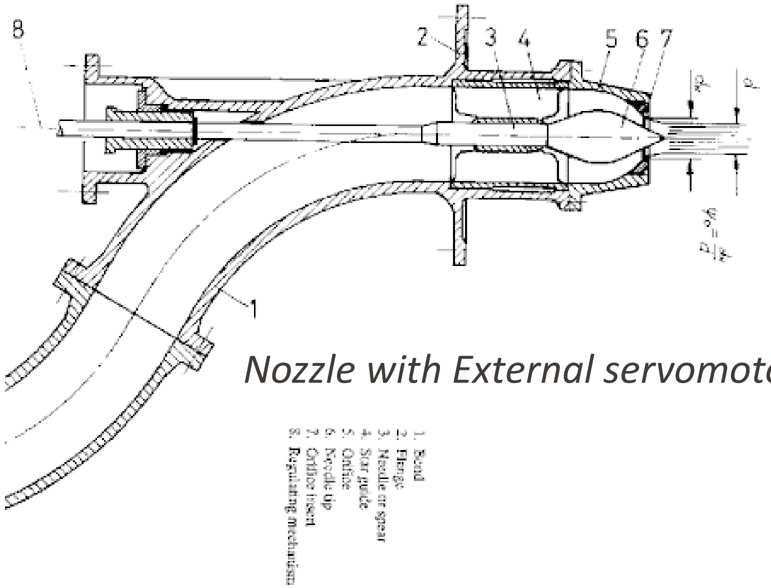
Pelton turbines

- **The nozzle:**
  - Key element of the design: must produce a jet of a “high quality”
    - Straight jet
    - Minimum atomization (formation of small droplets)

What happens when the nozzle closes suddenly?



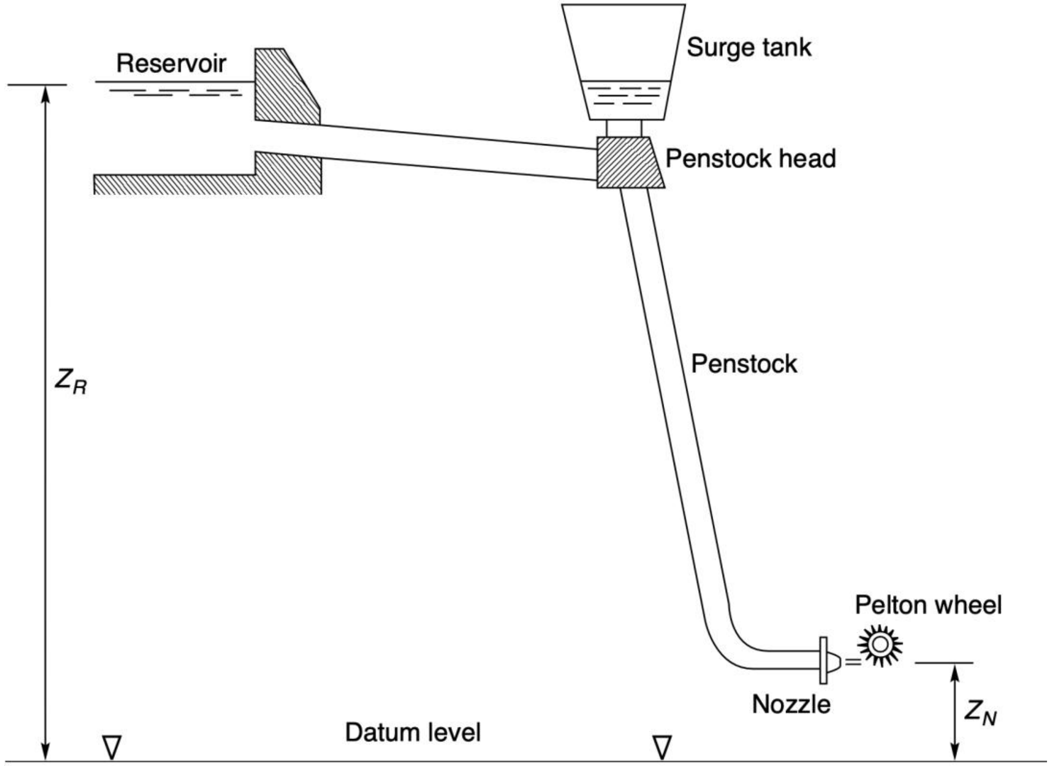
Nozzle with Internal servomotor

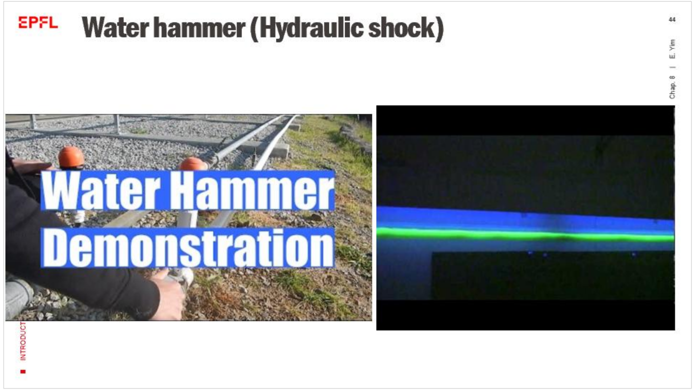


Nozzle with External servomotor

Pressure surges

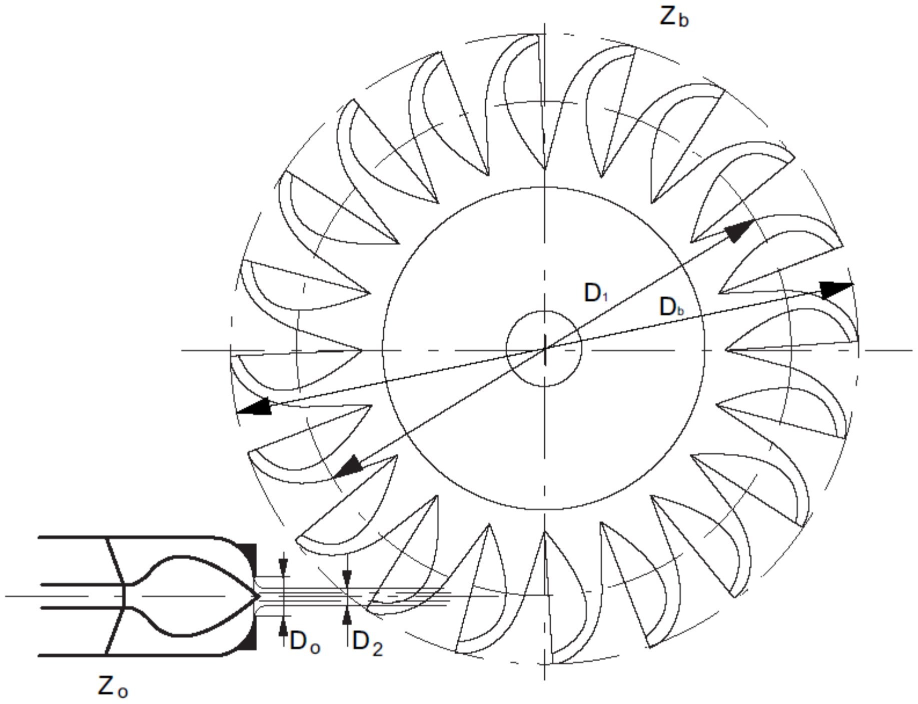
- Spear valve must move slowly: sudden reduction in flow rate may result in serious damage from **pressure surges** (called water hammer)
- If the spear valve closes quickly: all the kinetic energy of the water in the penstock would be absorbed by the elasticity of the supply pipeline (penstock) and the water, creating very large stresses, which would reach their greatest intensity at the turbine inlet where the pipeline is already heavily stressed.
- The surge chamber has the function of absorbing and dissipating some of the pressure and energy fluctuations created by too rapid a closure of the needle valve.

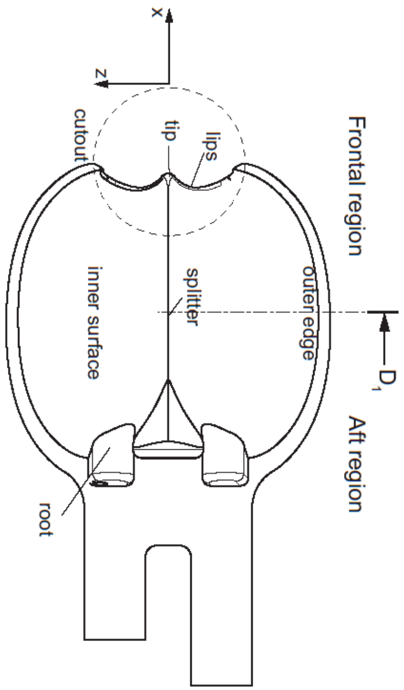


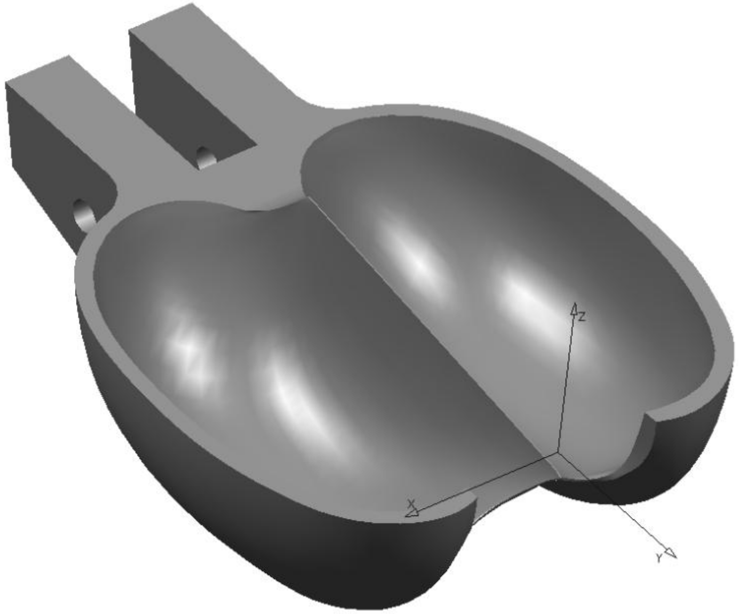


Pelton turbines

- The buckets:
  - Must deviate “smoothly” the flow and allow for its evacuation with a minimum interference with each other

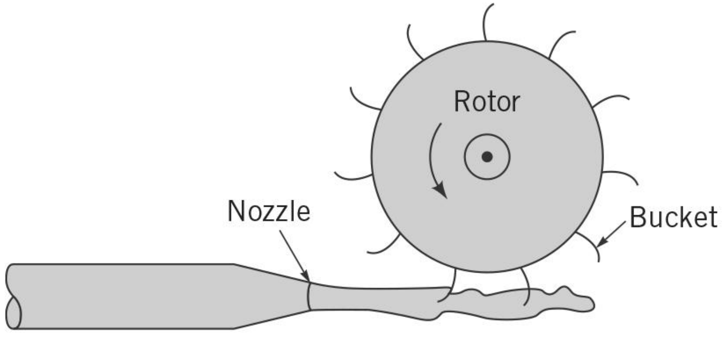




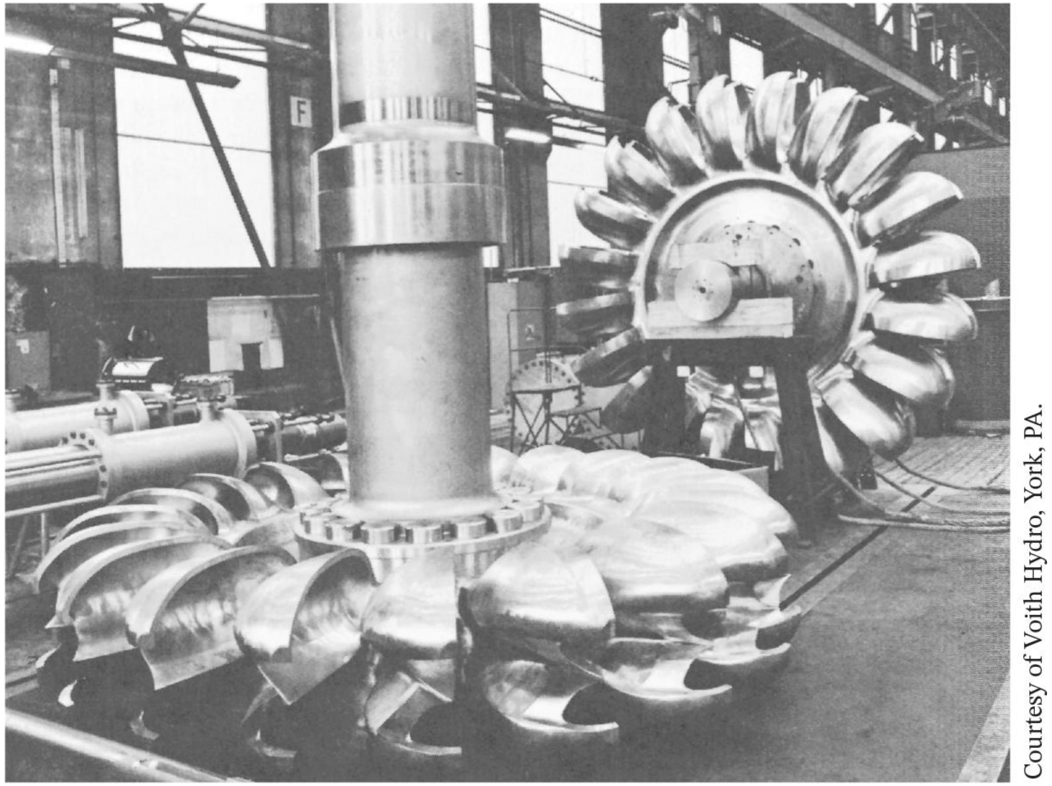


EPFL

Pelton turbines



(*a*)



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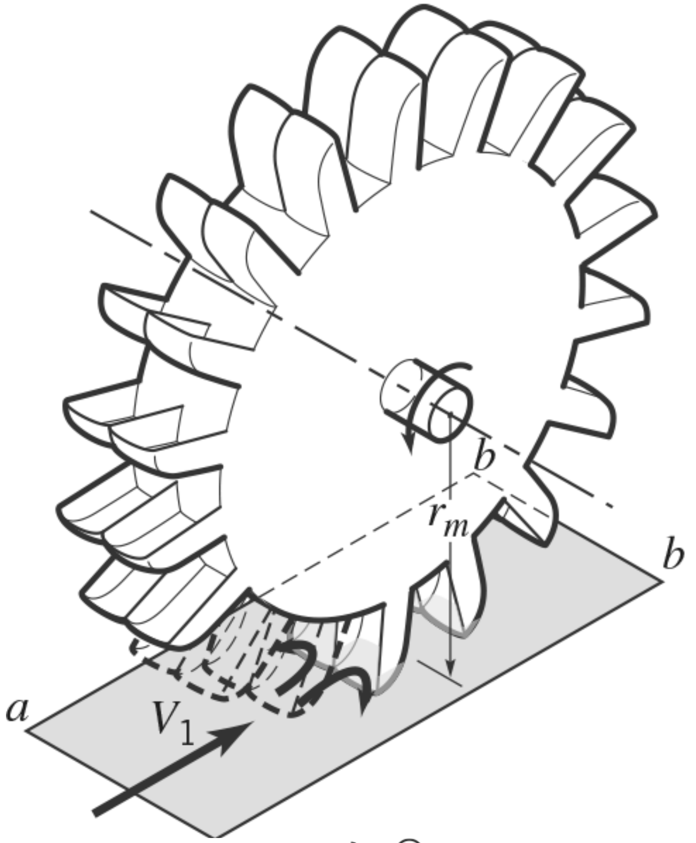
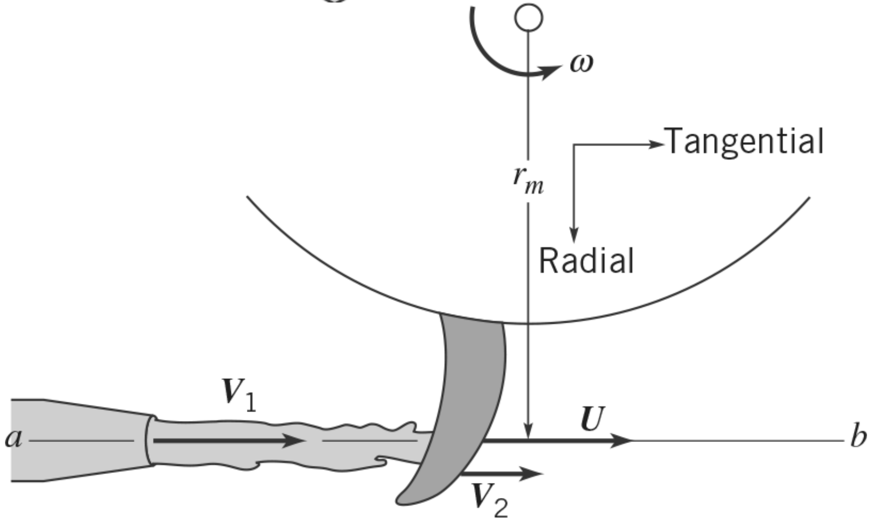
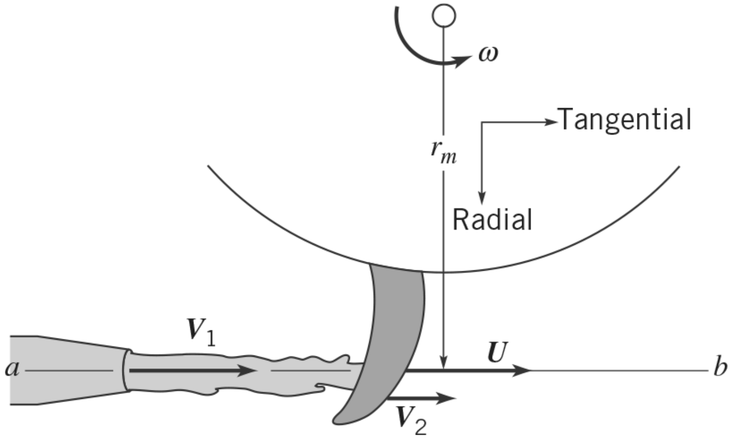
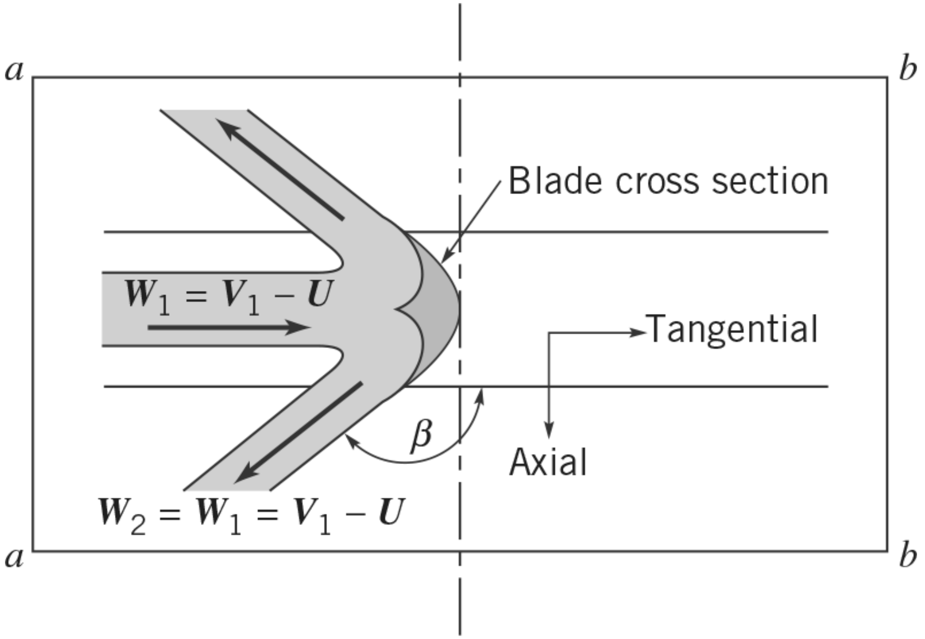




Pelton turbines





Assumptions

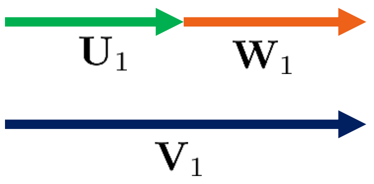
- no radial component of flow
- $W_2 \simeq W_1$ (otherwise $W_2 = C_v W_1$ with $C_v < 1$ velocity coefficient) → exercise 2

$$T_{\text{shaft}} = -\dot{m}_1 (r_1 V_{\theta 1}) + \dot{m}_2 (r_2 V_{\theta 2})$$

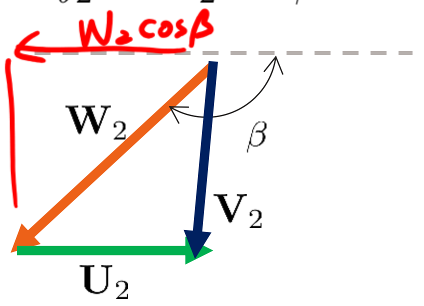
$$\dot{m}_1 = \dot{m}_2 = \dot{m}, \; r_1 = r_2 = r_m, \; \mathbf{U}_1 = \mathbf{U}_2 = U$$

$$T_{shaft} = \dot{m} r_m (V_{\theta 2} - V_{\theta 1})$$

$$V_{\theta 1} = V_1 = W_1 + U$$



$$V_{\theta 2} = W_2 \cos\beta + U$$



$W_2 = W_1$

$= V_1 - U$

$$V_{\theta 2} - V_{\theta 1} = (V_1 - U)\cos\beta + U - V_1$$
$$= (U - V_1)(1 - \cos\beta)$$

$$T_{shaft} = \dot{m} r_m \left((U - V_1)(1 - \cos\beta) \right)$$

$$\dot{W}_{\text{shaft}} = T_{\text{shaft}}\,\omega = \dot{m} U (U - V_1)(1 - \cos\beta)$$

↑ $r\omega$

If $V_1 > U$ (jet impacting bucket), $\dot{W}_{\text{shaft}} < 0$ the turbine extracts power from the fluid

Pelton turbines

$$\dot{W}_{shaft} = \dot{m}U(U - V_1)(1 - \cos\beta)$$

- Effect of β : maximum when β = 180° (cos π=-1)

$$\dot{W}_{shaft} = 2\dot{m}(U^2 - UV_1)$$

Maximum power with respect to U

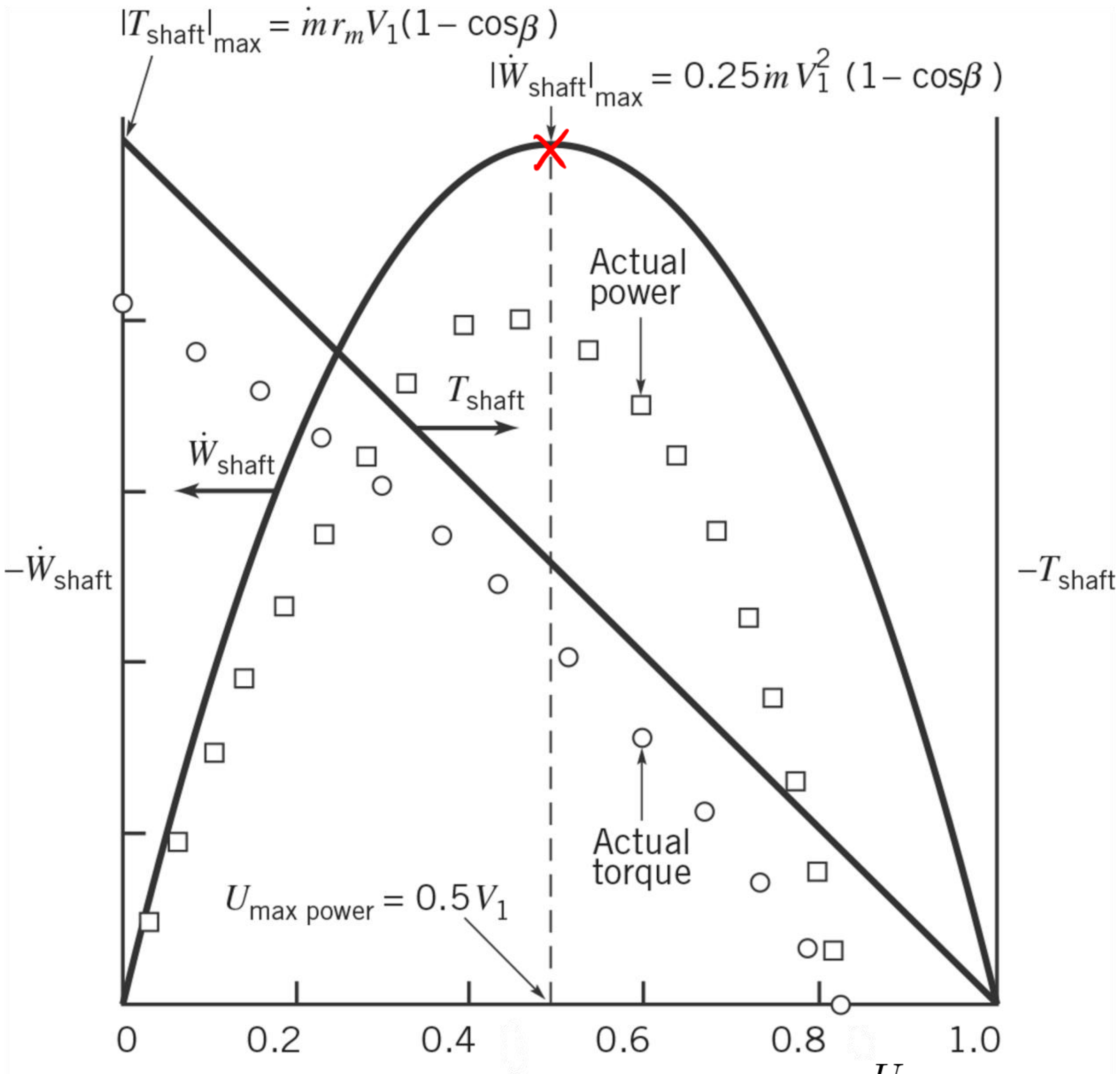
$$\frac{d\dot{W}_{shaft}}{dU} = 2\dot{m}(2U - V_1) = 0$$

$$U_{max\ power} = \frac{V_1}{2}$$

A bucket speed one-half the speed of the fluid coming from the nozzle gives maximum power.

$$T_{shaft} = \dot{m}r_m(U - V_1)(1 - \cos\beta)$$

- Shaft torque = 0 when U = V_1



blade speed to jet-speed ratio $\lambda = \frac{U}{V_1}$

Pelton turbines – Nozzle speed

$$\frac{p_{atm}}{\gamma} + z_0 = \frac{p_{atm}}{\gamma} + \frac{V_1^2}{2g} + z_1 + \sum h_L$$

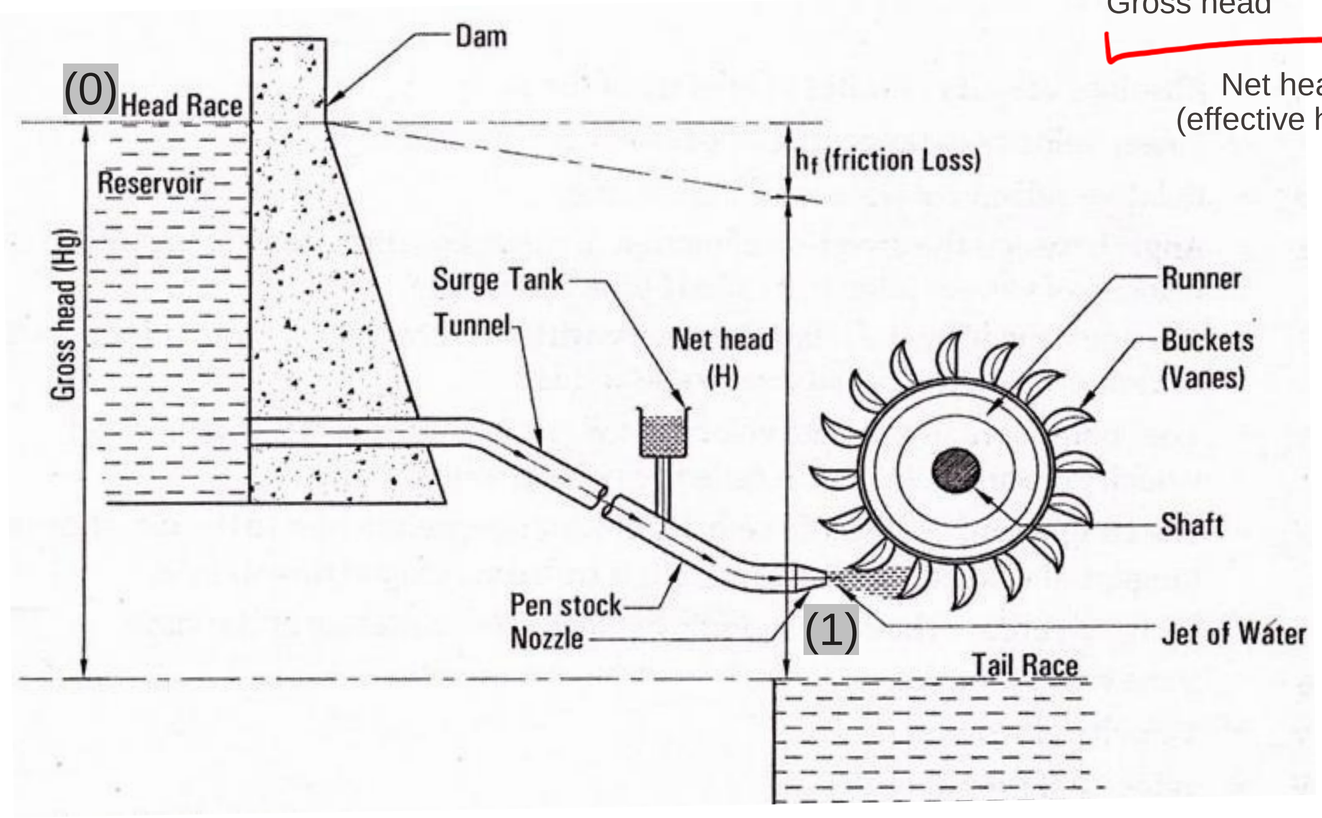
$$z_0 - z_1 - \Sigma h_L = \frac{V_1^2}{2g}$$

Gross head ($z_0 - z_1$)

Net head h_a (effective head)

$$h_a = \frac{V_1^2}{2g}$$

$$V_1 = \sqrt{2gh_a} \approx V_{nozzle}$$



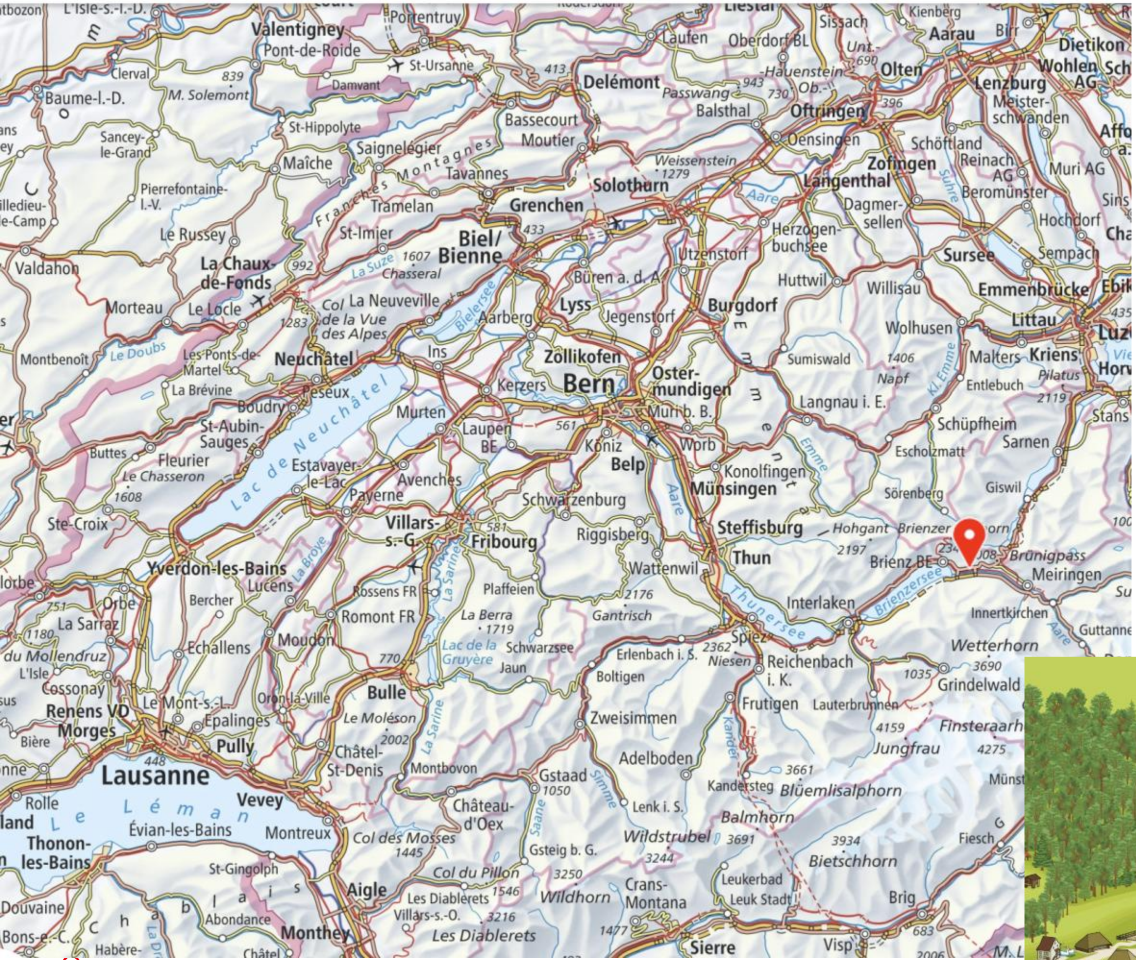
Or $c_N V_1 = V_{\text{nozzle}}$ Nozzle coefficient, c_N <1

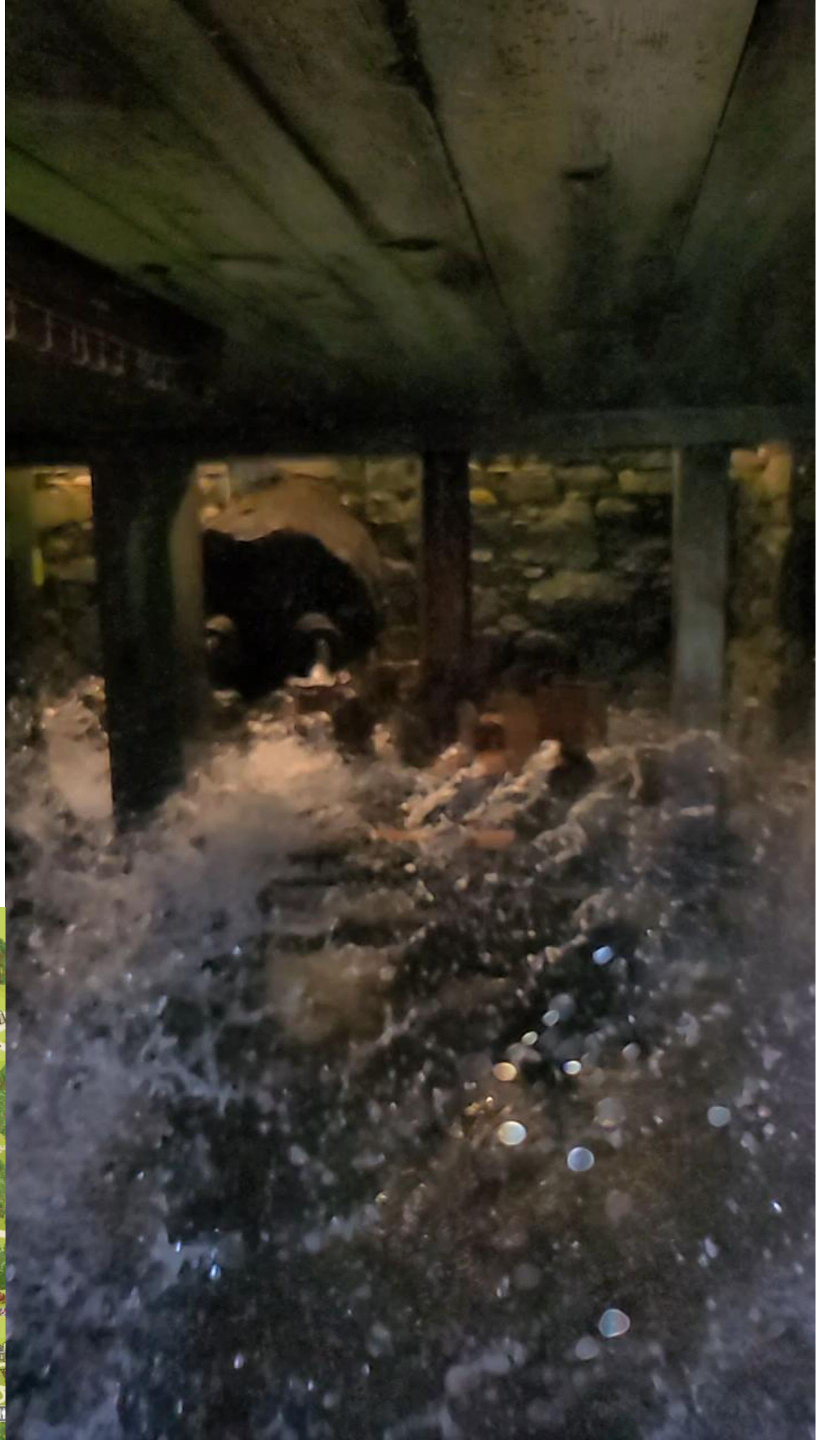
Tygun's empirical formula for the number of buckets (Z) in the wheel

$$Z = \frac{D_{\text{wheel}}}{2D_{\text{nozzle}}} + 15$$

Pelton turbines in Switzerland

SwitzerlandMobility



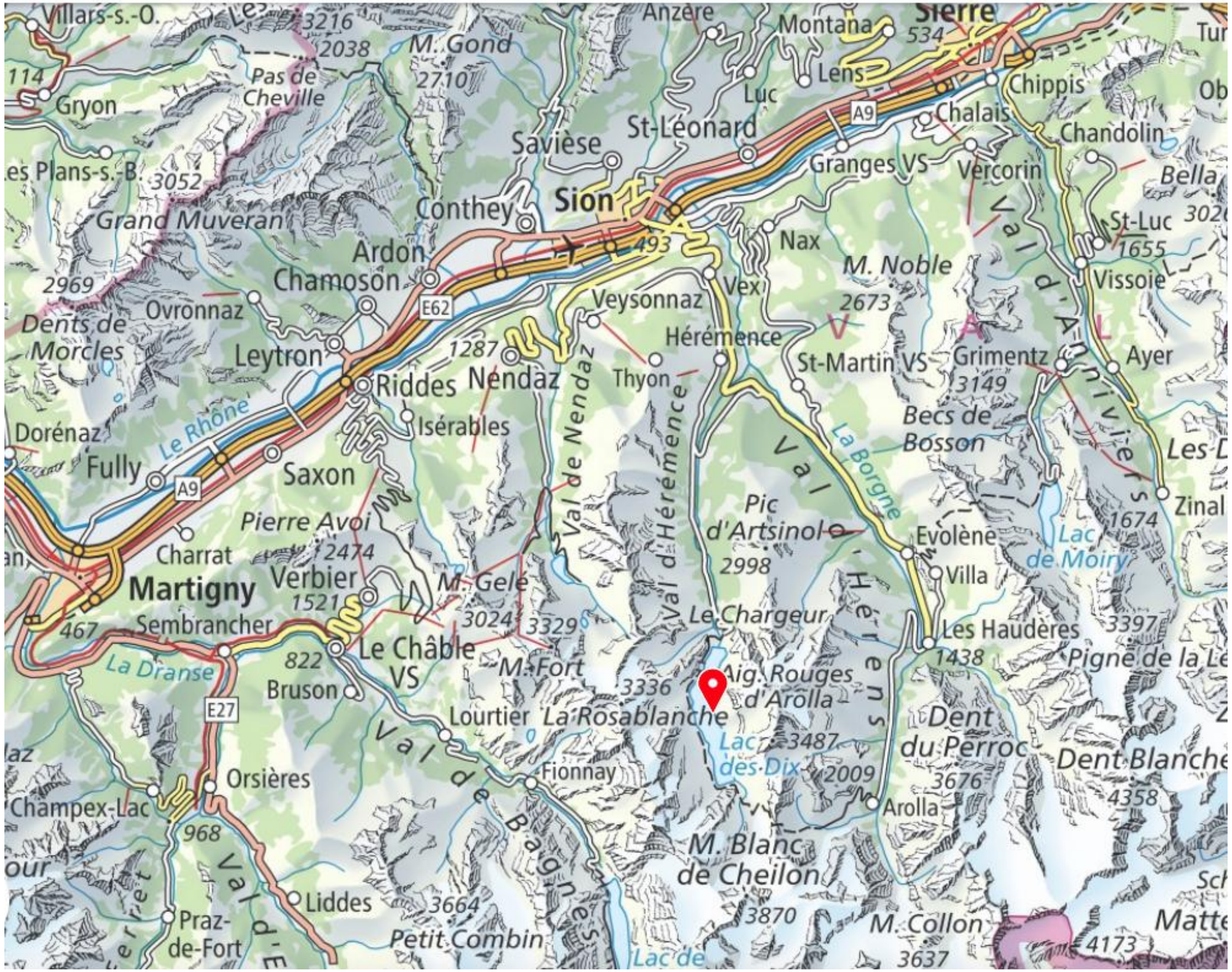


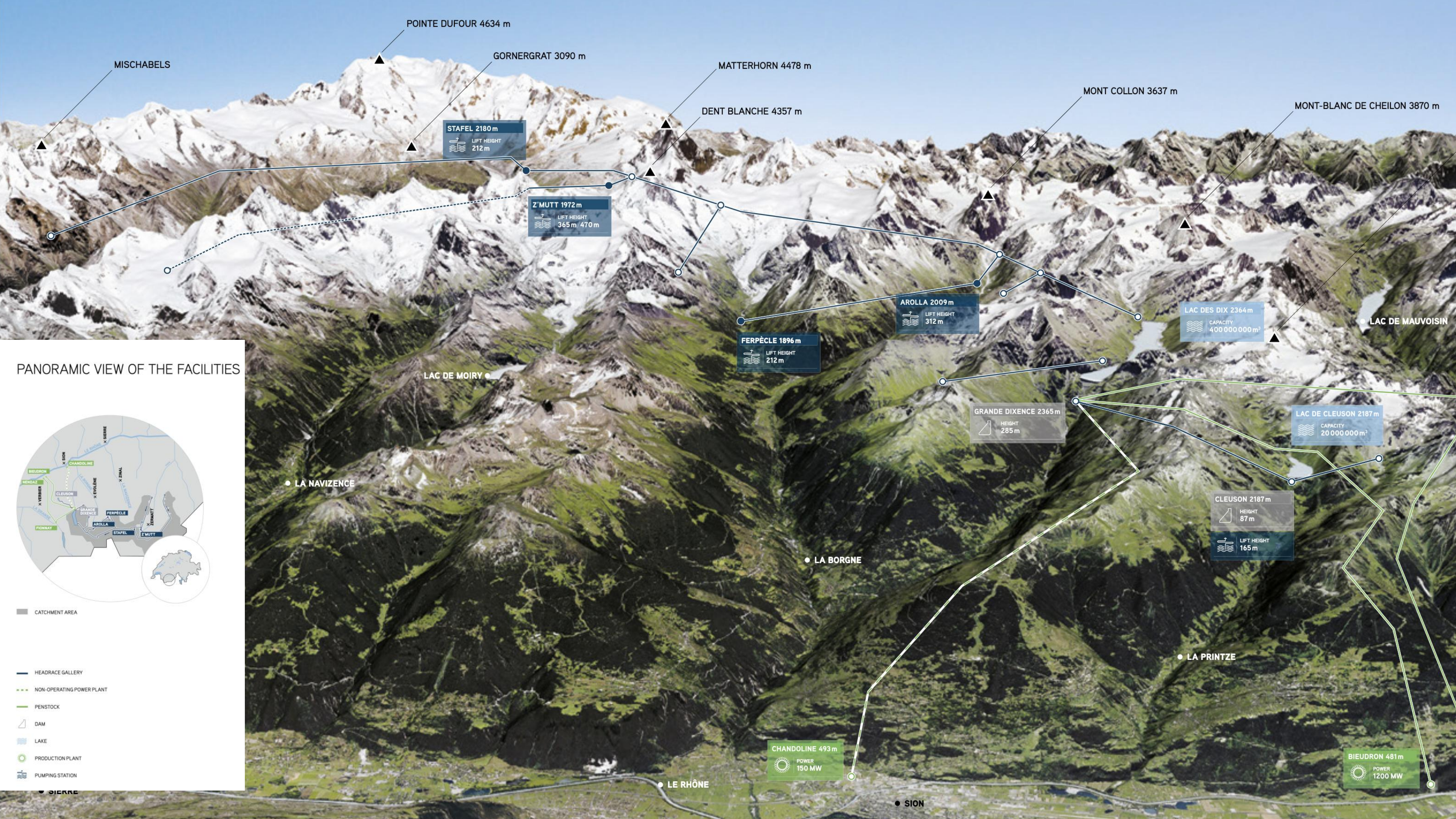
Ballenberg, Swiss Open-Air Museum

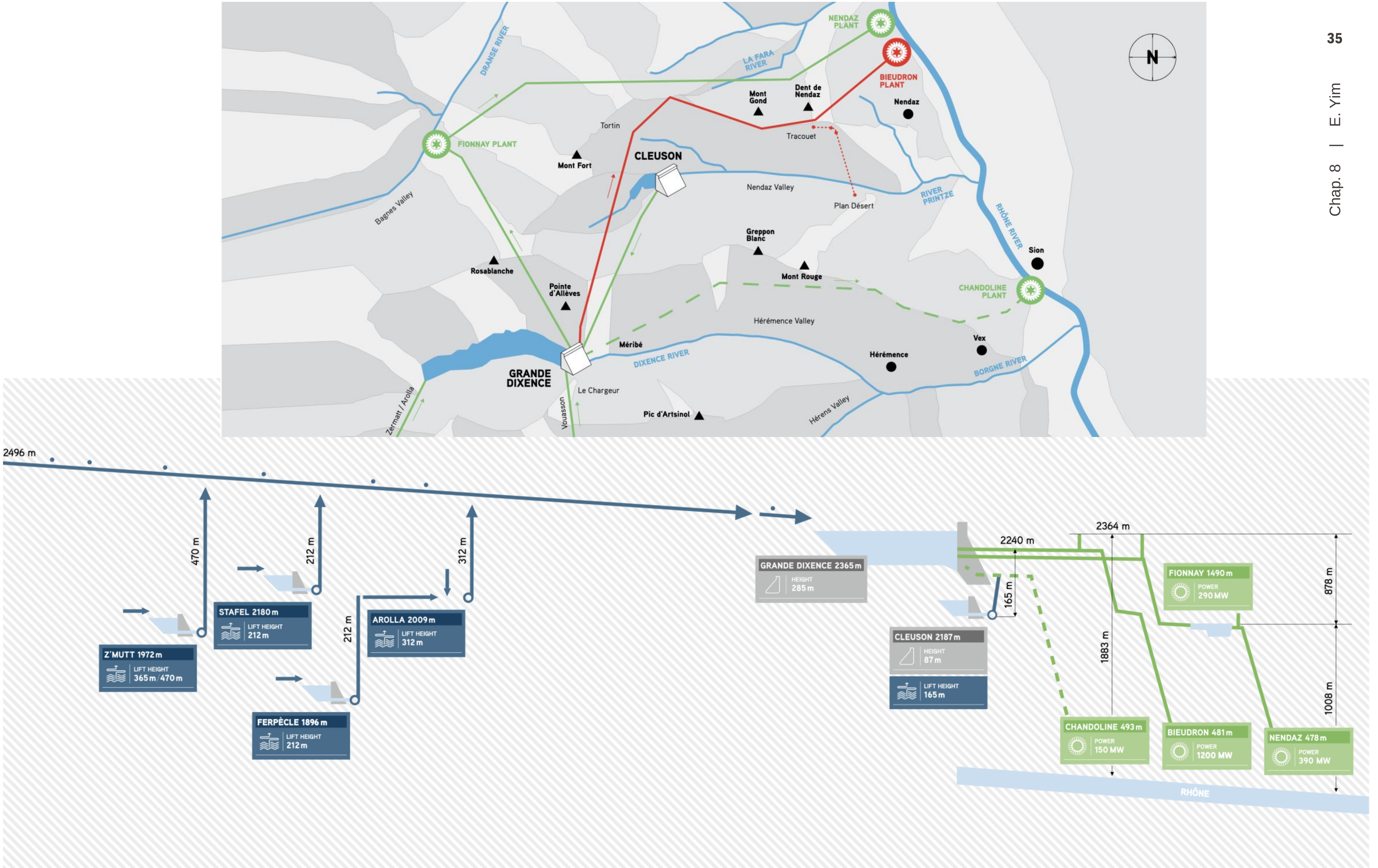
INTRODUC

Pelton turbines in Switzerland

SwitzerlandMobility




PANORAMIC VIEW OF THE FACILITIES
POINTE DUFOUR 4634 m
MISCHABELS
GORNERGRAT 3090 m
MATTERHORN 4478 m
DENT BLANCHE 4357 m
MONT COLLON 3637 m
MONT-BLANC DE CHEILON 3870 m
STAFEL 2180 m
LIFT HEIGHT 212 m
Z'MUTT 1972 m
LIFT HEIGHT 365 m 470 m
AROLLA 2009 m
LIFT HEIGHT 312 m
FERPÈCLE 1896 m
LIFT HEIGHT 212 m
LAC DES DIX 2364 m
CAPACITY 400 000 000 m³
LAC DE MAUVOISIN
LAC DE MOIRY
GRANDE DIXENCE 2365 m
HEIGHT 285 m
LAC DE CLEUSON 2187 m
CAPACITY 20 000 000 m³
LA NAVIZENCE
CLEUSON 2187 m
HEIGHT 87 m
LIFT HEIGHT 165 m
LA BORGNE
LA PRINTZE
CHANDOLINE 493 m
POWER 150 MW
BIEUDRON 481 m
POWER 1200 MW
LE RHÔNE
SION
SIERRE
CHANDOLINE
BIEUDRON
NENDAZ
VERBIER
CLEUSON
GRANDE DIXENCE
FIONNAY
EVOLÈNE
ZINAL
FERPÈCLE
AROLLA
STAFEL
Z'MUTT
ZERMATT
CATCHMENT AREA
HEADRACE GALLERY
NON-OPERATING POWER PLANT
PENSTOCK
DAM
LAKE
PRODUCTION PLANT
PUMPING STATION


NENDAZ PLANT
BIEUDRON PLANT
FIONNAY PLANT
CHANDOLINE PLANT
DRANSE RIVER
LA FARA RIVER
RIVER PRINTZE
RHÔNE RIVER
DIXENCE RIVER
BORGNE RIVER
Mont Gond
Dent de Nendaz
Nendaz
Tortin
Tracouet
Mont Fort
CLEUSON
Nendaz Valley
Plan Désert
Bagnes Valley
Greppon Blanc
Mont Rouge
Sion
Rosablanche
Pointe d'Allèves
Hérémence Valley
Méribé
Vex
Hérémence
GRANDE DIXENCE
Le Chargeur
Zermatt / Arolla
Vouasson
Pic d'Artsinol
Hérens Valley
N
2496 m
470 m
212 m
312 m
212 m
Z'MUTT 1972 m
LIFT HEIGHT 365 m / 470 m
STAFEL 2180 m
LIFT HEIGHT 212 m
FERPÈCLE 1896 m
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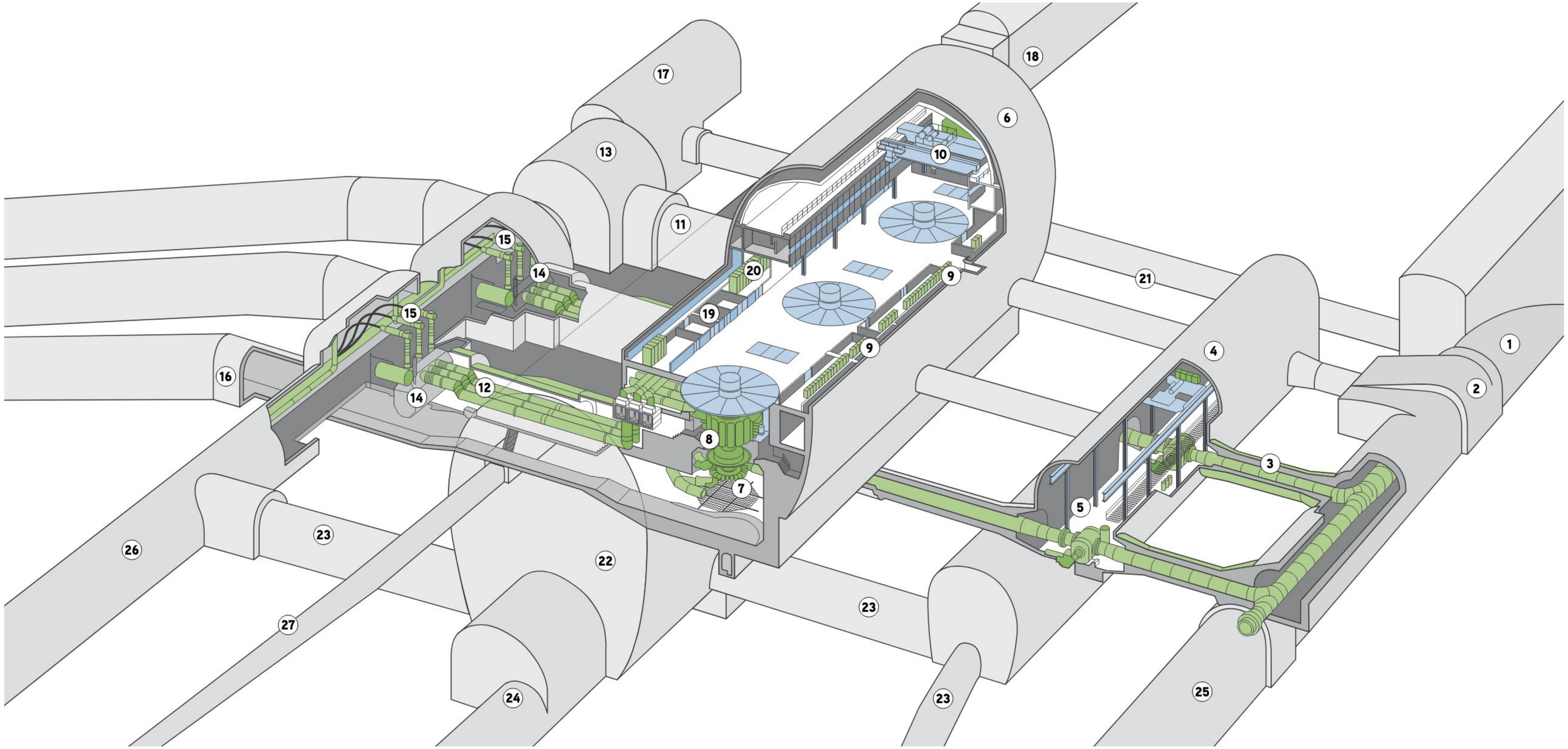
HEIGHT 285 m
CLEUSON 2187 m
HEIGHT 87 m
LIFT HEIGHT 165 m
165 m
2240 m
2364 m
1883 m
878 m
1008 m
FIONNAY 1490 m
POWER 290 MW
CHANDOLINE 493 m
POWER 150 MW
BIEUDRON 481 m
POWER 1200 MW
NENDAZ 478 m
POWER 390 MW
RHÔNE

BIEUDRON POWER PLANT

OVERVIEW

1. PENSTOCK
2. DISTRIBUTOR
3. UNIT BRANCH (25 m³/s, PRESSURE 190 BAR)
4. VALVES CHAMBER
5. SPHERICAL VALVE (210 TONS PER UNIT)
6. MAIN CAVERN
7. 423 MW PELTON TURBINE
8. 465 MVA GENERATOR
9. CONTROL ROOM
10. 250 TONS OVERHEAD CRANE
11. BUSBAR TUNNELS
12. 21 kV 15'000 A BUSBARS
13. TRANSFORMER CELL
14. 465 MVA THREE-PHASE TRANSFORMERS
15. 230/410 kV ONE-PHASE CABLE OUTLET
16. TAILWATER BRANCH
17. MAINTENANCE AND STORAGE BUILDING
18. COOLING WATER RESERVOIR
19. 16/0.4 kV AUXILIARY TRANSFORMERS
20. 230/400 V SWITCHBOARDS
21. EMERGENCY TUNNEL
22. ASSEMBLY SITE
23. CONNECTING TUNNELS
24. ACCESS TUNNEL TO PLANT
25. ACCESS TUNNEL TO DISTRIBUTOR
26. TUNNEL FOR 410 kV CABLES AND ACCESS TO TRANSFORMERS
27. VENTILATION TUNNEL



Grande
Dixence

Exercise

A Pelton wheel operates with a gross head of 530 m and a flow rate of 9 m^3/s. The penstock length is 880 m, its diameter is 1.2 m, and its RMS roughness is 0.12 mm. The minor loss can be ignored. The hydraulic efficiency is η_h = 0.84, and the shaft speed is 650 rpm. Water kinematic viscosity is 1.02×10^{-6} m^2/s and density is 998 kg/m^3.

Find (a) the effective head and the power delivered by the turbine and (b) the specific speed and from it the recommended number of jets and the number of buckets in the wheel. The nozzle coefficient is C_N = 0.97, and the ratio of the blade speed to the discharge velocity is $\lambda = U/V_1 = 0.45$.

(a) $V_p = \frac{Q}{A} = \frac{4Q}{\pi D_p^2} = \frac{4}{\pi \cdot 1.2^2} 9 = 7.96 \, m/s$

$\frac{\varepsilon}{D_p} = \frac{0.12 \times 10^{-3}}{1.2} = 10^{-4}$

$Re = \frac{V_p D_p}{\nu} = \frac{7.96 \times 1.2}{1.02 \times 10^{-6}} = 9.36 \times 10^6$

→ see Next page for f

$f = 0.012$

$h_L = f \frac{l}{D} \frac{V_p^2}{2g} = 0.012 \frac{880}{1.2} \frac{7.96^2}{2 \cdot 9.81} = 28.4 \, m$

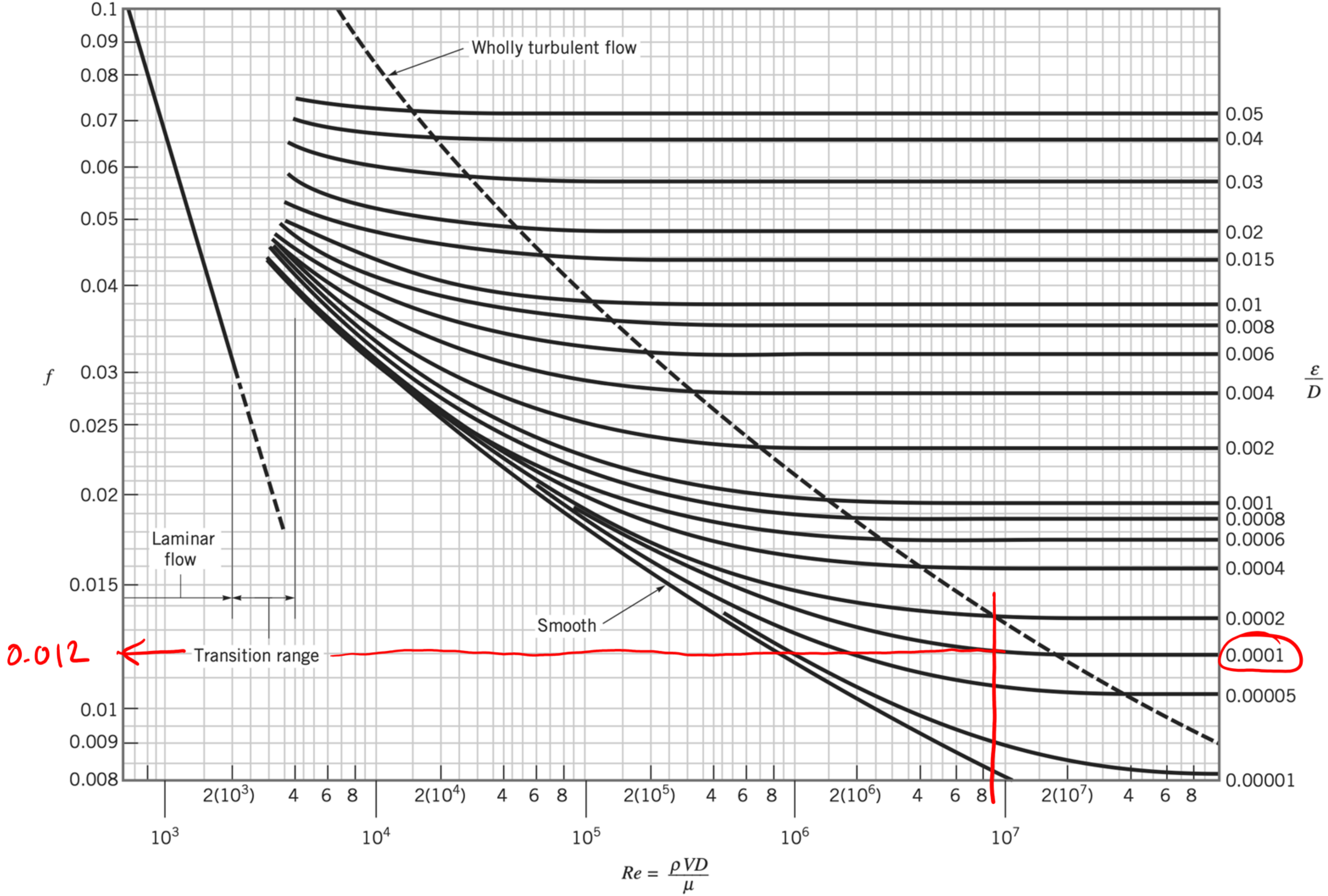
$h_a = h_g - h_L = 530 - 28.4 = 501.6 \, m$

$\dot{W}_{shaft} = \eta \cdot Q \cdot h_a \cdot \gamma$

$= 0.84 \cdot 9 \cdot 501.6 \cdot (9.81 \times 998)$

$= 37.1 \, MW$

Moody diagram – Chapter 3



Exercise

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Find (a) the effective head and the power delivered by the turbine and (b) the specific speed and from it the recommended number of jets and the number of buckets in the wheel. The nozzle coefficient is C_N = 0.97, and the ratio of the blade speed to the discharge velocity is $\lambda = U/V_1$ = 0.45.

$$(b)\quad N_s' = \frac{\omega\sqrt{\dot{W}_{shaft}/\rho}}{(g h_a)^{5/4}} = \frac{650\cdot 2\pi}{60}\cdot\frac{\sqrt{37.1\times10^6/998}}{(9.81\cdot 501.6)^{5/4}} = 0.318$$

Four jets recommended.

for the number of buckets we need D_{wheel} and D_{nozzle}

For 4-jets: $Q_j = \frac{Q}{4} = 2.25\ m^3/s$ per nozzle

$$V_1 = C_N\sqrt{2 g h_a} = 0.97\sqrt{2\cdot 9.81\cdot 501.6} = 96.23\ m/s$$

$$Q_j = V_1\cdot A_j = V_1\cdot\frac{\pi D_{nozzle}^2}{4} \rightarrow D_{nozzle} = \sqrt{\frac{4 Q_j}{V_1\cdot\pi}} = 0.173\ m$$

speed ratio $\lambda = \frac{U}{V_1}$

$\rightarrow U = \lambda\cdot V_1 = 43.3\ m/s$

$U = \omega R \rightarrow R = 0.635\ m$

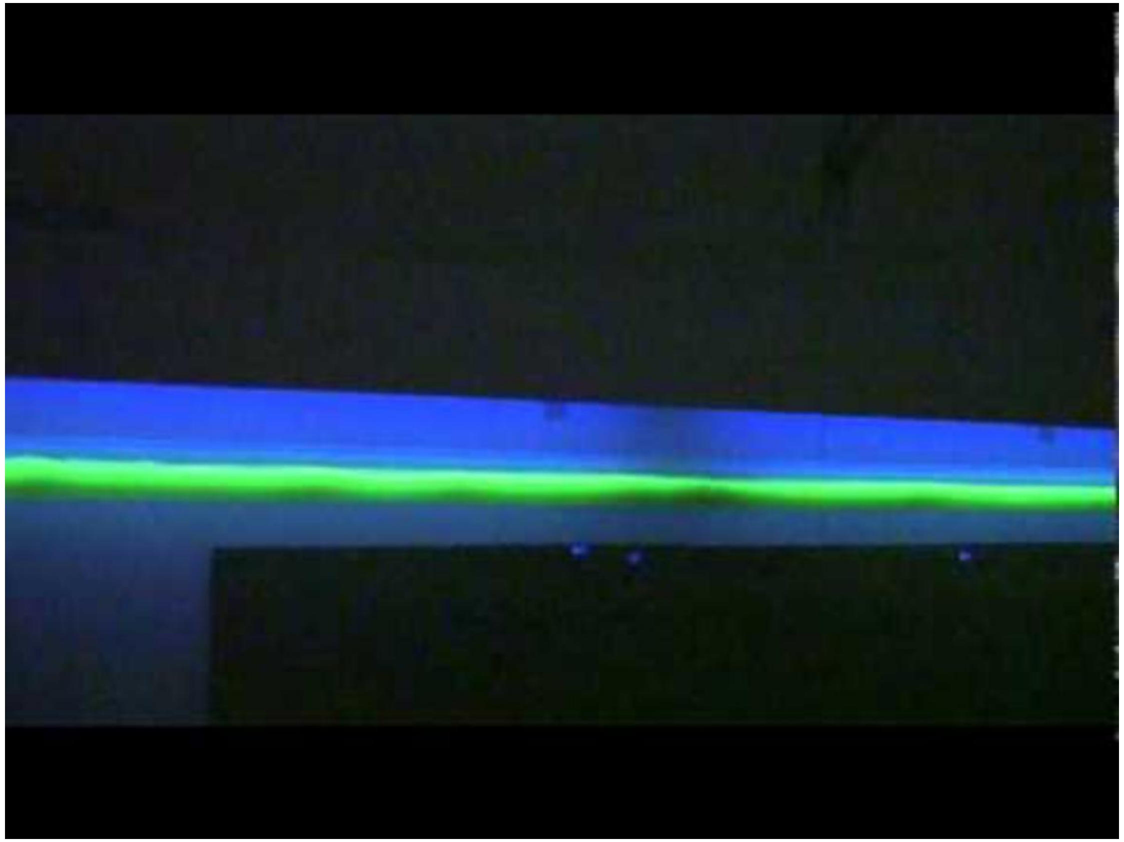
$D_{wheel} = 2R = 1.27\ m$

$$, \quad Z = \frac{D_{wheel}}{2\cdot D_{nozzle}} + 15 = 18.67 \rightarrow 19$$

Type		N_s'	η %
Pelton wheel			
	Single jet	0.02 – 0.18	88 – 90
	Twin jet	0.09 – 0.26	89 – 92
	Three jet	0.10 – 0.30	89 – 92
	Four jet	0.12 – 0.36	86
Francis			
	Low-speed	0.39 – 0.65	90 – 92
	Medium-speed	0.65 – 1.2	93
	High-speed	1.2 – 1.9	93 – 96
	Extreme-speed	1.9 – 2.3	89 – 91
Kaplan turbine		1.55 – 5.17	87 – 94
Bulb turbine		3 – 8	

Appendix

Water hammer (Hydraulic shock)



Recall – chapter 4

3-Energy equation

- cs: control **surface**
- cv: control **volume**

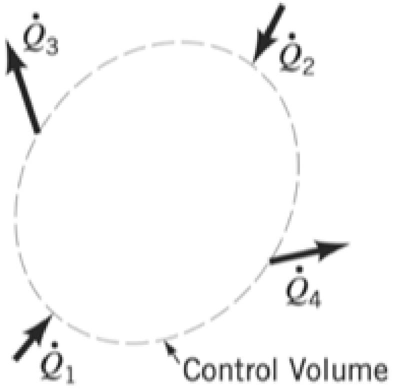
The **first law of thermodynamics**

$$\frac{D}{Dt}\int_{\text{sys}} e\rho d\mathcal{V} = \frac{\partial}{\partial t}\int_{\text{cv}} e\rho d\mathcal{V} + \int_{\text{cs}} e\rho \mathbf{V}\cdot\hat{\mathbf{n}}dA$$

Time rate of increase of the total stored energy of the system = time rate of increase of the total stored energy of the contents of the control volume + net rate of flow of the total stored energy out of the control volume through the control surface

where $e = \check{u} + \frac{V^2}{2} + gz$ total stored energy per unit mass

($\check{u}$: internal energy)



$$\dot{Q}_{\substack{\text{net}\\\text{in}}} = \dot{Q}_1 + \dot{Q}_2 - \dot{Q}_3 - \dot{Q}_4$$

$$\dot{Q}_{\substack{\text{net}\\\text{in}}} + \dot{W}_{\substack{\text{shaft}\\\text{net in}}} = \frac{\partial}{\partial t}\int_{\text{cv}} e\rho d\mathcal{V} + \int_{\text{cs}}\left(\check{u} + \frac{p}{\rho} + \frac{V^2}{2} + gz\right)\rho\mathbf{V}\cdot\hat{\mathbf{n}}dA$$

Heat transfer ratio; Work transfer rate, power

INTRODUCTION TO TURBOMACHINERY

Recall- chap 5 Basic governing equations for turbomachine

- **Shaft torque**

$$T_{\text{shaft}} = -\dot{m}_1 \left(r_1 V_{\theta 1}\right) + \dot{m}_2 \left(r_2 V_{\theta 2}\right)$$

- **Shaft power**

$$\dot{W}_{\text{shaft}} = T_{\text{shaft}}\, \omega = -\dot{m} r_1 V_{\theta 1} \omega + \dot{m} r_2 V_{\theta 2} \omega$$

($r_1 \omega = U_1$, $r_2 \omega = U_2$)

$$\dot{W}_{\text{shaft}} = \left(-\dot{m}_1\right)\left(U_1 V_{\theta 1}\right) + \dot{m}_2 \left(U_2 V_{\theta 2}\right)$$ [W] = [kg·m^2/s^3]

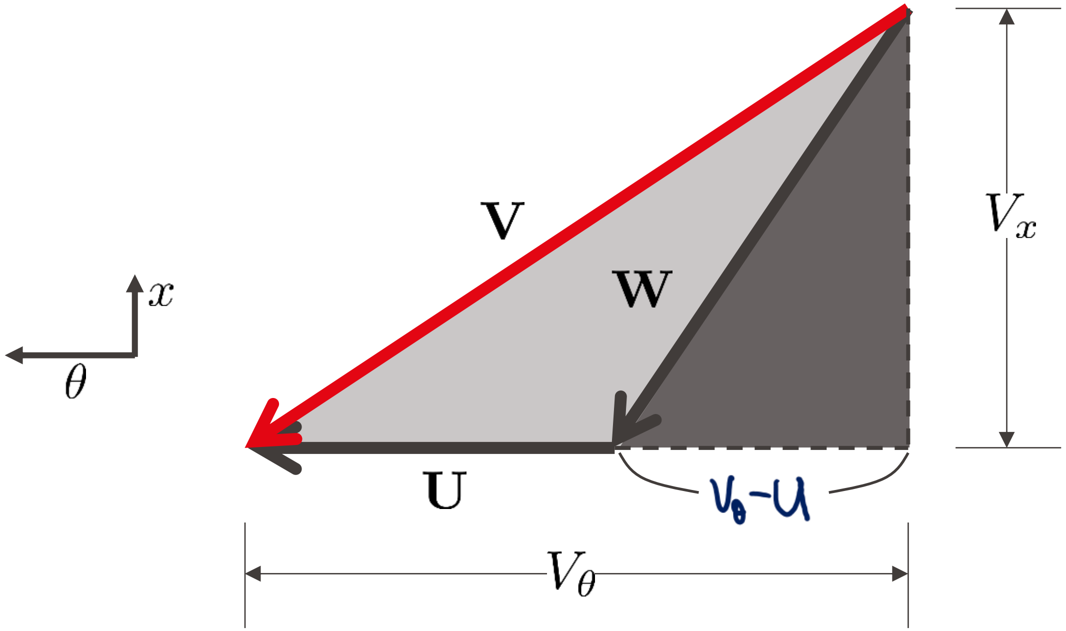
- **Shaft work per unit mass (shaft power per unit mass flow rate),** $\dot{m}_1 = \dot{m}_2$

$$w_{\text{shaft}} = -\left(U_1 V_{\theta 1}\right) + \left(U_2 V_{\theta 2}\right)$$ [m^2/s^2]

- Basic governing equations for pumps or turbines whether the machines are radial-, mixed-, or axial-flow devices and for compressible and incompressible flows
- Note it is only the function of tangential component of velocity, no V_r, V_x

Recall- chap 5 Basic governing equations for turbomachine

$$\mathbf{V} = \mathbf{W} + \mathbf{U}$$



Velocity triangle: **V** absolute velocity, **W** relative velocity, **U** blade velocity

- From the big triangle (grey)

$$V^2 = V_\theta^2 + V_x^2 \quad \text{OR} \quad V_x^2 = V^2 - V_\theta^2$$

- From the small triangle (dark grey)

$$W^2 = (V_\theta - U)^2 + V_x^2$$

$$= V_\theta^2 - 2V_\theta U + U^2 + V_x^2$$

$$W^2 = V_\theta^2 - 2V_\theta U + U^2 + V^2 - V_\theta^2$$

$$V_\theta U = \frac{-W^2 + U^2 + V^2}{2}$$

$$w_{\text{shaft}} = -(U_1 V_{\theta 1}) + (U_2 V_{\theta 2})$$

$$w_{\text{shaft}} = \frac{V_2^2 - V_1^2 + U_2^2 - U_1^2 - (W_2^2 - W_1^2)}{2}$$

Turbomachine work is related to changes in absolute, relative, and blade velocities.

Incidence and deviation angles

The **incidence** is the difference between the inlet flow angle and the blade inlet angle:

$$i = \alpha_1 - \alpha_1'$$

The change in angle of the flow is called **deflection**:

$$\varepsilon = \alpha_1 - \alpha_2$$

Howell (1945) defined a *nominal fluid deflection* as

$$\varepsilon^* = 0.8\varepsilon_s$$