

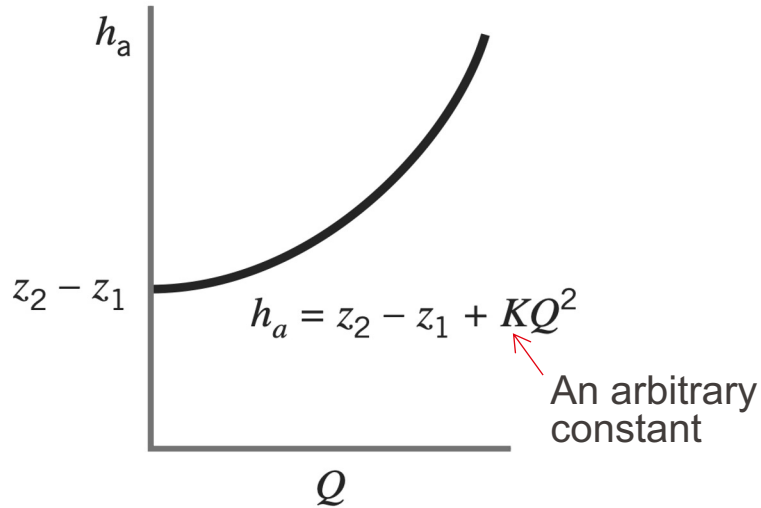
# Chapter 8: Hydraulic Turbines

ME-342 Introduction to  
turbomachinery

Prof. Eunok Yim, HEAD-lab.

- **K vs.  $K_L$**  ↖ Loss coefficient for Minor loss

System equation



## EPFL Exercise

Pump water with  $0.014 \text{ m}^3/\text{s}$  the required NPSH is  $4.5 \text{ m}$  specified pump manufacturer. The water temperature is  $30^\circ\text{C}$   $101.3 \text{ kPa}$ . The loss occurs mainly due to the filter at inlet with  $K_L = 20$ . Friction loss is neglected. Pipe is with diameter of  $10 \text{ cm}$ . Determine the max height  $z$  the pump can be located without the cavitation.

Cavitation occurs when  $\text{NPSH}_R = \text{NPSH}_A$

$$\text{NPSH}_A = \frac{p_{\text{atm}}}{\gamma} - \Delta z - \sum h_L - \frac{p_v}{\gamma}$$

$\uparrow \frac{V^2}{2g}$   $\uparrow K_L \frac{V^2}{2g}$

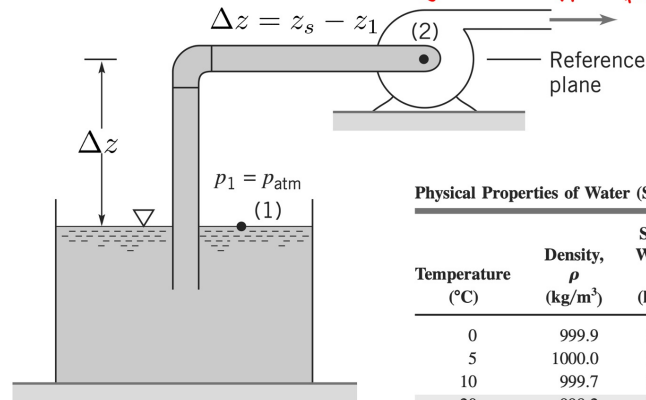
$\text{NPSH}_A = \text{NPSH}_R = 4.5 \text{ m} = \frac{101.3 \times 10^3 \text{ Pa}}{9.765 \times 10^3 \text{ N/m}^3} - \Delta z - 20 \cdot \frac{V^2}{2g} - \frac{4.243 \times 10^3}{9.765 \times 10^3}$

$V = \frac{Q}{A} = \frac{0.014}{\pi \cdot 0.05^2} = 1.78 \text{ m/s}$

$= 10.37 - \Delta z - 3.23 - 0.43$

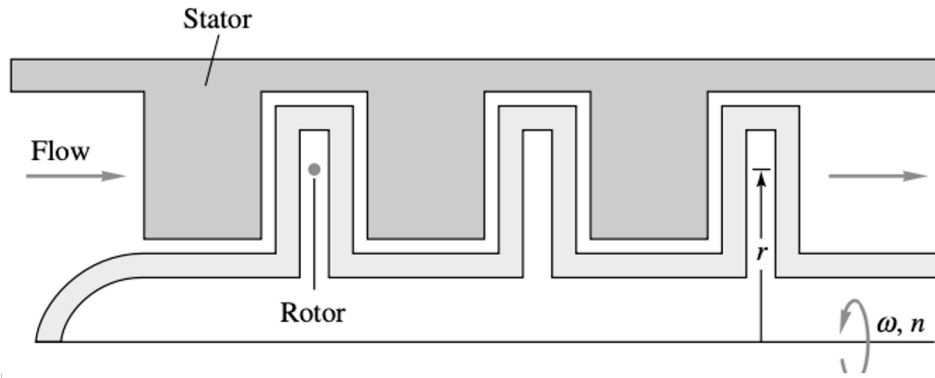
$\Delta z = 10.37 - 4.5 - 3.23 - 0.43$

$= 2.21 \text{ m}$

Physical Properties of Water (SI Units)<sup>a</sup>

Temperature (°C)	Density, $\rho$ (kg/m <sup>3</sup> )	Specific Weight <sup>b</sup> , $\gamma$ (kN/m <sup>3</sup> )	Dynamic Viscosity, $\mu$ (N·s/m <sup>2</sup> )	Kinematic Viscosity, $\nu$ (m <sup>2</sup> /s)	Surface Tension <sup>c</sup> , $\sigma$ (N/m)	Vapor Pressure, $p_v$ [N/m <sup>2</sup> (abs)]
0	999.9	9.806	1.787 E-3	1.787 E-6	7.56 E-2	6.105 E+2
5	1000.0	9.807	1.519 E-3	1.519 E-6	7.49 E-2	8.722 E+2
10	999.7	9.804	1.307 E-3	1.307 E-6	7.42 E-2	1.228 E+3
20	998.2	9.789	1.002 E-3	1.004 E-6	7.28 E-2	2.338 E+3
30	995.7	9.765	7.975 E-4	8.009 E-7	7.12 E-2	4.243 E+3
40	992.2	9.731	6.529 E-4	6.580 E-7	6.96 E-2	7.376 E+3

# Last week - Axial pump

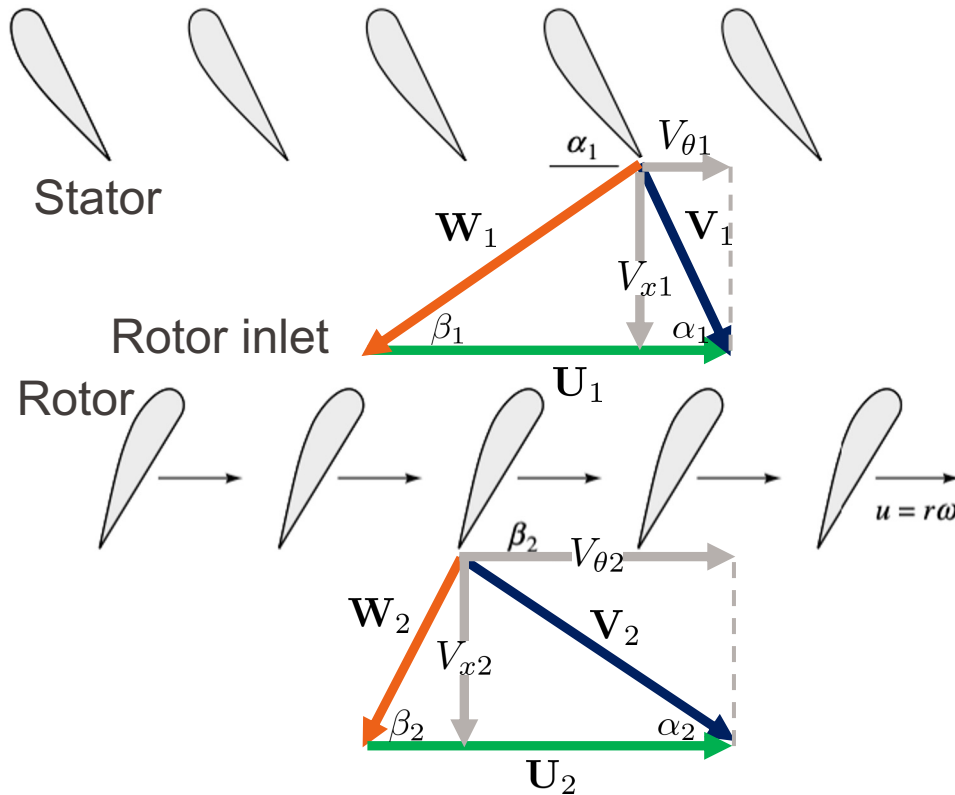


Since the stator is fixed, ideally the absolute velocity  $V_1$  is parallel to the trailing edge of the blade

$$V_{x1} = V_{x2} = V_x = \frac{Q}{A} = \text{const}$$

The ideal head expressed stator angle  $\alpha_1$  and rotor angle  $\beta_2$

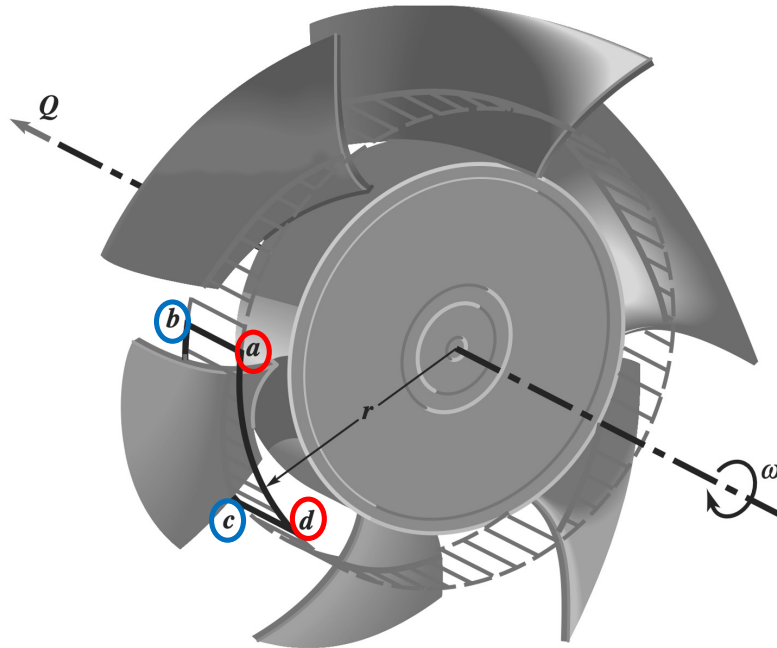
$$h_i = \frac{U_2 V_{\theta 2} - U_1 V_{\theta 1}}{g}$$



Strictly speaking, this applies only to a single streamtube of radius  $r$ , but it is a good approximation for very short blades if  $r$  denotes the average radius.

## Basic Energy Considerations

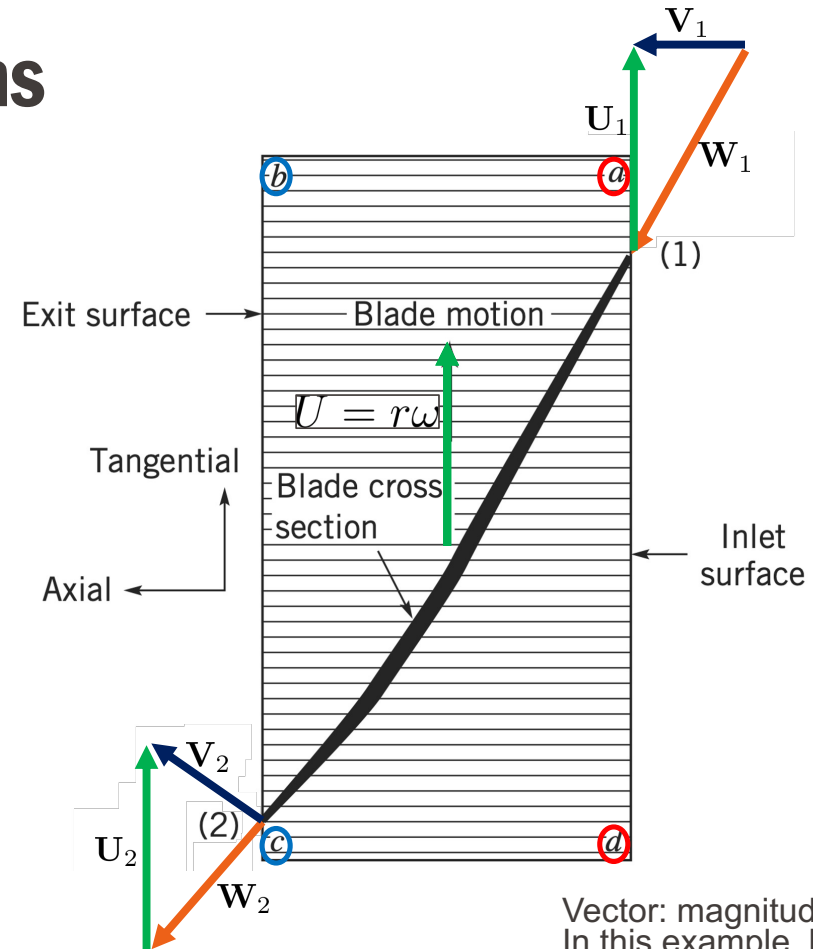
- Velocity diagram (fan)



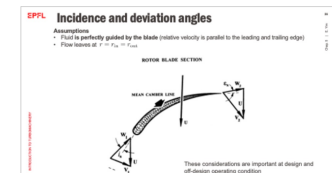
## Assumptions

- Fluid is **perfectly guided by the blade** (relative velocity is parallel to the leading and trailing edge)\*
- Flow leaves at  $r = r_{in} = r_{out}$

\*sometimes leading edge (inlet)  $V$  is prescribed



Vector: magnitude and angle  
In this example, let's assume we know the magnitude of  $W$

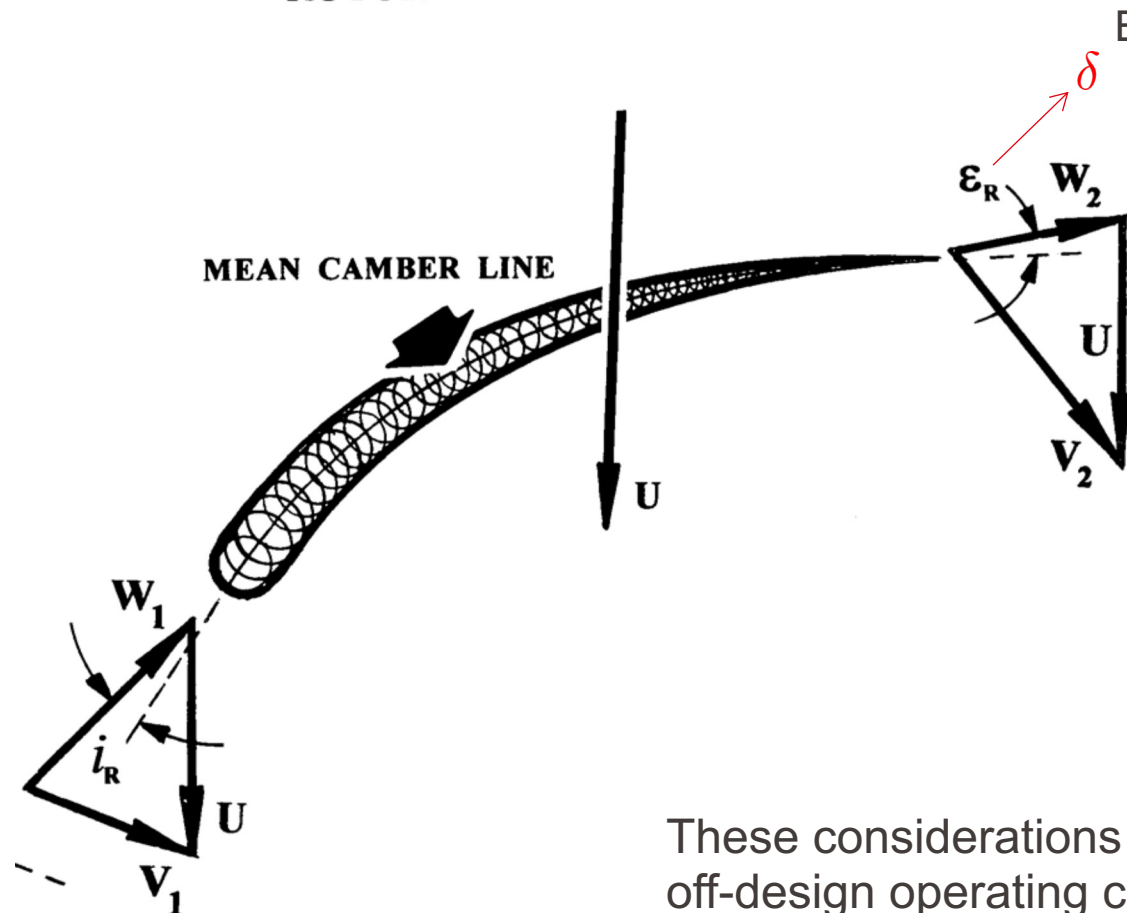


# Incidence and deviation angles

## Assumptions

- Fluid is perfectly guided by the blade (relative velocity is parallel to the leading and trailing edge)
- Flow leaves at  $r = r_{\text{in}} = r_{\text{out}}$

## ROTOR BLADE SECTION



Both  $\epsilon$  and  $\delta$  are used..

These considerations are important at design and off-design operating condition

The **incidence** is the difference between the inlet flow angle ( $\alpha$ ) and the blade **inlet** angle ( $\alpha'$ ):

$$i = \alpha_1 - \alpha'_1$$

The **deviation** is the difference between the exit flow angle and the blade **exit** angle:

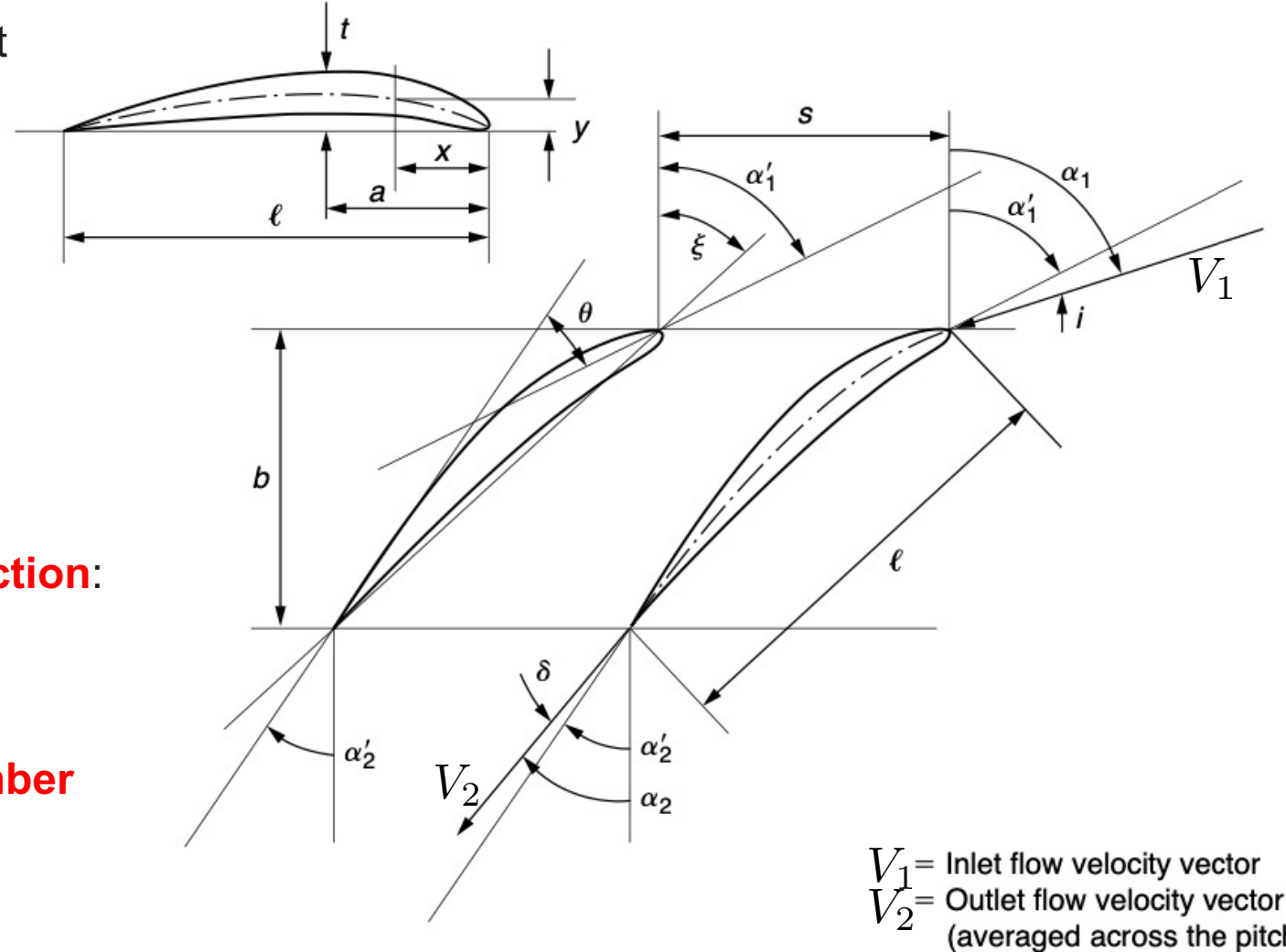
$$\delta = \alpha_2 - \alpha'_2$$

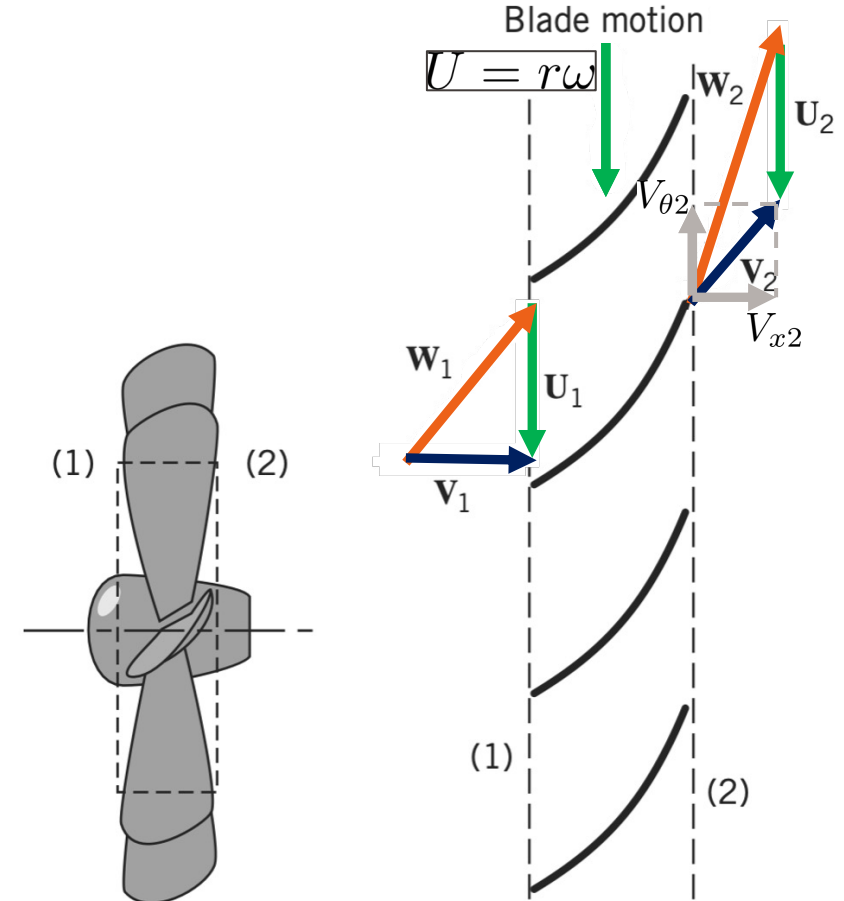
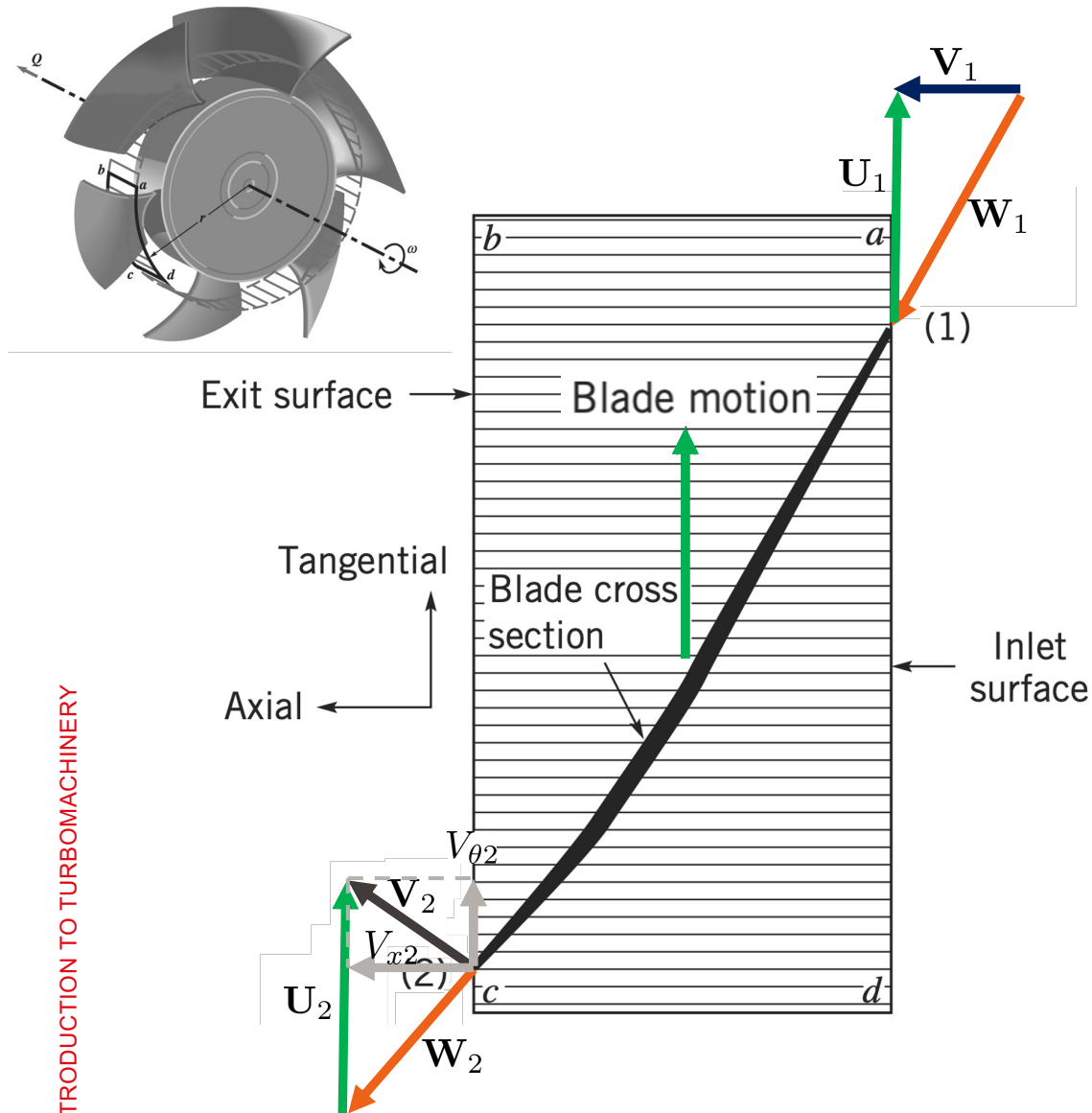
The change in angle of the **flow** is called **deflection**:

$$\varepsilon = \alpha_1 - \alpha_2$$

The change in angle of the **blade** is called **camber angle**:

$$\theta = \alpha'_1 - \alpha'_2$$

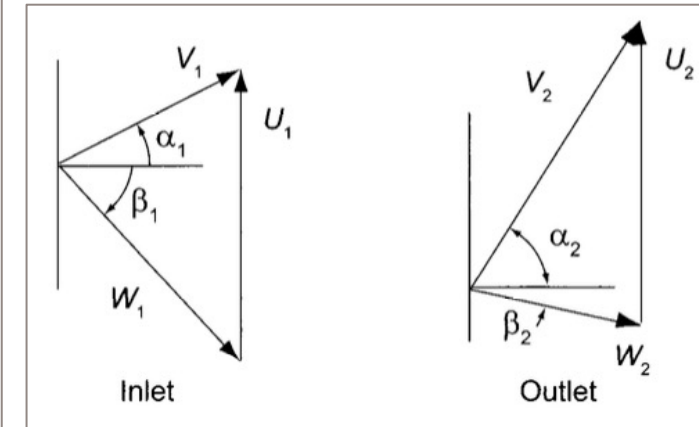
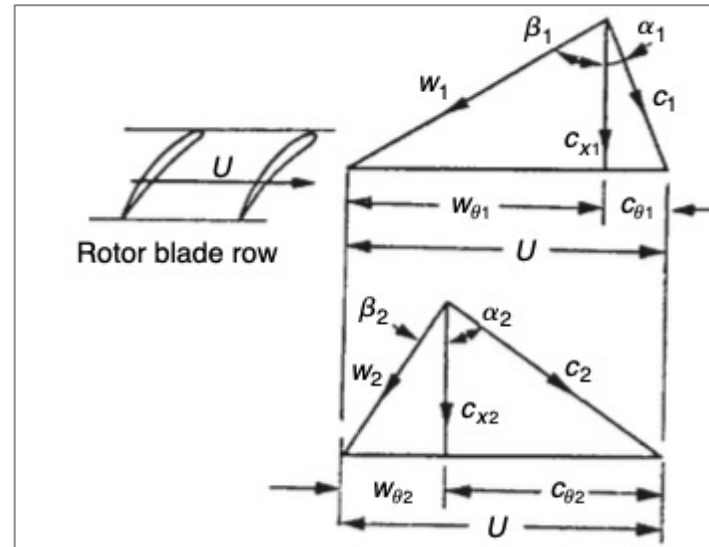
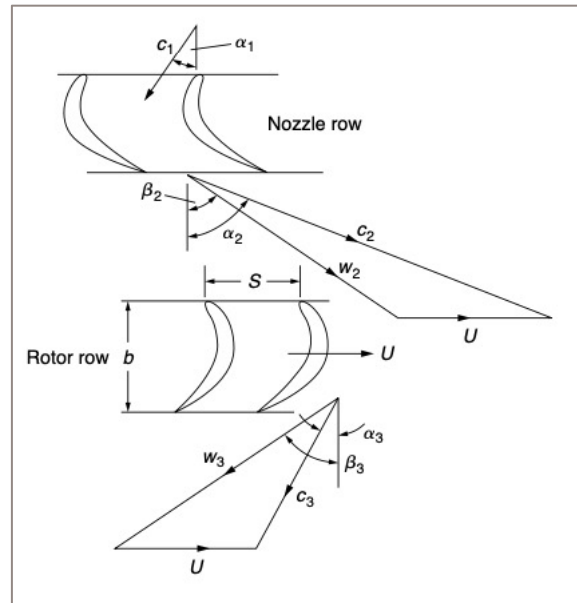
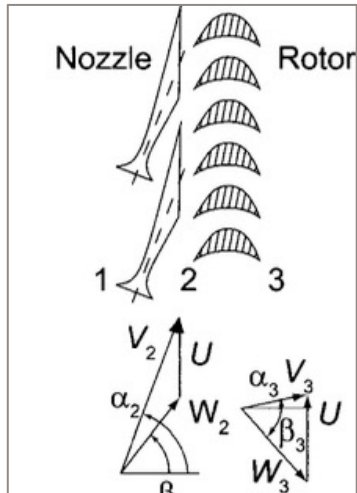
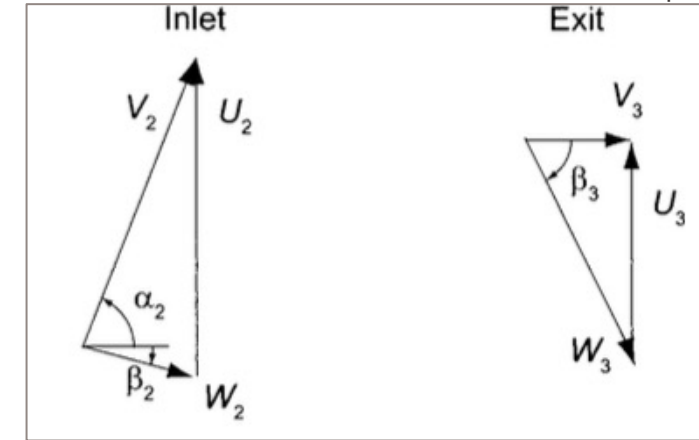
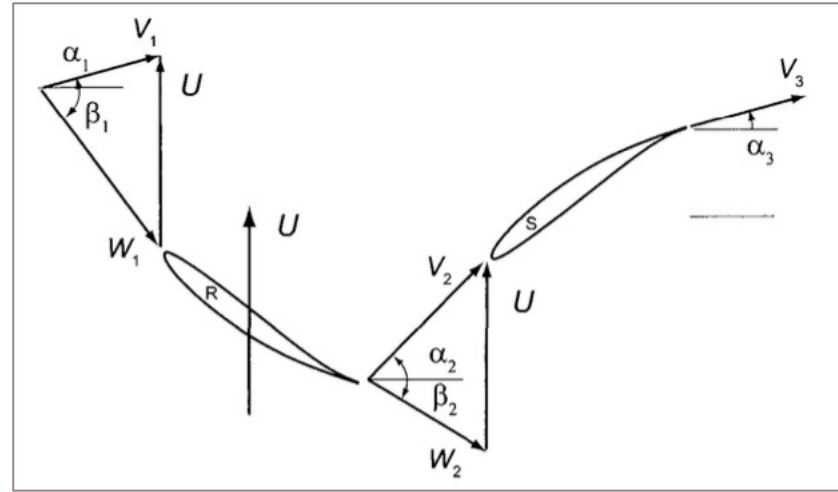
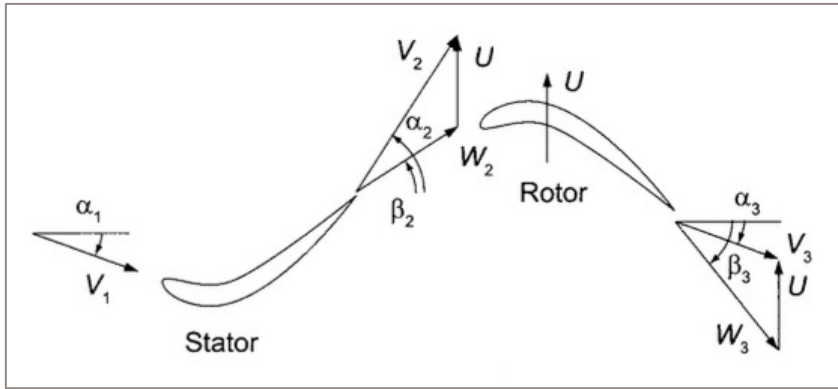




$$w_{\text{shaft}} = \frac{\dot{W}_{\text{shaft}}}{\rho Q} = U_2 V_{\theta 2} - U_1 V_{\theta 1}$$

- The direction argument is valid only when  $V_{\theta 1} = 0$
- For nonzero  $V_{\theta 1} \neq 0$ , compare the magnitudes of  $U_1 V_{\theta 1}$  and  $U_2 V_{\theta 2}$ 
  - For  $U_1 = U_2$ , compare only  $V_{\theta}$

# Pump or turbine? – when inlet $V_{\theta 1}$ is non-zero



Commonly  
used notations

Relative	<b>W</b>	<b>w</b>
Rotation	<b>U</b>	<b>u</b>
Absolute	<b>V</b>	<b>c</b>

# Hydraulic Turbines

**Impulse or reaction** : No matter the working fluid, turbines can be broadly classified into two types based on the mechanism of the fluid interaction

**Impulse turbine**: the force on the blades is produced solely by turning the fluid, without appreciable pressure drop in the blade passage, with all of the pressure drop occurring in a fixed nozzle.

**Reaction turbine**: some of the fluid-vane force is from fluid turning and some of the force is a reaction to acceleration of the fluid relative to the vane. In reaction blading, a pressure drop occurs in both a fixed nozzle and the moving vane.

Turbine blading is characterized by the **degree of reaction (R)**, which is the ratio of the drop in static pressure (or enthalpy) across the moving blade to the overall drop in static pressure (or enthalpy) across the fixed nozzle plus the moving blade. Impulse turbines have  $R = 0$  while reaction turbines typically have  $0.1 < R < 0.7$ .

# Degree of reaction or Reaction (R)

Change in **static enthalpy** across the **rotor** divided by the static enthalpy change across the entire **stage**

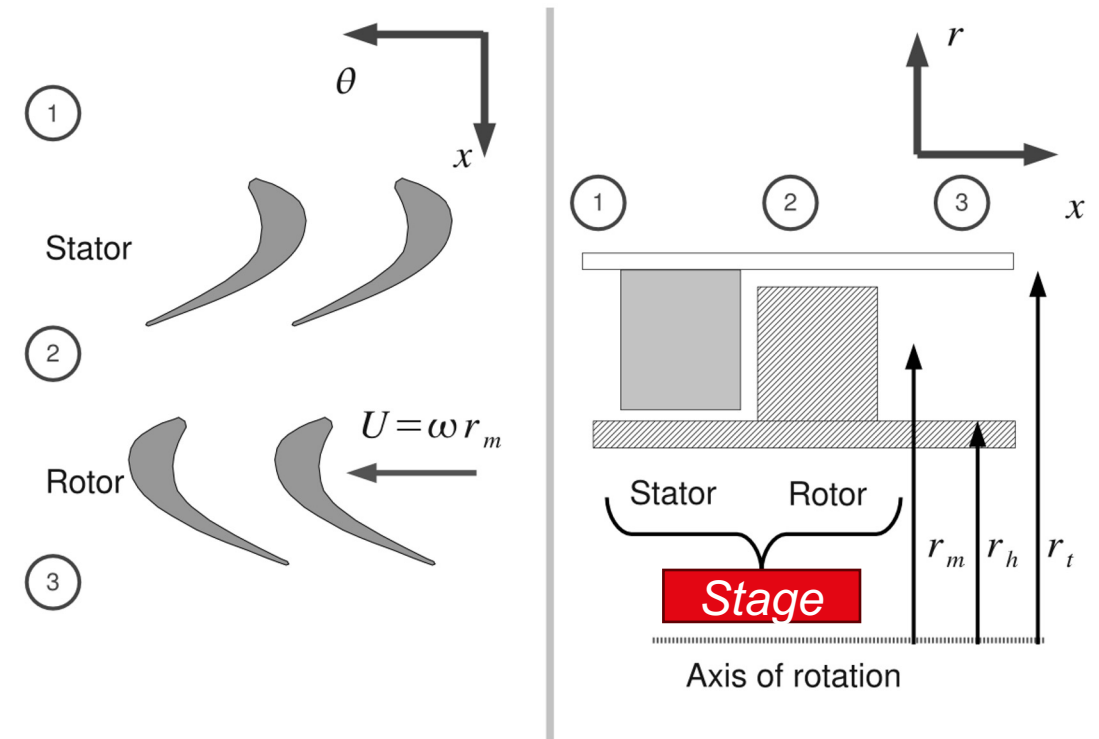
**(static) Enthalpy:** internal energy and flow work

$$\check{h} = \check{u} + \frac{p}{\rho} \quad [\text{J/kg}]$$

$\text{Pa} \cdot \text{m}^3/\text{kg} \equiv \text{J/kg}$

**Stagnation enthalpy:** sum of enthalpy, kinetic energy and potential energy

$$\check{h}_0 = \check{h} + \frac{V^2}{2} + gz$$



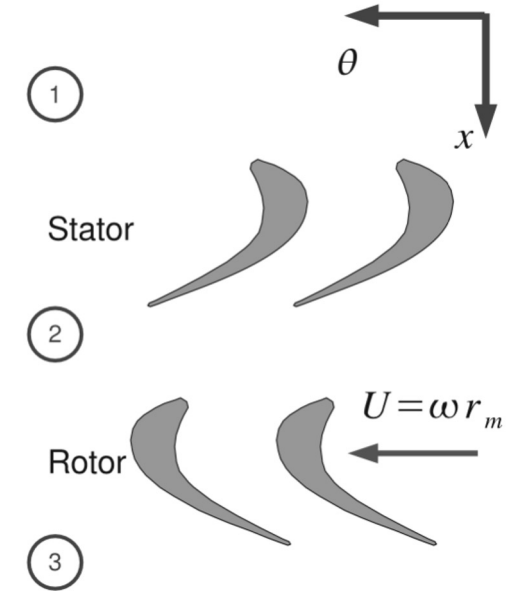
Achtung!  $\check{h}$  is enthalpy, not head  $h$  or  $h_a$

# Degree of reaction or Reaction (R)

This concept is much used in axial flow machines as a measure of the relative proportions of energy transfer obtained by static and dynamic pressure change.

**(static) Enthalpy:** internal energy and flow work

$$\check{h} = \check{u} + \frac{p}{\rho} \quad [\text{J/kg}]$$



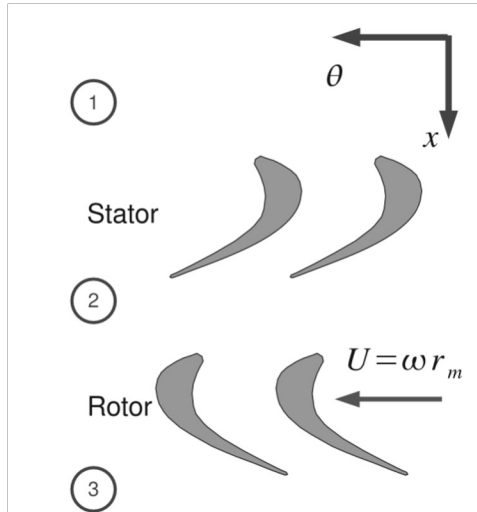
$$R = \frac{\text{energy change due to, or resulting from, static pressure change in the **rotor**}}{\text{total energy change for a **stage**}}$$

$$R = \frac{\text{static enthalpy change in **rotor**}}{\text{stage static enthalpy change}}$$

$$= \frac{\check{h}_2 - \check{h}_3}{\check{h}_1 - \check{h}_3}$$

If no internal energy is changed, incompressible,  $R \simeq \frac{p_2 - p_3}{p_1 - p_3}$

# Degree of reaction or Reaction (R)



$$R \simeq \frac{p_2 - p_3}{p_1 - p_3}$$

$$w_f = -w_{\text{shaft}}$$

$$R = \frac{\frac{1}{2} (V_3^2 - V_2^2) - w_{\text{shaft}}}{\frac{1}{2} (V_3^2 - V_1^2) - w_{\text{shaft}}} = \frac{\frac{1}{2} (V_3^2 - V_2^2) - \frac{1}{2} (V_3^2 - V_2^2 + U_3^2 - U_2^2 - (W_3^2 - W_2^2))}{\frac{1}{2} (V_3^2 - V_1^2) - \frac{1}{2} (V_3^2 - V_2^2 + U_3^2 - U_2^2 - (W_3^2 - W_2^2))}$$

$$R = \frac{U_2^2 - U_3^2 + W_3^2 - W_2^2}{V_2^2 - V_1^2 + U_2^2 - U_3^2 + W_3^2 - W_2^2}$$

- If  $U_2 = U_3$  (axial machine),  $W_3 = W_2 \rightarrow R = 0$
- If  $V_1 = V_2 \rightarrow R = 1$

## EPFL Recall -chapter 4

### EPFL 3-Energy equation

- cs: control surface
- cv: control volume

The first law of thermodynamics

$$\frac{D}{Dt} \int_{\text{sys}} e \rho dV = \frac{\partial}{\partial t} \int_{\text{cv}} e \rho dV + \int_{\text{cs}} e \rho \mathbf{V} \cdot \mathbf{n} dA$$

Time rate of increase of the total stored energy of the system = time rate of increase of the total stored energy of the control volume + net rate of flow of the total stored energy + out of the control volume through the control surface

where  $e = \hat{u} + \frac{V^2}{2} + gz$  total stored energy per unit mass

$$\dot{Q}_{\text{net}} + \dot{W}_{\text{shaft}} = \frac{\partial}{\partial t} \int_{\text{cv}} e \rho dV + \int_{\text{cs}} \left( \hat{u} + \frac{V^2}{2} + gz \right) \rho \mathbf{V} \cdot \mathbf{n} dA$$

Heat transfer into the control volume

## EPFL Recall- chap 5 Basic governing equations for turbomachine

- Shaft torque

$$T_{\text{shaft}} = -\dot{m}_1 (r_1 V_{\theta 1}) + \dot{m}_2 (r_2 V_{\theta 2})$$

- Shaft power

$$\dot{W}_{\text{shaft}} = T_{\text{shaft}} \omega = -\dot{m}_1 r_1 V_{\theta 1} \omega + \dot{m}_2 r_2 V_{\theta 2} \omega$$

$$\dot{W}_{\text{shaft}} = (-\dot{m}_1) (U_1 V_{\theta 1}) + \dot{m}_2 (U_2 V_{\theta 2}) \quad [\text{W}] = [\text{kg} \cdot \text{m}^2/\text{s}^2]$$

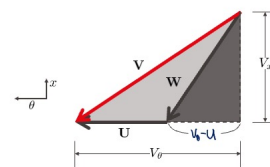
- Shaft work per unit mass (shaft power per unit mass flow rate),  $\dot{w}_{\text{shaft}} = \dot{W}_{\text{shaft}} / \dot{m}$

$$\dot{w}_{\text{shaft}} = -(U_1 V_{\theta 1}) + (U_2 V_{\theta 2}) \quad [\text{m}^2/\text{s}^2]$$

- Basic governing equations for pumps or turbines whether the machines are radial-, mixed-, or axial-flow devices and for compressible and incompressible flows
- Note it is only the function of tangential component of velocity, no  $V_r$ ,  $V_x$

## EPFL Recall- chap 5 Basic governing equations for turbomachine

$$\mathbf{V} = \mathbf{W} + \mathbf{U}$$



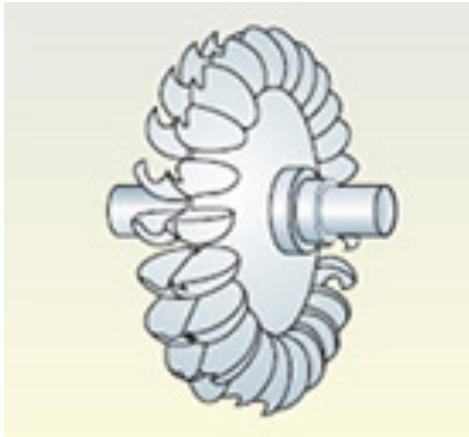
Velocity triangle: V absolute velocity, W relative velocity, U blade velocity

- From the big triangle (grey):  $V^2 = V_\theta^2 + V_r^2 \Rightarrow V_\theta^2 = V^2 - V_r^2$
- From the small triangle (dark grey):  $W^2 = (V_\theta - U)^2 + V_r^2$
- $W^2 = V_\theta^2 - 2V_\theta U + U^2 + V_r^2$
- $W^2 = V_\theta^2 - 2V_\theta U + U^2 + V^2 - V_\theta^2$
- $V_\theta U = \frac{W^2 - U^2 + V^2}{2}$
- $\dot{w}_{\text{shaft}} = -(U_1 V_{\theta 1}) + (U_2 V_{\theta 2})$
- $\dot{w}_{\text{shaft}} = \frac{V_2^2 - V_1^2 + U_2^2 - U_1^2 - (W_2^2 - W_1^2)}{2}$
- Turbomachine work is related to changes in absolute, relative, and blade velocities.

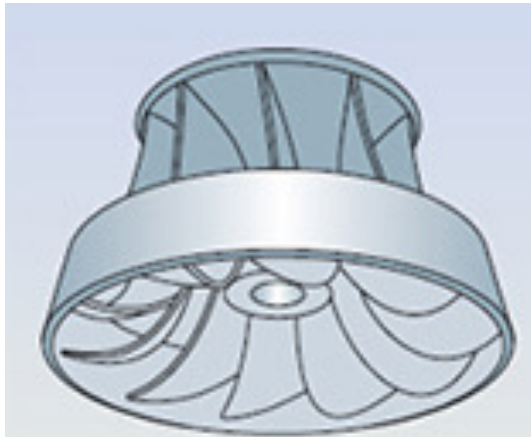
$$\dot{w}_{\text{shaft}} = \frac{V_3^2 - V_2^2 + U_3^2 - U_2^2 - (W_3^2 - W_2^2)}{2}$$

# Types of hydraulic turbines

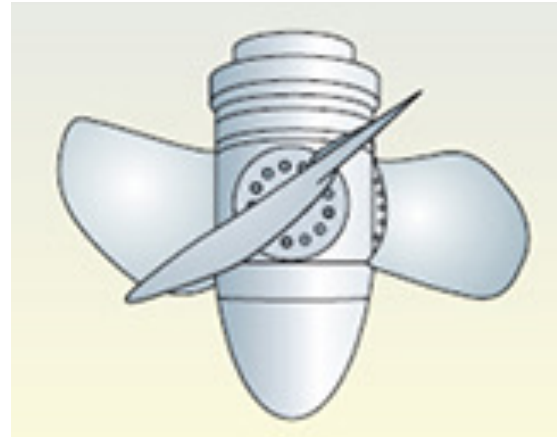
- Impulse (Action) turbines: Pelton turbines,  $R \sim 0$
- Reaction turbines:  $R \sim [0.1, 0.7]$ 
  - Francis turbines (radial et axial), Kaplan turbines (axial)
  - Propeller turbines (similar to Kaplan turbines with fixed pitch)



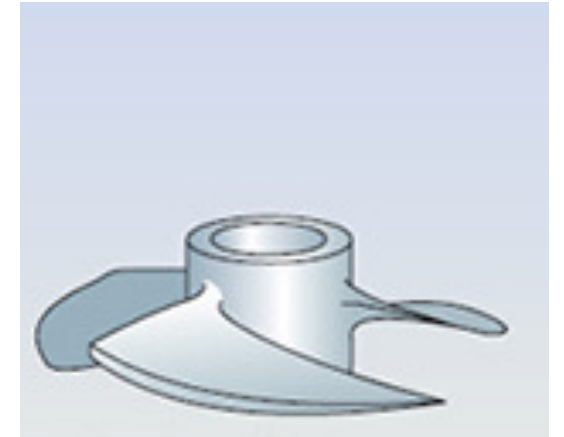
Pelton



Francis

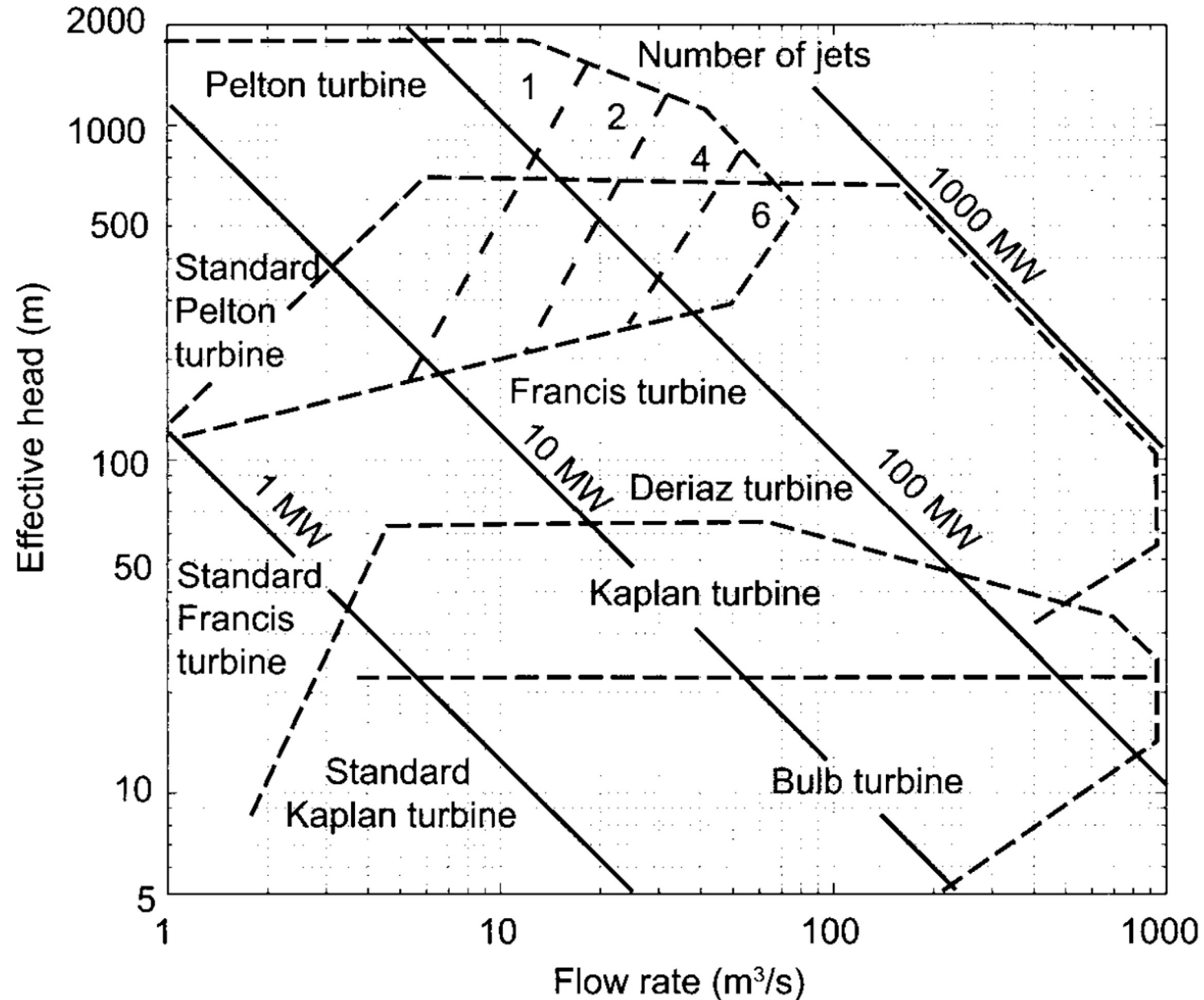


Kaplan & Bulb



Fixed pitch propeller

# Types of hydraulic turbines



For hydraulic turbines, the rotor diameter  $D$  is eliminated between the flow coefficient and the power coefficient to obtain the power-specific speed

**Power** specific speed (Hydraulic turbines)

$$N'_s = \frac{\omega \sqrt{\dot{W}_{\text{shaft}} / \rho}}{(gh_a)^{5/4}}$$

Specific speed

$$N_s = \frac{\omega \sqrt{Q}}{(gh_a)^{3/4}}$$

Commonly used, but not dimensionless, definition of power specific speed  $N'_{sd} = \frac{\omega(\text{rpm}) \sqrt{\dot{W}_{\text{shaft}}(\text{W})}}{[h_a(\text{m})]^{5/4}}$

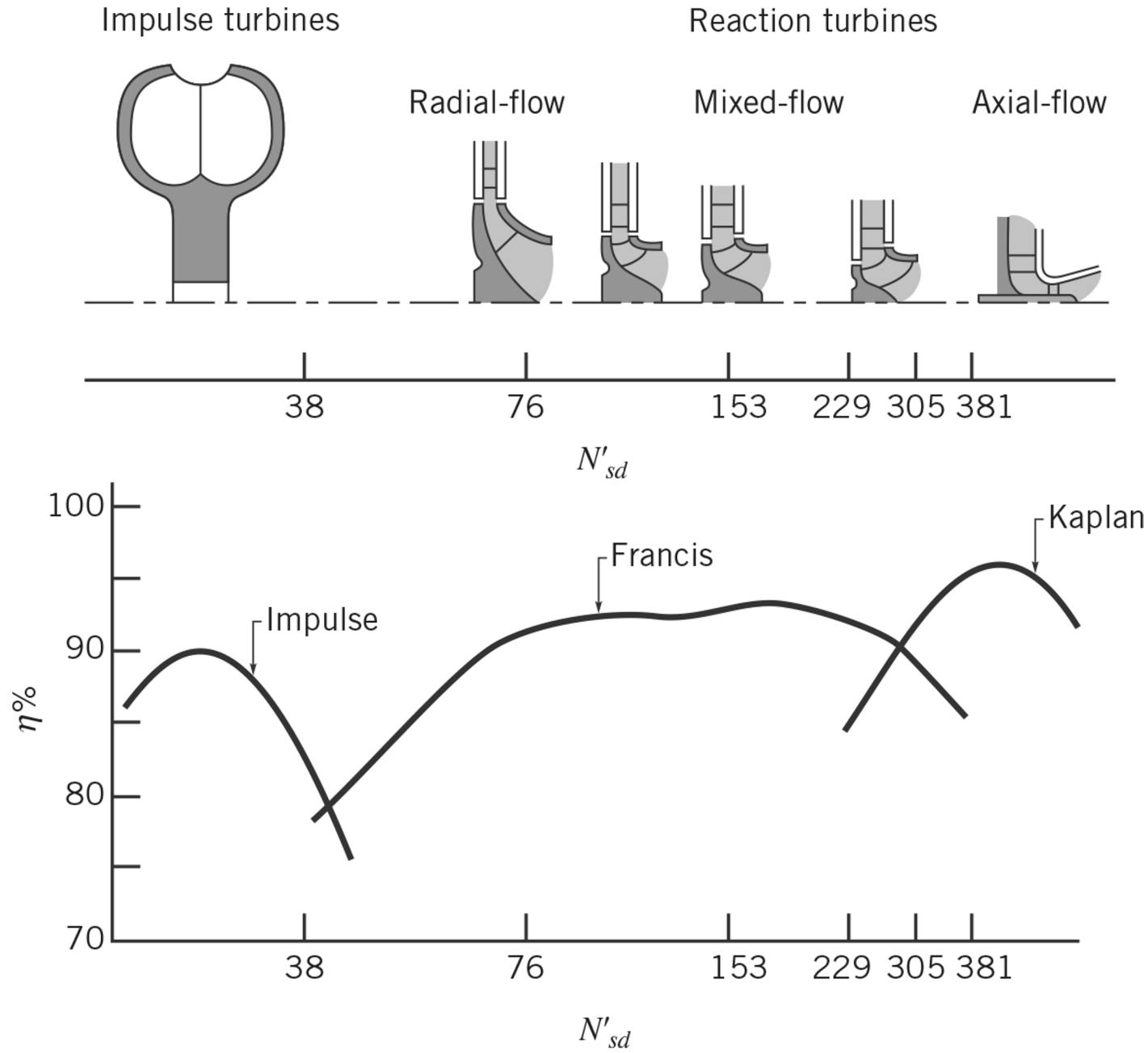
In hydraulic machine, the actual head (in pump)  $h_a$  is commonly called '**effective or net head**'.

The elevation head (physical difference between upper reservoir's surface and the one of the lower one) is called '**gross head**',  $h_g$ .

$$h_a = h_g - h_L$$

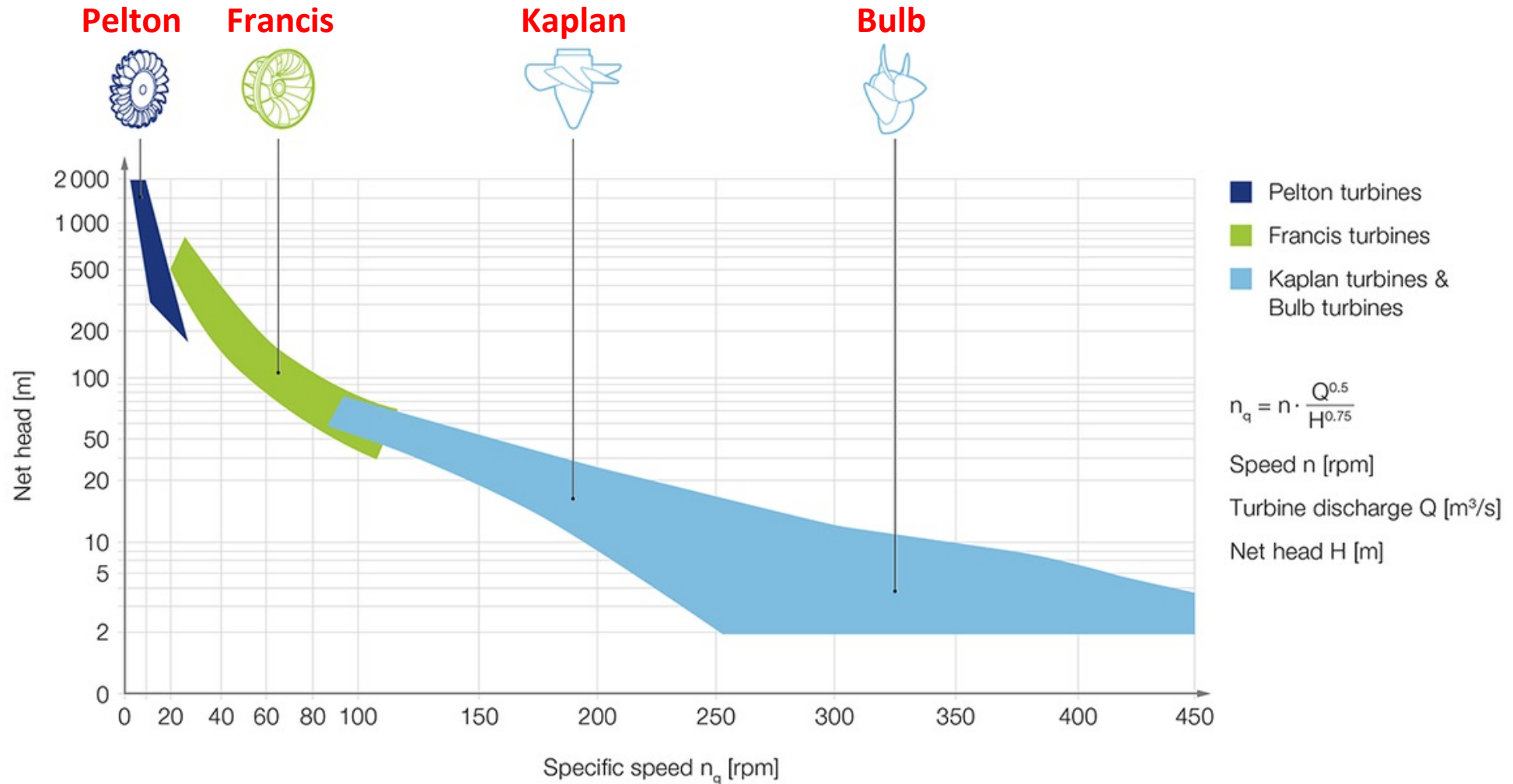
- Power gained by/extracted from the fluids,  $P_f = \gamma Q h_a$
- Efficiency pump  $\eta = \frac{P_f}{\dot{W}_{\text{shaft}}}$   $\longrightarrow \dot{W}_{\text{shaft}} = P_f / \eta = \gamma Q h_a / \eta$
- Efficiency turbine  $\eta = \frac{\dot{W}_{\text{shaft}}}{P_f}$   $\longrightarrow \dot{W}_{\text{shaft}} = \eta P_f = \eta \gamma Q h_a$

# EPFL Turbines



Type	$N'_s$	$\eta\%$
Pelton wheel	Single jet	0.02 – 0.18
	Twin jet	0.09 – 0.26
	Three jet	0.10 – 0.30
	Four jet	0.12 – 0.36
Francis	Low-speed	0.39 – 0.65
	Medium-speed	0.65 – 1.2
	High-speed	1.2 – 1.9
	Extreme-speed	1.9 – 2.3
Kaplan turbine	1.55 – 5.17	87 – 94
Bulb turbine	3 – 8	

- Classification of turbine types as a function of the head and unit specific speed



# Impulse turbines – Pelton turbine

- Impulse-type Turbines

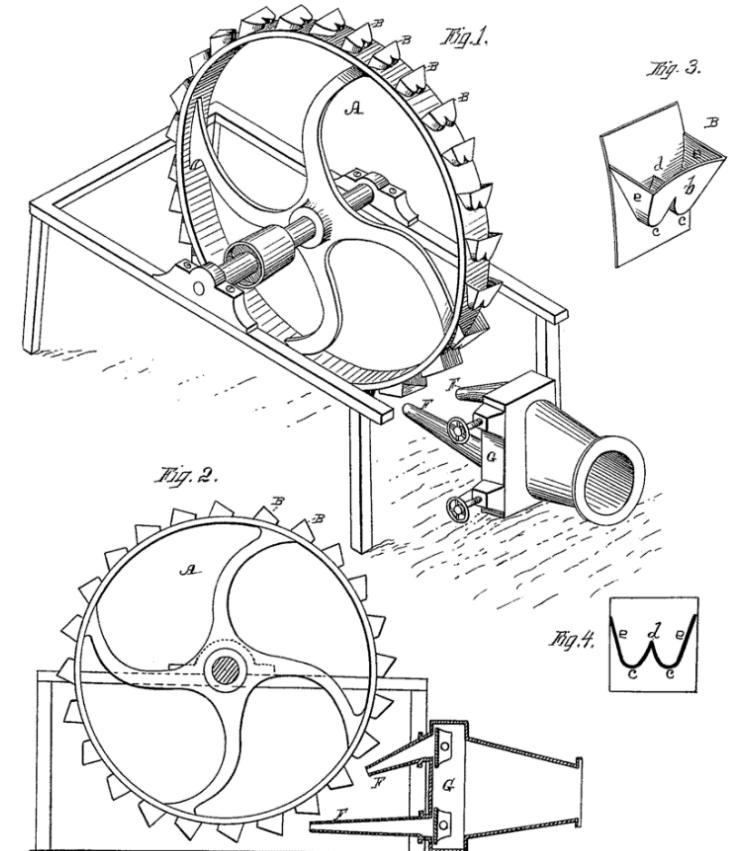
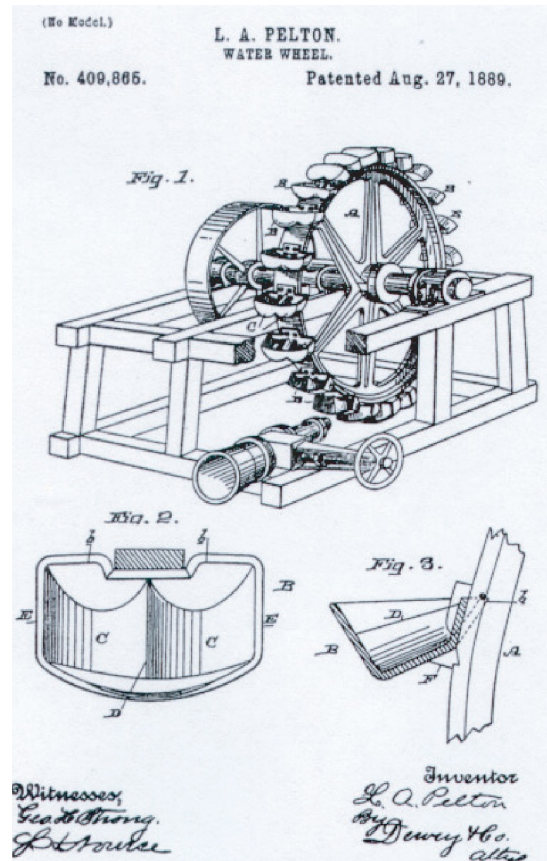
- Among several types of impulse turbines, the so-called Pelton turbine is the most used
- Patented by L. A. Pelton in 1889
- The rotor is made of several buckets and the motion is obtained by high-speed jet(s)



**Lester Allan Pelton**

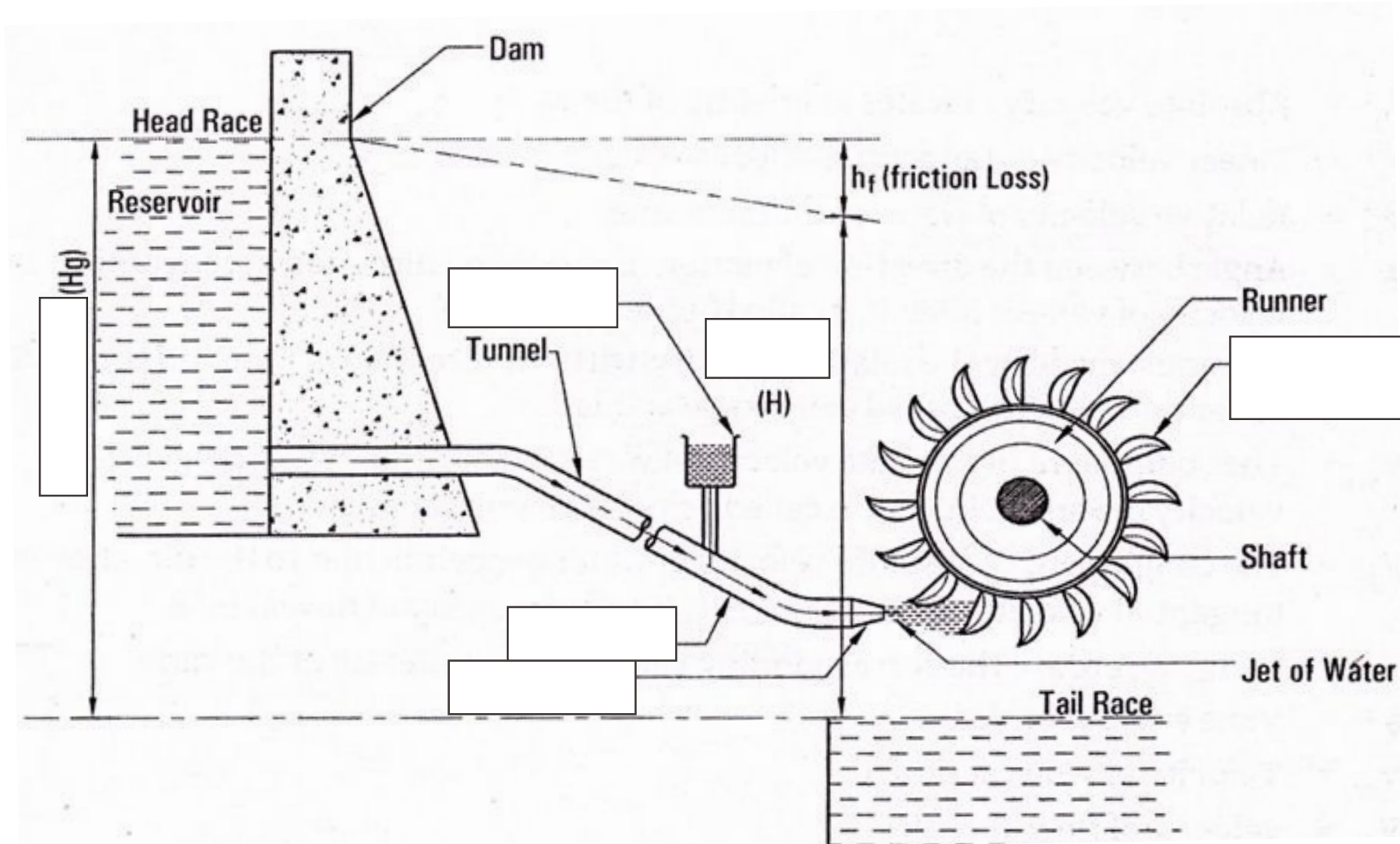
(Sept. 5, 1829 – Mar. 14, 1908)

Pelton made his living as a carpenter and a millwright. He created the most efficient form of impulse water turbine.



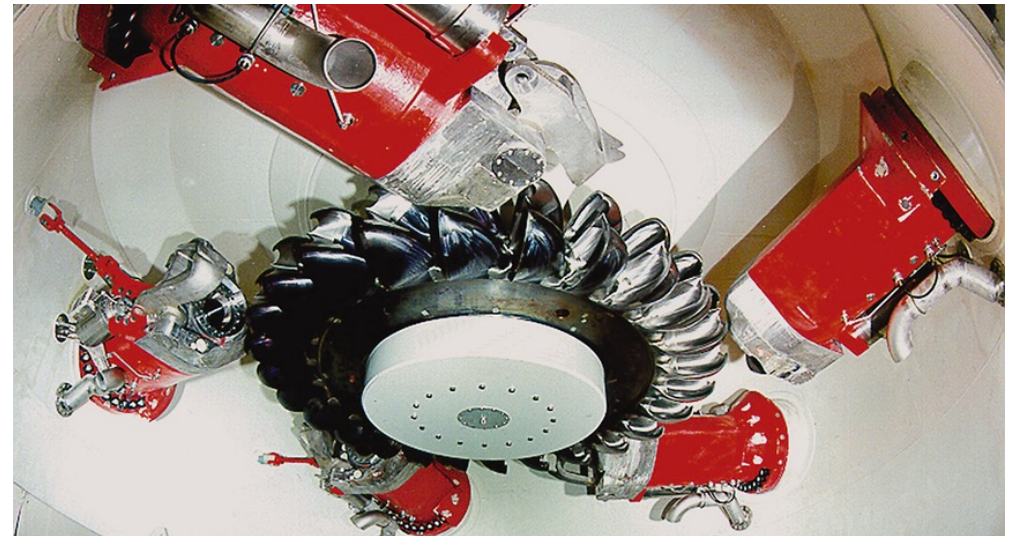
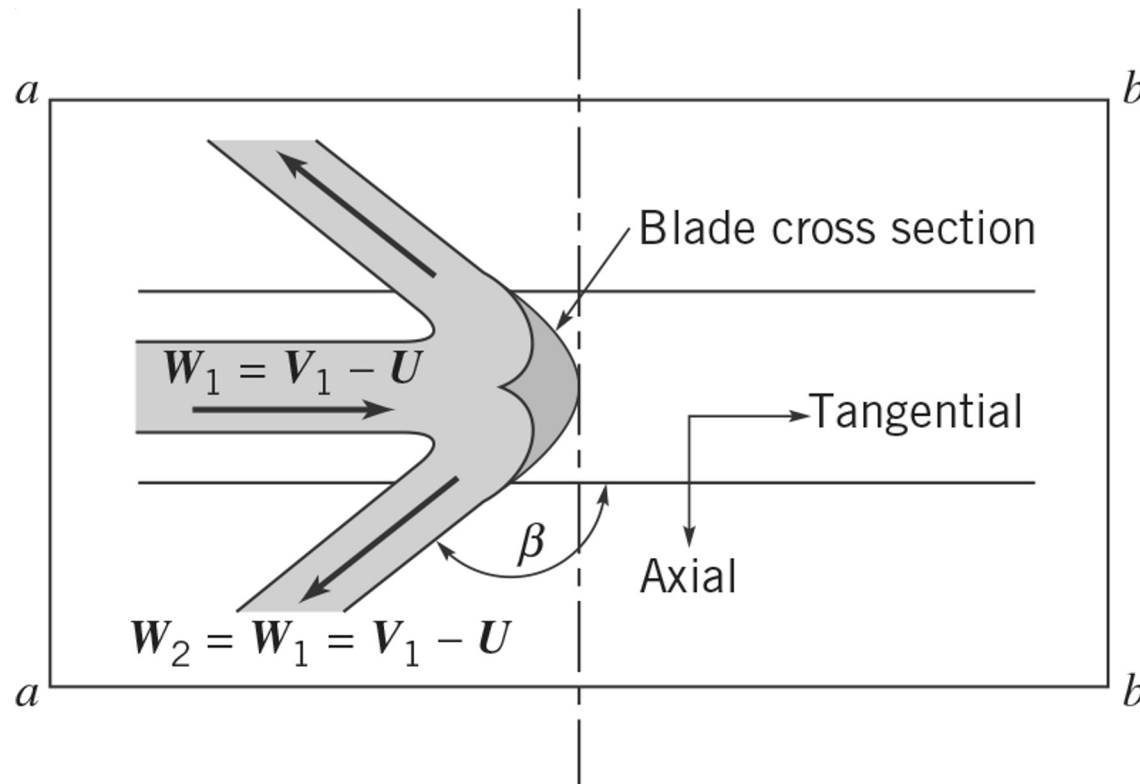
# Pelton turbines

- Pelton wheel is not typical turbomachines (no axial flow, no radial flow), tangential flow
  - Pelton turbines are the most used turbines in **Switzerland** in hydropower generation



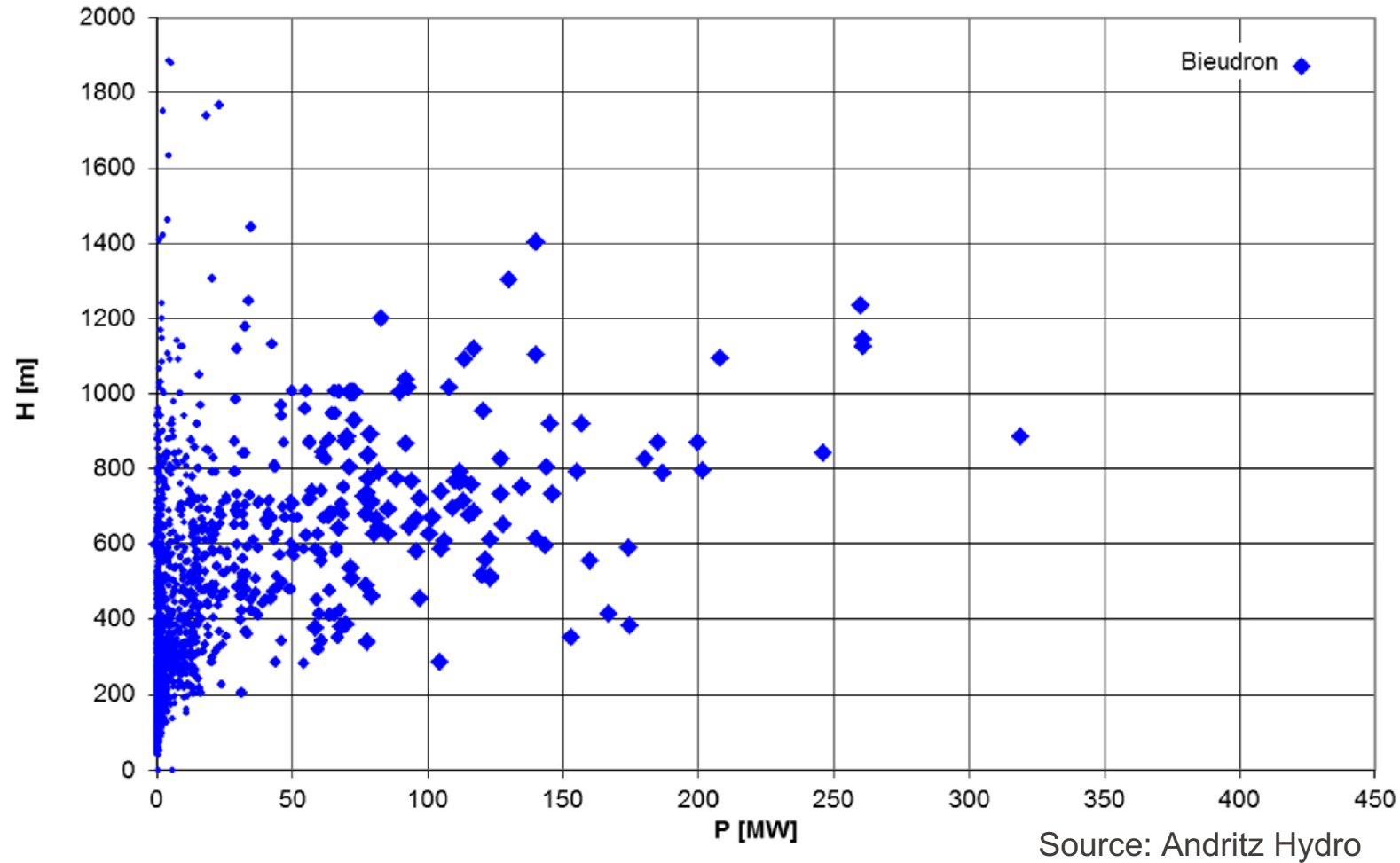
# Pelton turbines

- The water enters and leaves the control volume surrounding the wheel as free jets (at **atmospheric** pressure)
- A person riding on the bucket would note that the speed of the water does not change as it slides across the buckets (assuming viscous effects are negligible)
  - the magnitude of the relative velocity does not change, but its direction does.
- The change in direction of the velocity of the fluid jet causes a torque on the rotor, resulting in a power output from the turbine



# Pelton turbines

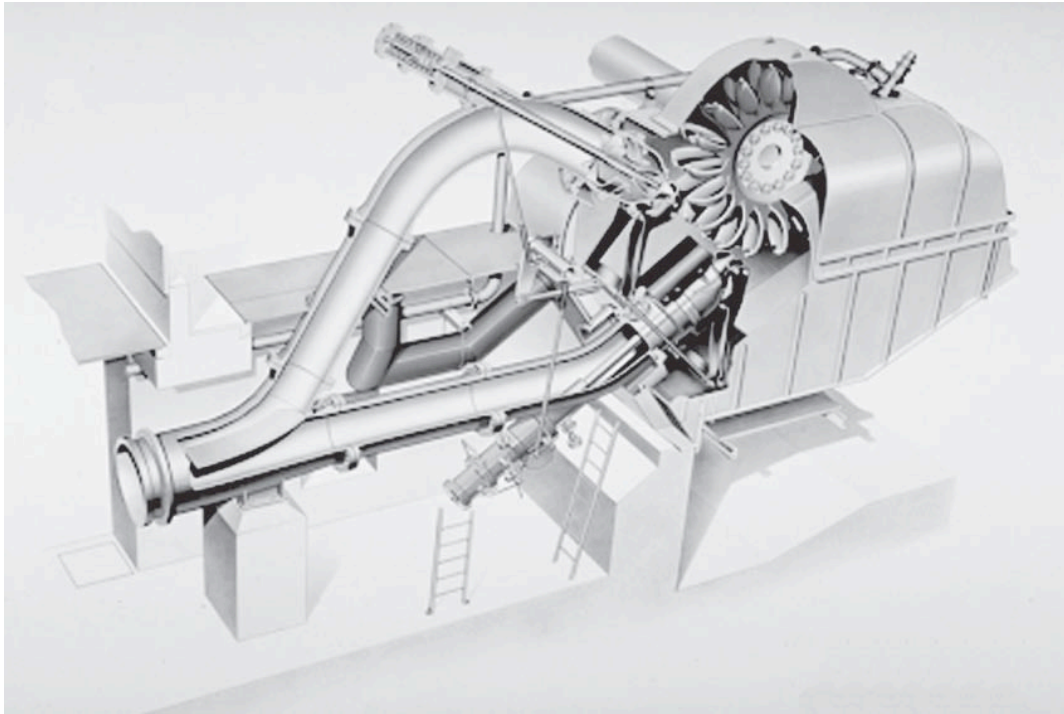
- Suitable for high heads (100's meter range):



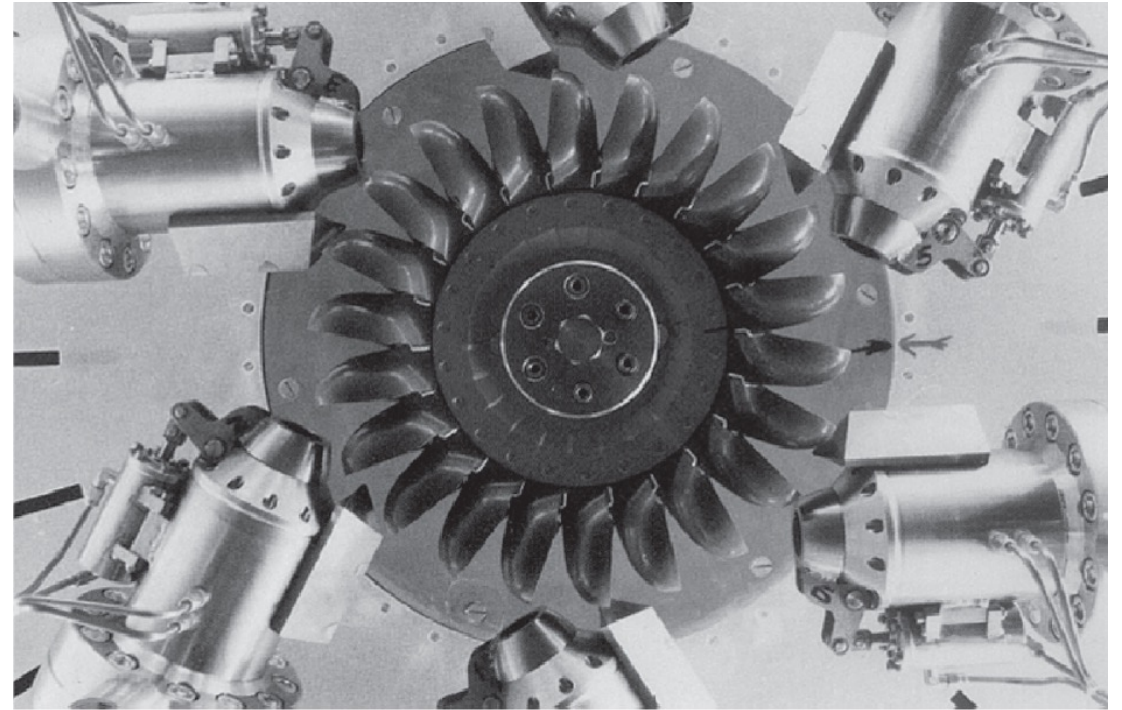
# Pelton turbines

- Types of Pelton Turbines:

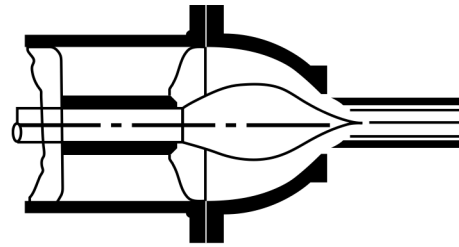
*Horizontal shaft: 1, 2 or 3 Jets*



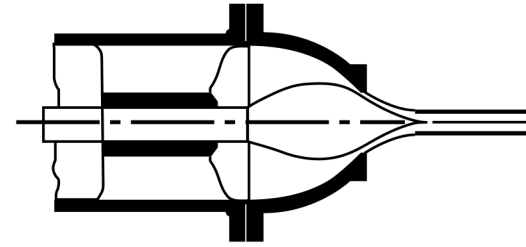
*Vertical shaft: 1 to 6 Jets*



- The nozzle:
  - Equipped with a needle for flowrate control (e.g. valve)
  - Actioned by a servomotor, which may be internal or external
  - Equipped with a deflector for emergency stops

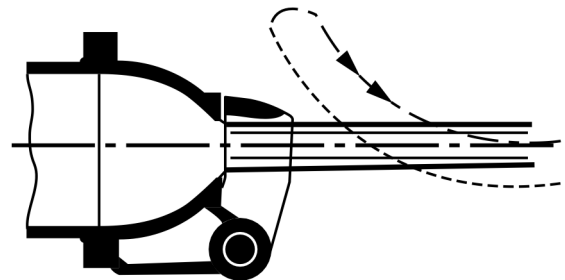


Full load

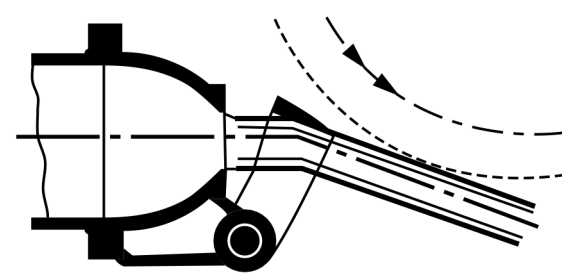


Part load

(a)



Deflector in normal position



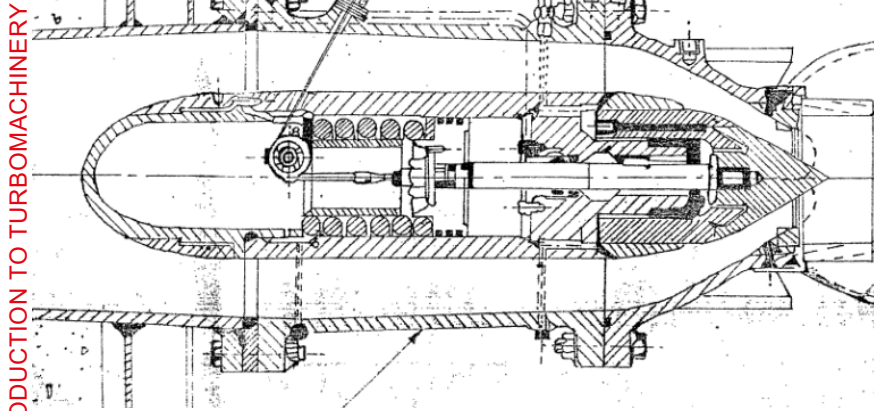
Fully deflected position

(b)

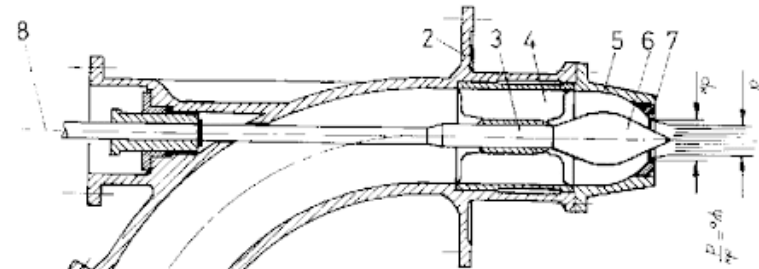
# Pelton turbines

- The nozzle:
  - Key element of the design: must produce a jet of a “high quality”
    - Straight jet
    - Minimum atomization (formation of small droplets)

*What happens when the nozzle closes suddenly?*



*Nozzle with Internal servomotor*



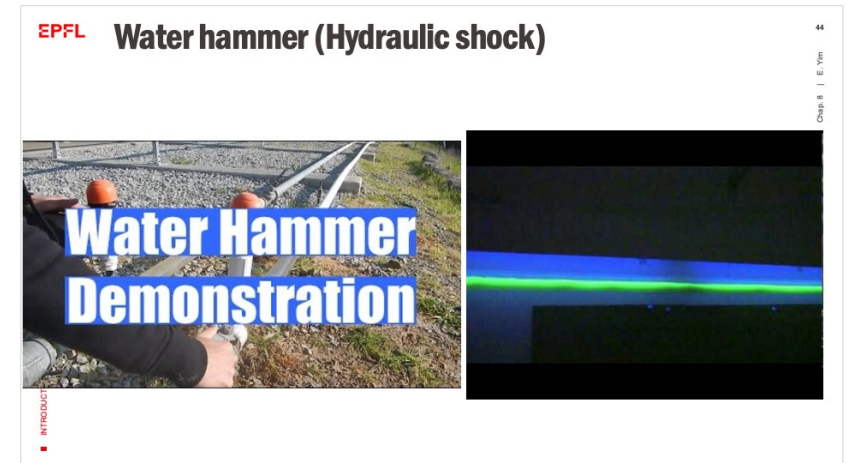
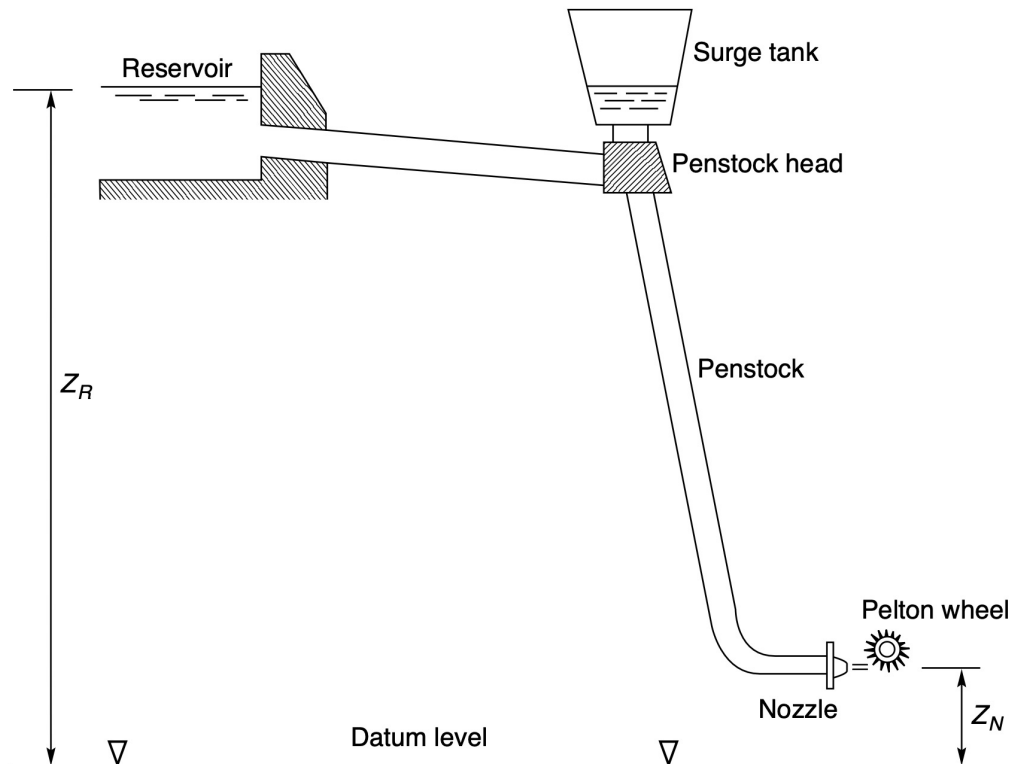
*Nozzle with External servomotor*

1. Needle
2. Needle or gear
3. Star guide
4. Needle up
5. Needle down
6. Needle up
7. Needle down
8. Regulating mechanism



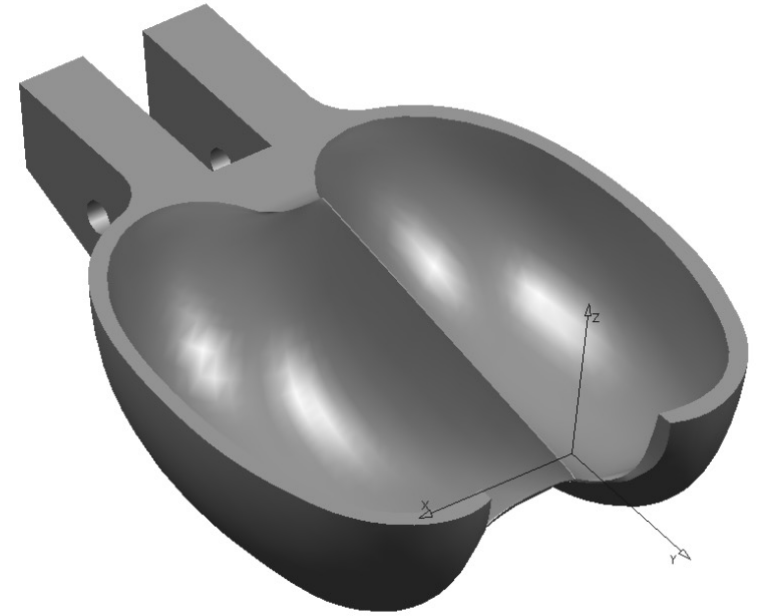
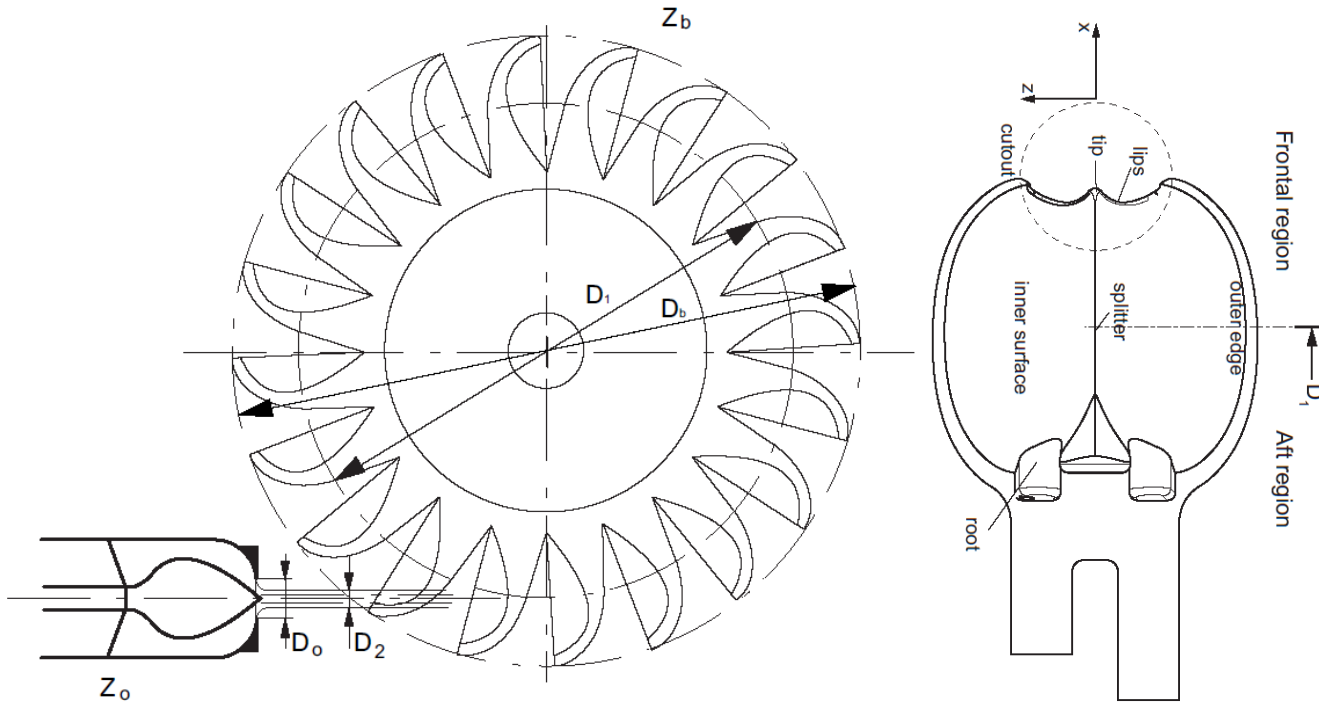
# Pressure surges

- Spear valve must move slowly: sudden reduction in flow rate may result in serious damage from **pressure surges** (called water hammer)
- If the spear valve closes quickly: all the kinetic energy of the water in the penstock would be absorbed by the elasticity of the supply pipeline (penstock) and the water, creating very large stresses, which would reach their greatest intensity at the turbine inlet where the pipeline is already heavily stressed.
- The surge chamber has the function of absorbing and dissipating some of the pressure and energy fluctuations created by too rapid a closure of the needle valve.

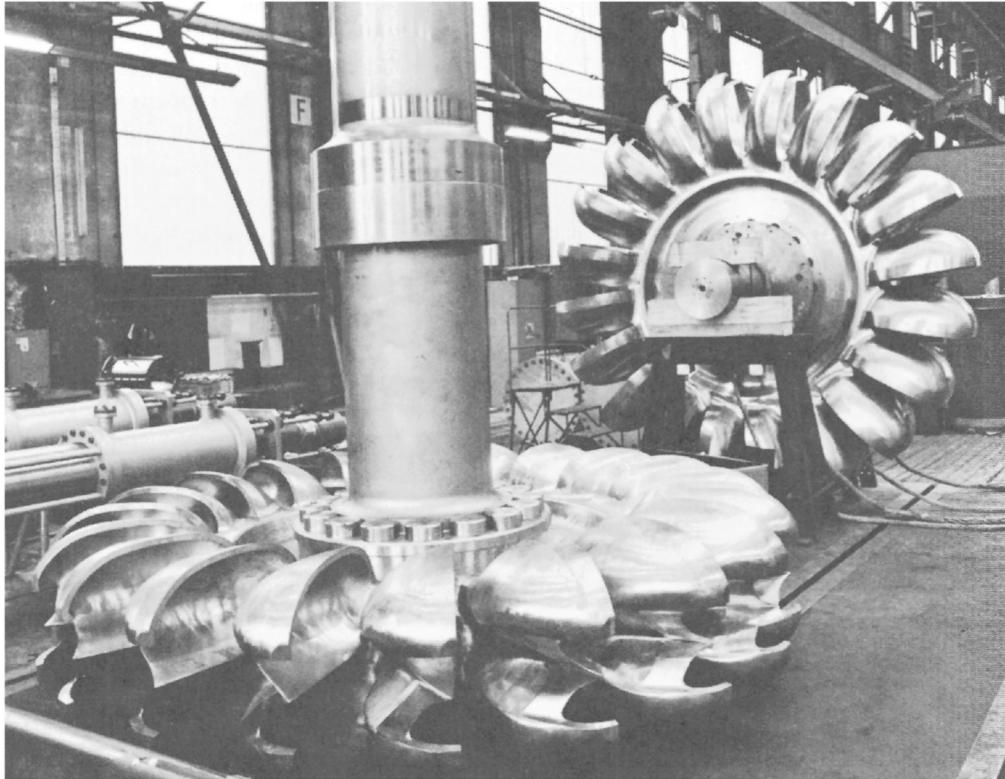
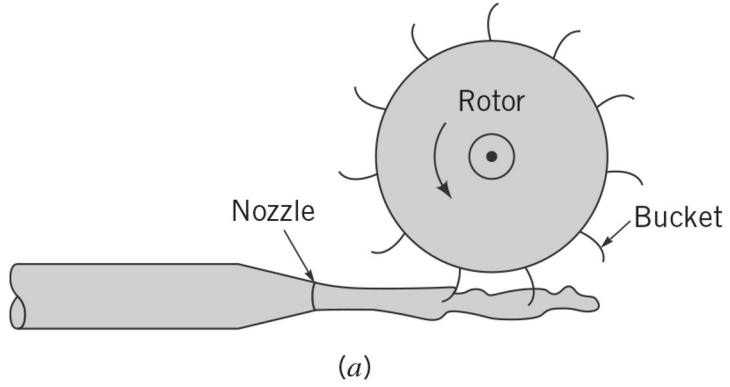


# Pelton turbines

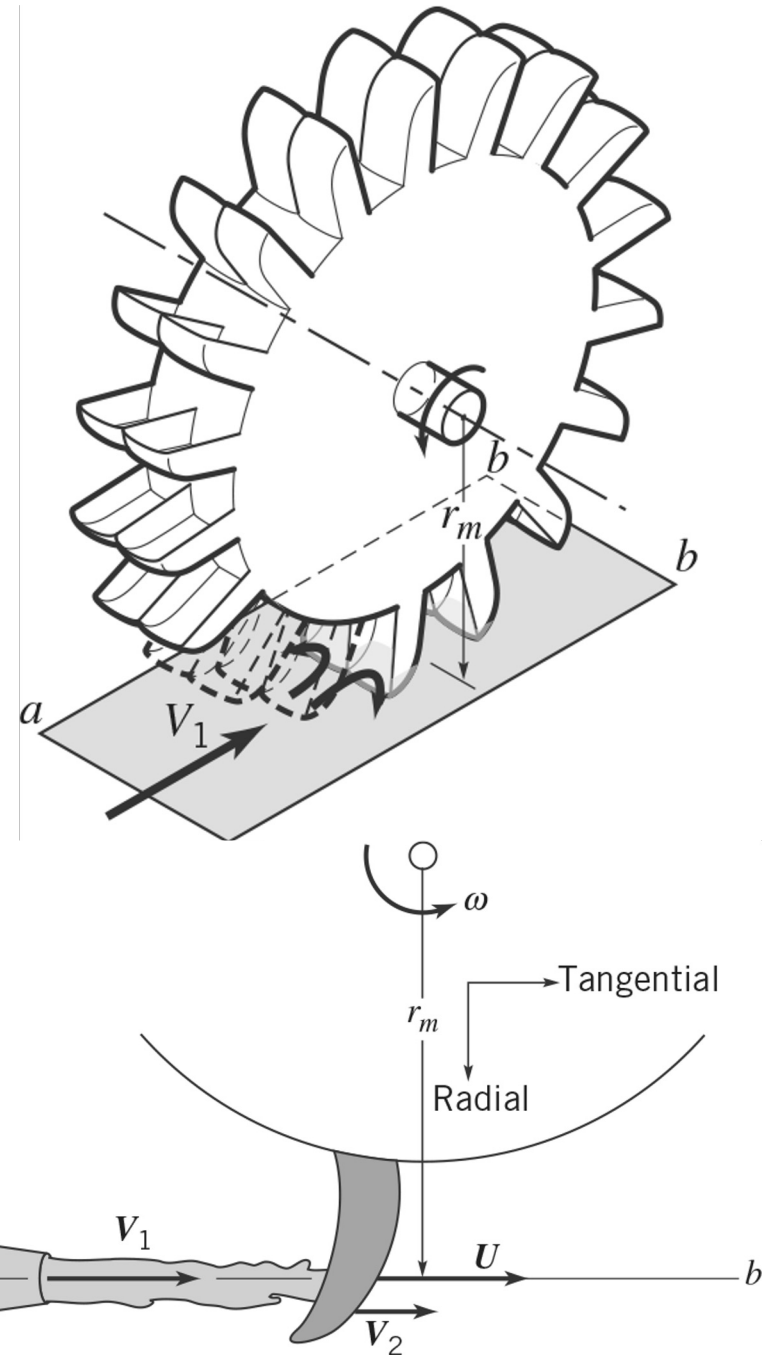
- The buckets:
  - Must deviate “smoothly” the flow and allow for its evacuation with a minimum interference with each other

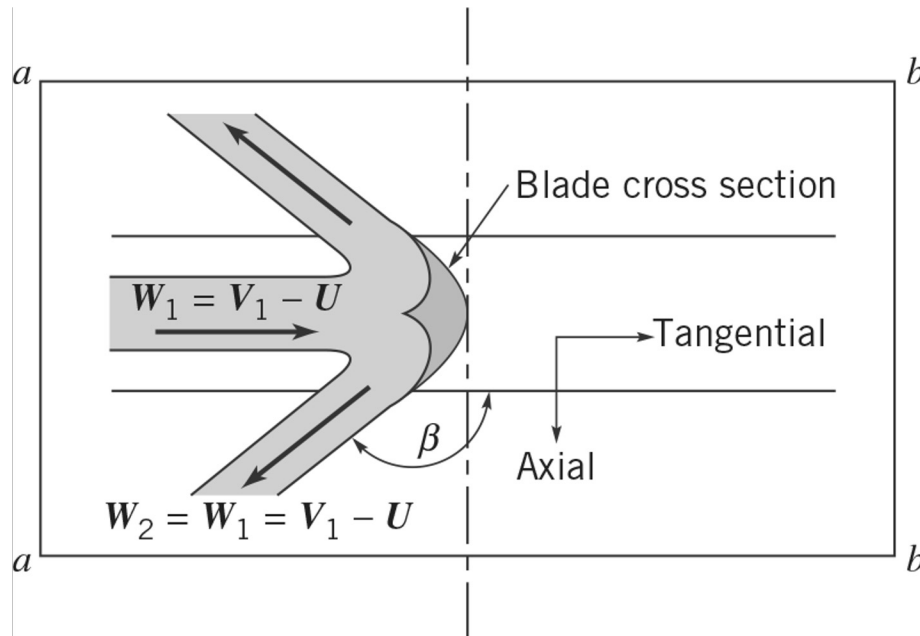
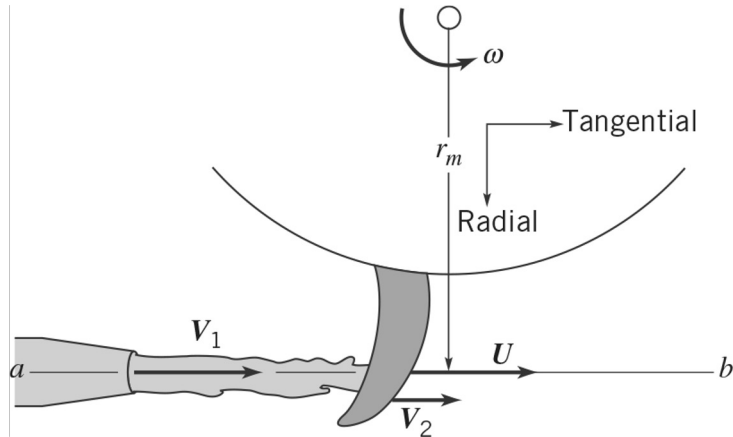


# Pelton turbines



Courtesy of Voith Hydro, York, PA.





## Assumptions

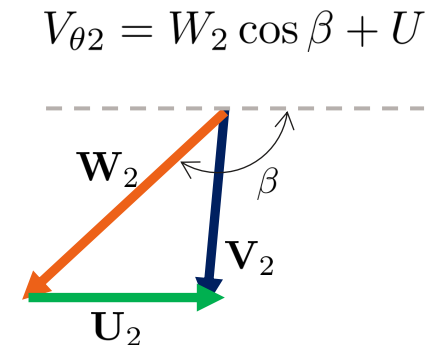
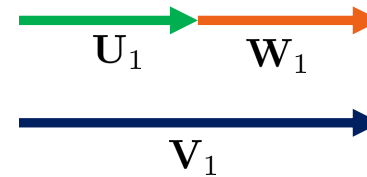
- no radial component of flow
- $W_2 \simeq W_1$  (otherwise  $W_2 = C_v W_1$  with  $C_v < 1$  velocity coefficient)  $\rightarrow$  exercise 2

$$T_{\text{shaft}} = -\dot{m}_1 (r_1 V_{\theta 1}) + \dot{m}_2 (r_2 V_{\theta 2})$$

$$\dot{m}_1 = \dot{m}_2 = \dot{m}, \quad r_1 = r_2 = r_m, \quad \mathbf{U}_1 = \mathbf{U}_2 = \mathbf{U}$$

$$V_{\theta 1} = V_1 = W_1 + U$$

$$V_{\theta 2} = W_2 \cos \beta + U$$



$$\dot{W}_{\text{shaft}} = T_{\text{shaft}} \omega = \dot{m} U (U - V_1) (1 - \cos \beta)$$

If  $V_1 > U$  (jet impacting bucket),  $\dot{W}_{\text{shaft}} < 0$  the turbine extracts power from the fluid

$$\dot{W}_{\text{shaft}} = \dot{m}U(U - V_1)(1 - \cos \beta)$$

- Effect of  $\beta$  : maximum when  $\beta = 180^\circ$  ( $\cos \pi = -1$ )

$$\dot{W}_{\text{shaft}} = 2\dot{m}(U^2 - UV_1)$$

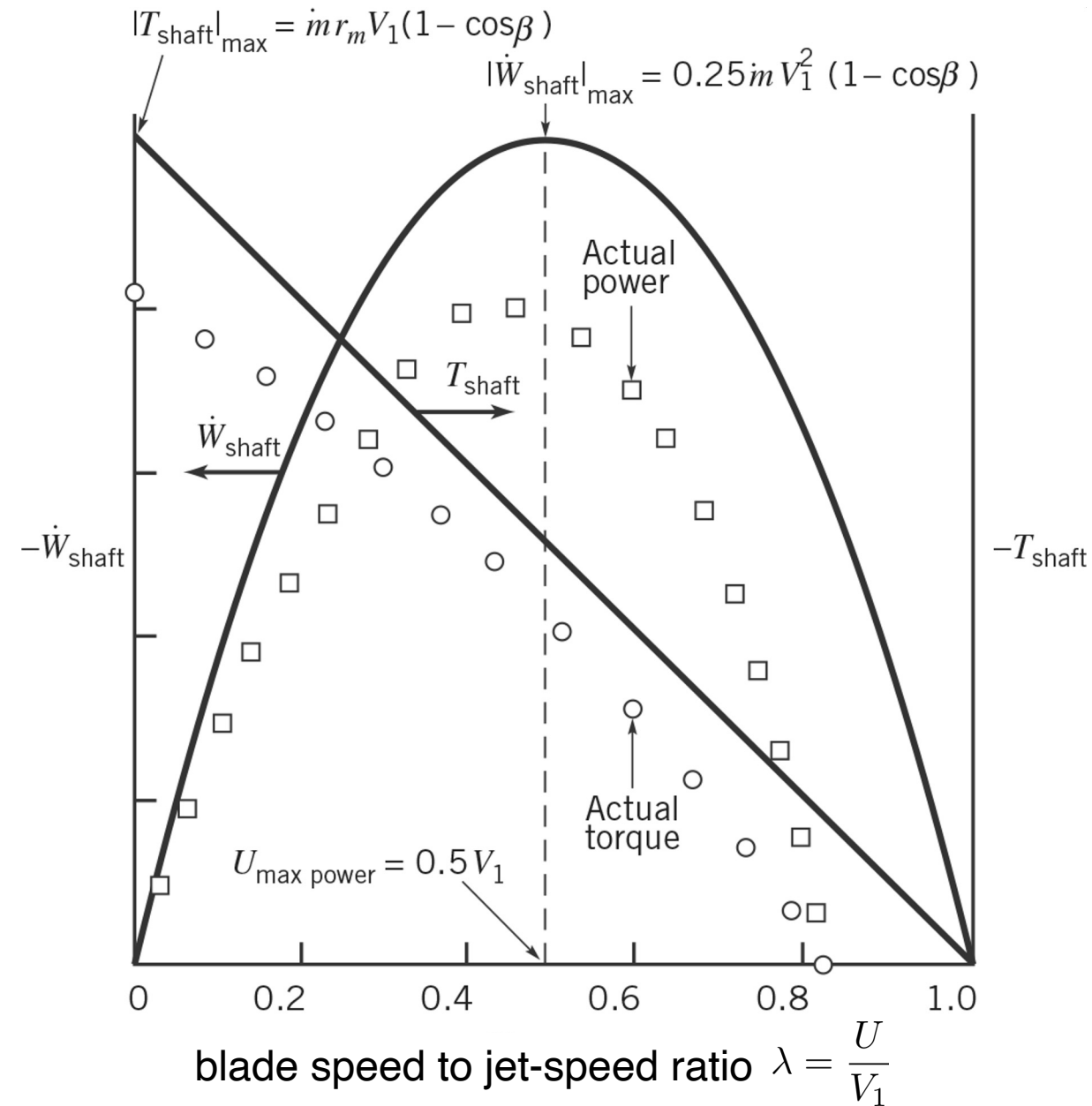
Maximum power  
with respect to U

$$U_{\text{max power}} = \frac{V_1}{2}$$

A bucket speed one-half the speed of the fluid coming from the nozzle gives maximum power.

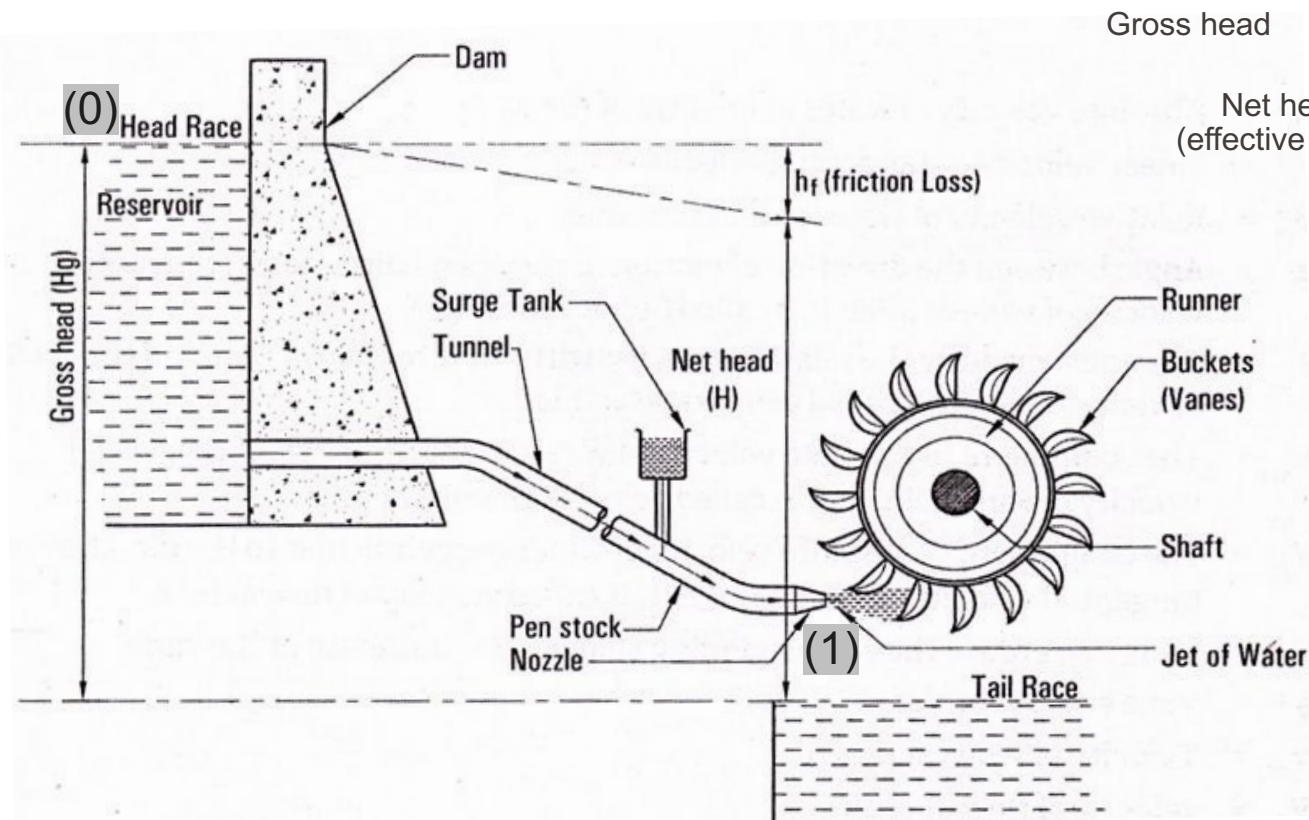
$$T_{\text{shaft}} = \dot{m}r_m(U - V_1)(1 - \cos \beta)$$

- Shaft torque = 0 when  $U = V_1$



# Pelton turbines – Nozzle speed

$$\frac{p_{\text{atm}}}{\gamma} + z_0 = \frac{p_{\text{atm}}}{\gamma} + \frac{V_1^2}{2g} + z_1 + \sum h_L$$



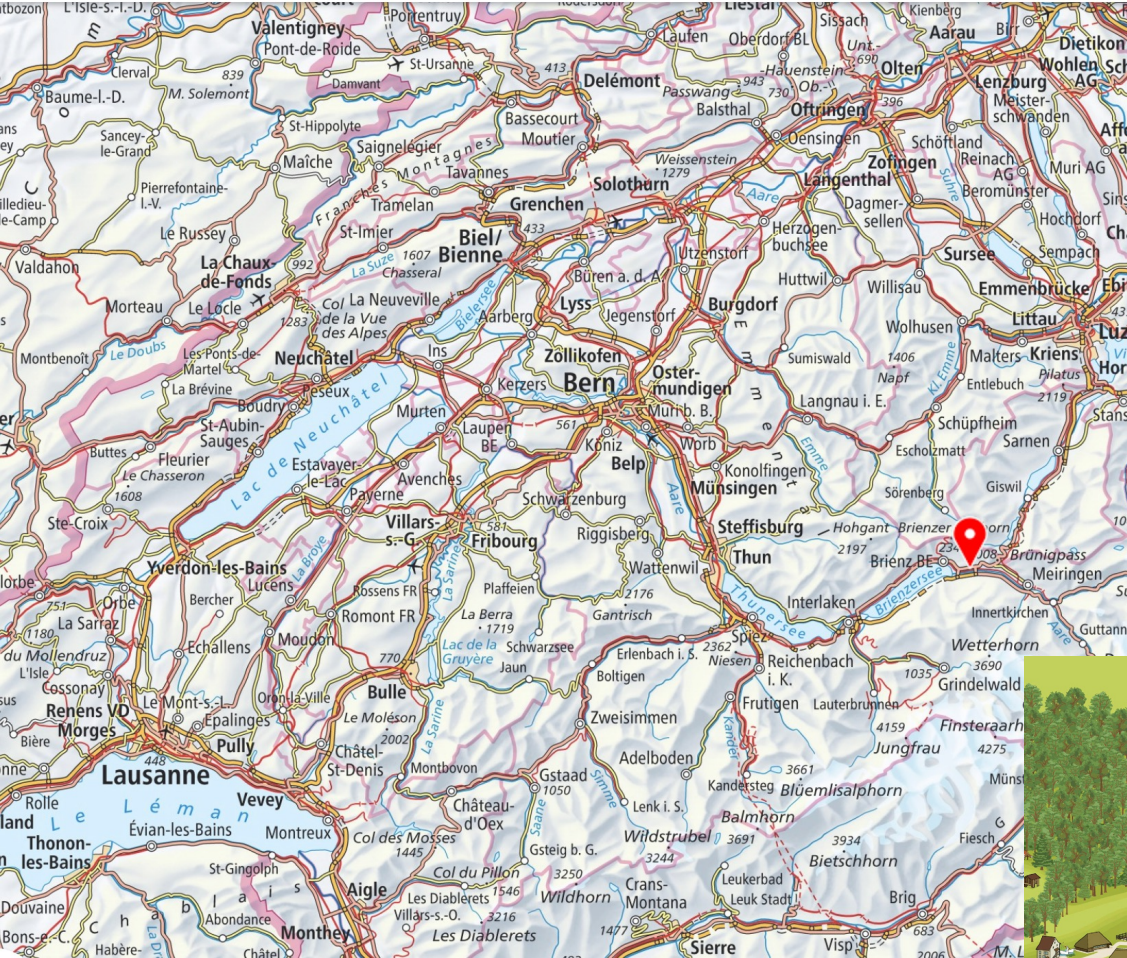
Or  $c_N V_1 = V_{\text{nozzle}}$  Nozzle coefficient,  $c_N < 1$

Tygun's empirical formula for the number of buckets ( $Z$ ) in the wheel

$$Z = \frac{D_{\text{wheel}}}{2D_{\text{nozzle}}} + 15$$

# Pelton turbines in Switzerland

Switzerland **Mobility** 

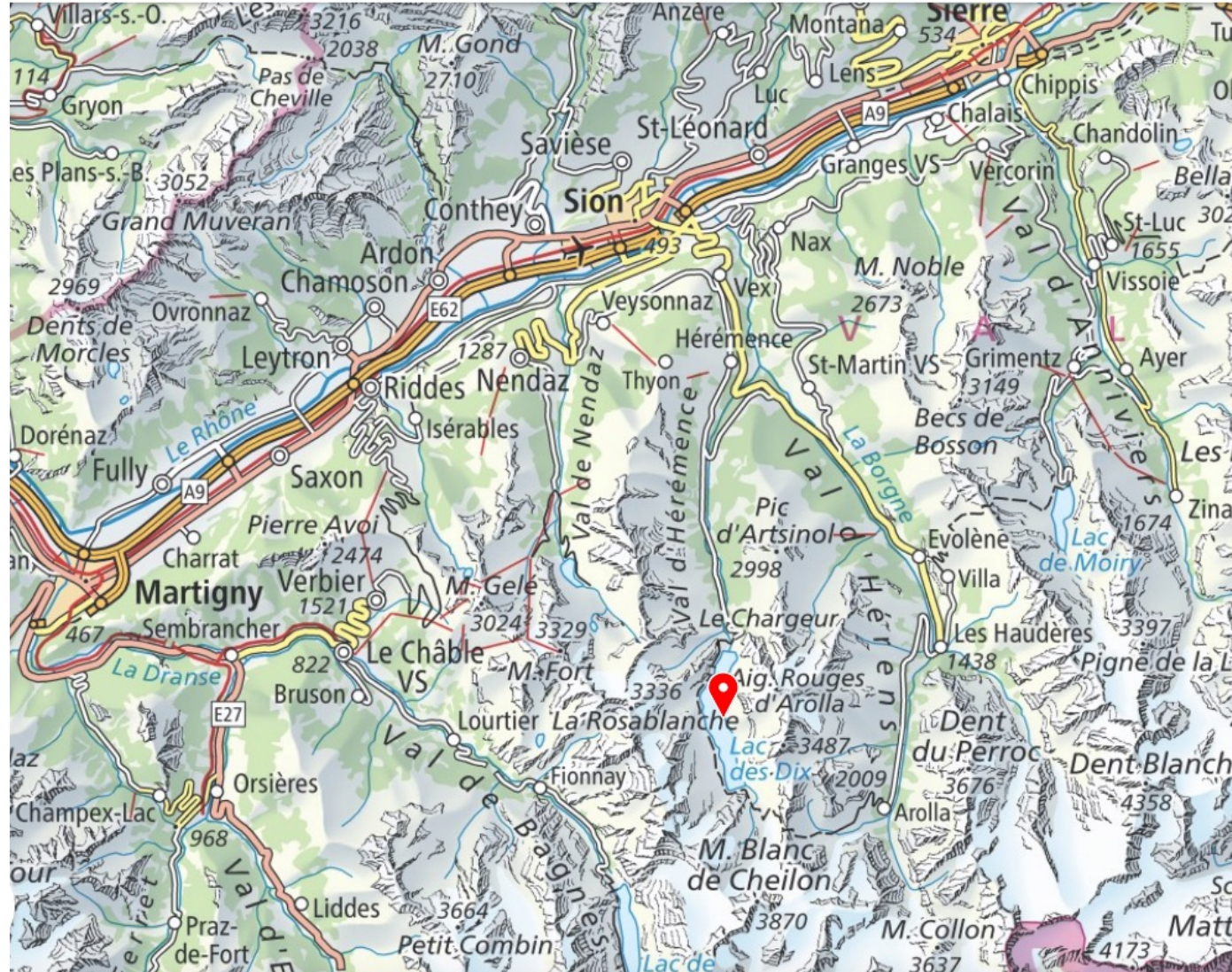


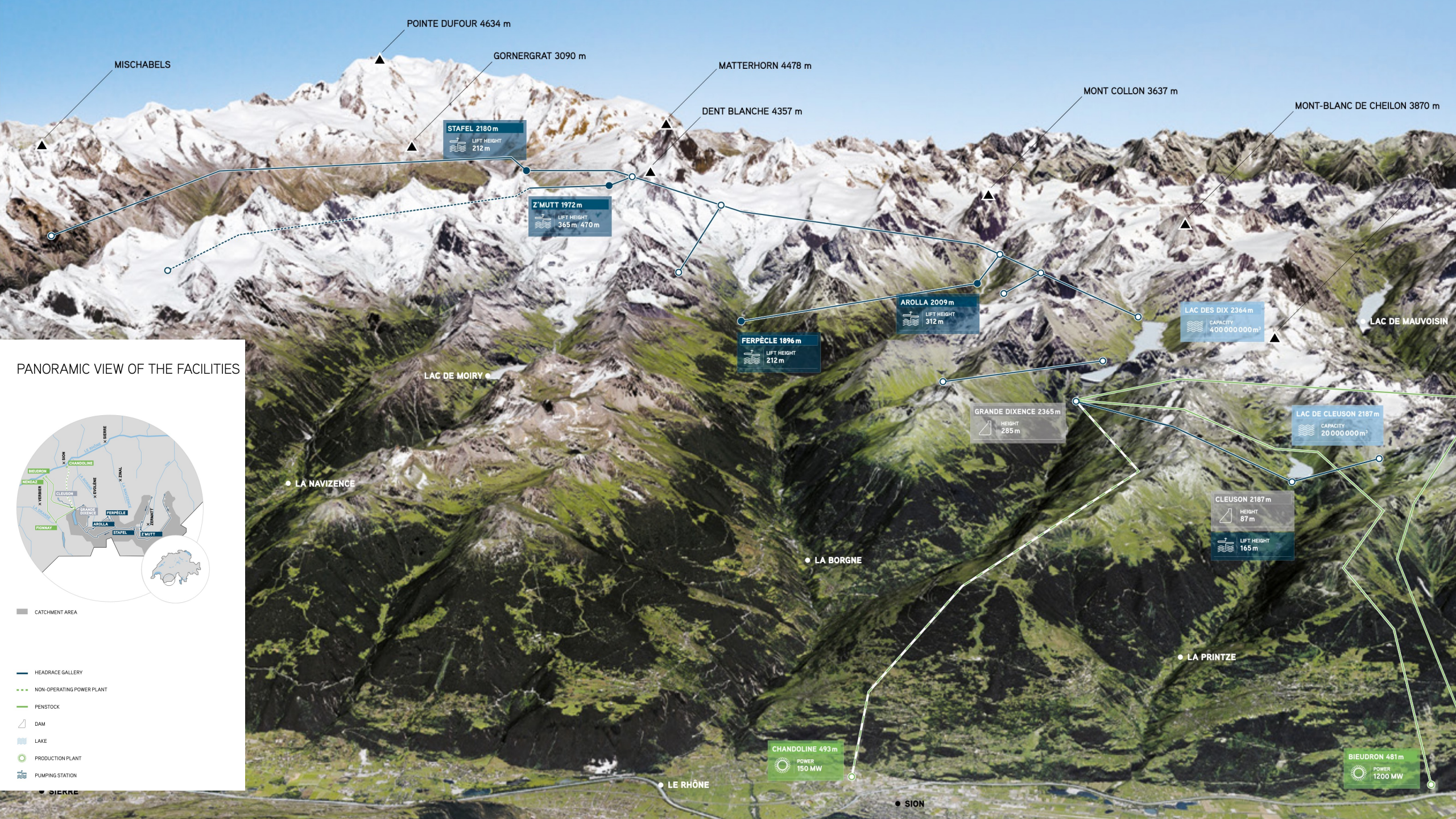
Ballenberg, Swiss  
Open-Air Museum



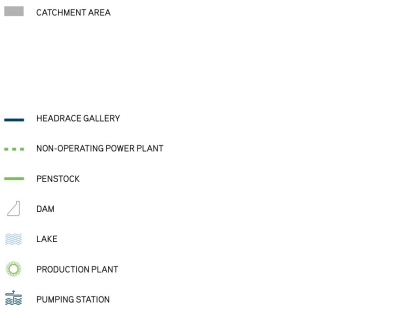
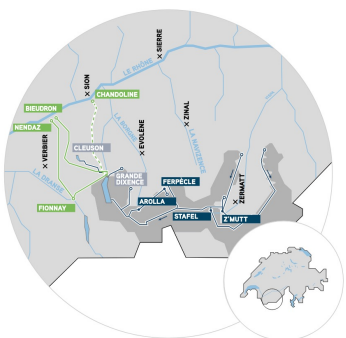
# Pelton turbines in Switzerland

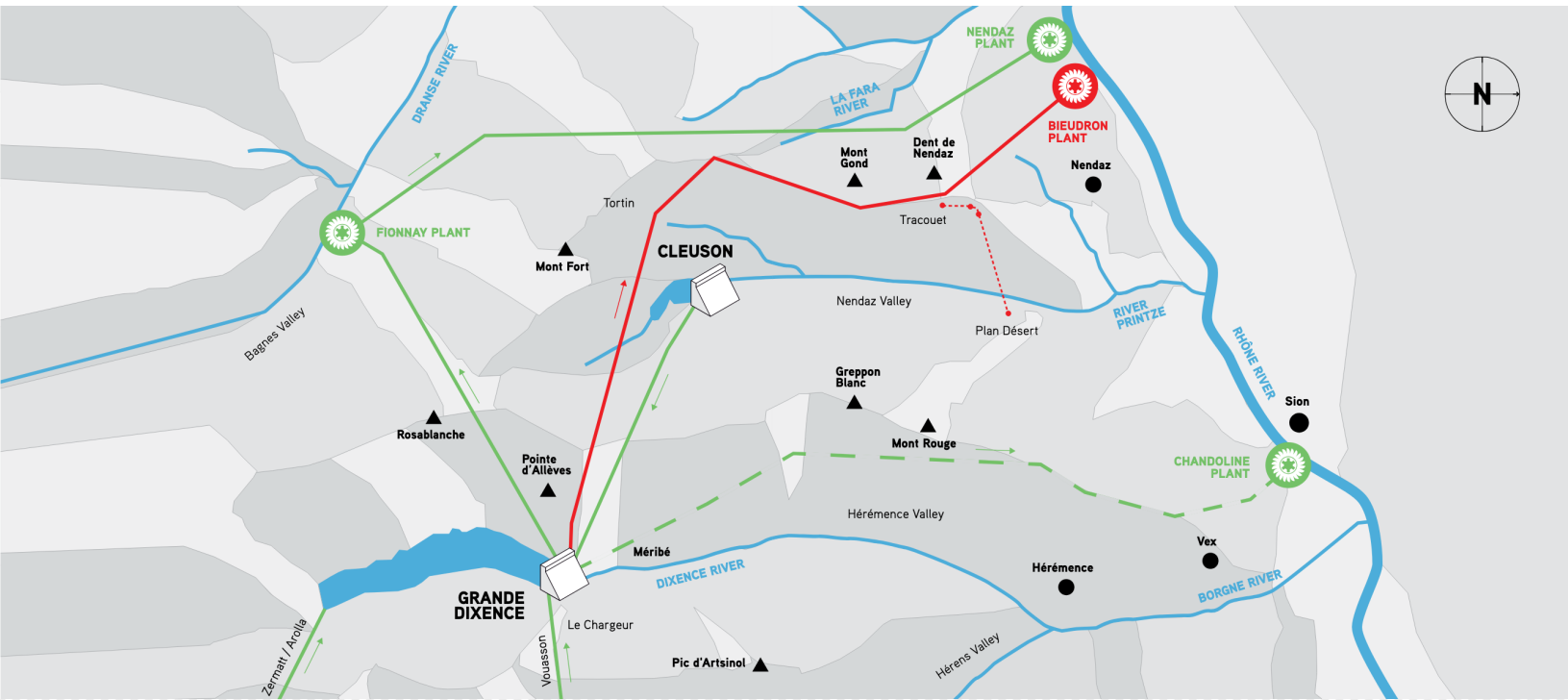
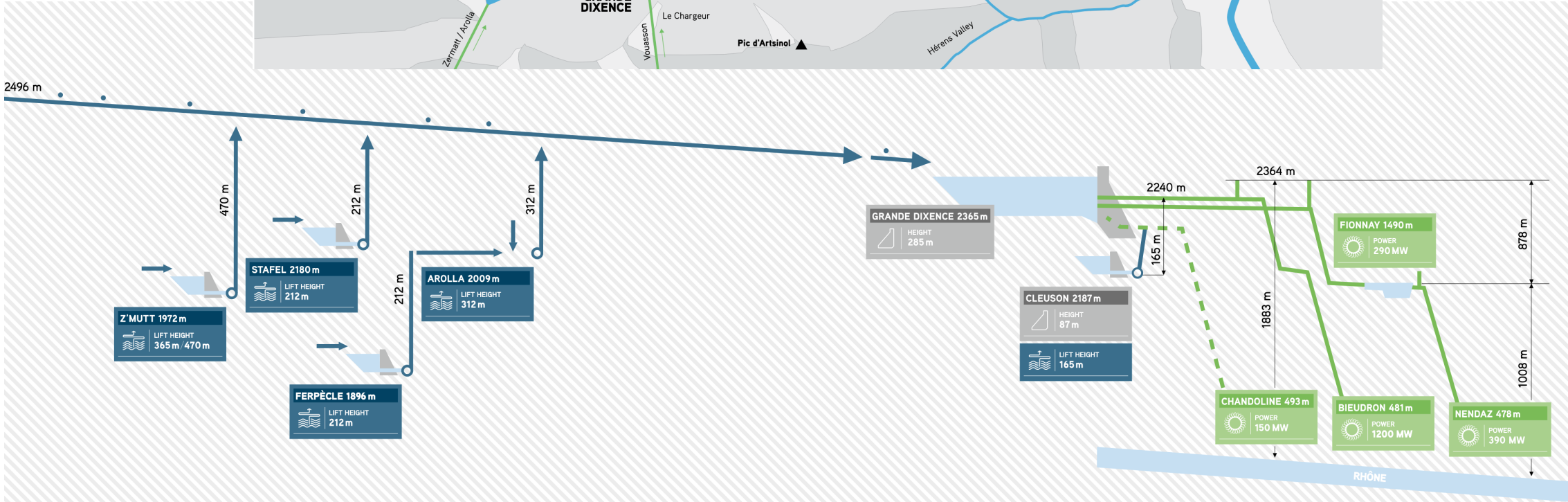
Switzerland **Mobility** 





PANORAMIC VIEW OF THE FACILITIES

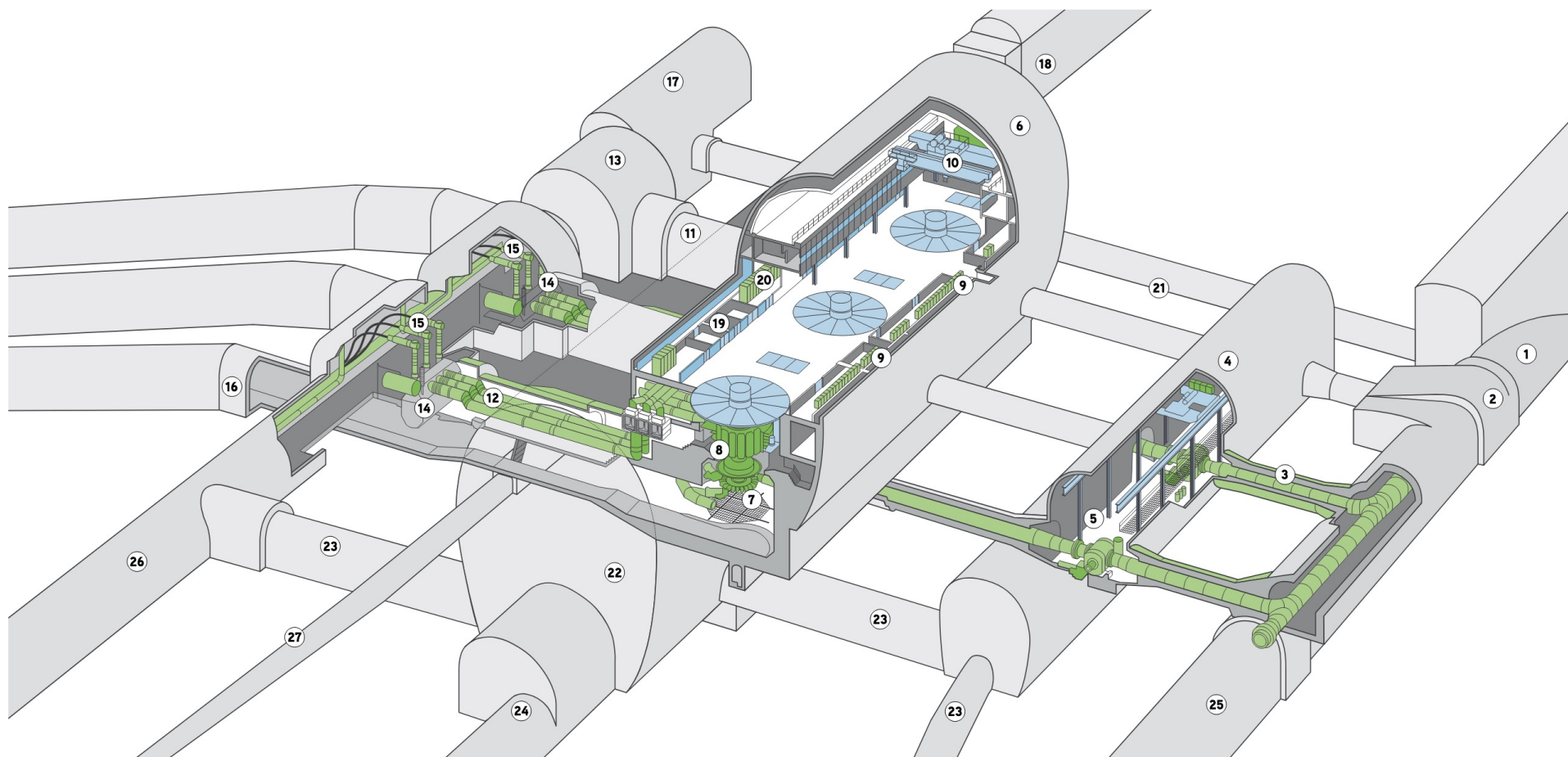




# BIEUDRON POWER PLANT

## OVERVIEW

- |  |  |
|--|--|
| 1 PENSTOCK   | 15 230/410 kV ONE-PHASE CABLE OUTLET                   |
| 2 DISTRIBUTOR  | 16 TAILWATER BRANCH                                    |
| 3 UNIT BRANCH (25 m <sup>3</sup> /s, PRESSURE 190 BAR) | 17 MAINTENANCE AND STORAGE BUILDING                    |
| 4 VALVES CHAMBER                                       | 18 COOLING WATER RESERVOIR                             |
| 5 SPHERICAL VALVE (210 TONS PER UNIT)                  | 19 16/0.4 kV AUXILIARY TRANSFORMERS                    |
| 6 MAIN CAVERN  | 20 230/400 V SWITCHBOARDS                              |
| 7 423 MW PELTON TURBINE                                | 21 EMERGENCY TUNNEL                                    |
| 8 465 MVA GENERATOR                                    | 22 ASSEMBLY SITE                                       |
| 9 CONTROL ROOM   | 23 CONNECTING TUNNELS                                  |
| 10 250 TONS OVERHEAD CRANE                             | 24 ACCESS TUNNEL TO PLANT                              |
| 11 BUSBAR TUNNELS                                      | 25 ACCESS TUNNEL TO DISTRIBUTOR                        |
| 12 21 kV 15'000 A BUSBARS                              | 26 TUNNEL FOR 410 kV CABLES AND ACCESS TO TRANSFORMERS |
| 13 TRANSFORMER CELL                                    | 27 VENTILATION TUNNEL                                  |
| 14 465 MVA THREE-PHASE TRANSFORMERS                    |  |





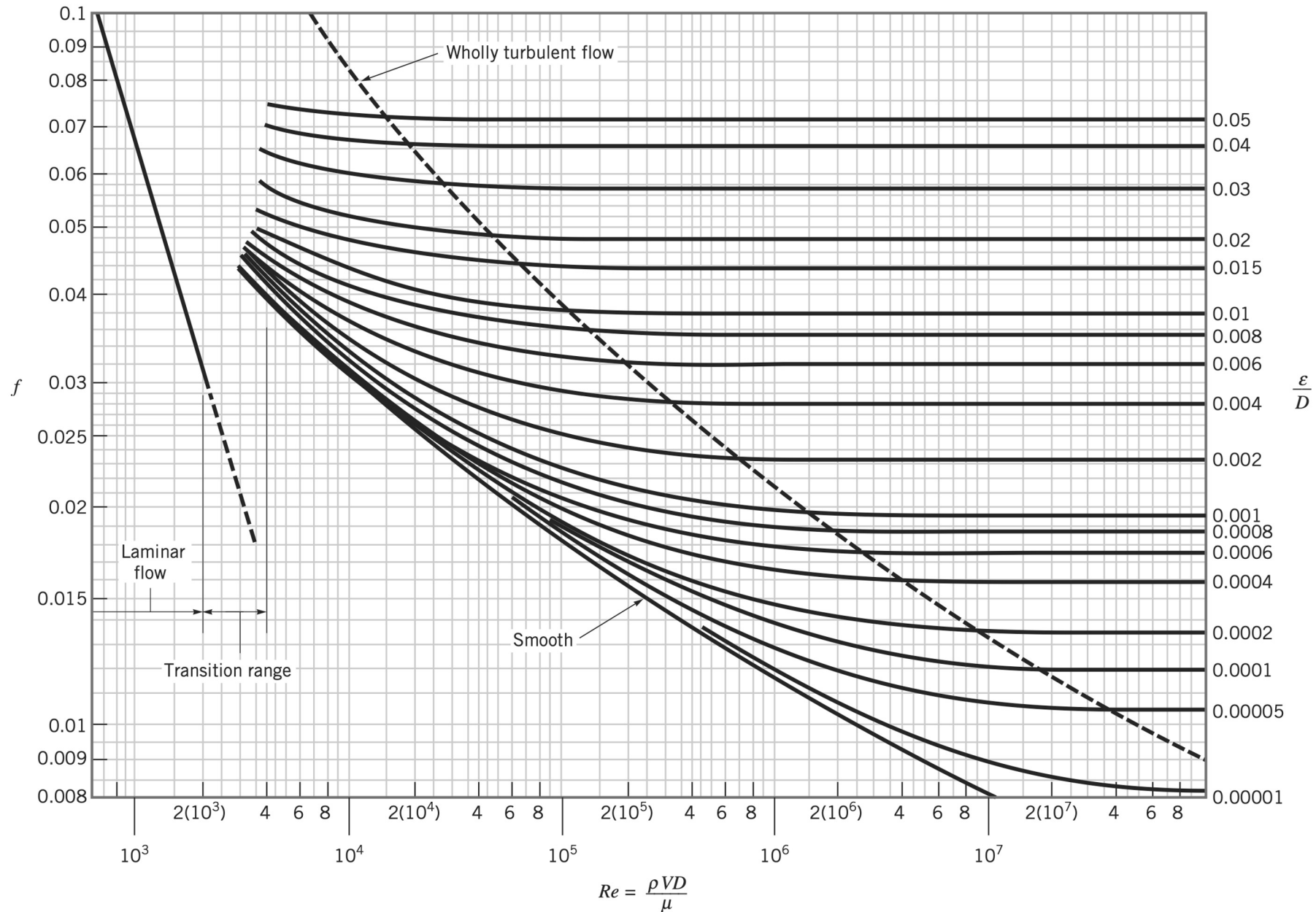




A Pelton wheel operates with a gross head of 530 m and a flow rate of 9 m<sup>3</sup>/s. The penstock length is 880 m, its diameter is 1.2 m, and its RMS roughness is 0.12 mm. The minor loss can be ignored. The hydraulic efficiency is  $\eta_h = 0.84$ , and the shaft speed is 650 rpm. Water kinematic viscosity is  $1.02 \times 10^{-6}$  m<sup>2</sup>/s and density is 998 kg/m<sup>3</sup>.

Find (a) the effective head and the power delivered by the turbine and (b) the specific speed and from it the recommended number of jets and the number of buckets in the wheel. The nozzle coefficient is  $C_N = 0.97$ , and the ratio of the blade speed to the discharge velocity is  $\lambda = U/V_1 = 0.45$ .

# Moody diagram – Chapter 3



A Pelton wheel operates with a gross head of 530 m and a flow rate of 9 m<sup>3</sup>/s. The penstock length is 880 m, its diameter is 1.2 m, and its RMS roughness is 0.12 mm. The minor loss can be ignored. The hydraulic efficiency is  $\eta_h = 0.84$ , and the shaft speed is 650 rpm. Water kinematic viscosity is  $1.02 \times 10^{-6}$  m<sup>2</sup>/s and density is 998 kg/m<sup>3</sup>.

Find (a) the effective head and the power delivered by the turbine and (b) the specific speed and from it the recommended number of jets and the number of buckets in the wheel. The nozzle coefficient is  $C_N = 0.97$ , and the ratio of the blade speed to the discharge velocity is  $\lambda = U/V_1 = 0.45$ .

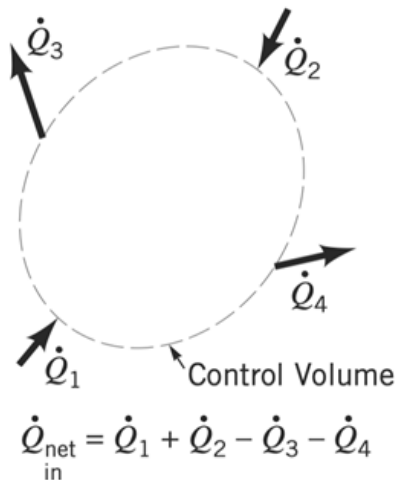
Type		$N'_s$	$\eta$ %
Pelton wheel	Single jet	0.02 – 0.18	88 – 90
	Twin jet	0.09 – 0.26	89 – 92
	Three jet	0.10 – 0.30	89 – 92
	Four jet	0.12 – 0.36	86
Francis	Low-speed	0.39 – 0.65	90 – 92
	Medium-speed	0.65 – 1.2	93
	High-speed	1.2 – 1.9	93 – 96
	Extreme-speed	1.9 – 2.3	89 – 91
Kaplan turbine		1.55 – 5.17	87 – 94
Bulb turbine		3 – 8	

# Appendix



## 3-Energy equation

- cs: control **surface**
- cv: control **volume**



The **first law of thermodynamics**

$$\frac{D}{Dt} \int_{\text{sys}} e \rho dV = \frac{\partial}{\partial t} \int_{\text{cv}} e \rho dV + \int_{\text{cs}} e \rho \mathbf{V} \cdot \hat{\mathbf{n}} dA$$

Time rate  
of increase  
of the total  
stored energy  
of the system

= time rate of increase  
of the total stored  
energy of the contents  
of the control volume

+ net rate of flow  
of the total stored energy  
out of the control  
volume through the  
control surface

where  $e = \underbrace{\check{u}}_{\text{internal energy}} + \frac{V^2}{2} + gz$  total stored energy per unit mass

$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft net in}} = \frac{\partial}{\partial t} \int_{\text{cv}} e \rho dV + \int_{\text{cs}} \left( \check{u} + \frac{p}{\rho} + \frac{V^2}{2} + gz \right) \rho \mathbf{V} \cdot \hat{\mathbf{n}} dA$$

Heat  
transfer ratio

Work transfer  
rate, power

- Shaft torque

$$T_{\text{shaft}} = -\dot{m}_1 (r_1 V_{\theta 1}) + \dot{m}_2 (r_2 V_{\theta 2})$$

- Shaft power

$$\dot{W}_{\text{shaft}} = T_{\text{shaft}} \omega = -\dot{m} \underbrace{r_1 V_{\theta 1}}_{U_1} \omega + \dot{m} \underbrace{r_2 V_{\theta 2}}_{U_2} \omega$$

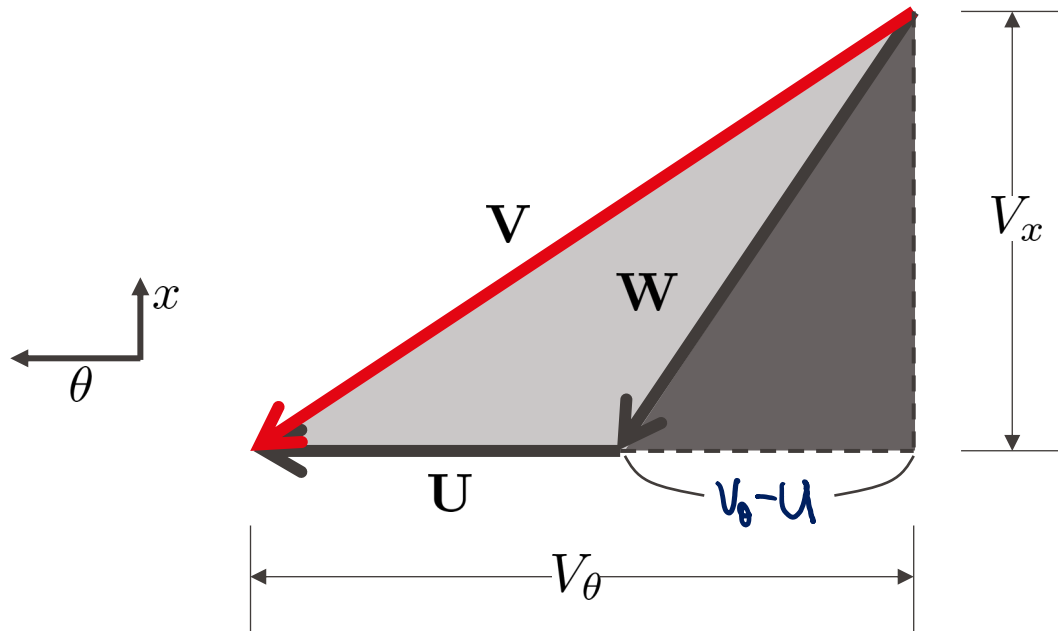
$$\dot{W}_{\text{shaft}} = (-\dot{m}_1) (U_1 V_{\theta 1}) + \dot{m}_2 (U_2 V_{\theta 2}) \quad [W] = [\text{kg} \cdot \text{m}^2 / \text{s}^3]$$

- Shaft work per unit mass (shaft power per unit mass flow rate),  $\dot{m}_1 = \dot{m}_2$

$$w_{\text{shaft}} = - (U_1 V_{\theta 1}) + (U_2 V_{\theta 2}) \quad [\text{m}^2 / \text{s}^2]$$

- Basic governing equations for pumps or turbines whether the machines are radial-, mixed-, or axial-flow devices and for compressible and incompressible flows
- Note it is only the function of tangential component of velocity, no  $V_r$ ,  $V_x$

$$\mathbf{V} = \mathbf{W} + \mathbf{U}$$



Velocity triangle:  $\mathbf{V}$  absolute velocity,  
 $\mathbf{W}$  relative velocity,  $\mathbf{U}$  blade velocity

- From the big triangle (grey)

$$V^2 = V_\theta^2 + V_x^2 \quad \text{or} \quad V_x^2 = V^2 - V_\theta^2$$

- From the small triangle (dark grey)

$$\begin{aligned} W^2 &= (V_\theta - U)^2 + V_x^2 \\ &= V_\theta^2 - 2V_\theta U + U^2 + V_x^2 \\ W^2 &= V_\theta^2 - 2V_\theta U + U^2 + V^2 - V_\theta^2 \\ V_\theta U &= \frac{-W^2 + U^2 + V^2}{2} \end{aligned}$$

$$w_{\text{shaft}} = -(U_1 V_{\theta 1}) + (U_2 V_{\theta 2})$$

$$w_{\text{shaft}} = \frac{V_2^2 - V_1^2 + U_2^2 - U_1^2 - (W_2^2 - W_1^2)}{2}$$

Turbomachine work is related to changes in absolute, relative, and blade velocities.