

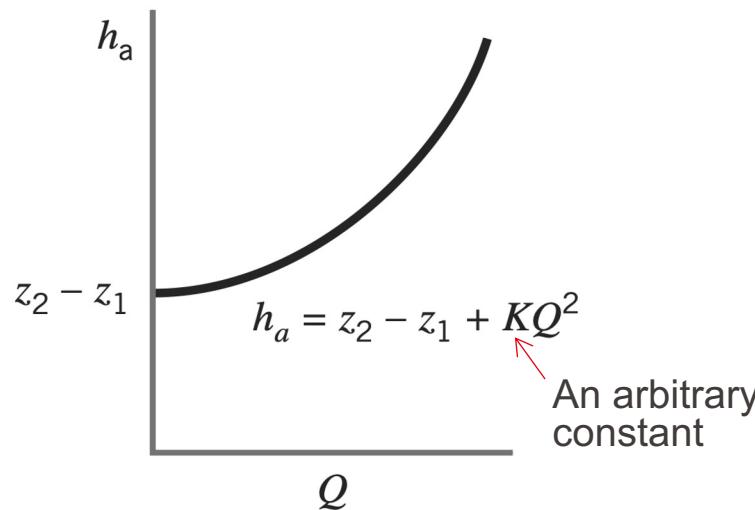
Chapter 8: Hydraulic Turbines

ME-342 Introduction to
turbomachinery

Prof. Eunok Yim, HEAD-lab.

- **K vs. K_L**
 - Loss coefficient for Minor loss

System equation



EPFL Exercise

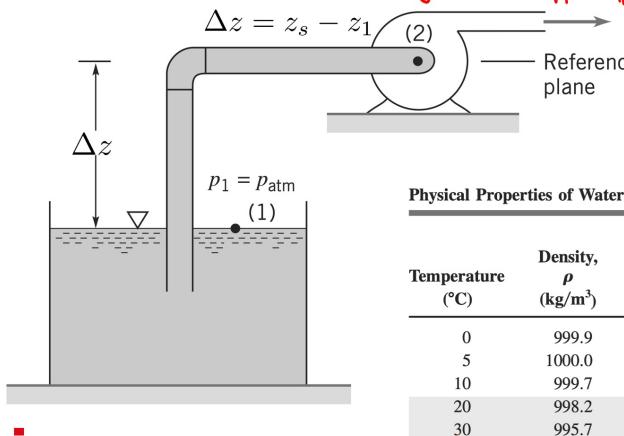
Pump water with $0.014 \text{ m}^3/\text{s}$ the required NPSH is 4.5 m specified pump manufacturer. The water temperature is 30°C 101.3 kPa . The loss occurs mainly due to the filter at inlet with $K_L = 20$. Friction loss is neglected. Pipe is with diameter of 10 cm . Determine the max height z the pump can be located without the cavitation.

$$\text{NPSH}_A = \frac{p_{\text{atm}}}{\gamma} - \Delta z - \sum h_L - \frac{p_v}{\gamma}$$

Cavitation occurs when $\text{NPSH}_R = \text{NPSH}_A$

$$\text{NPSH}_A = \text{NPSH}_R = 4.5 \text{ m} = \frac{101.3 \times 10^3 \text{ Pa}}{9.765 \times 10^3 \text{ N/m}^2} - \Delta z - 20 \cdot \frac{V^2}{2g} - \frac{4.243 \times 10^3}{9.765 \times 10^3}$$

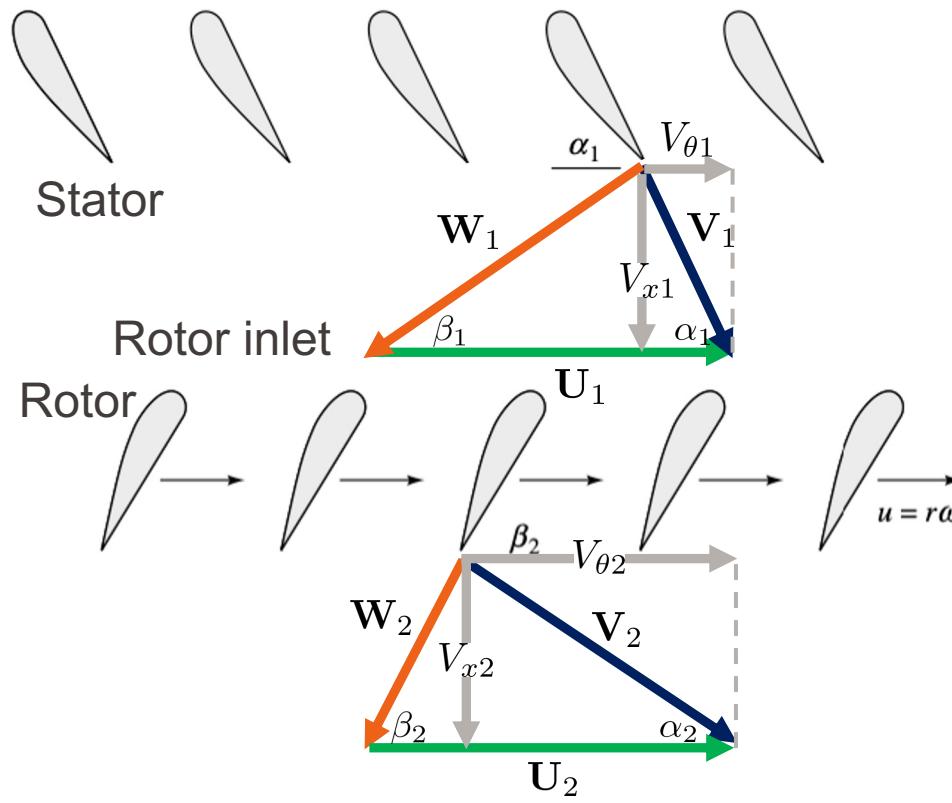
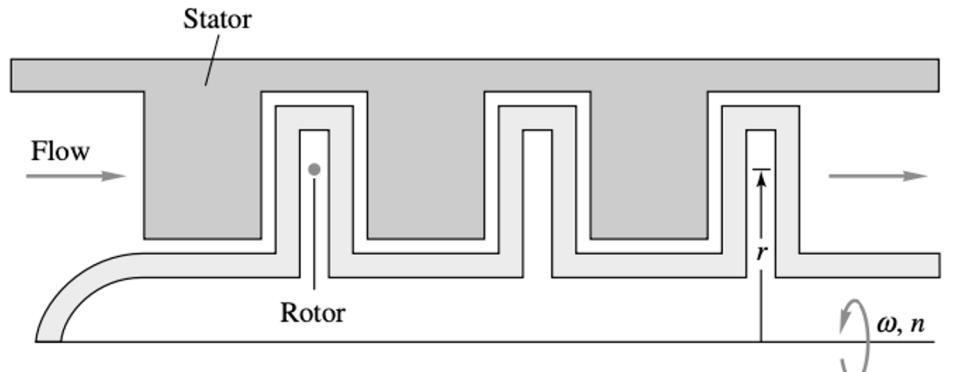
$$= 10.37 - \Delta z - 3.23 - 0.43$$



Physical Properties of Water (SI Units)^a

Temperature (°C)	Density, ρ (kg/m ³)	Specific Weight ^b , γ (kN/m ³)	Dynamic Viscosity, μ (N·s/m ²)	Kinematic Viscosity, ν (m ² /s)	Surface Tension ^c , σ (N/m)	Vapor Pressure, p_v [N/m ² (abs)]
0	999.9	9.806	1.787 E - 3	1.787 E - 6	7.56 E - 2	6.105 E + 2
5	1000.0	9.807	1.519 E - 3	1.519 E - 6	7.49 E - 2	8.722 E + 2
10	999.7	9.804	1.307 E - 3	1.307 E - 6	7.42 E - 2	1.228 E + 3
20	998.2	9.789	1.002 E - 3	1.004 E - 6	7.28 E - 2	2.338 E + 3
30	995.7	9.765	7.975 E - 4	8.009 E - 7	7.12 E - 2	4.243 E + 3
40	992.2	9.731	6.529 E - 4	6.580 E - 7	6.96 E - 2	7.376 E + 3

Last week - Axial pump



Since the stator is fixed, ideally the absolute velocity V_1 is parallel to the trailing edge of the blade

$$V_{x1} = V_{x2} = V_x = \frac{Q}{A} = \text{const}$$

The ideal head expressed stator angle α_1 and rotor angle β_2

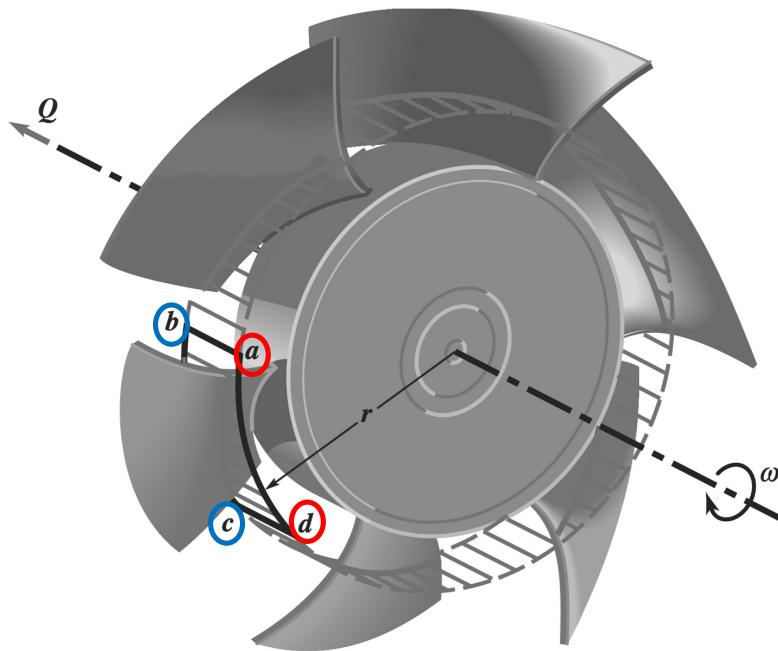
$$h_i = \frac{U_2 V_{\theta 2} - U_1 V_{\theta 1}}{g}$$

Strictly speaking, this applies only to a single streamtube of radius r , but it is a good approximation for very short blades if r denotes the average radius.

EPFL

Basic Energy Considerations

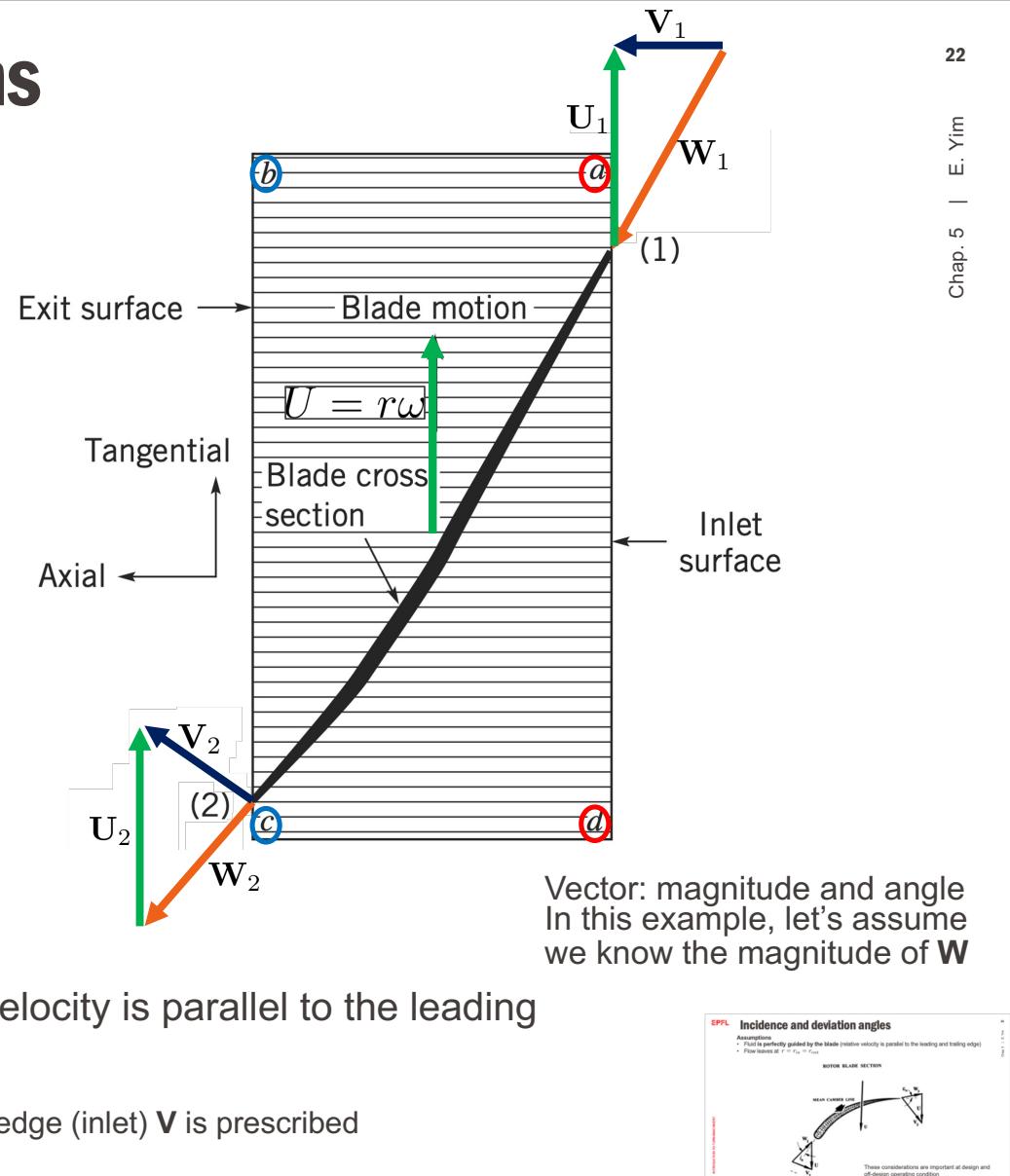
- Velocity diagram (fan)



Assumptions

- Fluid is **perfectly guided by the blade** (relative velocity is parallel to the leading and trailing edge)*
- Flow leaves at $r = r_{in} = r_{out}$

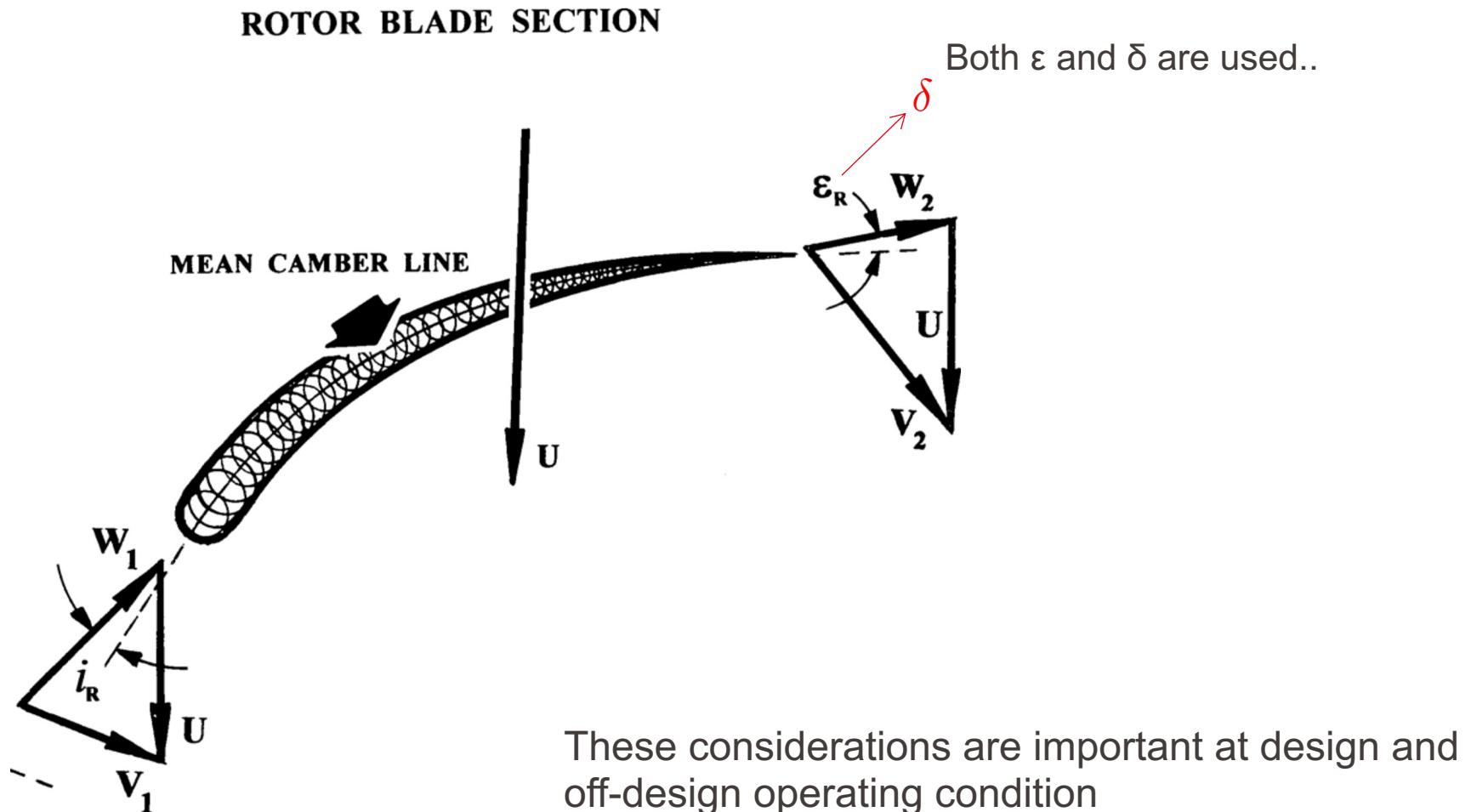
*sometimes leading edge (inlet) \mathbf{V} is prescribed



Incidence and deviation angles

Assumptions

- Fluid is perfectly guided by the blade (relative velocity is parallel to the leading and trailing edge)
- Flow leaves at $r = r_{\text{in}} = r_{\text{out}}$



Incidence and deviation angles

The **incidence** is the difference between the inlet flow angle (α) and the blade **inlet angle** (α'):

$$i = \alpha_1 - \alpha'_1$$

The **deviation** is the difference between the exit flow angle and the blade **exit angle**:

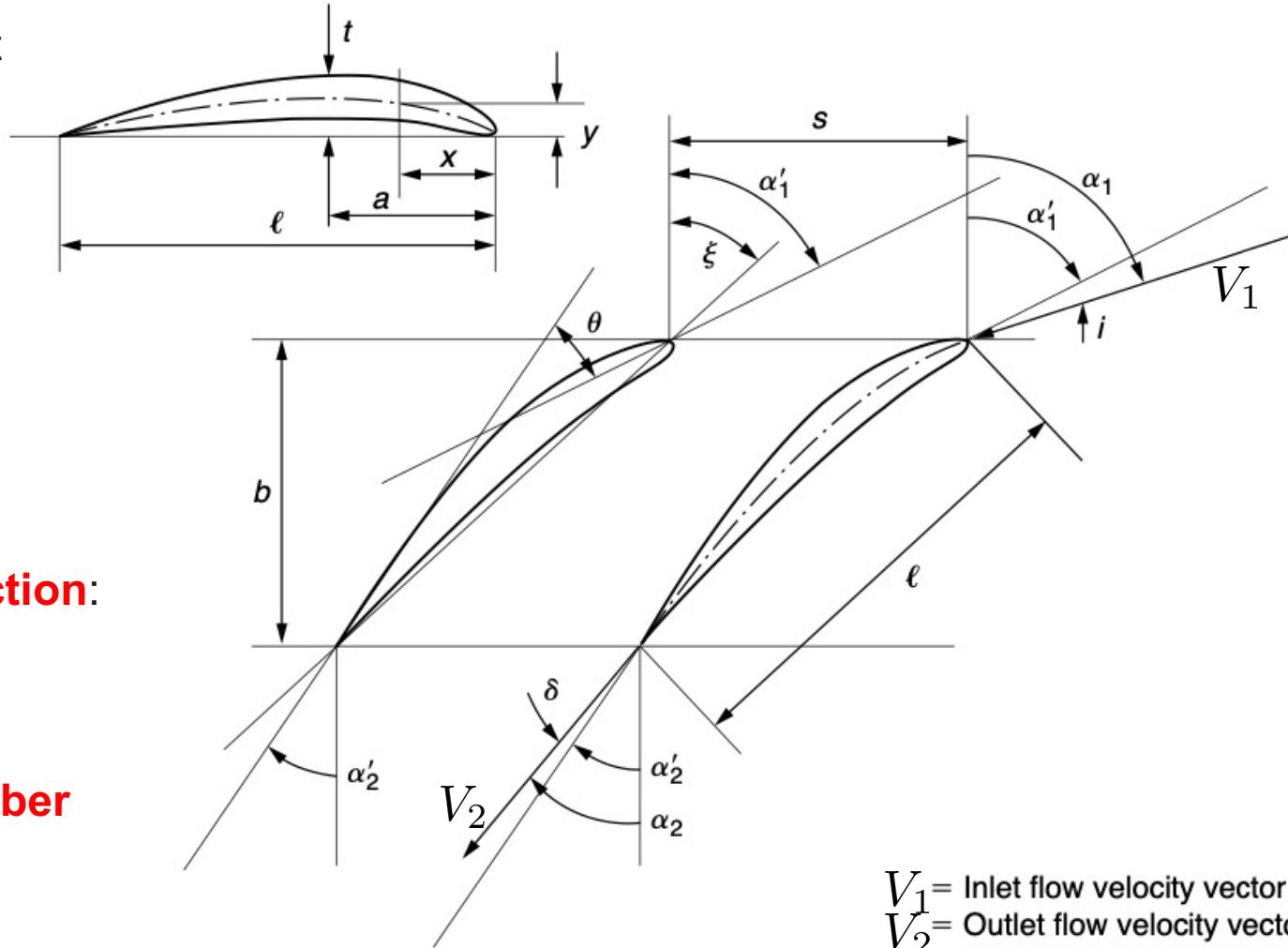
$$\delta = \alpha_2 - \alpha'_2$$

The change in angle of the **flow** is called **deflection**:

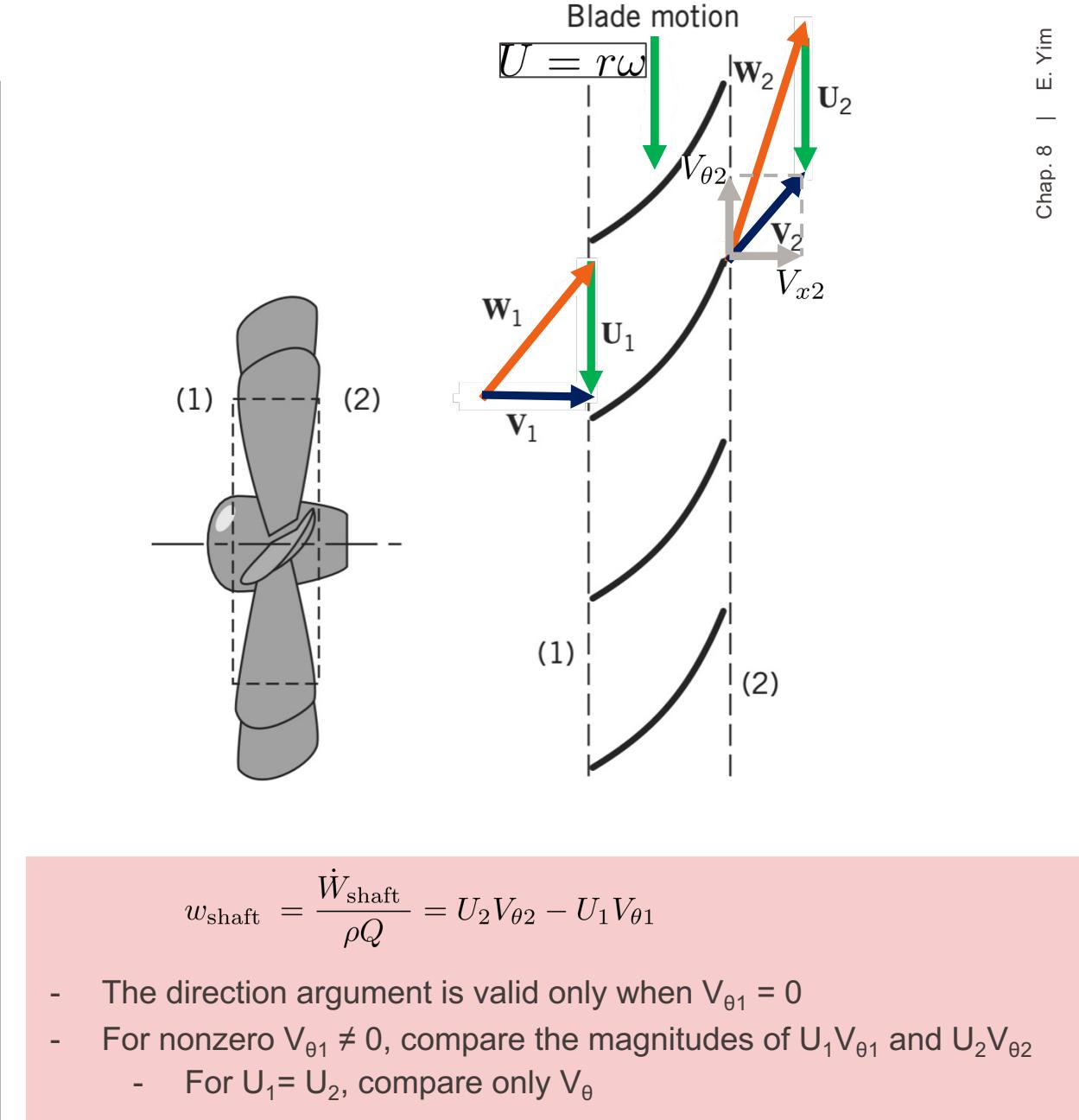
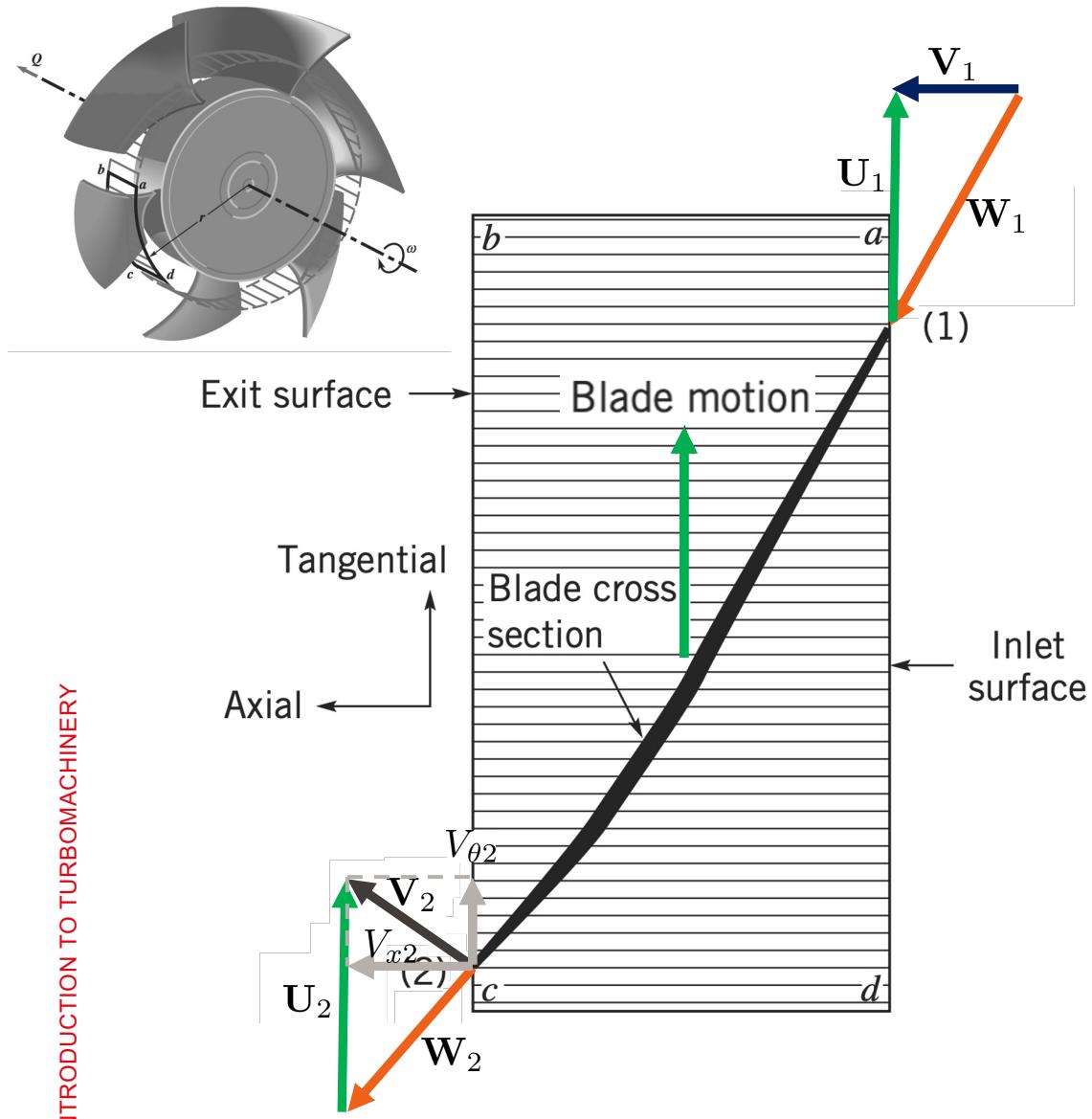
$$\varepsilon = \alpha_1 - \alpha_2$$

The change in angle of the **blade** is called **camber angle**:

$$\theta = \alpha'_1 - \alpha'_2$$



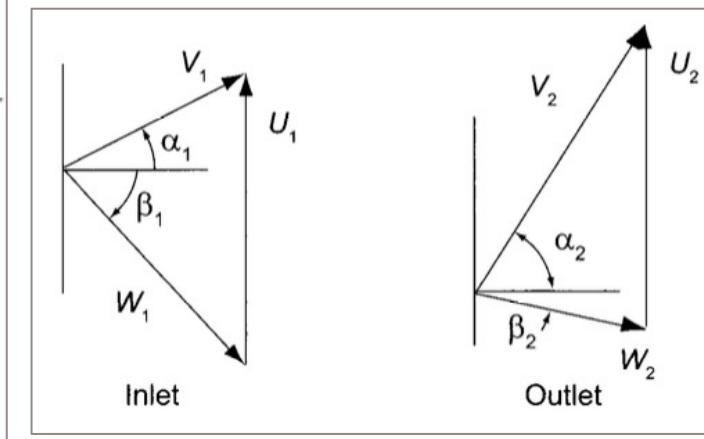
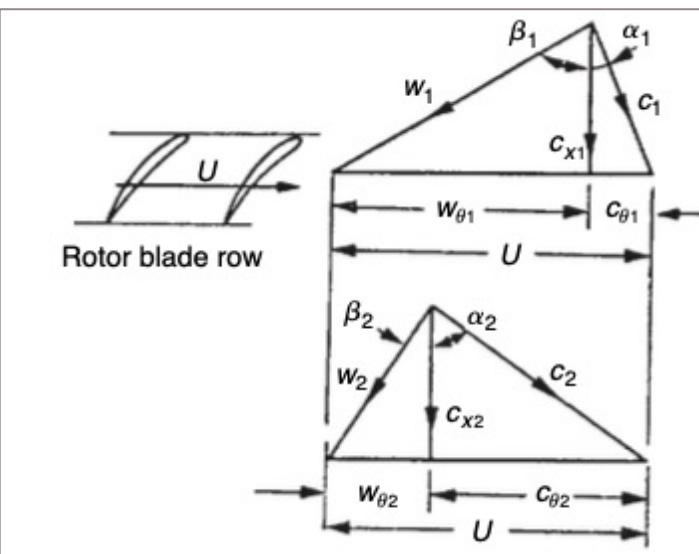
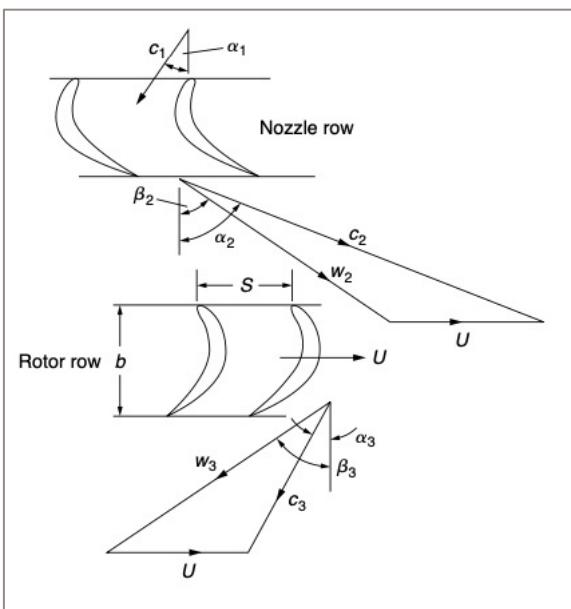
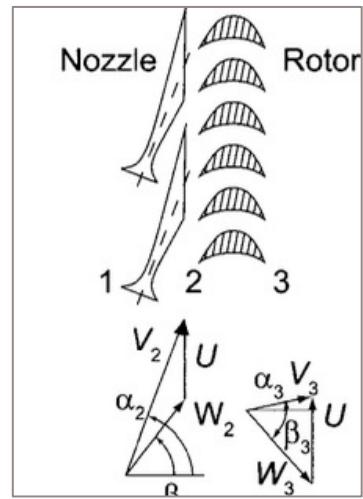
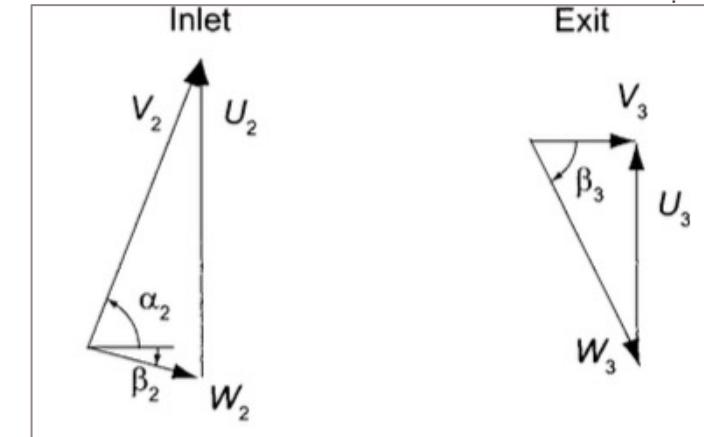
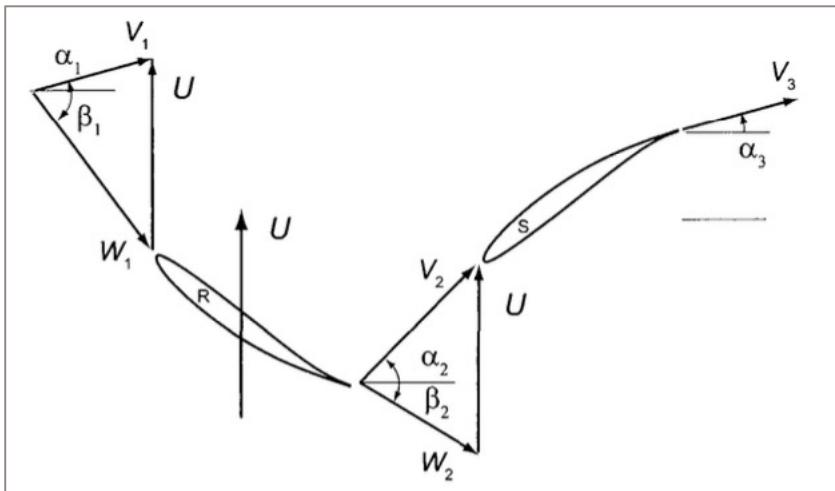
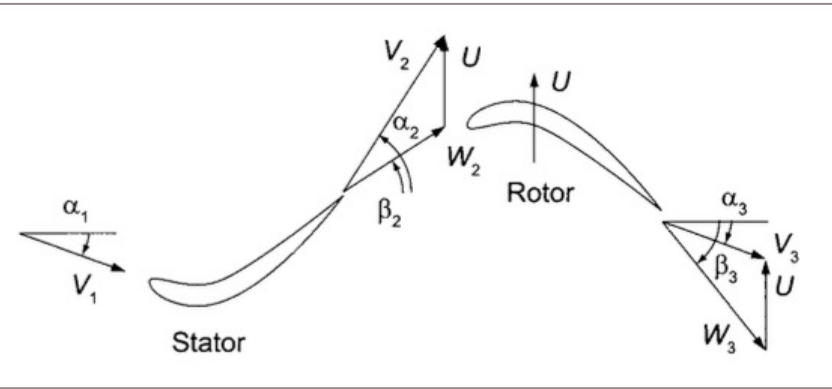
V_1 = Inlet flow velocity vector
 V_2 = Outlet flow velocity vector
 (averaged across the pitch)



$$w_{\text{shaft}} = \frac{\dot{W}_{\text{shaft}}}{\rho Q} = U_2 V_{\theta 2} - U_1 V_{\theta 1}$$

- The direction argument is valid only when $V_{\theta 1} = 0$
- For nonzero $V_{\theta 1} \neq 0$, compare the magnitudes of $U_1 V_{\theta 1}$ and $U_2 V_{\theta 2}$
 - For $U_1 = U_2$, compare only V_{θ}

Pump or turbine? – when inlet $V_{\theta 1}$ is non-zero



Commonly used notations

Relative	W	w
Rotation	U	u
Absolute	V	c

Hydraulic Turbines

Impulse or reaction : No matter the working fluid, turbines can be broadly classified into two types based on the mechanism of the fluid interaction

Impulse turbine: the force on the blades is produced solely by turning the fluid, without appreciable pressure drop in the blade passage, with all of the pressure drop occurring in a fixed nozzle.

Reaction turbine: some of the fluid-vane force is from fluid turning and some of the force is a reaction to acceleration of the fluid relative to the vane. In reaction blading, a pressure drop occurs in both a fixed nozzle and the moving vane.

Turbine blading is characterized by the **degree of reaction (R)**, which is the ratio of the drop in static pressure (or enthalpy) across the moving blade to the overall drop in static pressure (or enthalpy) across the fixed nozzle plus the moving blade. Impulse turbines have $R = 0$ while reaction turbines typically have $0.1 < R < 0.7$.

Degree of reaction or Reaction (R)

Change in **static enthalpy** across the **rotor** divided by the static enthalpy change across the entire **stage**

(static) Enthalpy: internal energy and flow work

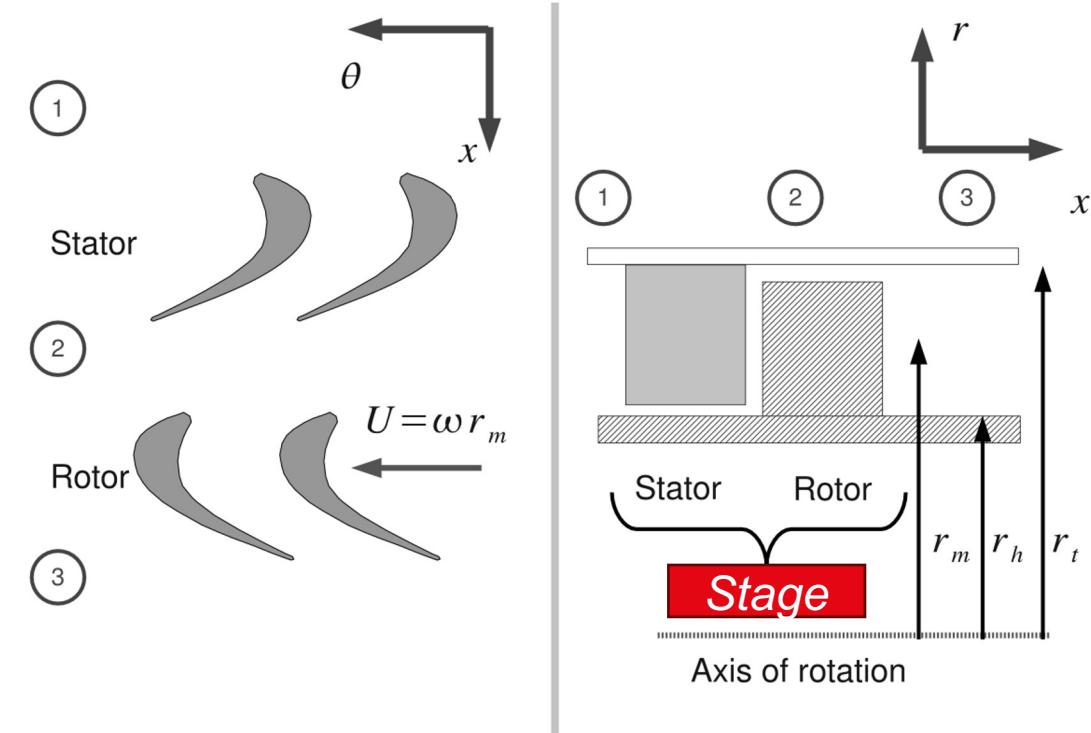
$$\check{h} = \check{u} + \frac{p}{\rho} \quad [\text{J/kg}]$$

Pa · m³/kg \equiv J/kg

Stagnation enthalpy: sum of enthalpy, kinetic energy and potential energy

$$\check{h}_0 = \check{h} + \frac{V^2}{2} + gz$$

Achtung! \check{h} is enthalpy, not head h or h_a



Degree of reaction or Reaction (R)

This concept is much used in axial flow machines as a measure of the relative proportions of energy transfer obtained by static and dynamic pressure change.

(static) Enthalpy: internal energy and flow work

$$\check{h} = \check{u} + \frac{p}{\rho} \quad [\text{J/kg}]$$

energy change due to, or resulting from,
static pressure change in the **rotor**

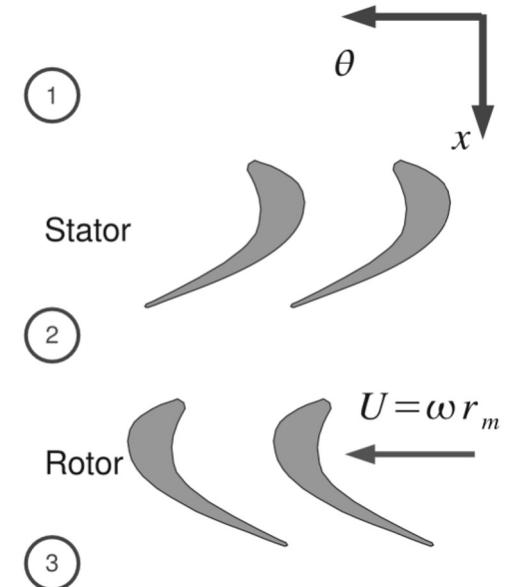
$$R = \frac{\text{static pressure change in the rotor}}{\text{total energy change for a stage}}$$

$$R = \frac{\text{static enthalpy change in rotor}}{\text{stage static enthalpy change}}$$

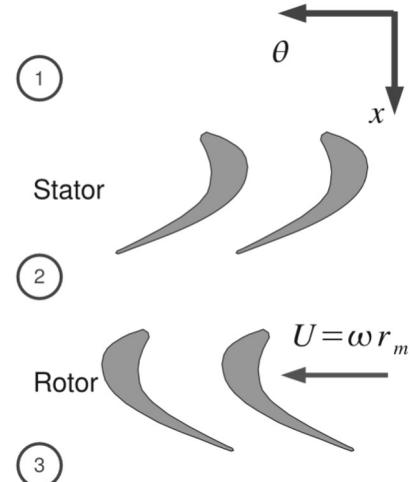
$$= \frac{\check{h}_2 - \check{h}_3}{\check{h}_1 - \check{h}_3}$$

If no internal energy is changed, incompressible,

$$R \simeq \frac{p_2 - p_3}{p_1 - p_3}$$



Degree of reaction or Reaction (R)



$$R \simeq \frac{p_2 - p_3}{p_1 - p_3}$$

$$w_f = -w_{\text{shaft}}$$

$$R = \frac{\frac{1}{2} (V_3^2 - V_2^2) - w_{\text{shaft}}}{\frac{1}{2} (V_3^2 - V_1^2) - w_{\text{shaft}}} = \frac{\frac{1}{2} (V_3^2 - V_2^2) - \frac{1}{2} (V_3^2 - V_2^2 + U_3^2 - U_2^2 - (W_3^2 - W_2^2))}{\frac{1}{2} (V_3^2 - V_1^2) - \frac{1}{2} (V_3^2 - V_2^2 + U_3^2 - U_2^2 - (W_3^2 - W_2^2))}$$

$$R = \frac{U_2^2 - U_3^2 + W_3^2 - W_2^2}{V_2^2 - V_1^2 + U_2^2 - U_3^2 + W_3^2 - W_2^2}$$

EPFL 3-Energy equation

- cs: control surface
- cv: control volume

$$\frac{D}{Dt} \int_{\text{sys}} e \rho dV = \frac{\partial}{\partial t} \int_{\text{cv}} e \rho dV + \int_{\text{cs}} e \rho V \cdot \dot{n} dA$$

time rate of increase of the total stored energy of the system
+ or of the control volume through the control surface

$$e = \dot{q} + \frac{V^2}{2} + gz$$

where e = total stored energy per unit mass

$$\dot{Q}_{\text{net}} + \dot{W}_{\text{shaft}} = \frac{\partial}{\partial t} \int_{\text{cv}} e \rho dV + \int_{\text{cs}} (\dot{q} + \frac{V^2}{2} + gz) \rho V \cdot \dot{n} dA$$

Heat transfer rate Work transfer rate power

EPFL Recall- chap 5 Basic governing equations for turbomachinery

- Shaft torque $T_{\text{shaft}} = -\dot{m}_1 (r_1 V_{01}) + \dot{m}_2 (r_2 V_{02})$

- Shaft power $\dot{W}_{\text{shaft}} = T_{\text{shaft}} \omega = -\dot{m}_1 r_1 V_{01} \omega + \dot{m}_2 r_2 V_{02} \omega$

$$\dot{W}_{\text{shaft}} = (-\dot{m}_1) (U_1 V_{01}) + \dot{m}_2 (U_2 V_{02}) \quad [\text{W}] = [\text{kg} \cdot \text{m}^2/\text{s}^3]$$

- Shaft work per unit mass (shaft power per unit mass flow rate), $\dot{w}_{\text{shaft}} = \dot{w}_1 = \dot{w}_2$

$$\dot{w}_{\text{shaft}} = -(U_1 V_{01}) + (U_2 V_{02}) \quad [\text{m}^2/\text{s}^2]$$

- Basic governing equations for pumps or turbines whether the machines are radial-, mixed-, or axial-flow devices and for compressible and incompressible flows
- Note it is only the function of tangential component of velocity, no V_r , V_x

EPFL Recall- chap 5 Basic governing equations for turbomachine

$$\mathbf{V} = \mathbf{W} + \mathbf{U}$$

From the big triangle (grey) $V^t = V_b^t + V_r^t \quad \text{or} \quad V_x^t = V^t - V_b^t$

From the small triangle (dark grey) $W^t = (V_b - U)^2 + V_x^t$

$$= V_b^t - 2V_b U + U^t + V_x^t$$

$$W^t = V_b^t - 2V_b U + U^t + V^t - V_b$$

$$V_b U = \frac{-W^t + U^t + V^t}{2}$$

$$\dot{w}_{\text{shaft}} = -(U_1 V_{01}) + (U_2 V_{02})$$

$$\dot{w}_{\text{shaft}} = \frac{V_2^2 - V_1^2 + U_2^2 - U_1^2 - (W_2^2 - W_1^2)}{2}$$

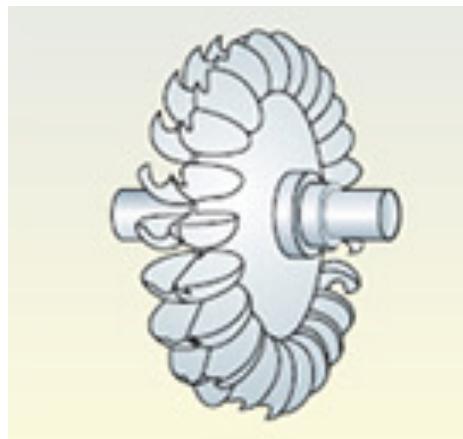
Turbomachine work is related to changes in absolute, relative, and blade velocities.

$$w_{\text{shaft}} = \frac{V_2^2 - V_1^2 + U_2^2 - U_1^2 - (W_2^2 - W_1^2)}{2}$$

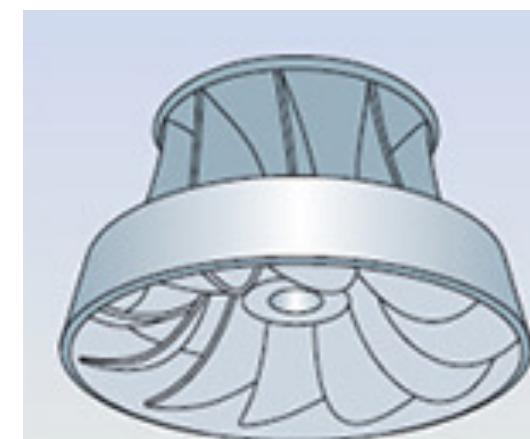
- If $U_2 = U_3$ (axial machine), $W_3 = W_2 \rightarrow R=0$
- If $V_1 = V_2 \rightarrow R=1$

Types of hydraulic turbines

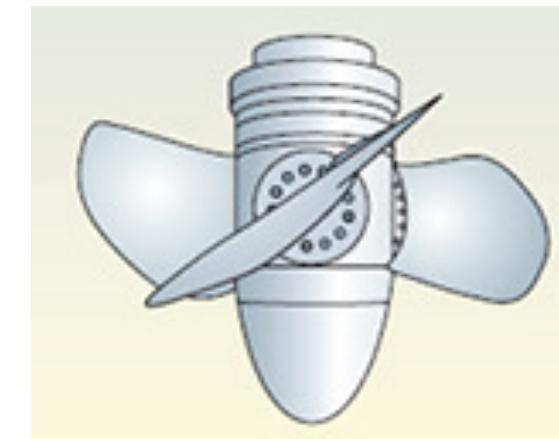
- Impulse (Action) turbines: Pelton turbines, $R \sim 0$
- Reaction turbines: $R \sim [0.1, 0.7]$
 - Francis turbines (radial et axial), Kaplan turbines (axial)
 - Propeller turbines (similar to Kaplan turbines with fixed pitch)



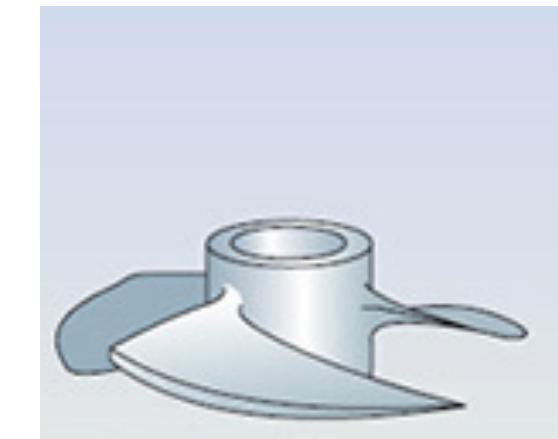
Pelton



Francis

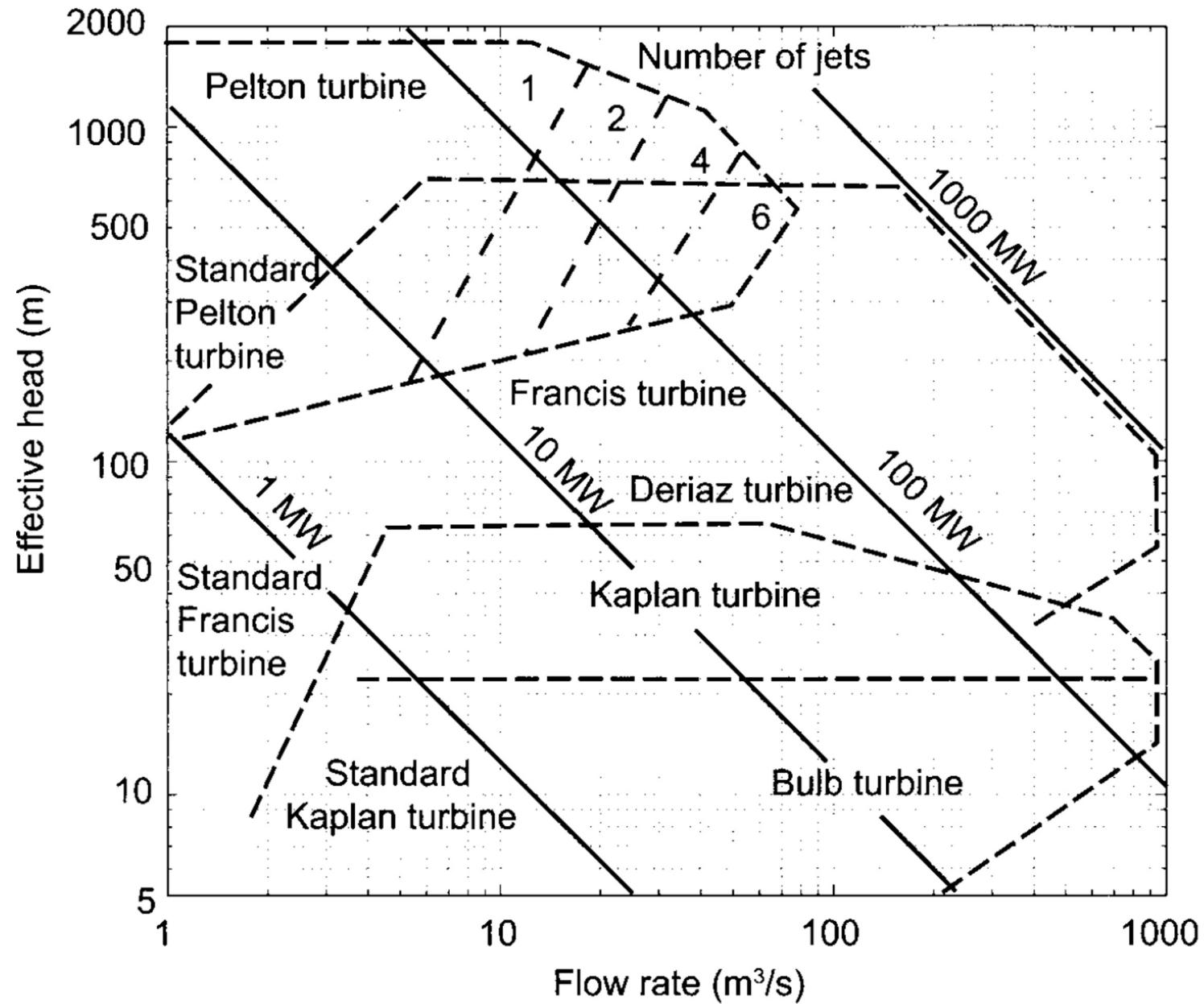


Kaplan & Bulb



Fixed pitch propeller

Types of hydraulic turbines



Power specific speed

For hydraulic turbines, the rotor diameter D is eliminated between the flow coefficient and the power coefficient to obtain the power-specific speed

Power specific speed (Hydraulic turbines)

$$N'_s = \frac{\omega \sqrt{\dot{W}_{\text{shaft}} / \rho}}{(gh_a)^{5/4}}$$

Specific speed

$$N_s = \frac{\omega \sqrt{Q}}{(gh_a)^{3/4}}$$

Commonly used, but not dimensionless, definition of power specific speed

$$N'_{sd} = \frac{\omega(\text{rpm}) \sqrt{\dot{W}_{\text{shaft}}(\text{W})}}{[h_a(\text{m})]^{5/4}}$$

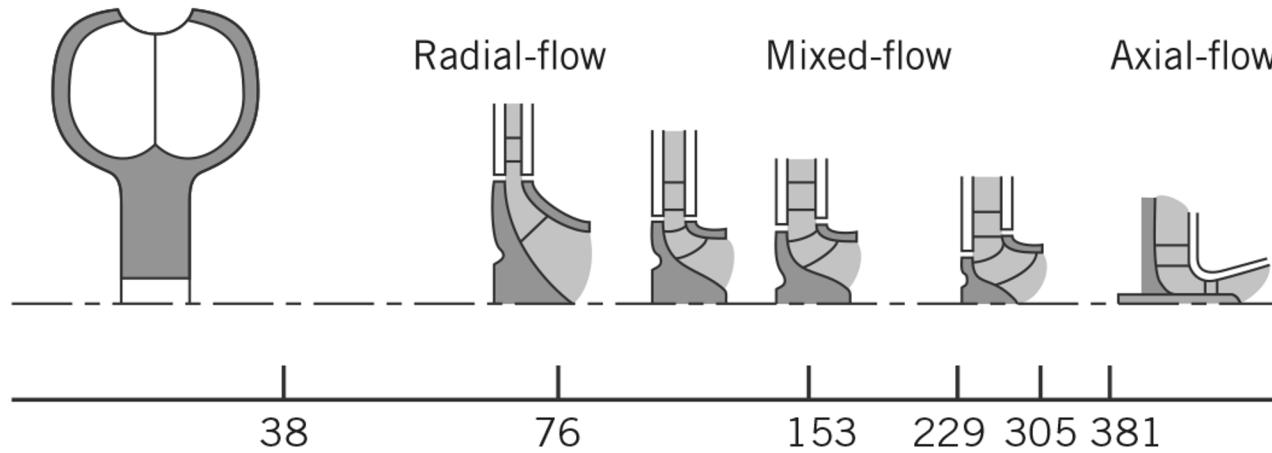
In hydraulic machine, the actual head (in pump) h_a is commonly called '**effective or net head**'.

The elevation head (physical difference between upper reservoir's surface and the one of the lower one) is called '**gross head**', h_g .

$$h_a = h_g - h_L$$

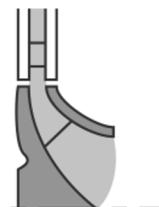
- Power gained by/extracted from the fluids, $P_f = \gamma Q h_a$
- Efficiency pump $\eta = \frac{P_f}{\dot{W}_{\text{shaft}}} \longrightarrow \dot{W}_{\text{shaft}} = P_f / \eta = \gamma Q h_a / \eta$
- Efficiency turbine $\eta = \frac{\dot{W}_{\text{shaft}}}{P_f} \longrightarrow \dot{W}_{\text{shaft}} = \eta P_f = \eta \gamma Q h_a$

Impulse turbines

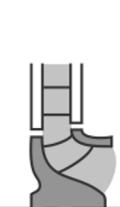


Reaction turbines

Radial-flow



Mixed-flow

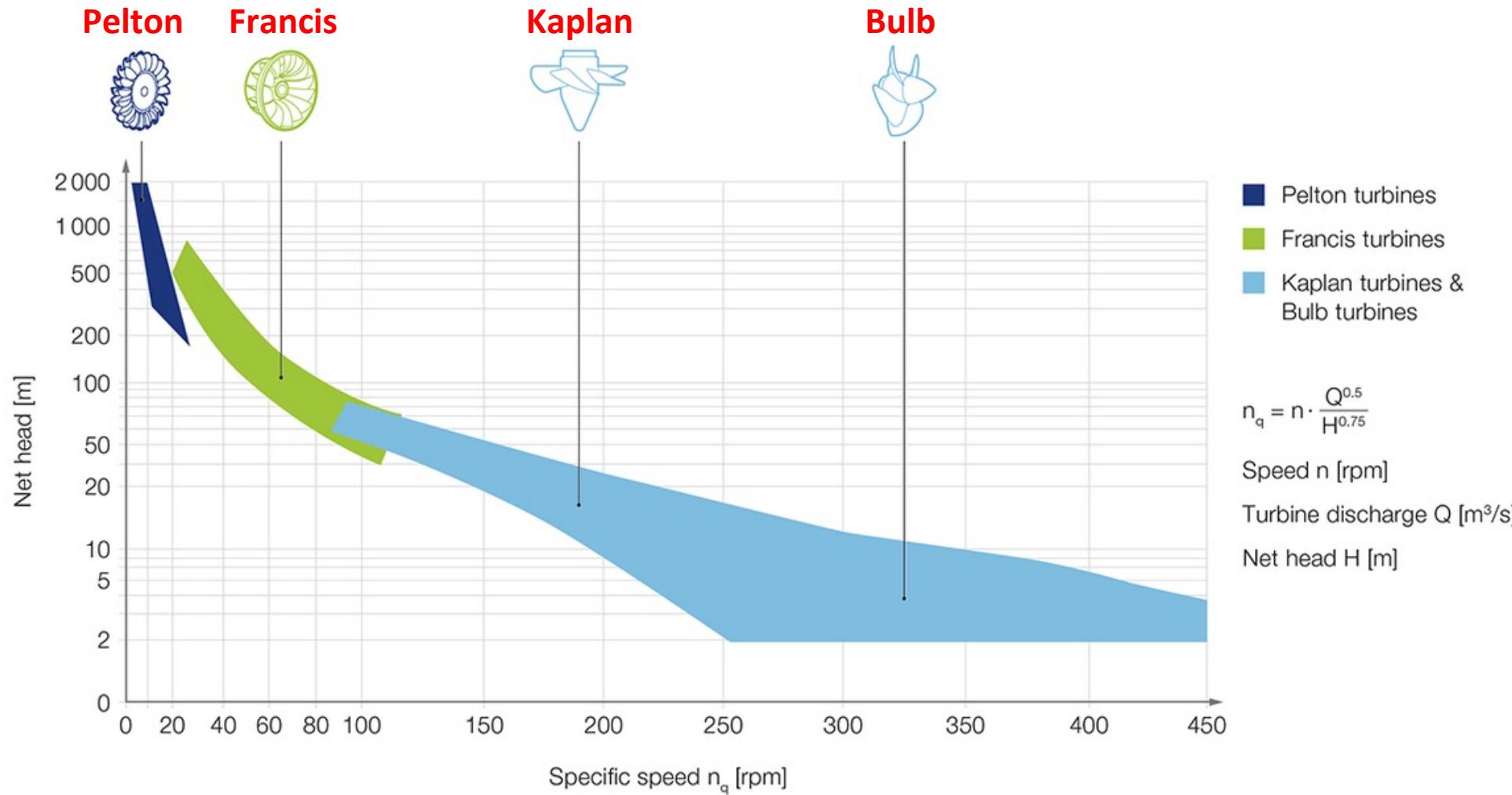


Axial-flow



Type	N'_{sd}	η %
Pelton wheel	Single jet	0.02 – 0.18
	Twin jet	0.09 – 0.26
	Three jet	0.10 – 0.30
	Four jet	0.12 – 0.36
Francis	Low-speed	0.39 – 0.65
	Medium-speed	0.65 – 1.2
	High-speed	1.2 – 1.9
	Extreme-speed	1.9 – 2.3
Kaplan turbine	1.55 – 5.17	87 – 94
Bulb turbine	3 – 8	

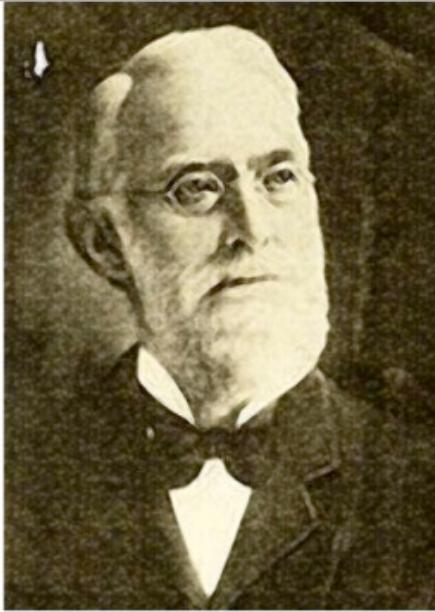
- Classification of turbine types as a function of the head and unit specific speed



Impulse turbines – Pelton turbine

- **Impulse-type Turbines**

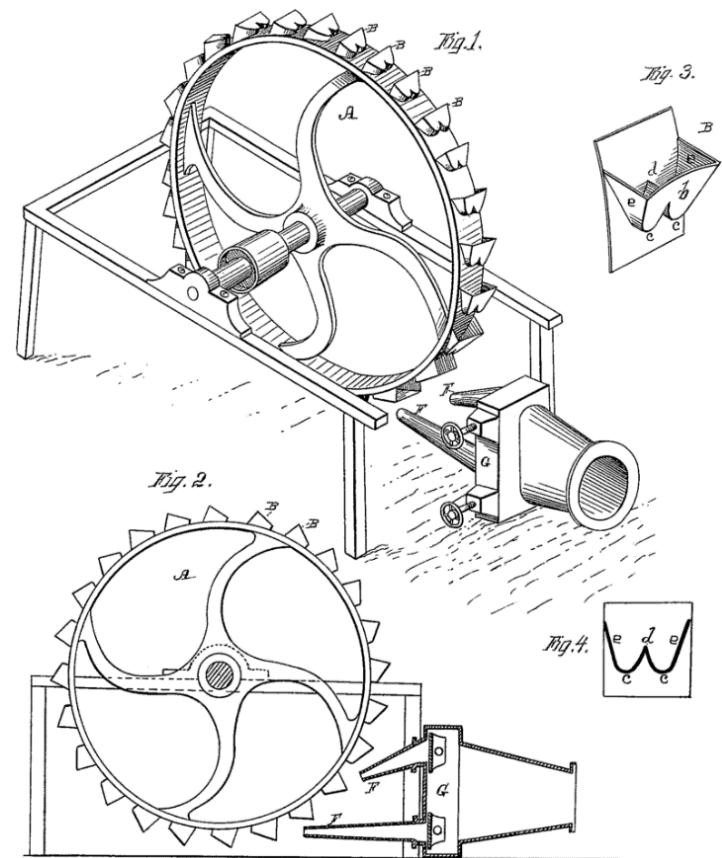
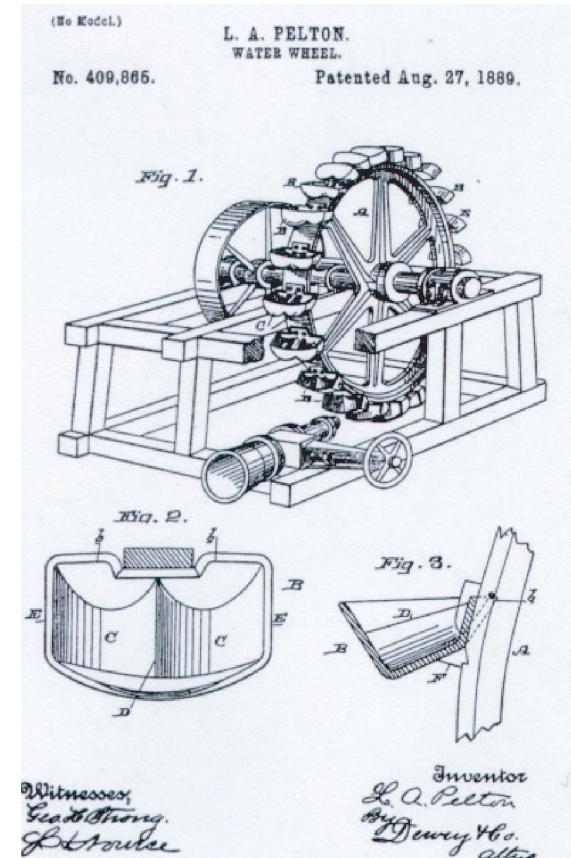
- Among several types of impulse turbines, the so-called Pelton turbine is the most used
- Patented by L. A. Pelton in 1889
- The rotor is made of several buckets and the motion is obtained by high-speed jet(s)



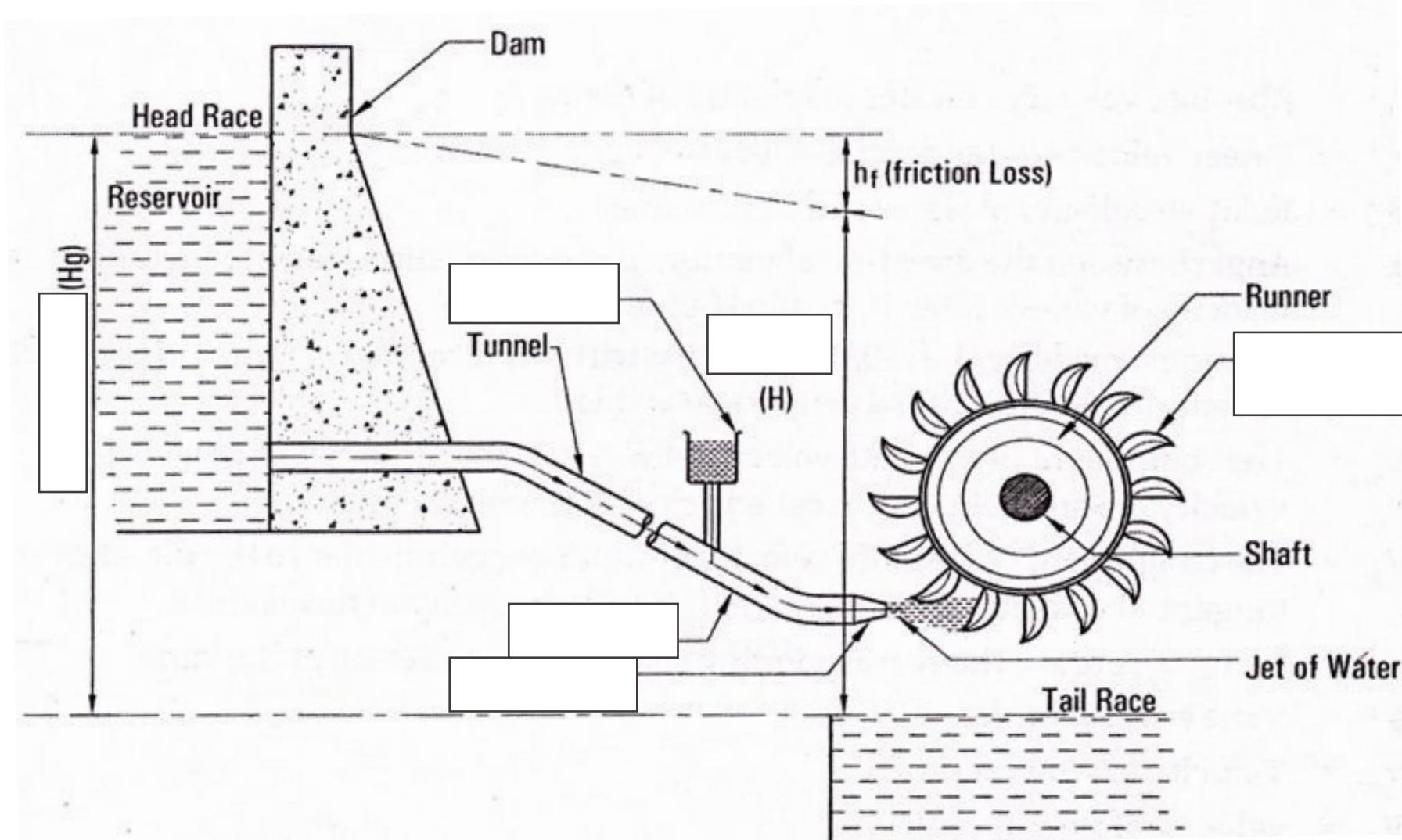
Lester Allan Pelton

(Sept. 5, 1829 – Mar. 14, 1908)

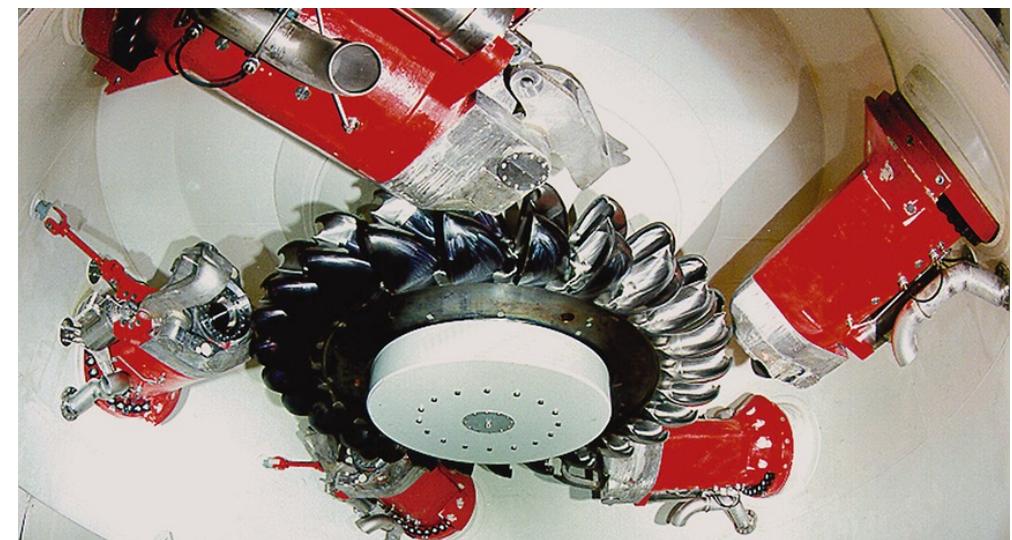
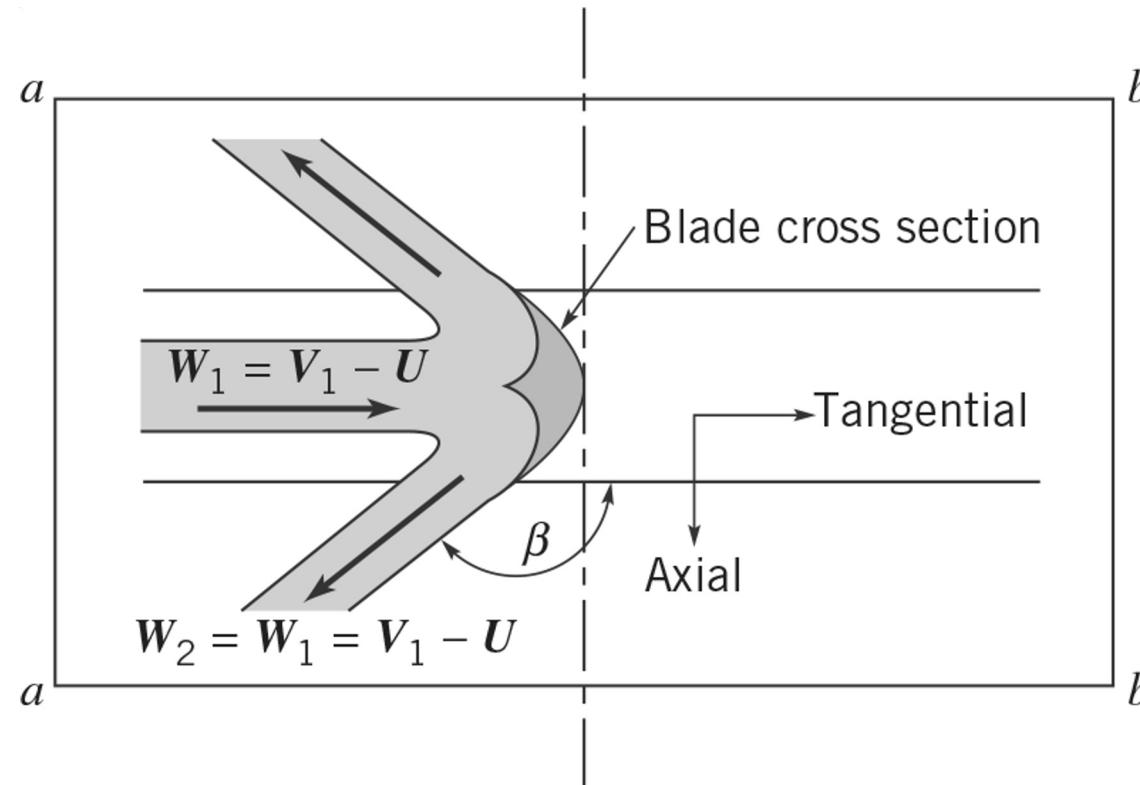
Pelton made his living as a carpenter and a millwright. He created the most efficient form of impulse water turbine.



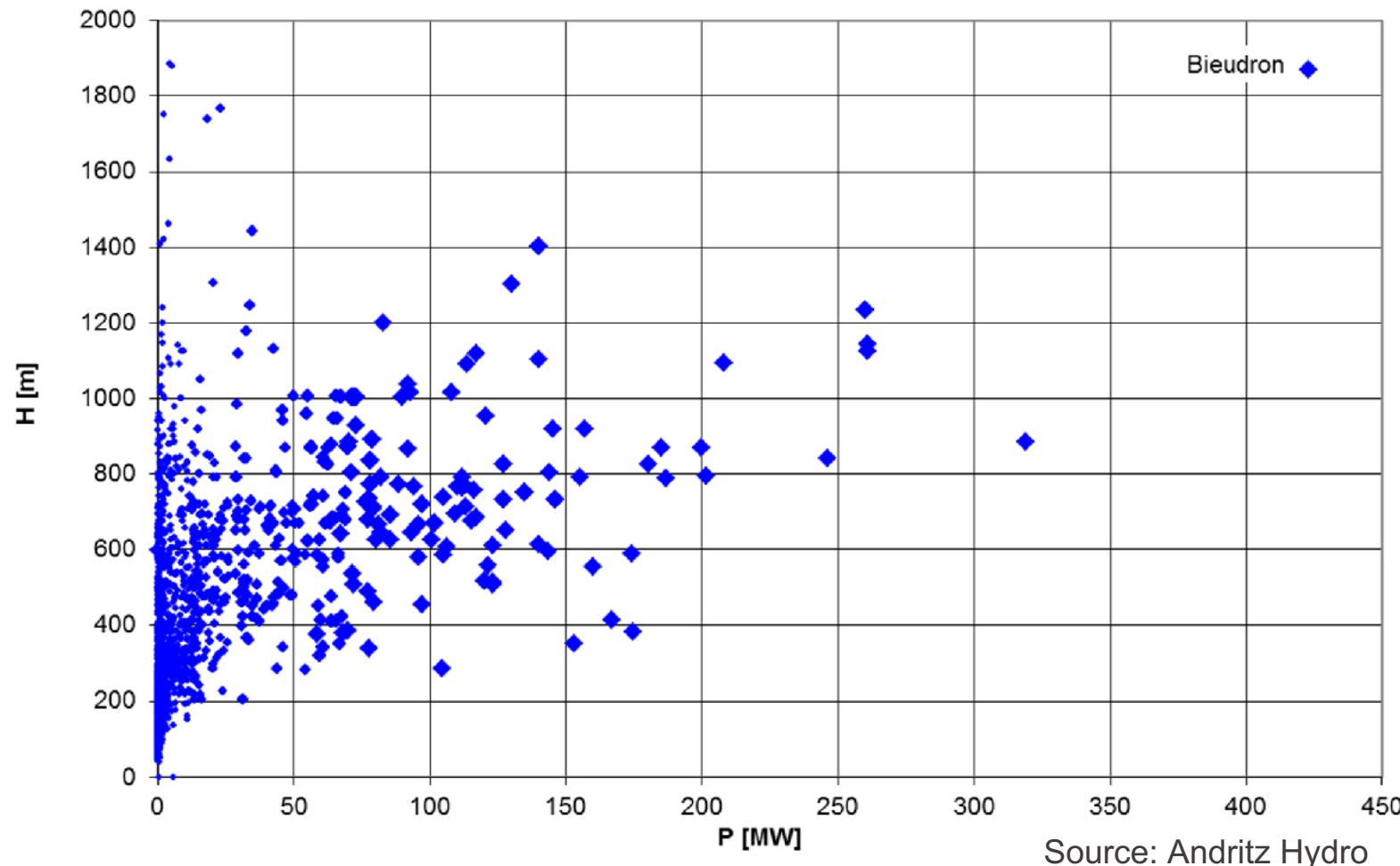
- Pelton wheel is not typical turbomachines (no axial flow, no radial flow), tangential flow
 - Pelton turbines are the most used turbines in **Switzerland** in hydropower generation



- The water enters and leaves the control volume surrounding the wheel as free jets (at **atmospheric pressure**)
- A person riding on the bucket would note that the speed of the water does not change as it slides across the buckets (assuming viscous effects are negligible)
 - the magnitude of the relative velocity does not change, but its direction does.
- The change in direction of the velocity of the fluid jet causes a torque on the rotor, resulting in a power output from the turbine



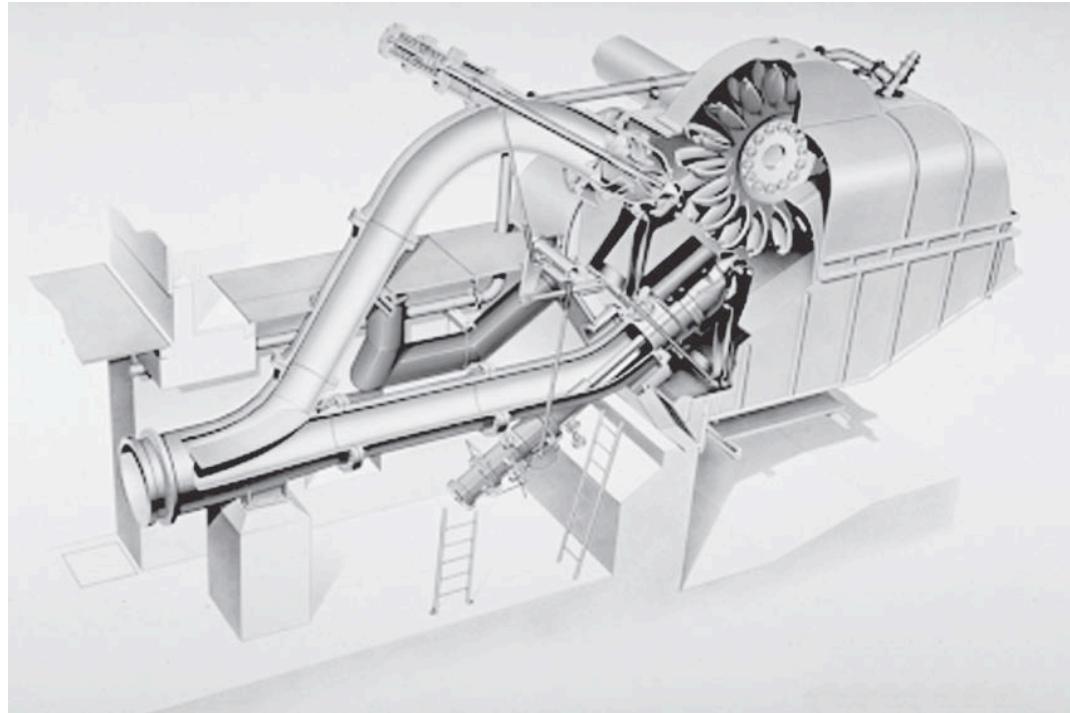
- Suitable for high heads (100's meter range):



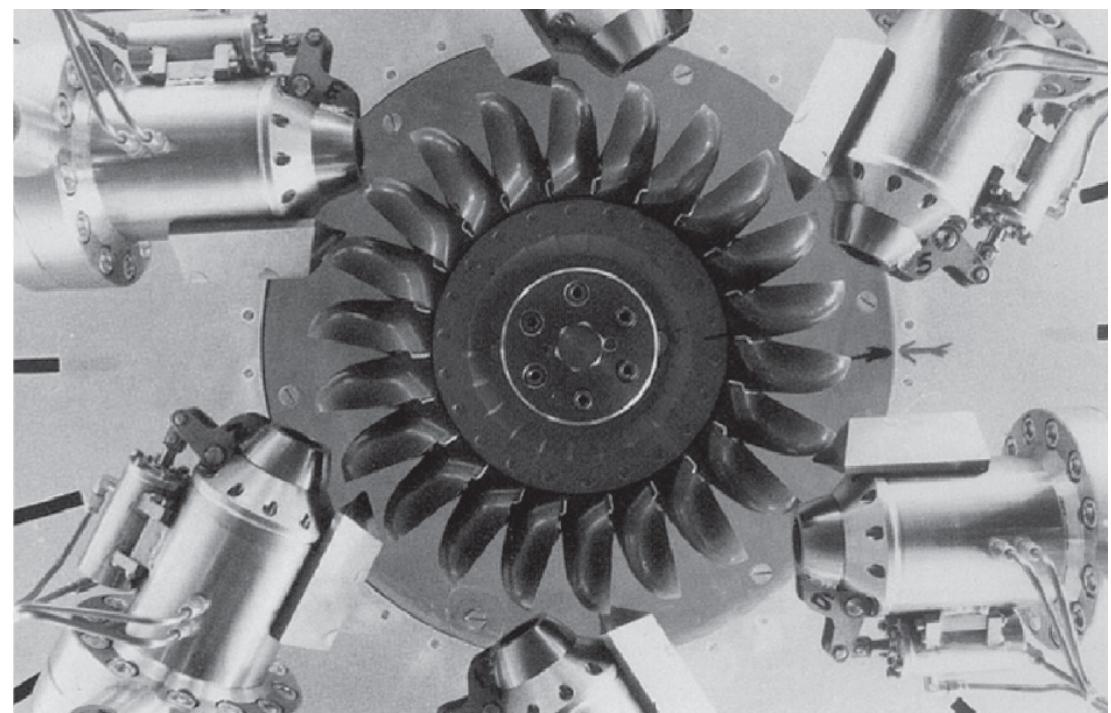
Pelton turbines

- Types of Pelton Turbines:

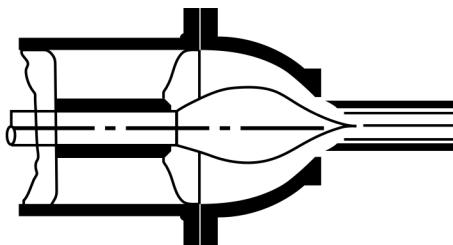
Horizontal shaft: 1, 2 or 3 Jets



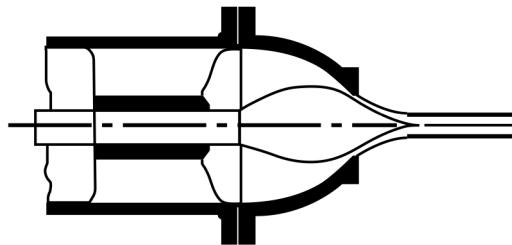
Vertical shaft: 1 to 6 Jets



- The nozzle:
 - Equipped with a needle for flowrate control (e.g. valve)
 - Actioned by a servomotor, which may be internal or external
 - Equipped with a deflector for emergency stops

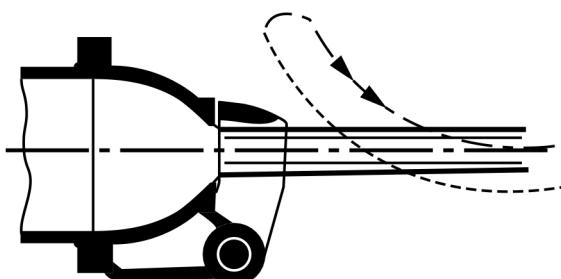


Full load

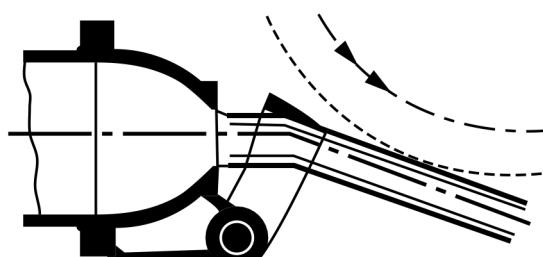


Part load

(a)



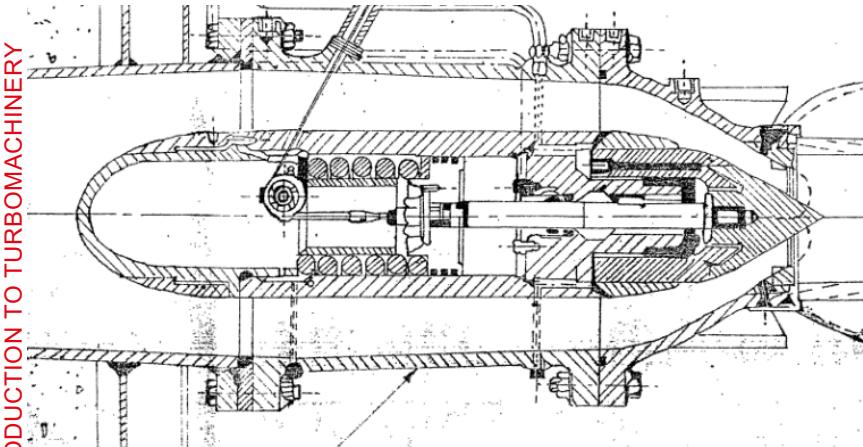
Deflector in normal position



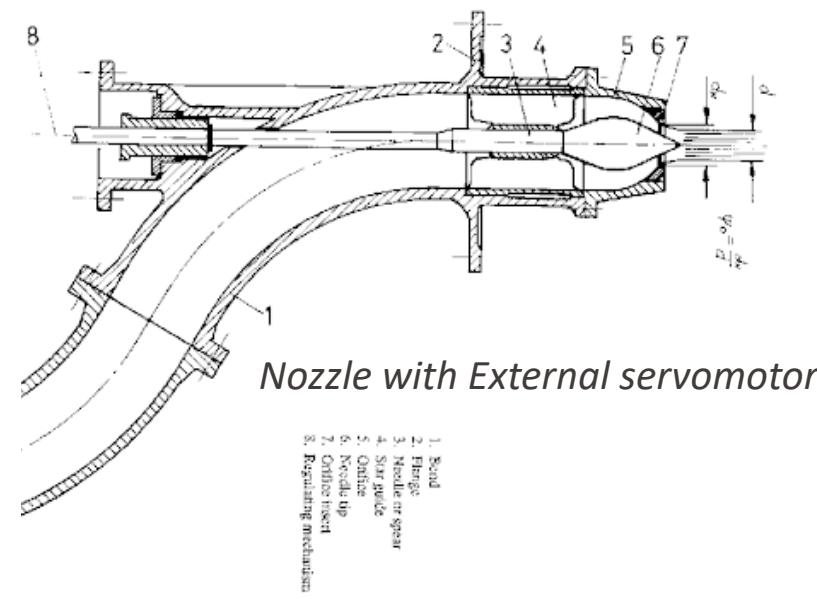
Fully deflected position

(b)

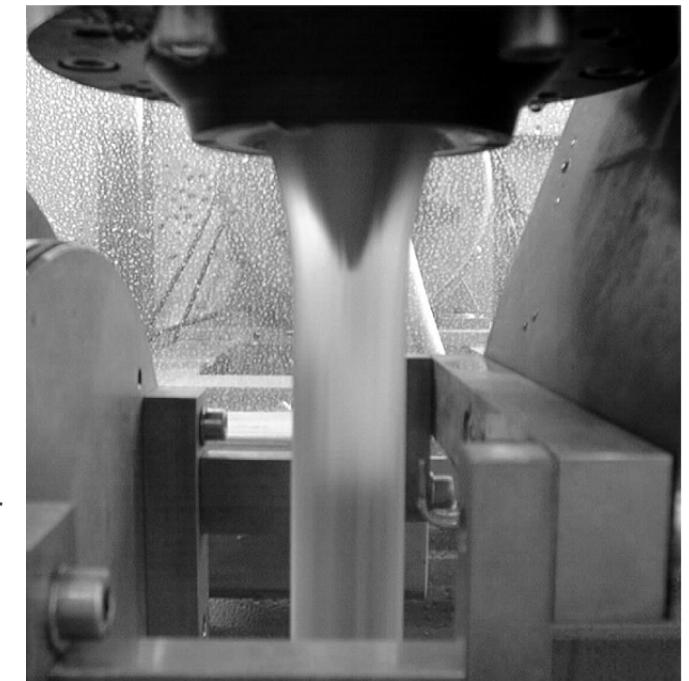
- The nozzle:
 - Key element of the design: must produce a jet of a “high quality”
 - Straight jet
 - Minimum atomization (formation of small droplets)



Nozzle with Internal servomotor



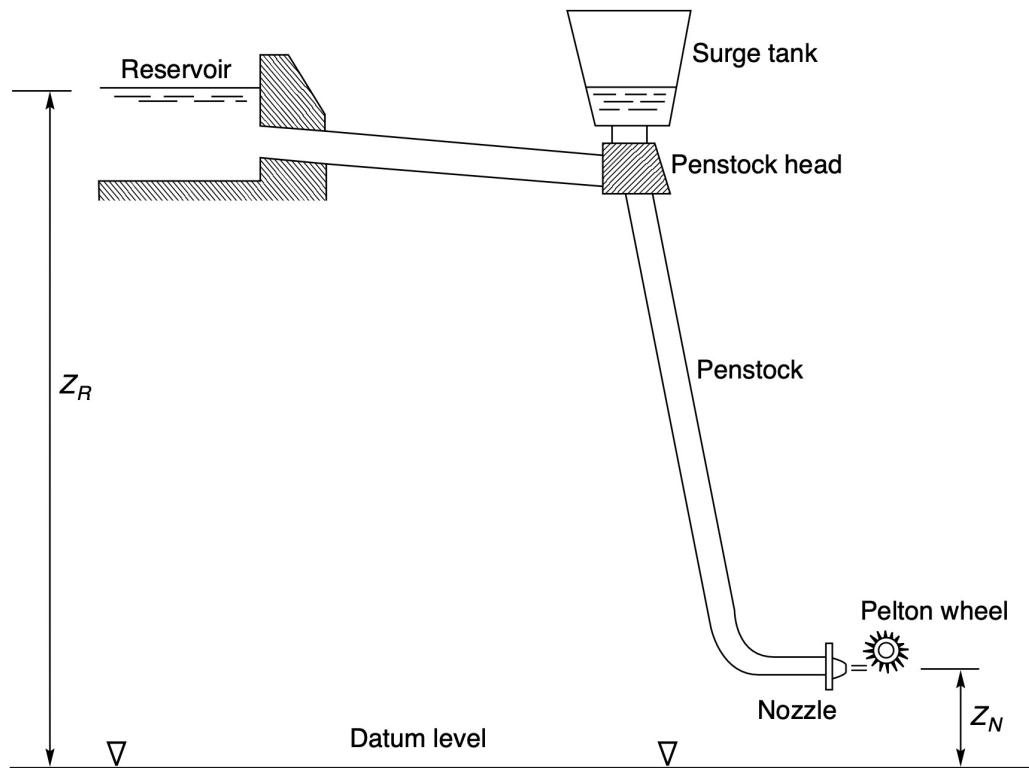
Nozzle with External servomotor



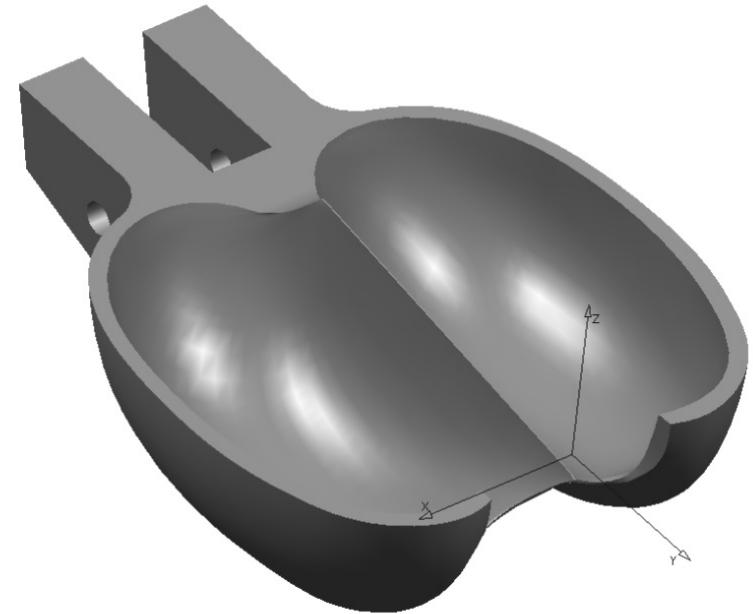
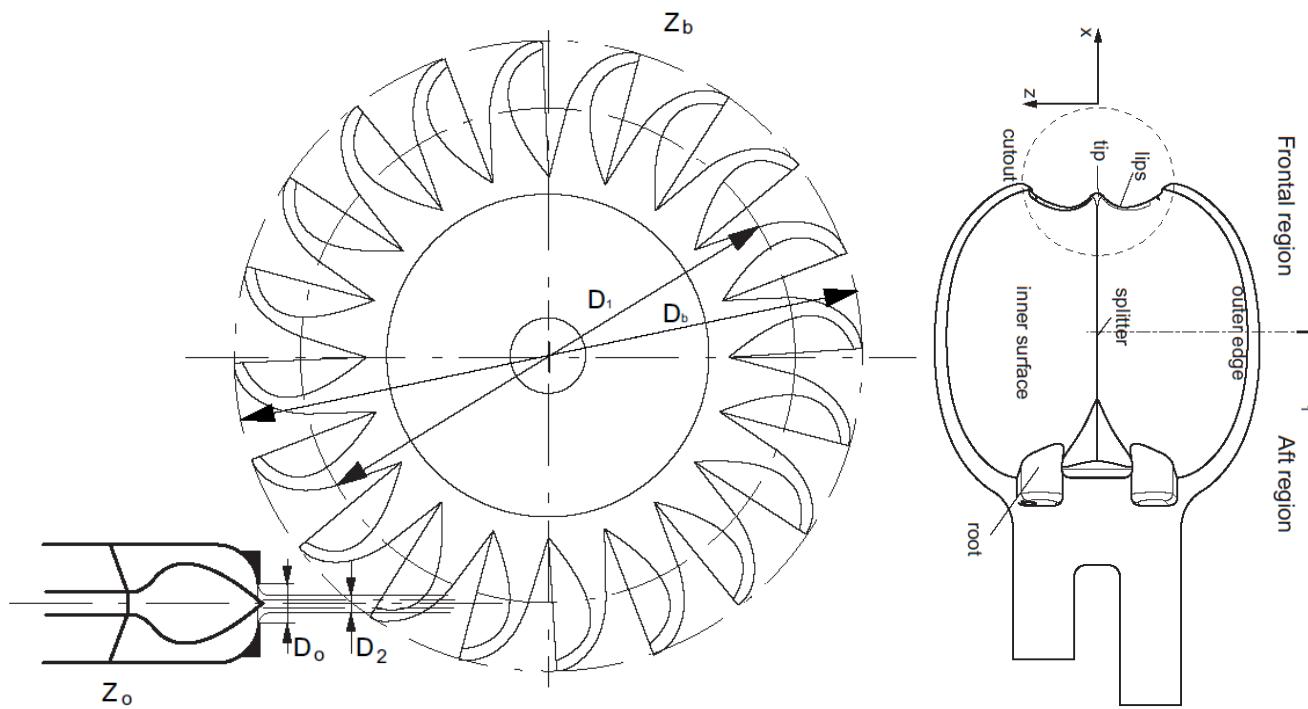
What happens when the nozzle closes suddenly?

Pressure surges

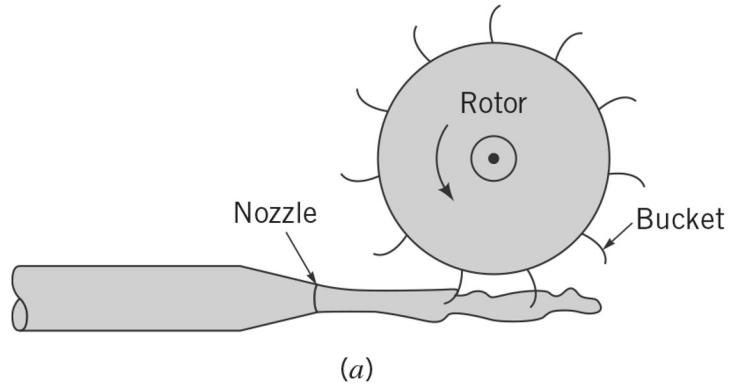
- Spear valve must move slowly: sudden reduction in flow rate may result in serious damage from **pressure surges** (called water hammer)
- If the spear valve closes quickly: all the kinetic energy of the water in the penstock would be absorbed by the elasticity of the supply pipeline (penstock) and the water, creating very large stresses, which would reach their greatest intensity at the turbine inlet where the pipeline is already heavily stressed.
- The surge chamber has the function of absorbing and dissipating some of the pressure and energy fluctuations created by too rapid a closure of the needle valve.



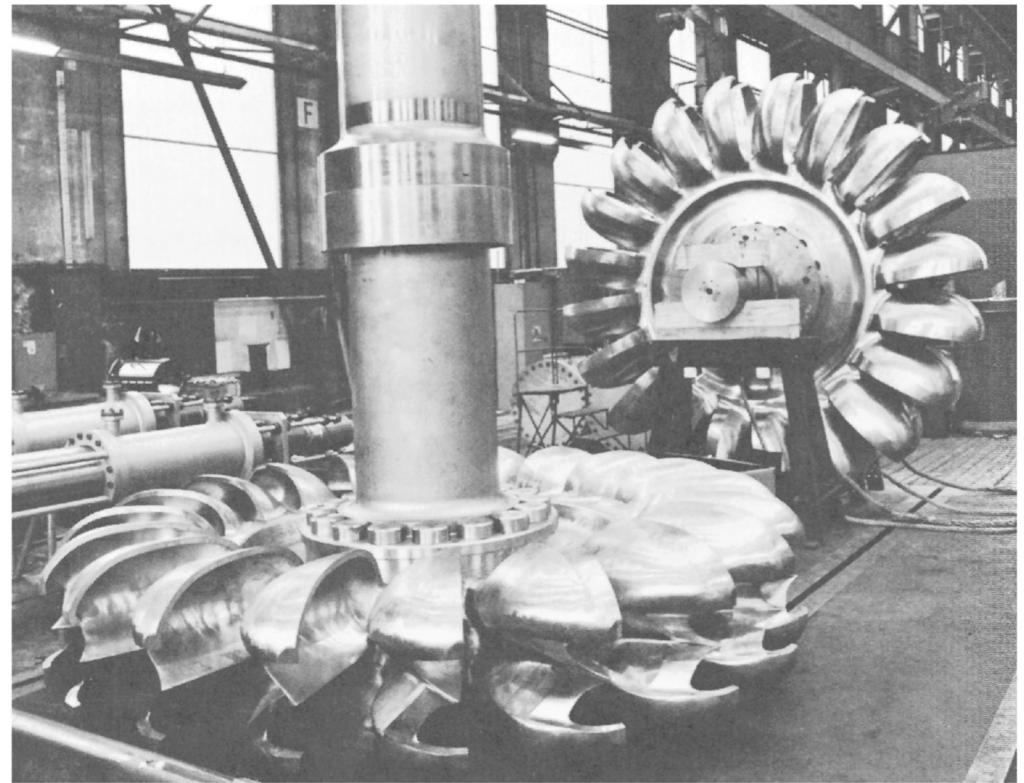
- The buckets:
 - Must deviate “smoothly” the flow and allow for its evacuation with a minimum interference with each other



Pelton turbines

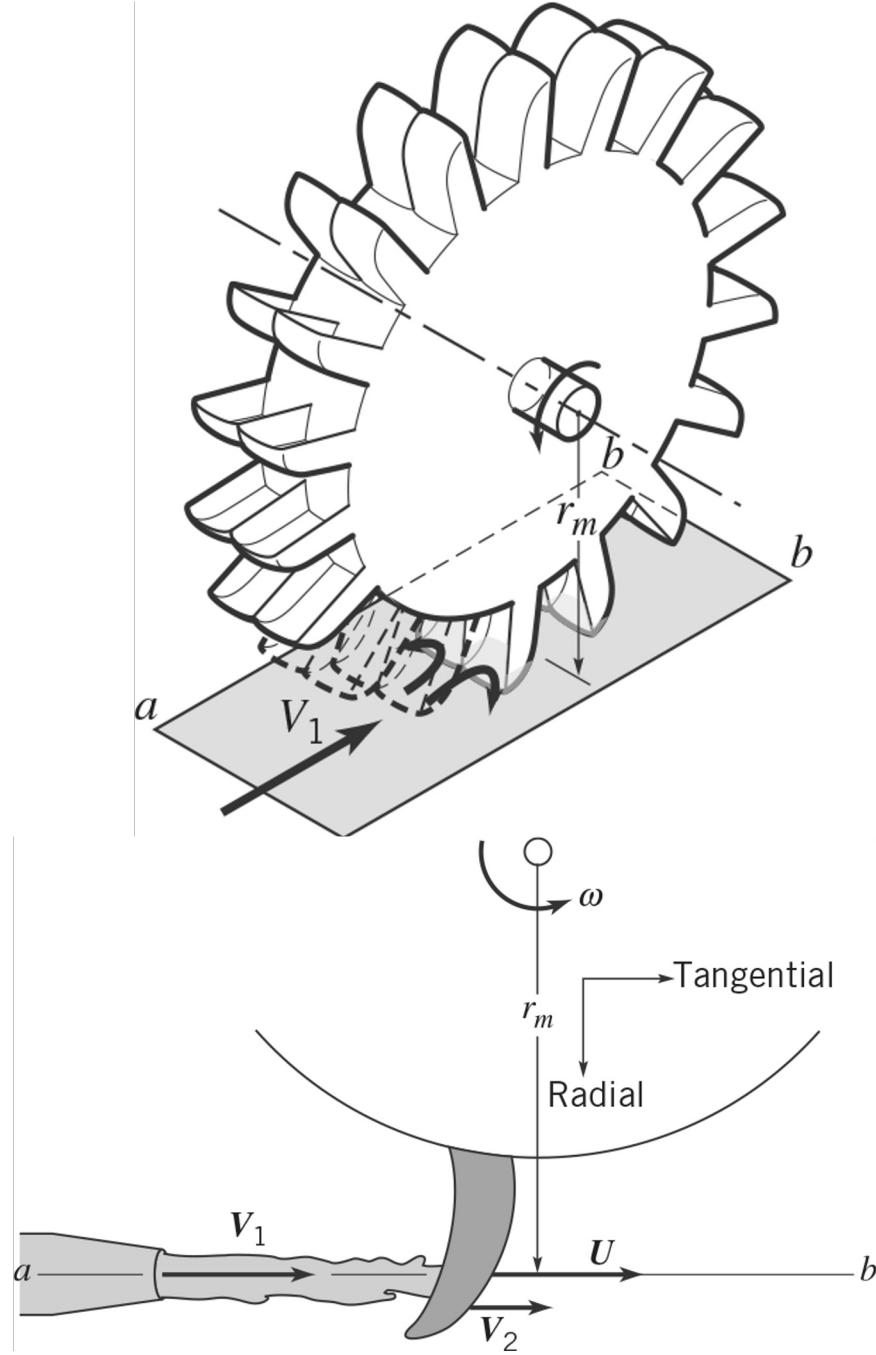


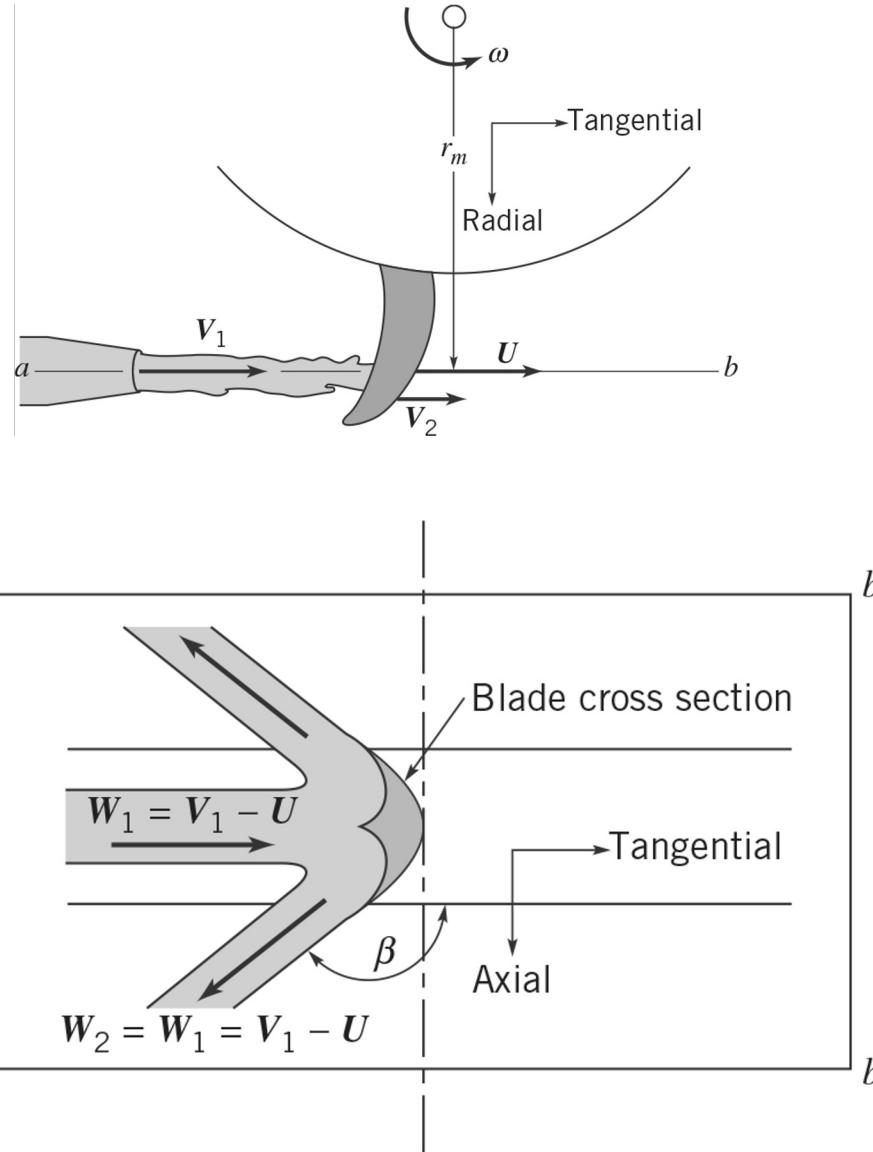
(a)



(b)

Courtesy of Voith Hydro, York, PA.





Assumptions

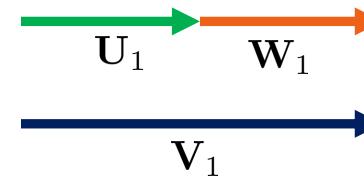
- no radial component of flow
- $W_2 \simeq W_1$ (otherwise $W_2 = C_v W_1$ with $C_v < 1$ velocity coefficient) \rightarrow exercise 2

$$T_{\text{shaft}} = -\dot{m}_1 (r_1 V_{\theta 1}) + \dot{m}_2 (r_2 V_{\theta 2})$$

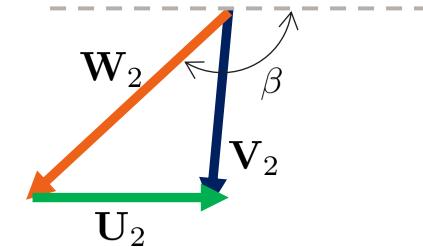
$$\dot{m}_1 = \dot{m}_2 = \dot{m}, \quad r_1 = r_2 = r_m, \quad \mathbf{U}_1 = \mathbf{U}_2 = \mathbf{U}$$



$$V_{\theta 1} = V_1 = W_1 + U$$



$$V_{\theta 2} = W_2 \cos \beta + U$$



$$\dot{W}_{\text{shaft}} = T_{\text{shaft}} \omega = \dot{m} U (U - V_1) (1 - \cos \beta)$$

If $V_1 > U$ (jet impacting bucket), $\dot{W}_{\text{shaft}} < 0$ the turbine extracts power from the fluid

$$\dot{W}_{\text{shaft}} = \dot{m}U(U - V_1)(1 - \cos\beta)$$

- Effect of β : maximum when $\beta = 180^\circ$ ($\cos \pi = -1$)

$$\dot{W}_{\text{shaft}} = 2\dot{m}(U^2 - UV_1)$$

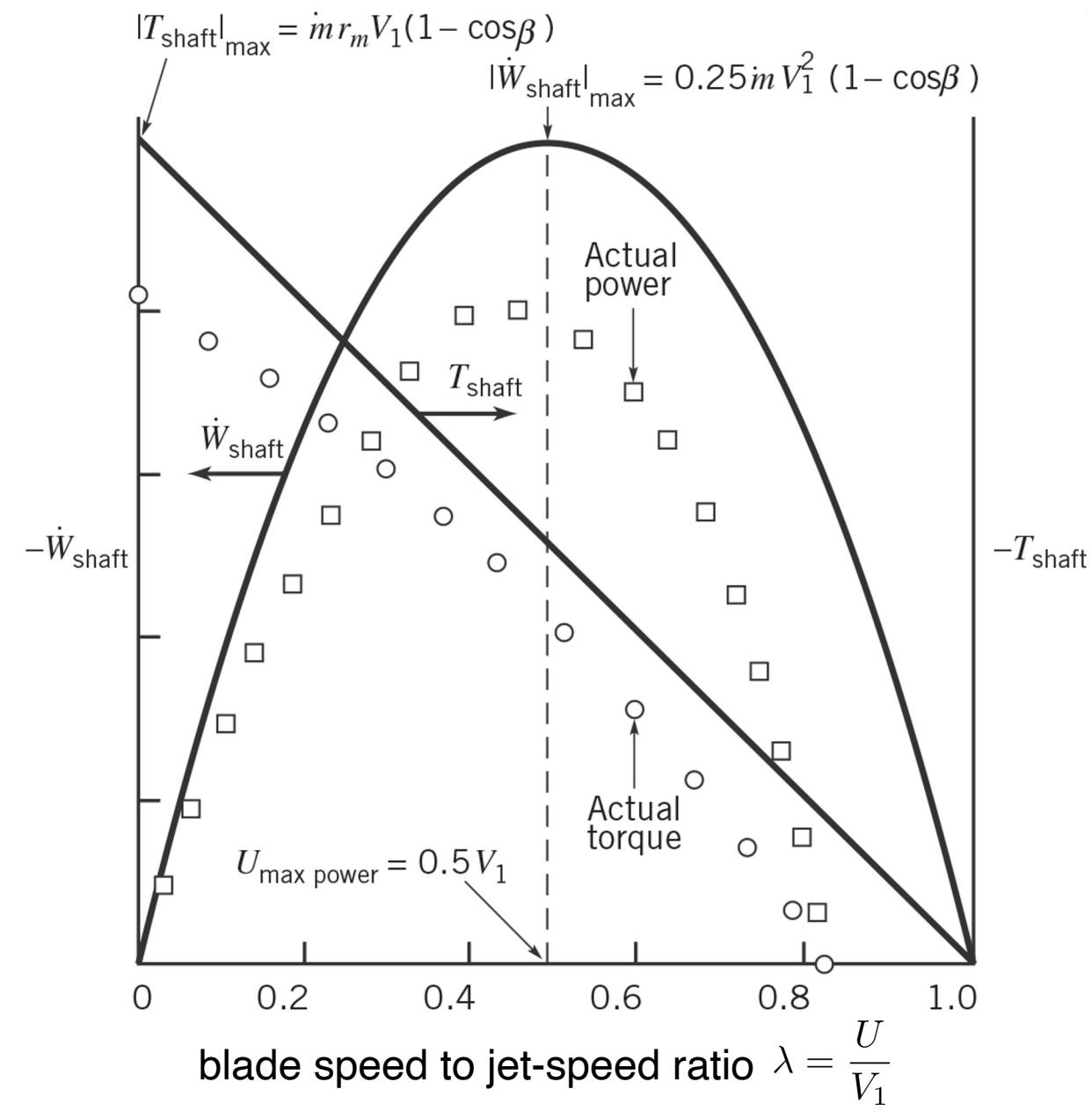
Maximum power
with respect to U

$$U_{\text{max power}} = \frac{V_1}{2}$$

A bucket speed one-half the speed of the fluid coming from the nozzle gives maximum power.

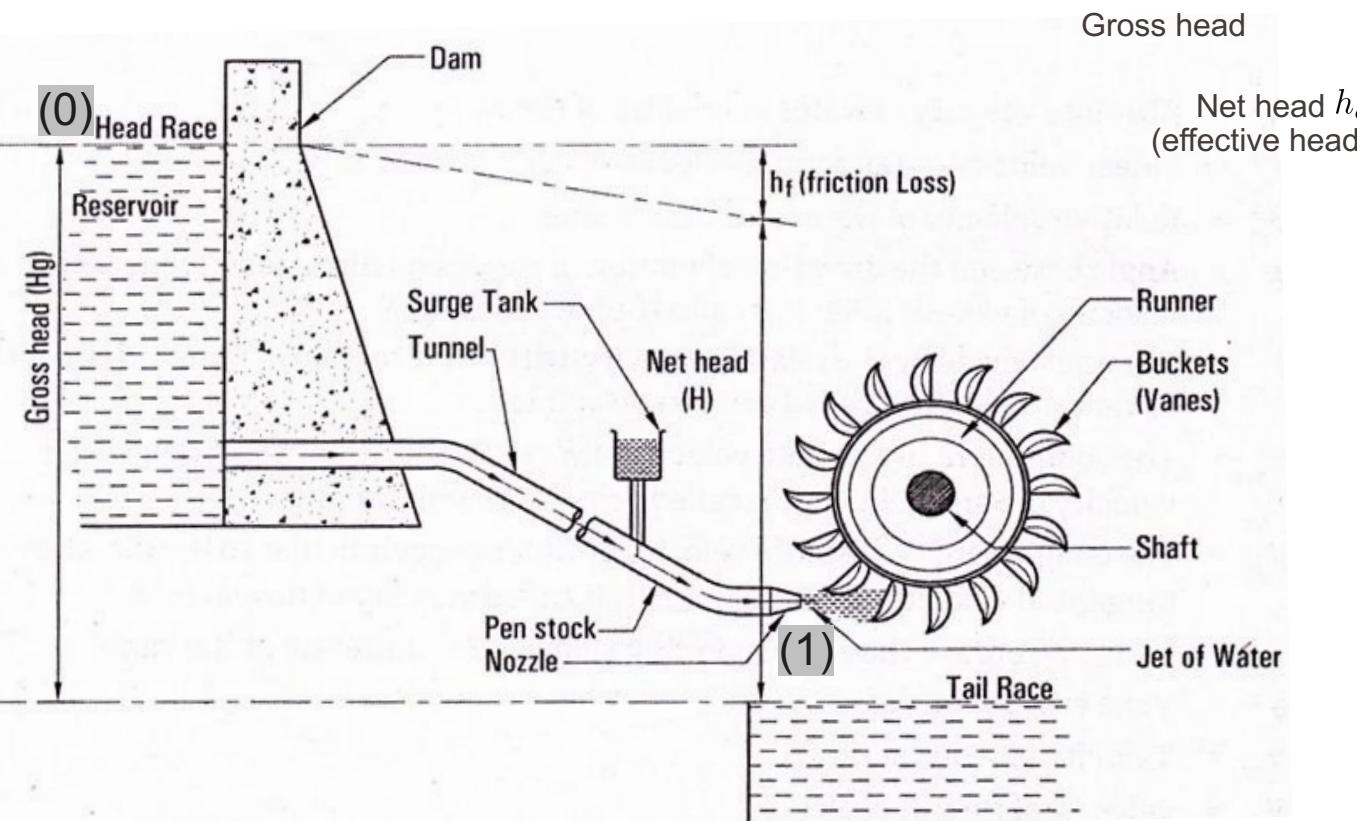
$$T_{\text{shaft}} = \dot{m}r_m(U - V_1)(1 - \cos\beta)$$

- Shaft torque = 0 when $U = V_1$



Pelton turbines – Nozzle speed

$$\frac{p_{\text{atm}}}{\gamma} + z_0 = \frac{p_{\text{atm}}}{\gamma} + \frac{V_1^2}{2g} + z_1 + \sum h_L$$



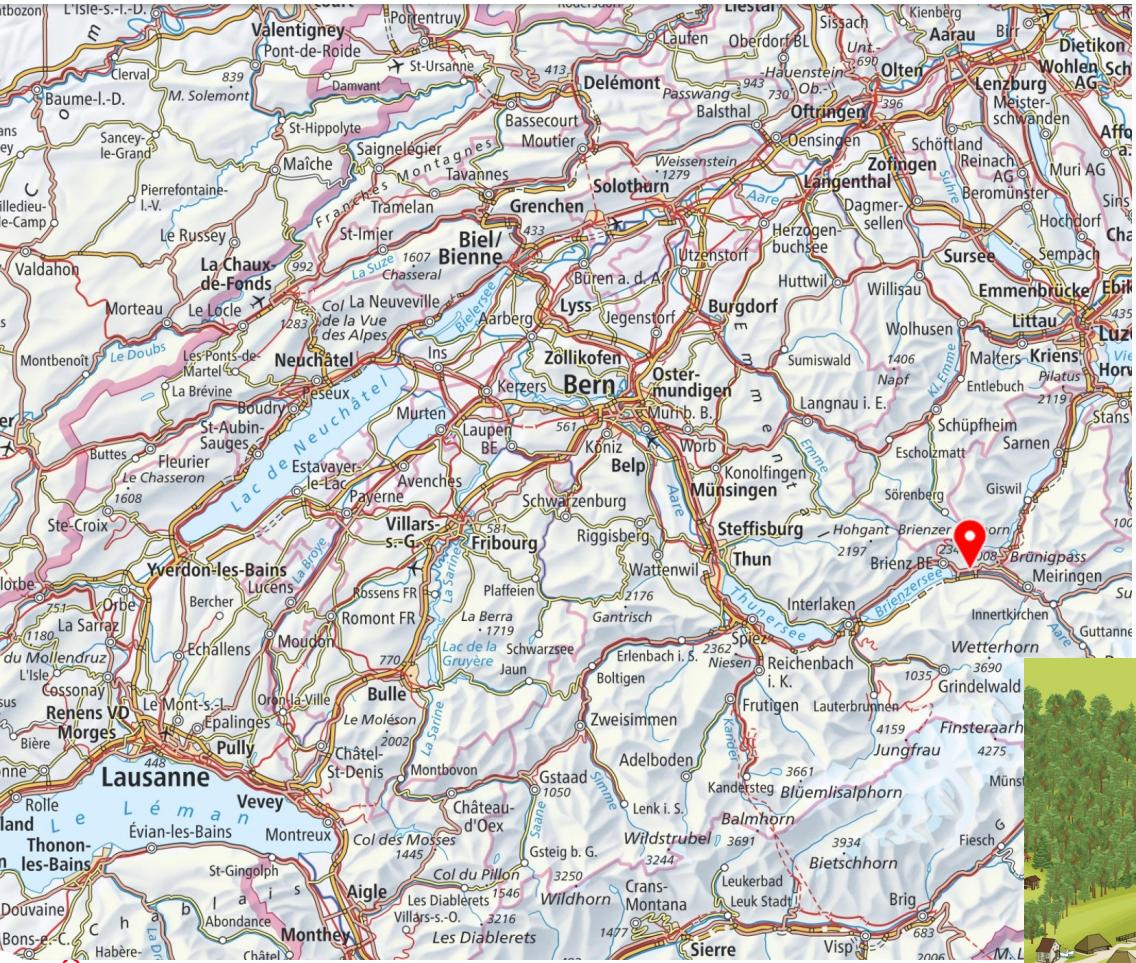
Or $c_N V_1 = V_{\text{nozzle}}$ Nozzle coefficient, $c_N < 1$

Tygun's empirical formula for the number of buckets (Z) in the wheel

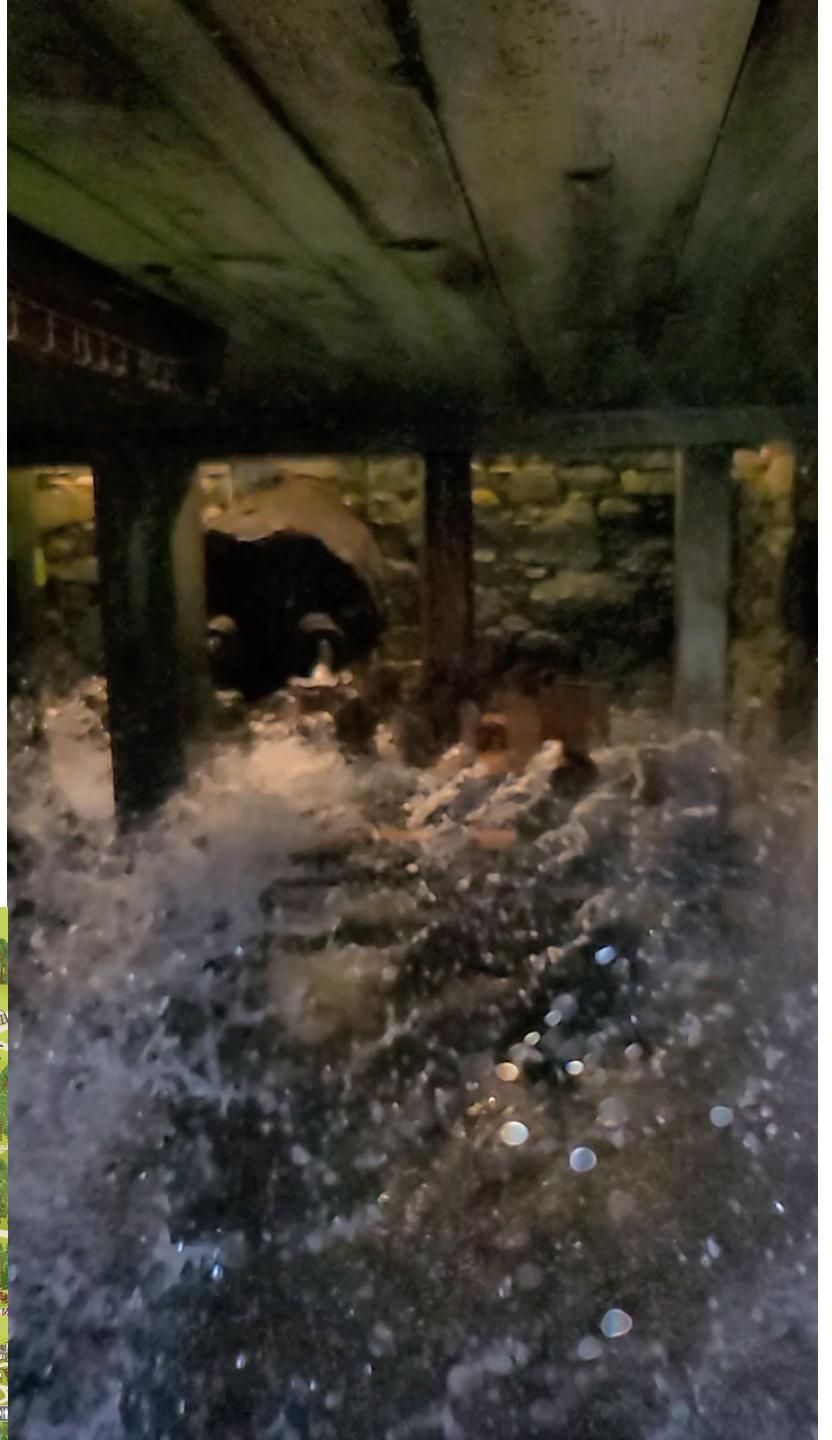
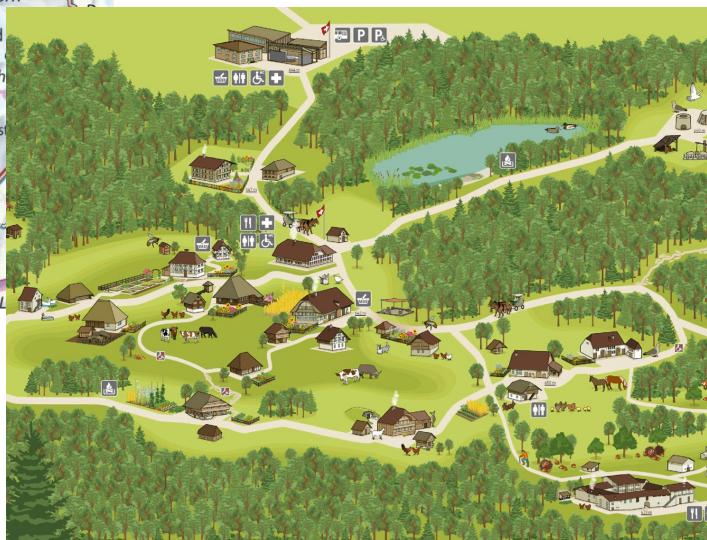
$$Z = \frac{D_{\text{wheel}}}{2D_{\text{nozzle}}} + 15$$

Pelton turbines in Switzerland

SwitzerlandMobility *

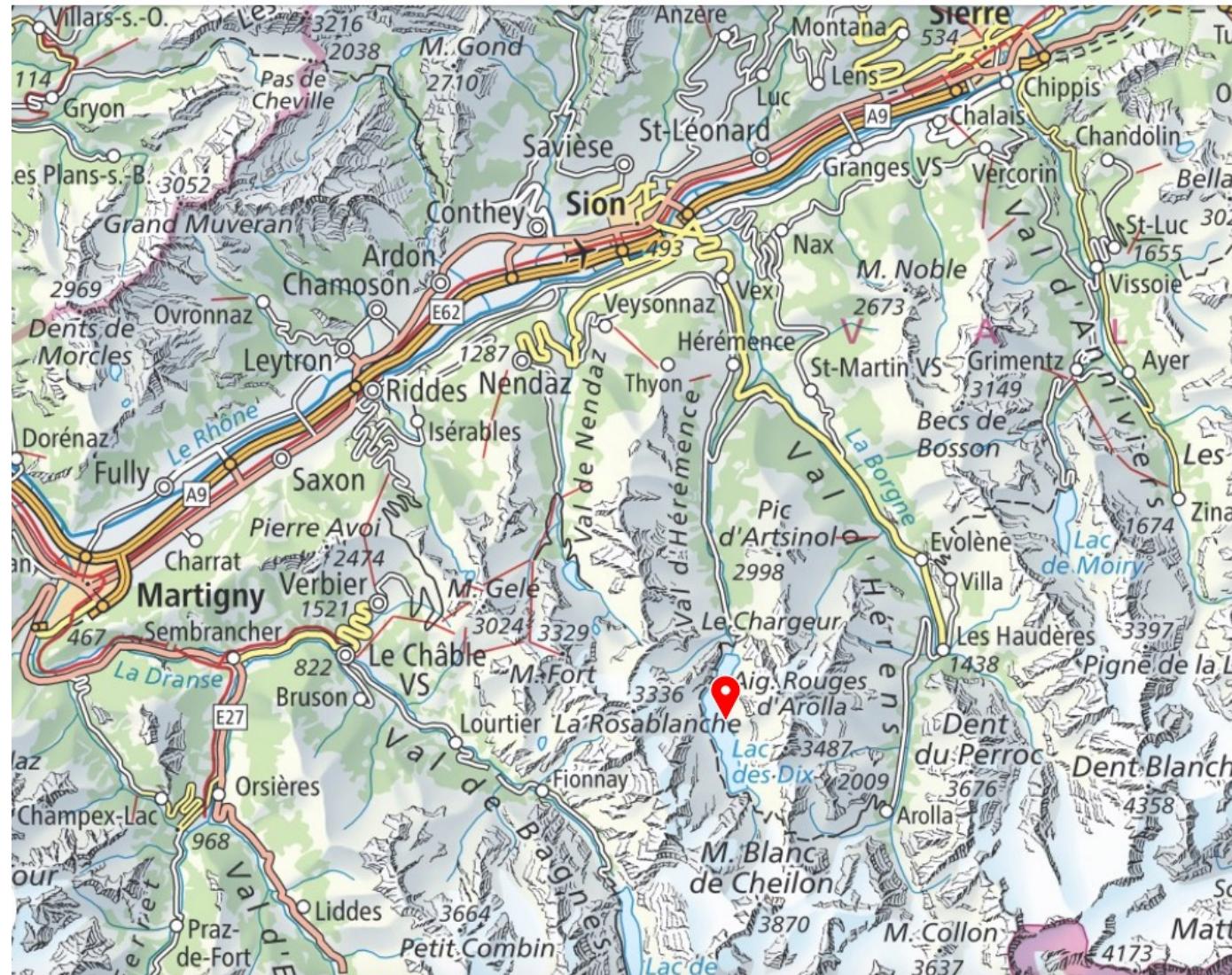


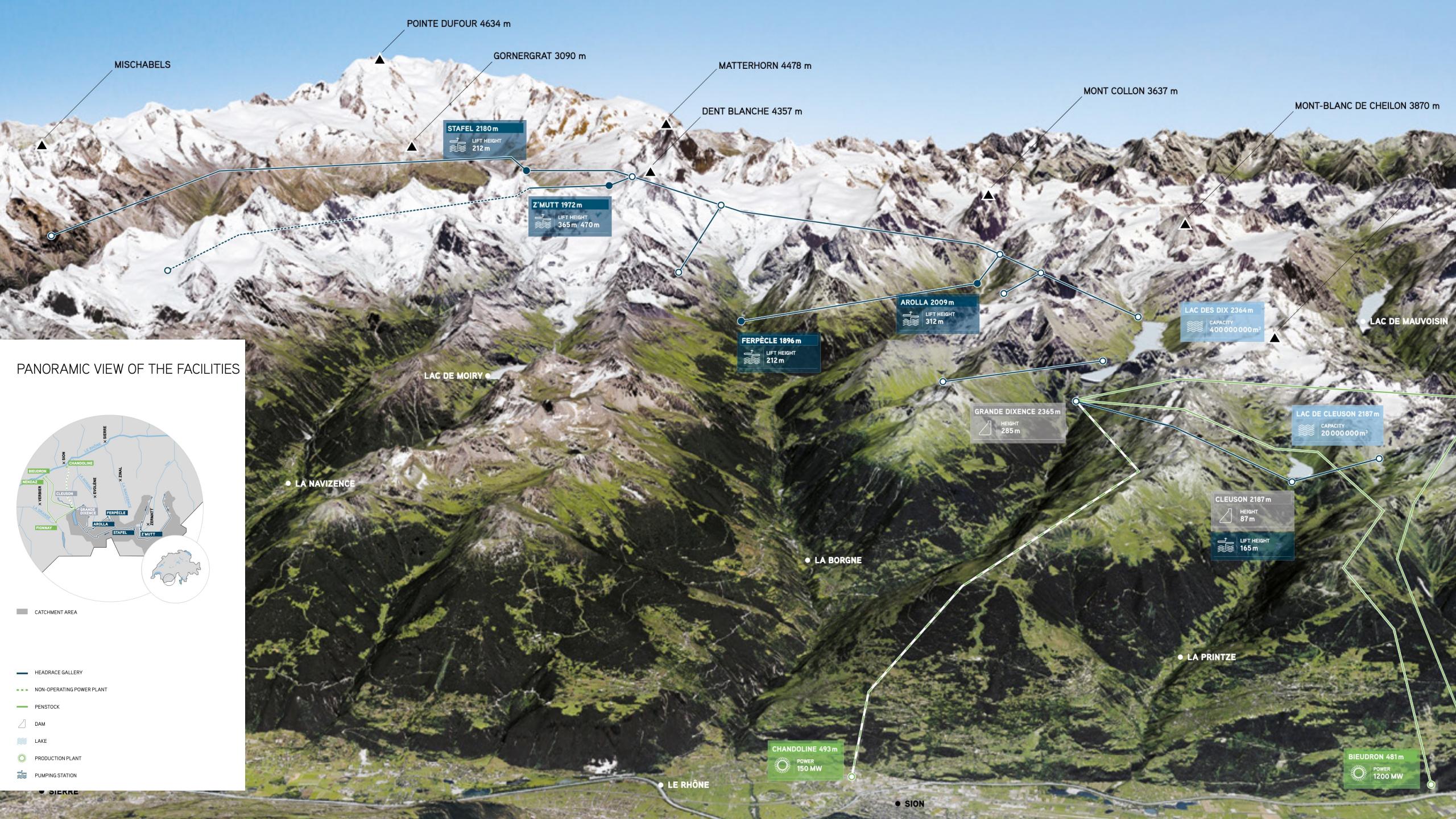
Ballenberg, Swiss
Open-Air Museum

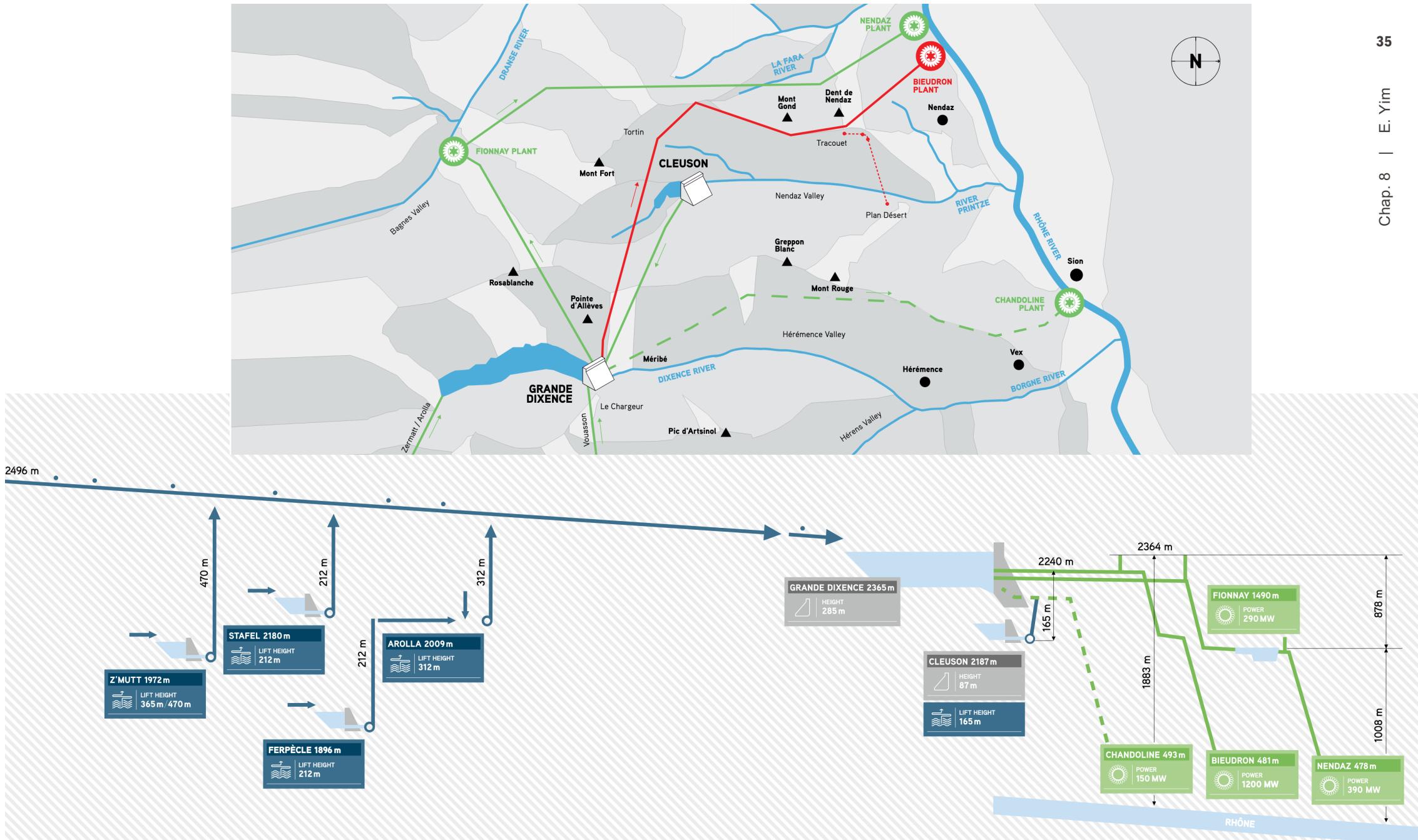


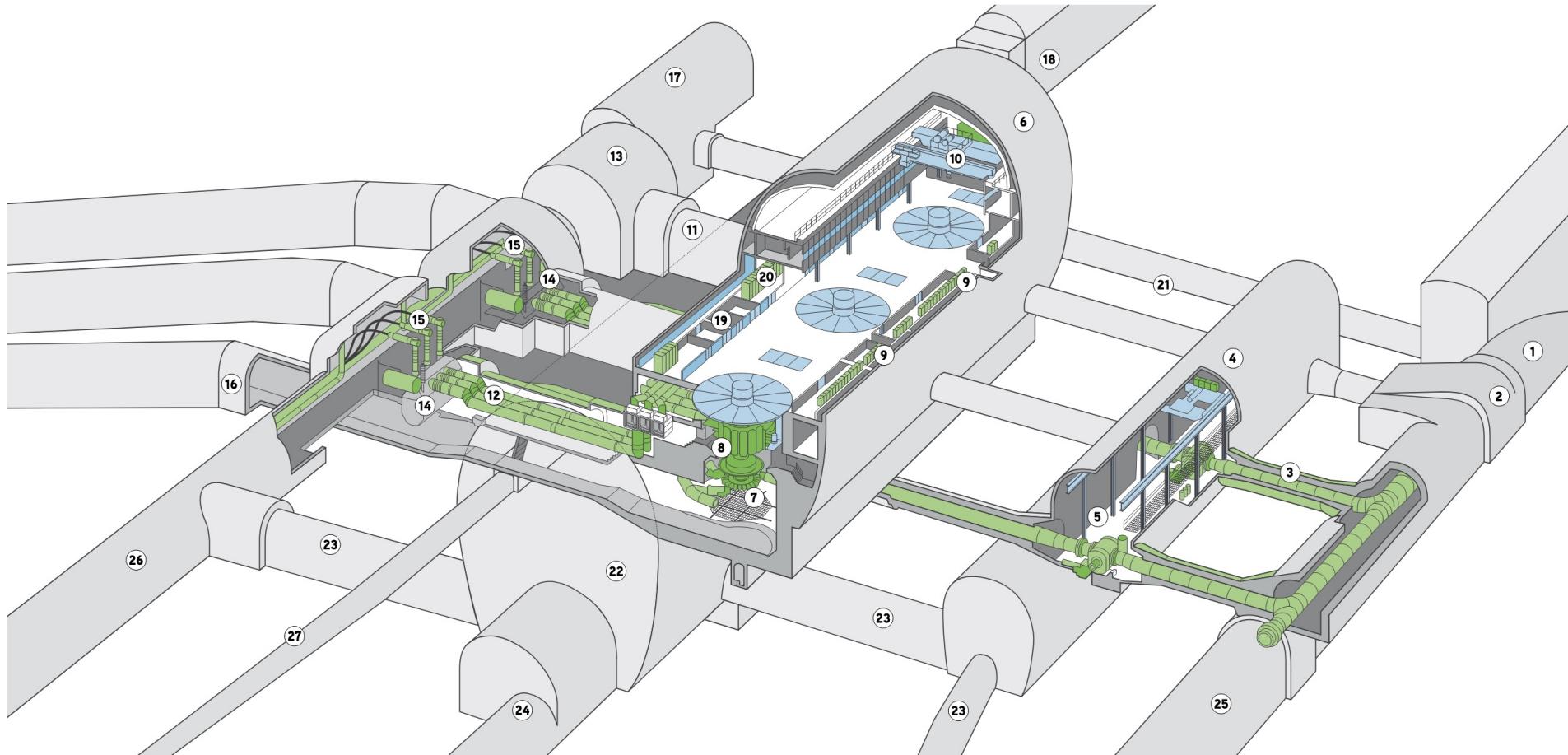
Pelton turbines in Switzerland

Switzerland**Mobility** *









1	PENSTOCK	15	230/410 kV ONE-PHASE CABLE OUTLET
2	DISTRIBUTOR	16	TAILWATER BRANCH
3	UNIT BRANCH (25 m ³ /s, PRESSURE 190 BAR)	17	MAINTENANCE AND STORAGE BUILDING
4	VALVES CHAMBER	18	COOLING WATER RESERVOIR
5	SPHERICAL VALVE (210 TONS PER UNIT)	19	16/0.4 kV AUXILIARY TRANSFORMERS
6	MAIN CAVERN	20	230/400 V SWITCHBOARDS
7	423 MW PELTON TURBINE	21	EMERGENCY TUNNEL
8	465 MVA GENERATOR	22	ASSEMBLY SITE
9	CONTROL ROOM	23	CONNECTING TUNNELS
10	250 TONS OVERHEAD CRANE	24	ACCESS TUNNEL TO PLANT
11	BUSBAR TUNNELS	25	ACCESS TUNNEL TO DISTRIBUTOR
12	21 kV 15'000 A BUSBARS	26	TUNNEL FOR 410 kV CABLES AND ACCESS TO TRANSFORMERS
13	TRANSFORMER CELL	27	VENTILATION TUNNEL
14	465 MVA THREE-PHASE TRANSFORMERS		



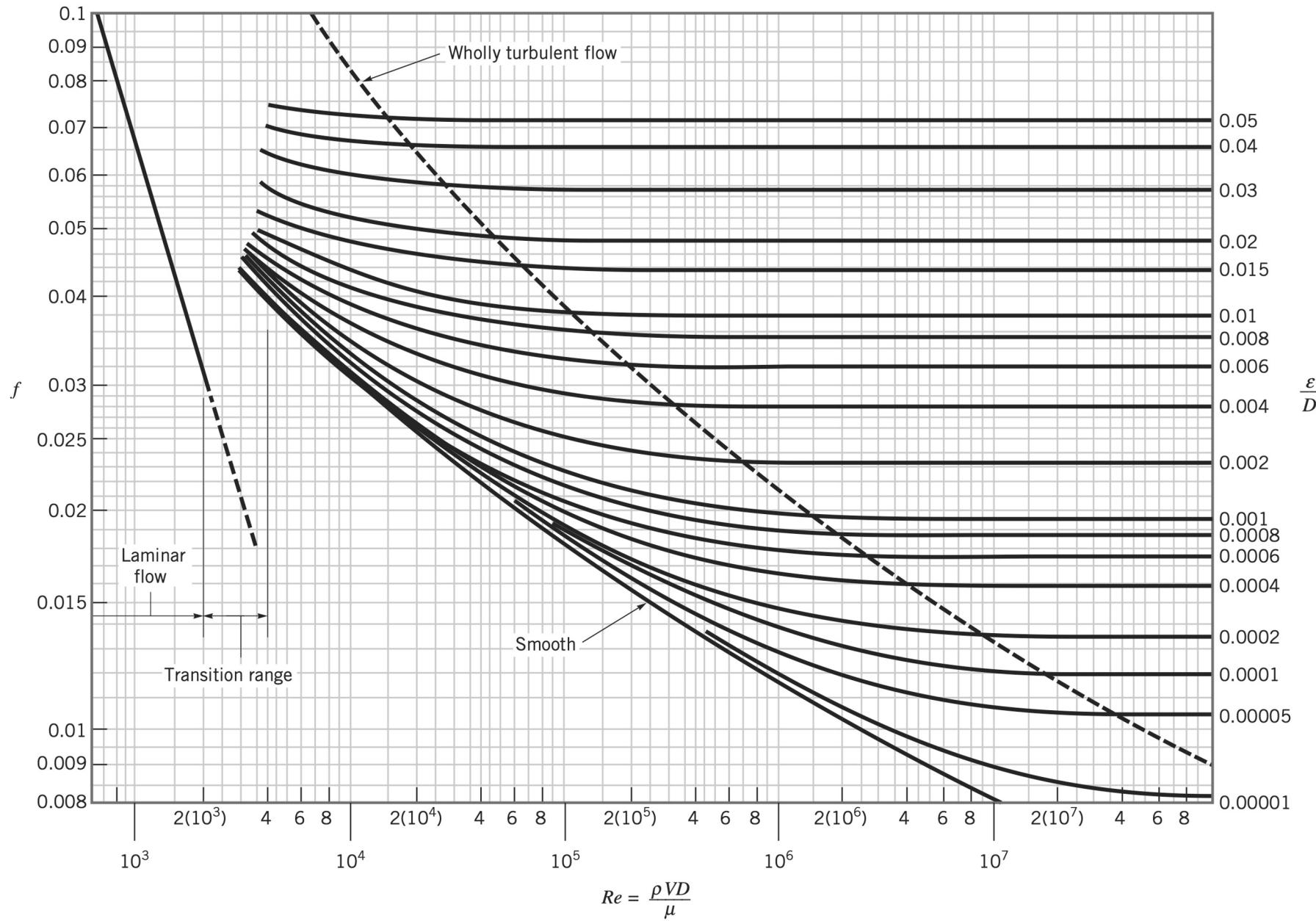




A Pelton wheel operates with a gross head of 530 m and a flow rate of 9 m³/s. The penstock length is 880 m, its diameter is 1.2 m, and its RMS roughness is 0.12 mm. The minor loss can be ignored. The hydraulic efficiency is $\eta_h = 0.84$, and the shaft speed is 650 rpm. Water kinematic viscosity is 1.02×10^{-6} m²/s and density is 998 kg/m³.

Find (a) the effective head and the power delivered by the turbine and (b) the specific speed and from it the recommended number of jets and the number of buckets in the wheel. The nozzle coefficient is $C_N = 0.97$, and the ratio of the blade speed to the discharge velocity is $\lambda = U/V_1 = 0.45$.

Moody diagram – Chapter 3



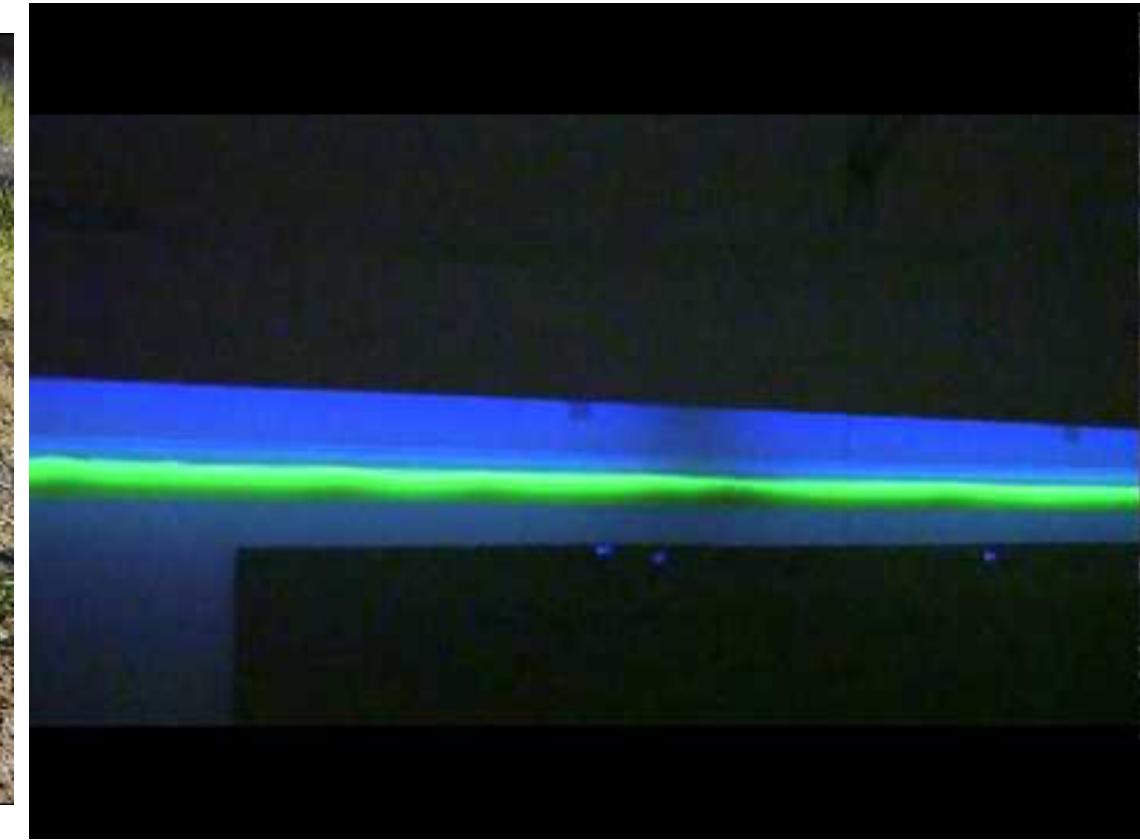
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Type	N'_s	$\eta \%$
Pelton wheel	Single jet	0.02 – 0.18
	Twin jet	0.09 – 0.26
	Three jet	0.10 – 0.30
	Four jet	0.12 – 0.36
Francis	Low-speed	0.39 – 0.65
	Medium-speed	0.65 – 1.2
	High-speed	1.2 – 1.9
	Extreme-speed	1.9 – 2.3
Kaplan turbine		1.55 – 5.17
Bulb turbine		3 – 8

Appendix

Water hammer (Hydraulic shock)



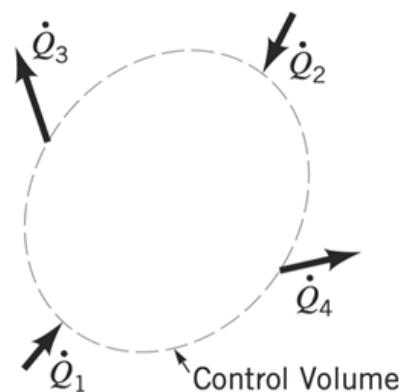
EPFL

3-Energy equation

13

- cs: control **surface**
- cv: control **volume**

Chap. 4 | E. Yim



$$\dot{Q}_{\text{net in}} = \dot{Q}_1 + \dot{Q}_2 - \dot{Q}_3 - \dot{Q}_4$$

■ INTRODUCTION TO TURBOMACHINERY

The first law of thermodynamics

$$\frac{D}{Dt} \int_{\text{sys}} e \rho dV = \frac{\partial}{\partial t} \int_{\text{cv}} e \rho dV + \int_{\text{cs}} e \rho \mathbf{V} \cdot \hat{\mathbf{n}} dA$$

Time rate of increase of the total stored energy of the system = time rate of increase of the total stored energy of the contents of the control volume + net rate of flow of the total stored energy out of the control volume through the control surface

where $e = \check{u} + \frac{V^2}{2} + gz$ total stored energy per unit mass
 internal energy

$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft net in}} = \frac{\partial}{\partial t} \int_{\text{cv}} e \rho dV + \int_{\text{cs}} \left(\check{u} + \frac{p}{\rho} + \frac{V^2}{2} + gz \right) \rho \mathbf{V} \cdot \hat{\mathbf{n}} dA$$

Heat transfer ratio

Work transfer rate, power

- **Shaft torque**

$$T_{\text{shaft}} = -\dot{m}_1 (r_1 V_{\theta 1}) + \dot{m}_2 (r_2 V_{\theta 2})$$

- **Shaft power**

$$\dot{W}_{\text{shaft}} = T_{\text{shaft}} \omega = -\dot{m} r_1 V_{\theta 1} \omega + \dot{m} r_2 V_{\theta 2} \omega$$

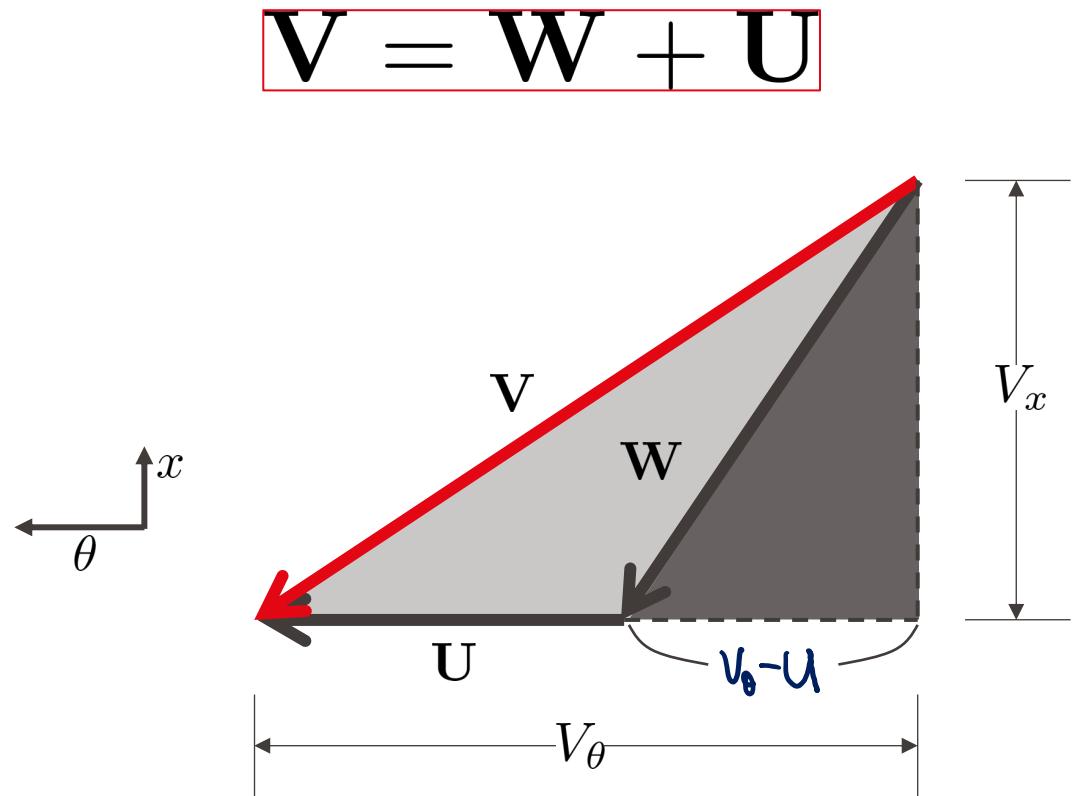
$U_1 \qquad \qquad \qquad U_2$

$$\dot{W}_{\text{shaft}} = (-\dot{m}_1) (U_1 V_{\theta 1}) + \dot{m}_2 (U_2 V_{\theta 2}) \quad [\text{W}] = [\text{kg} \cdot \text{m}^2/\text{s}^3]$$

- **Shaft work per unit mass (shaft power per unit mass flow rate), $\dot{m}_1 = \dot{m}_2$**

$$w_{\text{shaft}} = - (U_1 V_{\theta 1}) + (U_2 V_{\theta 2}) \quad [\text{m}^2/\text{s}^2]$$

- Basic governing equations for pumps or turbines whether the machines are radial-, mixed-, or axial-flow devices and for compressible and incompressible flows
- Note it is only the function of tangential component of velocity, no V_r , V_x



Velocity triangle: **V** absolute velocity,
W relative velocity, **U** blade velocity

- From the big triangle (grey)

$$V^2 = V_\theta^2 + V_x^2 \quad \text{or} \quad V_x^2 = V^2 - V_\theta^2$$

- From the small triangle (dark grey)

$$W^2 = (V_\theta - U)^2 + V_x^2$$

$$= V_\theta^2 - 2V_\theta U + U^2 + V_x^2$$

$$W^2 = V_\theta^2 - 2V_\theta U + U^2 + V^2 - V_\theta^2$$

$$V_\theta U = \frac{-W^2 + U^2 + V^2}{2}$$

$$w_{\text{shaft}} = -(U_1 V_{\theta 1}) + (U_2 V_{\theta 2})$$

$$w_{\text{shaft}} = \frac{V_2^2 - V_1^2 + U_2^2 - U_1^2 - (W_2^2 - W_1^2)}{2}$$

Turbomachine work is related to changes in absolute, relative, and blade velocities.