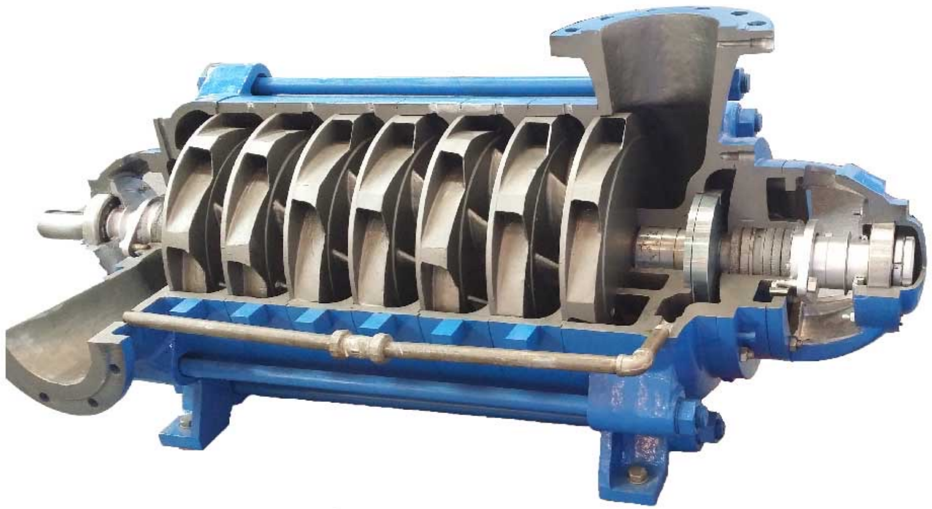
EPFL



Chapter 7: Centrifugal Pumps & Similarity rules

ME-342 Introduction to turbomachinery

Prof. Eunok Yim, HEAD-lab.

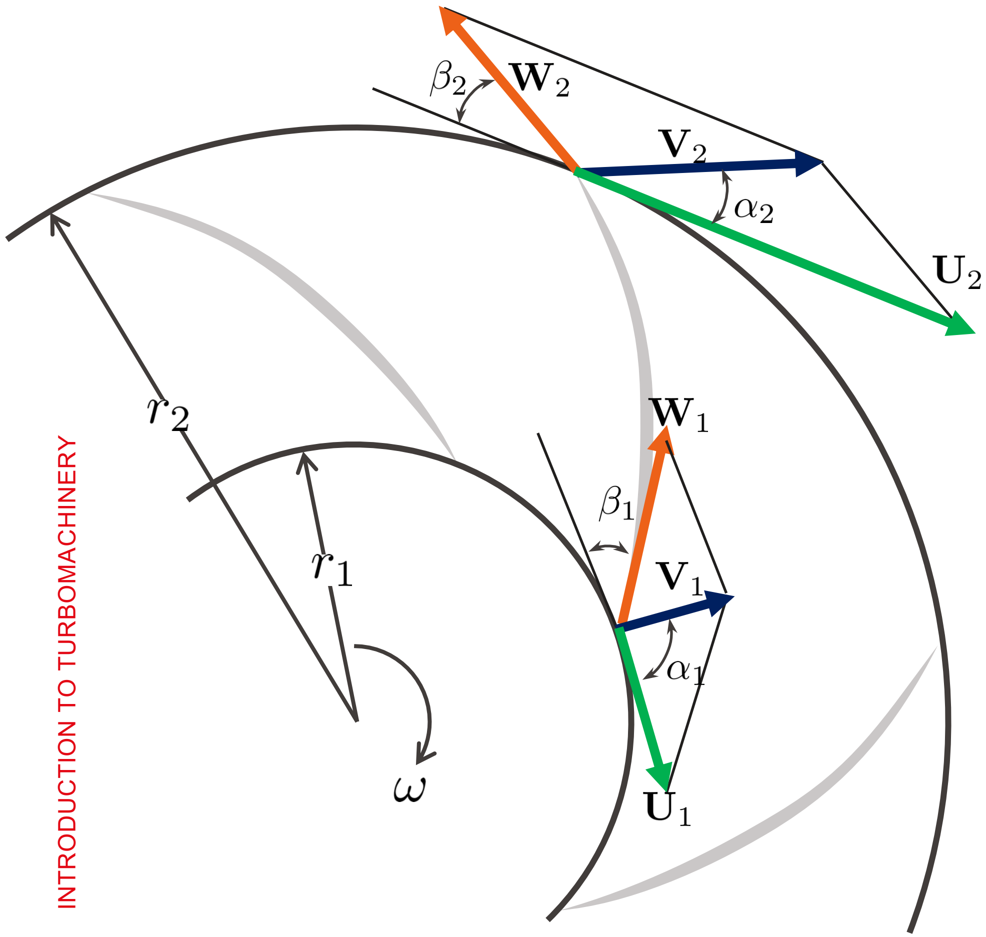
École polytechnique fédérale de Lausanne

Centrifugal Pump –con’d

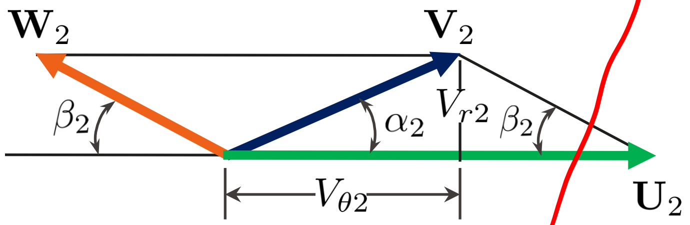
Theoretical consideration –recall

For centrifugal pump inlet with purely radial, (no $V_{\theta 1}$ component)

$$\alpha_1 = 90^\circ \quad \mathbf{V}_1 = [V_{r1}, 0, 0]$$



$$h_i = \frac{U_2 V_{\theta 2} - U_1 V_{\theta 1}}{g} = \frac{U_2 V_{\theta 2}}{g}$$



$$\tan\beta_2 = \quad \rightarrow \quad \cot\beta_2 = \frac{U_2 - V_{\theta 2}}{V_{r2}}$$

$$V_{\theta 2} = -V_{r2}\cot\beta_2 + U_2$$

$$h_i = \frac{U_2}{g}\left(U_2 - V_{r2}\cot\beta_2\right)$$

$$h_i = \frac{U_2^2}{g} - \frac{U_2 V_{r2}\cot\beta_2}{g} \qquad h_i = \frac{U_2^2}{g} - \frac{U_2\cot\beta_2}{2\pi r_2 b_2 g}Q$$

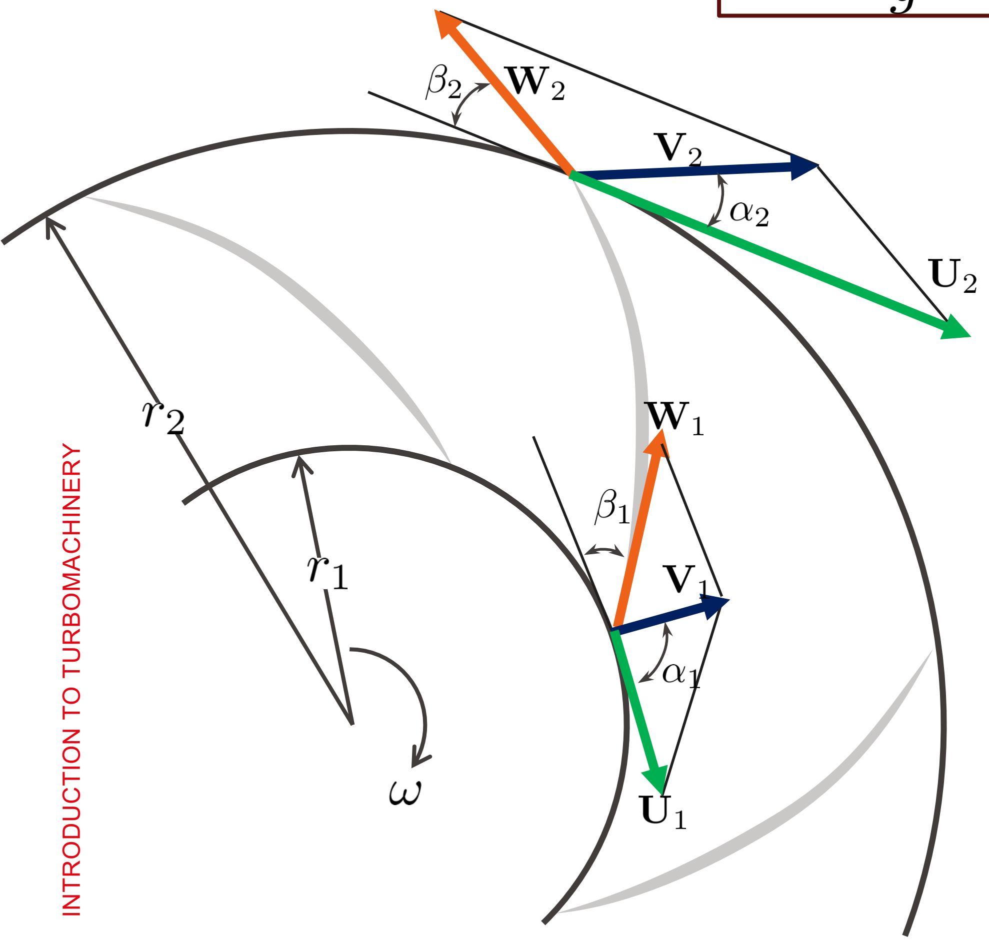
Flowrate, Q, is related to the **radial** component of the **absolute** velocity:

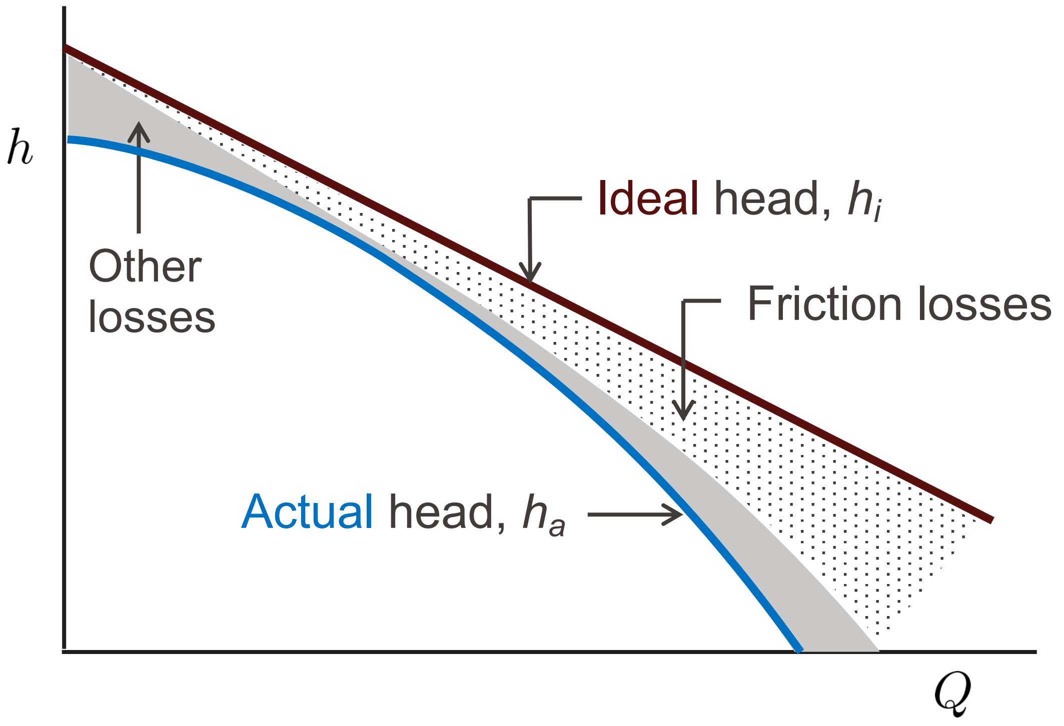
$$Q = 2\pi r_2 b_2 V_{r2}$$

Theoretical consideration

Ideal head rise can be represented as a function of flow rate and geometrical parameters of impeller only

$$h_i = \frac{U_2^2}{g} - \frac{U_2 \cot \beta_2}{2\pi r_2 b_2 g} Q$$





Losses: skin friction in the blade passages, which vary as Q^2, and other losses due to such factors as flow separation, impeller blade-casing clearance flows, and other three-dimensional flow effects

Pump performance characteristics

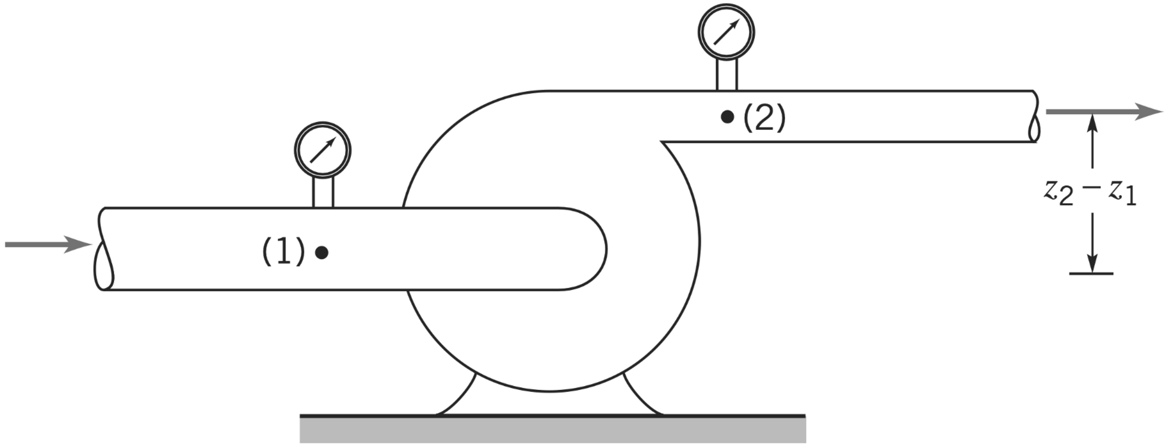
$h_a = H_2 - H_1$

$$H = \frac{p}{\rho g} + \frac{V^2}{2g} + z$$

$$h_a = \frac{p_2 - p_1}{\gamma} + z_2 - z_1 + \frac{V_2^2 - V_1^2}{2g}$$

When the differences in elevations (Δz) and velocities (ΔV^2) are small between pump inlet (1) and outlet (2)

$$h_a \approx \frac{p_2 - p_1}{\gamma}$$



- Power gained by the fluids, $P_f = \gamma Q h_a$

- Overall efficiency

$$\eta = \frac{\text{power gained by the fluid}}{\text{shaft power driving the pump}} = \frac{P_f}{\dot{W}_{\text{shaft}}}$$

Pump performance characteristics

- Overall efficiency $\eta = \dfrac{\text{power gained by the fluid}}{\text{shaft power driving the pump}} = \dfrac{P_f}{\dot{W}_{\text{shaft}}}$

  = ratio of power actually gained by the fluid to the shaft power supplied

- Hydraulic losses: Skin friction, flow separation, 3D and unsteady effects
  → Hydraulic efficiency η_h

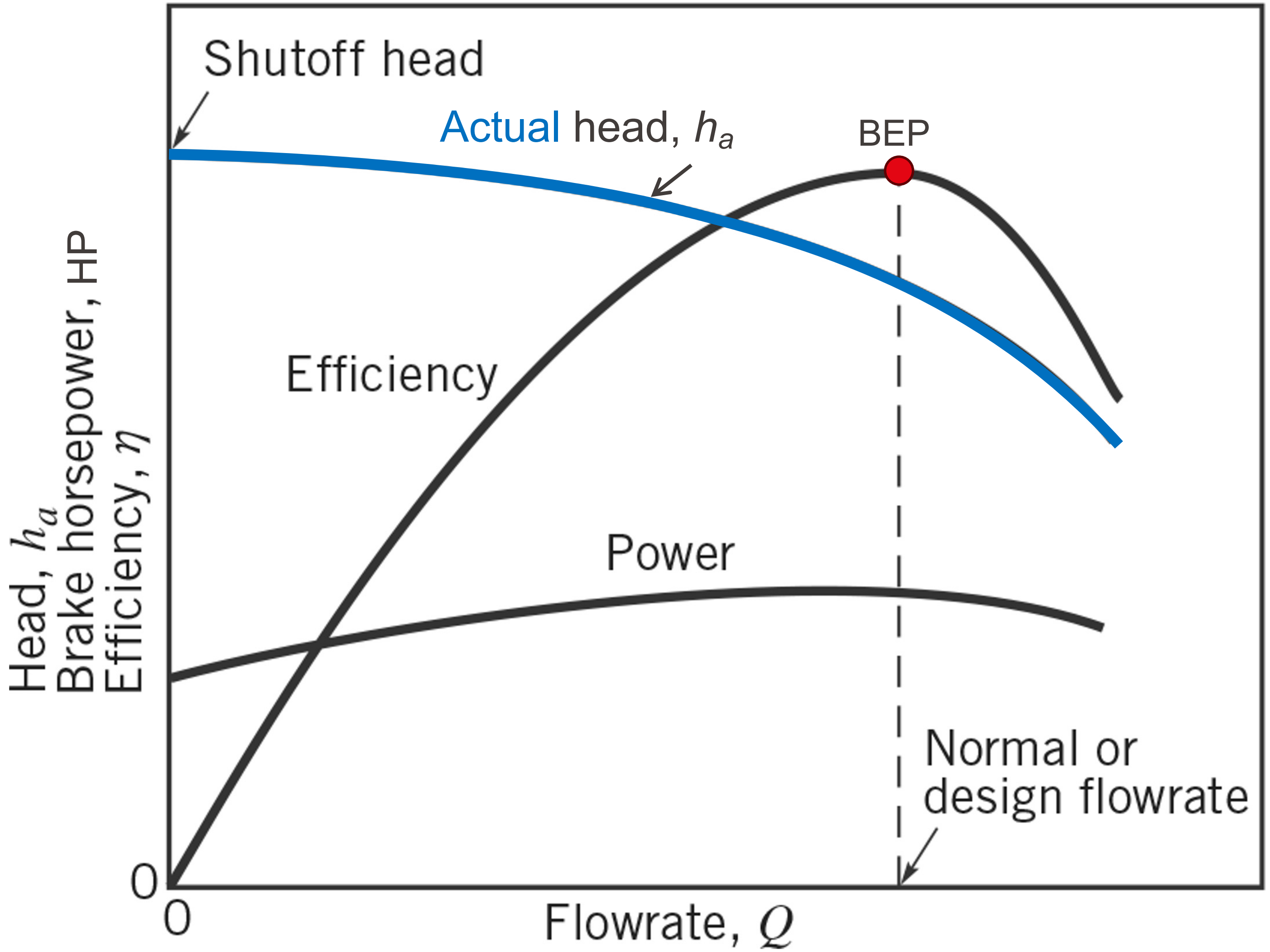
- Mechanical losses: bearing and sealing losses
  → Mechanical efficiency η_m

- Volumetric losses: flow leakage components
  → Volumetric efficiency η_v

$$\eta = \eta_h \eta_m \eta_v$$

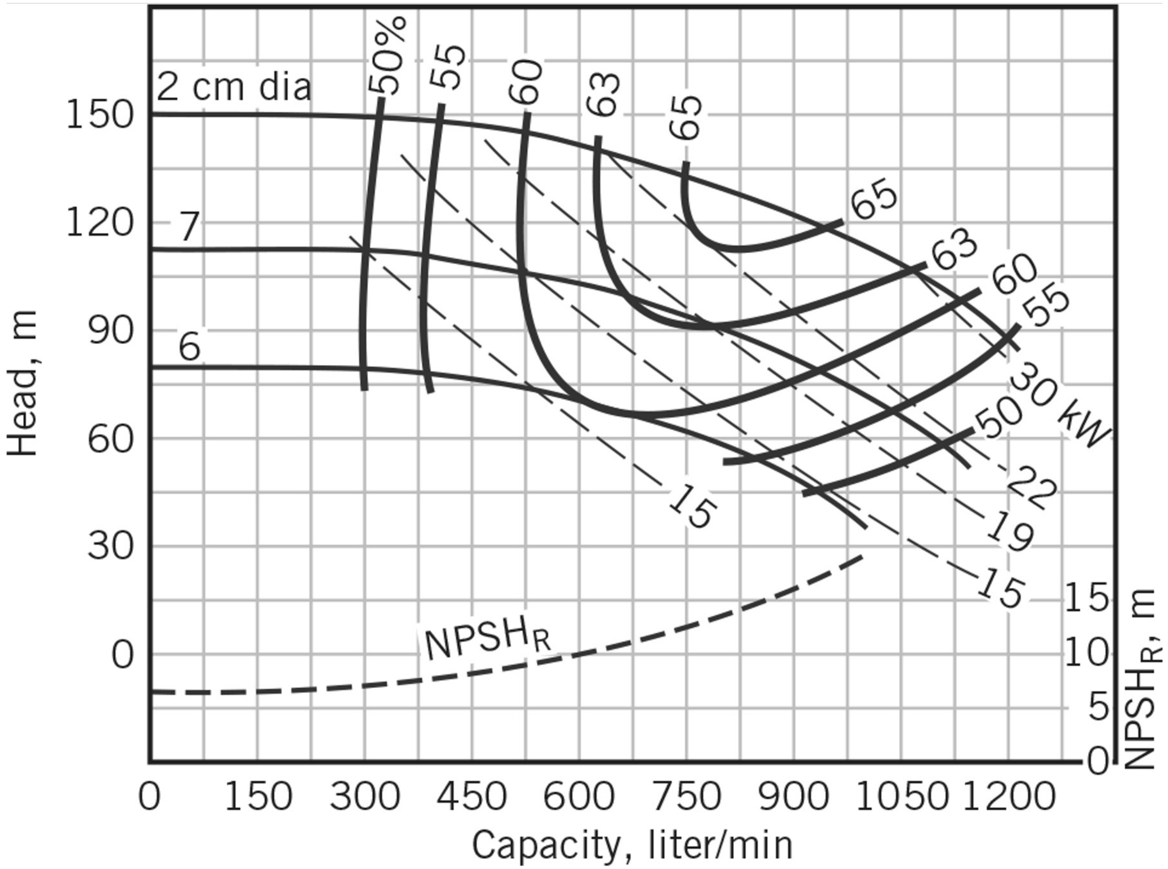
Pump performance characteristics

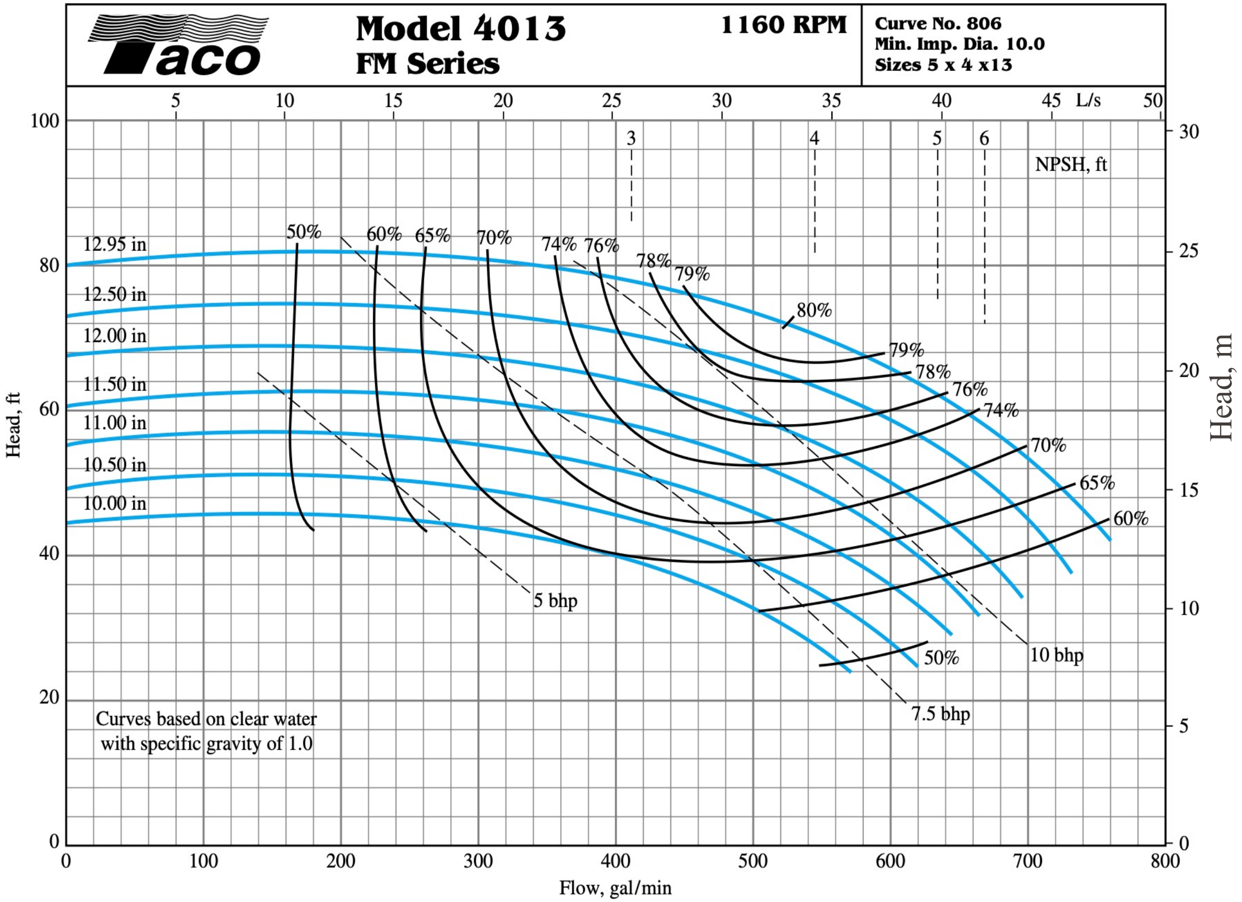
Typical performance characteristics for a centrifugal pump of a given size operating at a constant impeller speed



- **Normal, design flowrate or capacity of pump**
- **Shutoff head**: pressure head rise across the pump with the discharge valve closed. Since there is no flow with the valve closed, the related efficiency is zero, and the power supplied by the pump is simply dissipated as heat
- **Best efficiency point (BEP):** maximum efficiency at designed flowrate
- Brake horsepower (power as horsepower, hp unit): power required to drive the pump or brake power (in Watt) (1 horsepower = 746 watts)

Performance curve for centrifugal pump





Different impeller diameters are present for a given casing

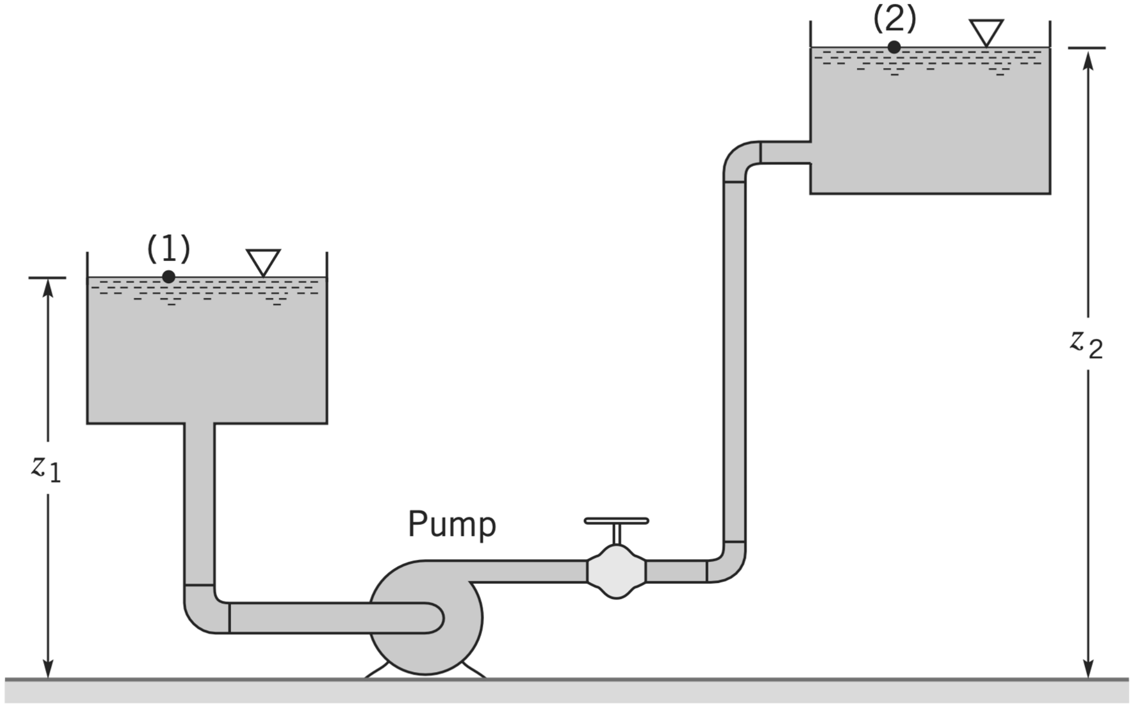
System Characteristics and Pump Selection

System head (h_{sys}) should match with the head that must be supplied by the pump (h_a), $h_a = h_{sys}$

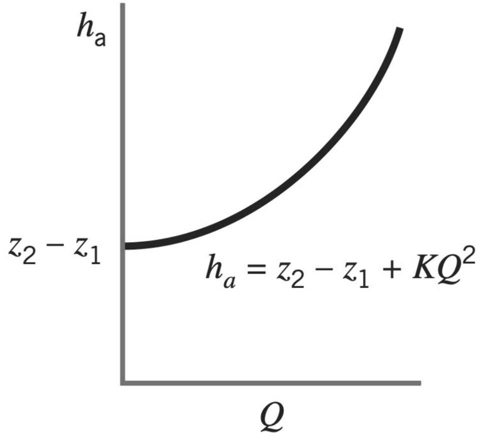
$$h_{sys} = \frac{p_2 - p_1}{\gamma} + z_2 - z_1 + \frac{V_2^2 - V_1^2}{2g} + \sum h_L$$

$$h_{sys} = z_2 - z_1 + \sum h_L$$

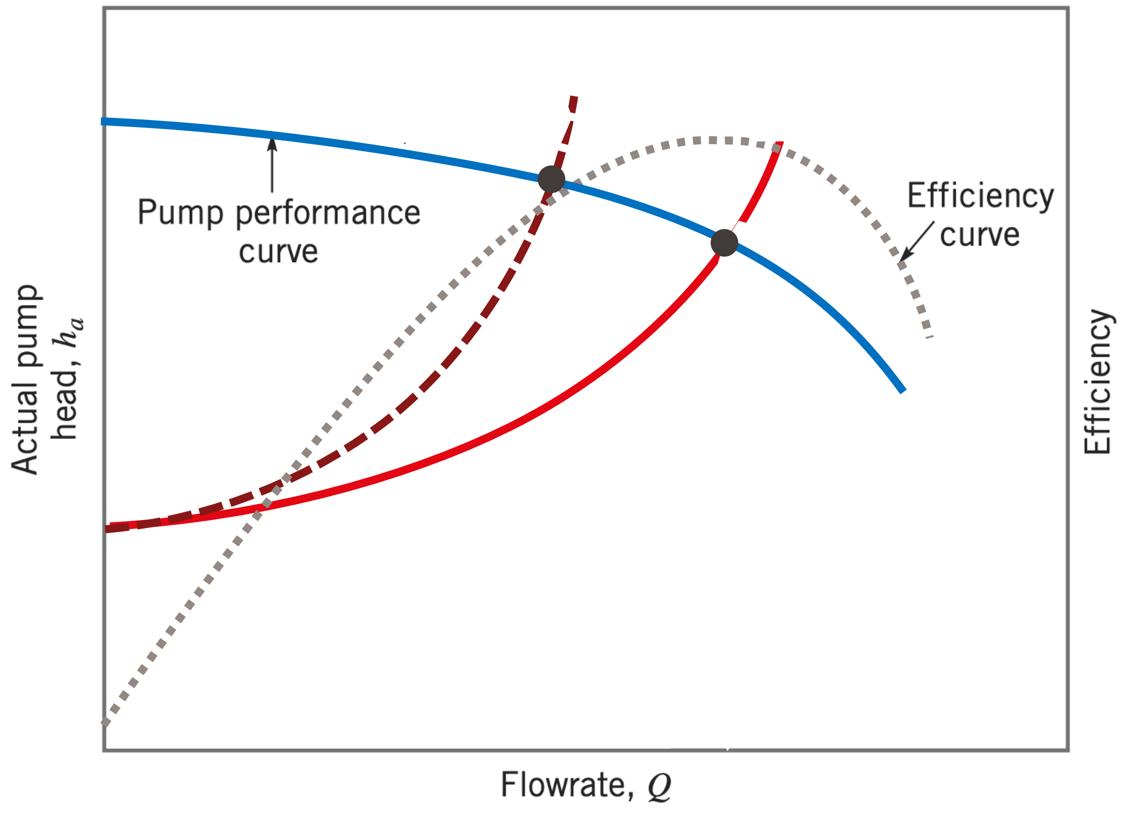
$$\sum h_L \propto V^2 \propto Q^2 \rightarrow KQ^2$$



System equation



System Characteristics and Pump Selection



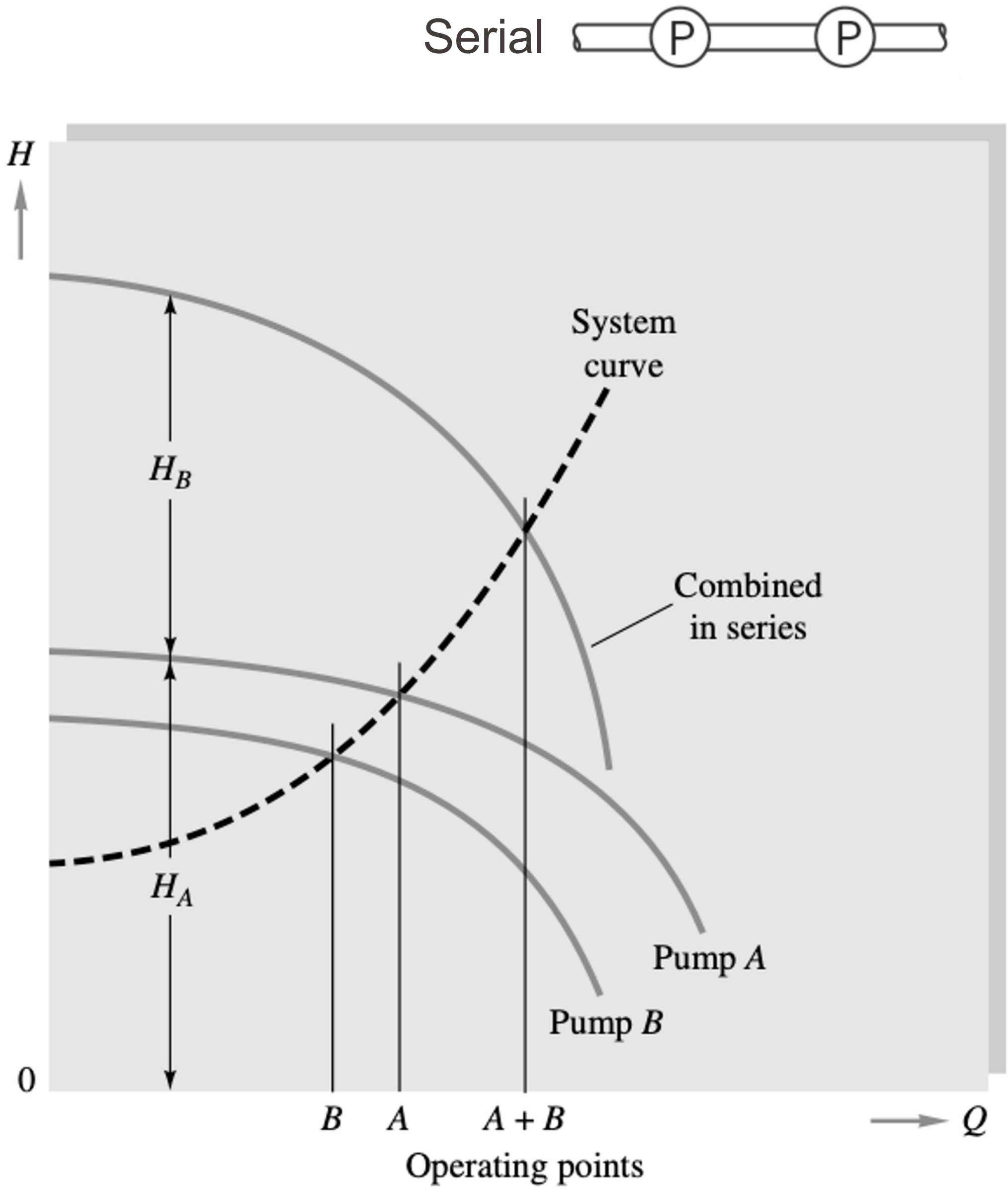
System equation:

$$h_a = z_2 - z_1 + KQ^2$$

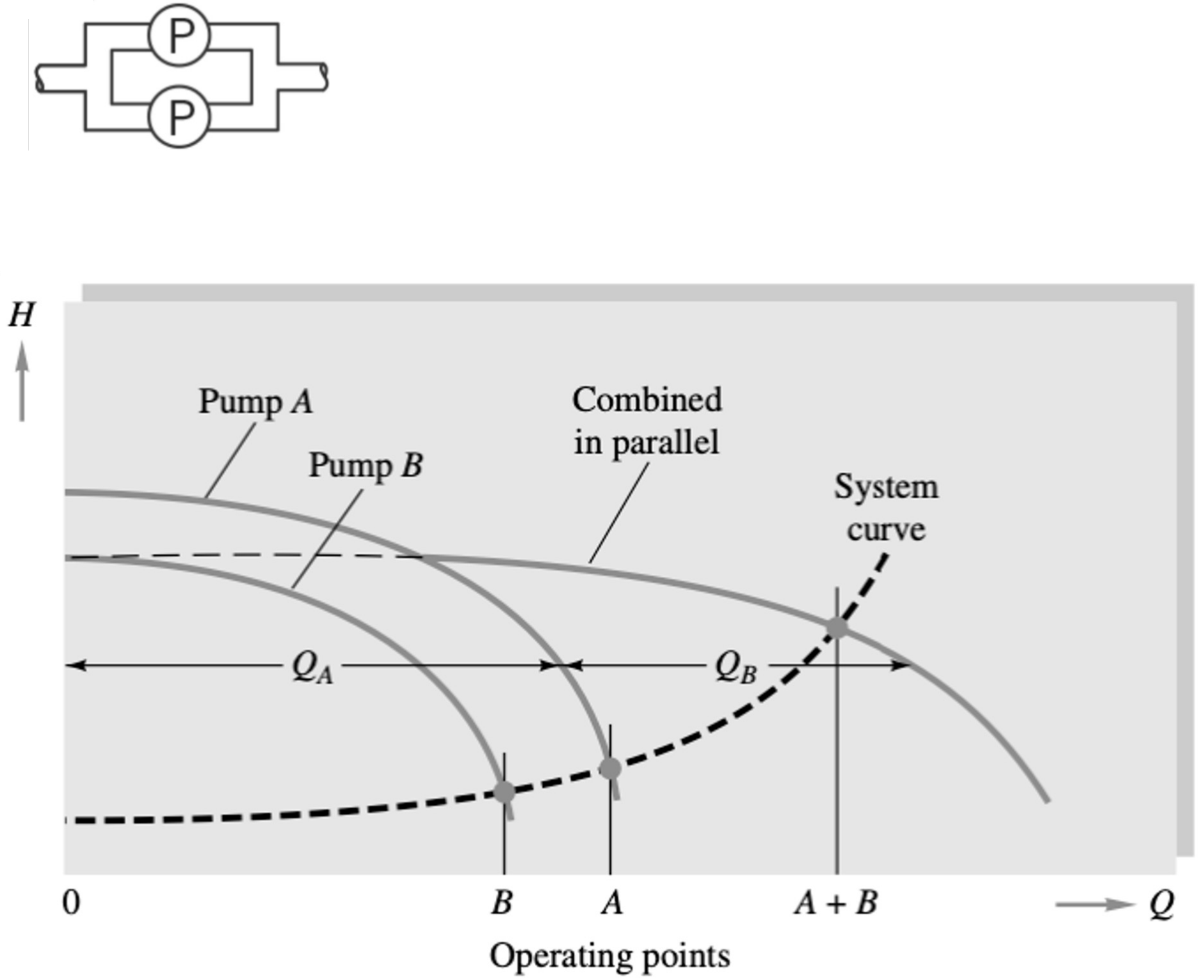
- The intersection point of performance curve and system curve: head and flowrate that satisfy both the *system equation* and the *pump equation*.
- Ideally, we want the operating point to be near the best efficiency point (BEP) for the pump.

Pump arrangement

Serial

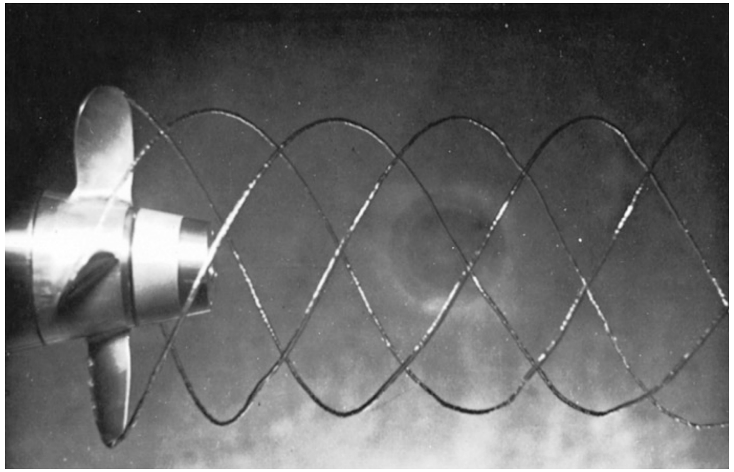


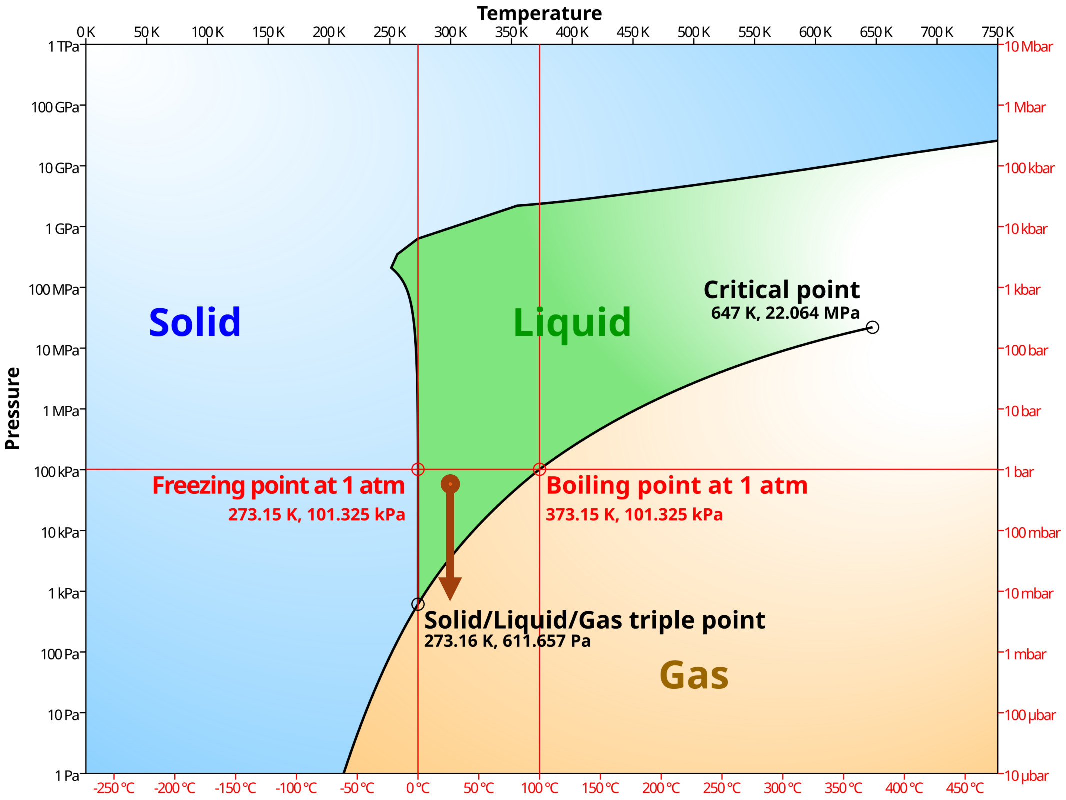
Parallel



Cavitation

- Cavitation: when liquid pressure at a given location is reduced to the vapor pressure, vapor form (the liquid starts to boil) → loss of efficiency and structural damage





Net Positive Suction Head (NPSH)

To characterize the potential for cavitation, the difference between the total head on the suction side, near the pump impeller inlet ($p_s/\gamma + V_s^2/2g + h_s$) and the **liquid vapor pressure** head, (p_v/γ) is used.

Net positive suction head
$$\mathrm{NPSH} = \frac{p_s}{\gamma} + \frac{V_s^2}{2g} + z_s - \frac{p_v}{\gamma}$$

The position reference for the elevation head usually passes through the centerline of the pump impeller inlet z_s=0

Required NPSH, NPSH_R must be maintained, or exceeded, so that cavitation will not occur

detecting cavitation or observing a change in the head–flowrate curve
pump providers give this from measurements

Available NPSH, NPSH_A represents the head that actually occurs for the particular flow system

NPSH

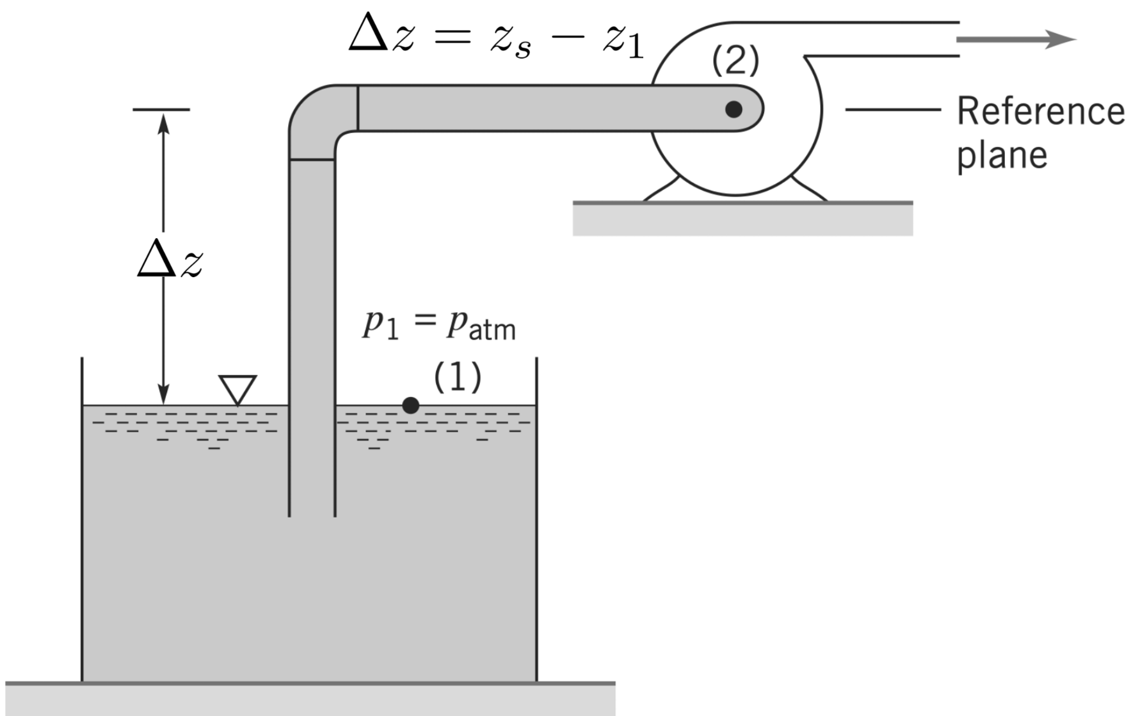
suspion side, near the pump impeller inlet

$$\frac{p_{\text{atm}}}{\gamma} + z_1 = \frac{p_s}{\gamma} + \frac{V_s^2}{2g} + z_s + \sum h_L$$

$$\frac{p_s}{\gamma} + \frac{V_s^2}{2g} = \frac{p_{\text{atm}}}{\gamma} - \Delta z - \sum h_L$$

Available NPSH,

$$\text{NPSH}_\text{A} = \frac{p_{\text{atm}}}{\gamma} - \Delta z - \sum h_L - \frac{p_v}{\gamma}$$



For proper pump operation, it is necessary:

$$\text{NPSH}_A \geq \text{NPSH}_\text{R}$$

If Δz increases, NPSH_A decreases

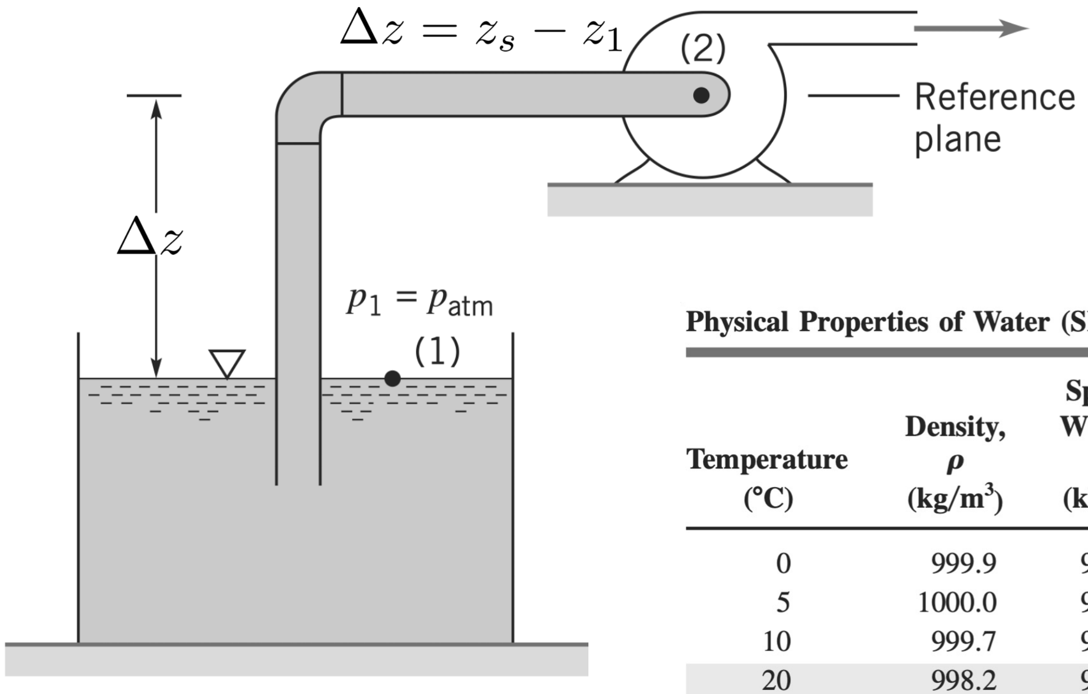
→ matching $\text{NPSH}_\text{A} = \text{NPSH}_\text{R}$

The pump operates with cavitation

Exercise

Pump water with 0.014 m^3/s the required NPSH is 4.5 m specified pump manufacturer. The water temperature is 30 °C, 101.3 kPa. The loss occurs mainly due to the filter at inlet with K_L = 20. Friction loss is neglected. Pipe is with diameter of 10 cm. Determine the max height z the pump can be located without the cavitation.

$$\mathrm{NPSH_A} = \frac{p_{\mathrm{atm}}}{\gamma} - \Delta z - \sum h_L - \frac{p_v}{\gamma}$$



Physical Properties of Water (SI Units)[a]

Temperature (°C)	Density, ρ (kg/m³)	Specific Weight[b], γ (kN/m³)	Dynamic Viscosity, μ (N·s/m²)	Kinematic Viscosity, ν (m²/s)	Surface Tension[c], σ (N/m)	Vapor Pressure, p_v [N/m²(abs)]
0	999.9	9.806	1.787 E − 3	1.787 E − 6	7.56 E − 2	6.105 E + 2
5	1000.0	9.807	1.519 E − 3	1.519 E − 6	7.49 E − 2	8.722 E + 2
10	999.7	9.804	1.307 E − 3	1.307 E − 6	7.42 E − 2	1.228 E + 3
20	998.2	9.789	1.002 E − 3	1.004 E − 6	7.28 E − 2	2.338 E + 3
30	995.7	9.765	7.975 E − 4	8.009 E − 7	7.12 E − 2	4.243 E + 3
40	992.2	9.731	6.529 E − 4	6.580 E − 7	6.96 E − 2	7.376 E + 3

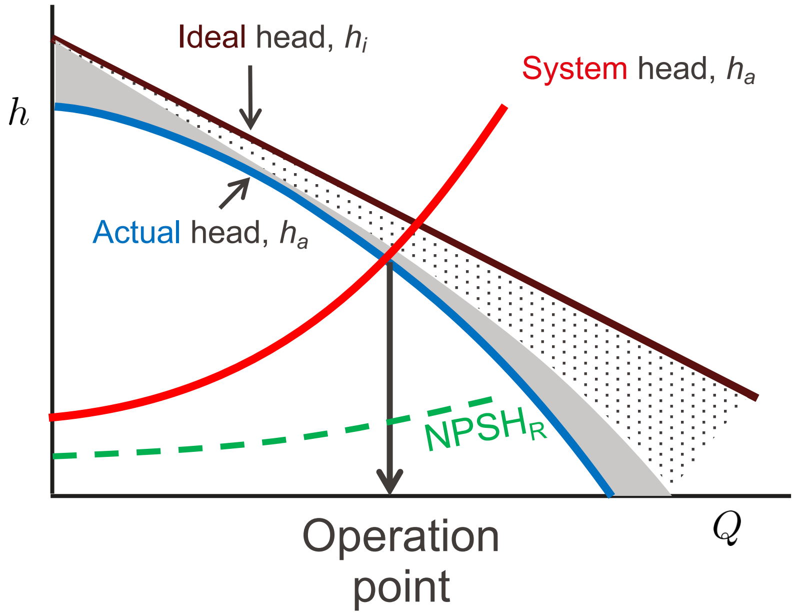
Recap

Ideal head rise of pump

$$h_i = \frac{U_2^2}{g} - \frac{U_2 \cot \beta_2}{2\pi r_2 b_2 g} Q$$

Depends only on the geometry and rotational speed of pump

Actual head rise of pump



System head

$$h_a = z_2 - z_1 + KQ^2$$

$NPSH_R$: Net positive suction head (maintained above to avoid cavitation)

Pump Similarity rule

Dimensionless Parameters and Similarity laws

Dependent pi term

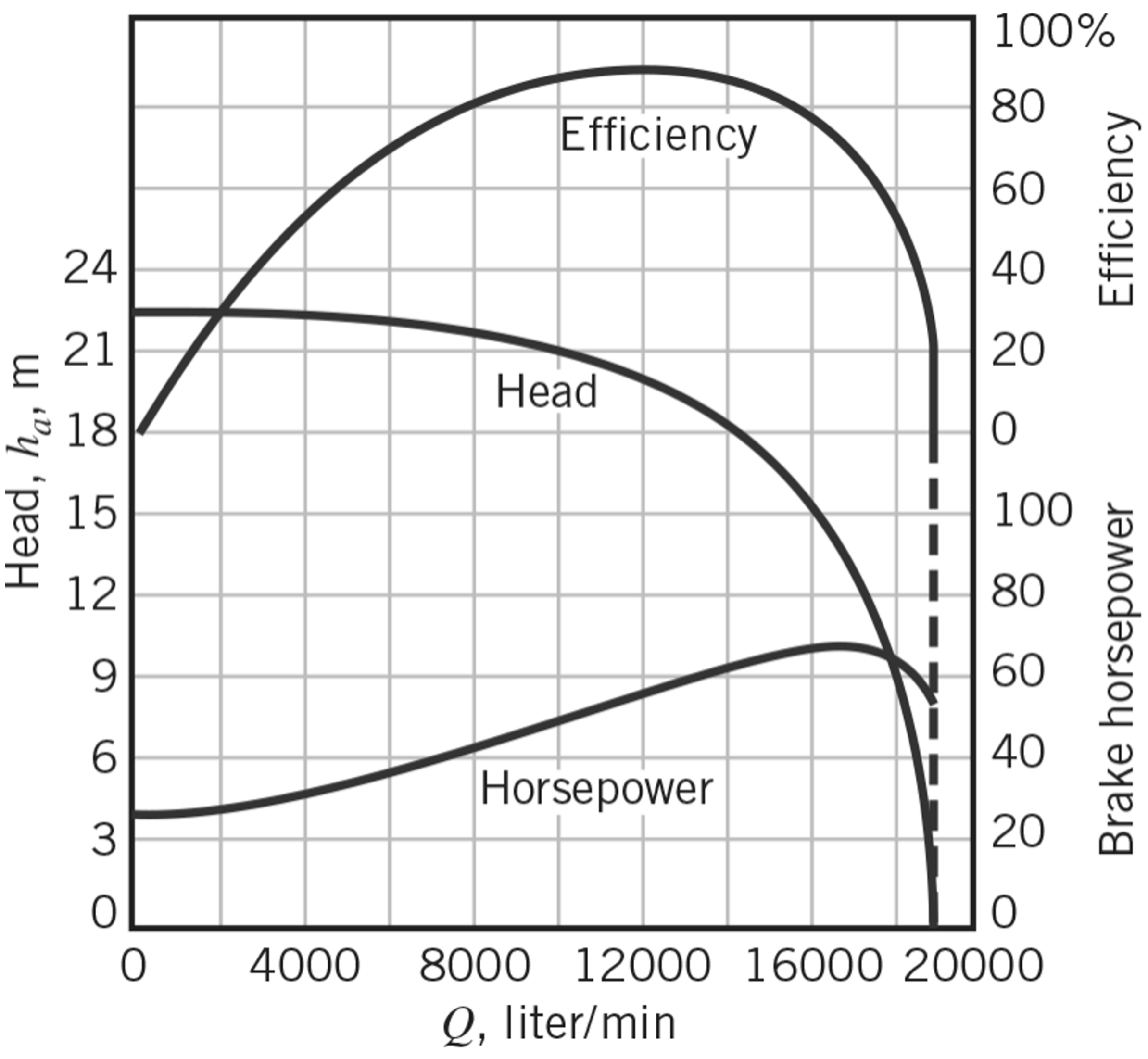
- Head rise coefficient $$C_H = \frac{gh_a}{\omega^2 D^2} = \phi_1\left(\cancel{\frac{\ell_i}{D}}, \cancel{\frac{\varepsilon}{D}}, \frac{Q}{\omega D^3}, \cancel{\frac{\rho\omega D^2}{\mu}}\right) \simeq \phi_1\left(\frac{Q}{\omega D^3}\right)$$
- Power coefficient $$C_{\mathscr{P}} = \frac{\dot{W}_{\text{shaft}}}{\rho\omega^3 D^5} = \phi_2\left(\cancel{\frac{\ell_i}{D}}, \cancel{\frac{\varepsilon}{D}}, \frac{Q}{\omega D^3}, \cancel{\frac{\rho\omega D^2}{\mu}}\right) \simeq \phi_2\left(\frac{Q}{\omega D^3}\right)$$
- Efficiency $$\eta = \frac{\rho g Q h_a}{\dot{W}_{\text{shaft}}} = \phi_3\left(\cancel{\frac{\ell_i}{D}}, \cancel{\frac{\varepsilon}{D}}, \frac{Q}{\omega D^3}, \cancel{\frac{\rho\omega D^2}{\mu}}\right) \simeq \phi_3\left(\frac{Q}{\omega D^3}\right)$$

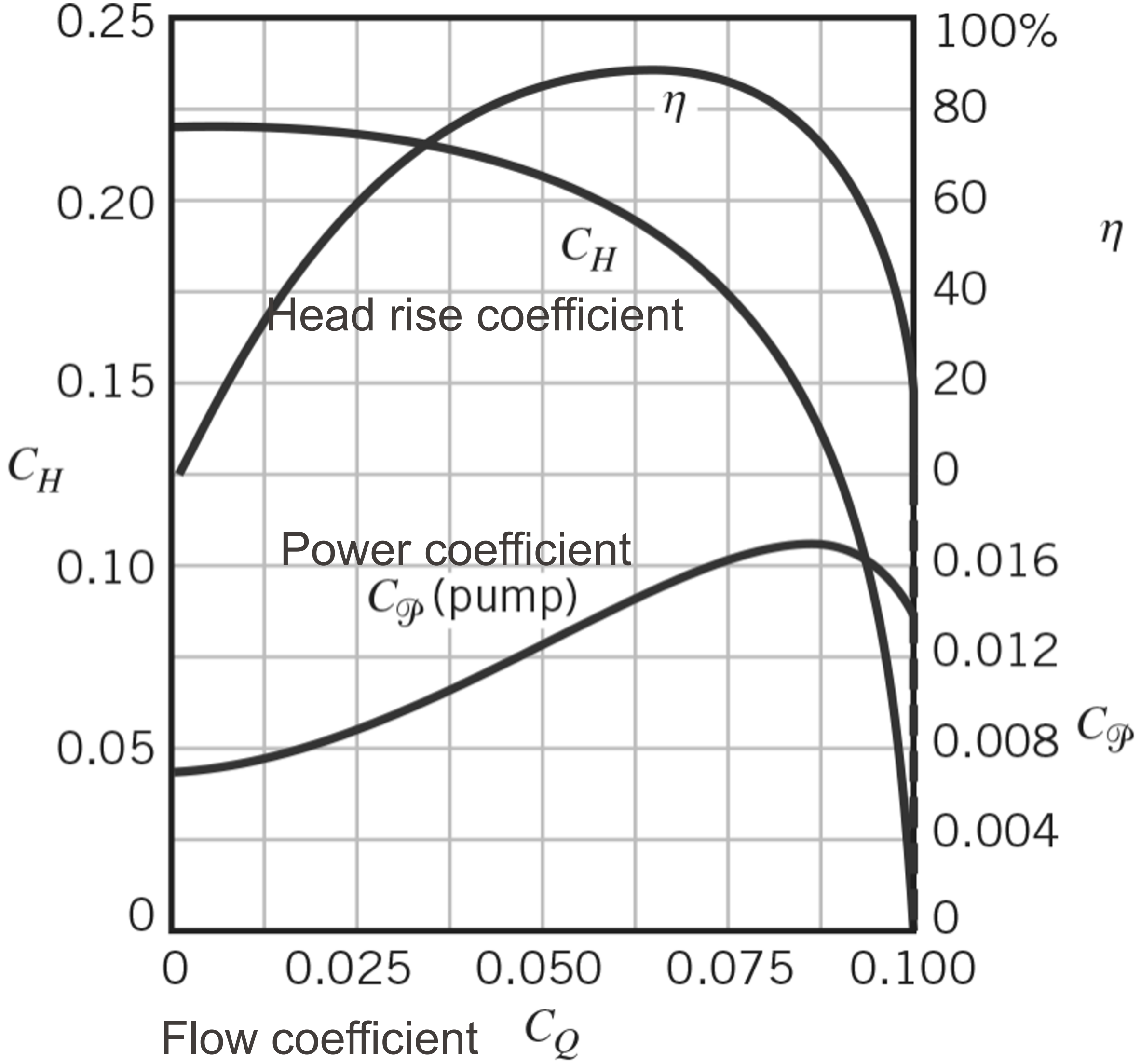
Flow coefficient $C_Q = Q/\omega D^3$

$$\eta = C_Q C_H C_{\mathscr{P}}^{-1}$$

- Reynolds effect: when Re very high, not relevant
- Roughness: pump chamber geometry is dominant
- Geometrically similar pump, (all pertinent dimensions l_i scaled with a common length scale)

Dimensionless Parameters and Similarity laws





E. Yim

Similarity relationship – pump scaling law

Among a family of geometrical similar pumps

If two pumps from the family are operated at the same value of flow coefficient

$$\left(\frac{Q}{\omega D^3}\right)_1 = \left(\frac{Q}{\omega D^3}\right)_2$$

$$\Downarrow$$

- Head rise coefficient C_H
$$\left(\frac{gh_a}{\omega^2 D^2}\right)_1 = \left(\frac{gh_a}{\omega^2 D^2}\right)_2$$
- Power coefficient $C_{\mathscr{P}}$
$$\left(\frac{\dot{W}_{\text{shaft}}}{\rho\omega^3 D^5}\right)_1 = \left(\frac{\dot{W}_{\text{shaft}}}{\rho\omega^3 D^5}\right)_2$$
- Efficiency
$$\eta_1 = \eta_2$$

Where the subscripts 1 and 2 refer to any two pumps from the family of **geometrically similar pumps**

Pump scaling laws: it is possible to experimentally determine the performance characteristics of one pump in the laboratory and then use these data to predict the corresponding characteristics for other pumps within the family under different operating conditions.

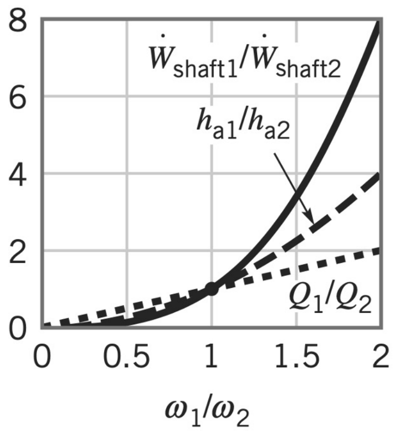
Special pump scaling laws – const. flow coefficient

$$\left(\frac{Q}{\omega D^3}\right)_1 = \left(\frac{Q}{\omega D^3}\right)_2$$

$$\left(\frac{gh_a}{\omega^2 D^2}\right)_1 = \left(\frac{gh_a}{\omega^2 D^2}\right)_2$$

$$\left(\frac{\dot{W}_{\text{shaft}}}{\rho\omega^3 D^5}\right)_1 = \left(\frac{\dot{W}_{\text{shaft}}}{\rho\omega^3 D^5}\right)_2$$

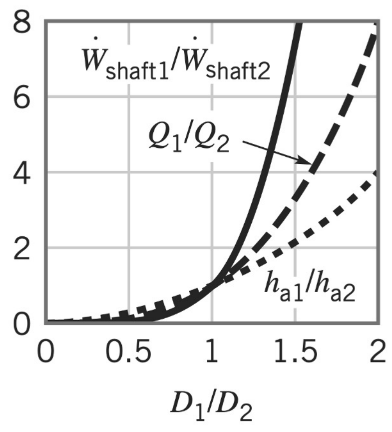
Speed change keeping $D_1 = D_2$



Flow varies proportional to the speed
Head varies as the speed squared
Power varies as the speed cubed

*All other important geometric variables are properly scaled to maintain geometric similarity.

Change in diameter keeping $\omega_1 = \omega_2$



Flow varies as the diameter cubed
Head varies as the diameter squared
Power varies as the diameter raised to the fifth power

Discussions

This type of geometric scaling is not always possible due to practical difficulties associated with manufacturing the pumps.
It is common practice for manufacturers to put impellers of different diameters in the same size pump casing. → Complete geometric similarity is not maintained, thus the similarity rule is not valid.

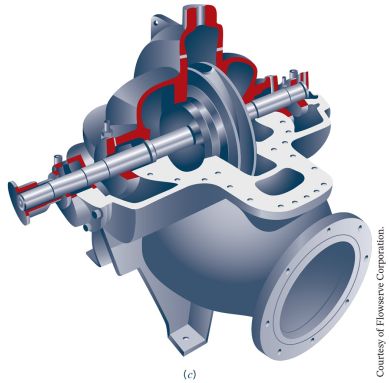
Similarity rule: effects of **viscosity** and **surface roughness** have been neglected

→ more significantly influence efficiency (because of smaller clearances and blade size)

Empirical relationship $$\frac{1-\eta_2}{1-\eta_1} \approx \left(\frac{D_1}{D_2}\right)^n, \quad n = \frac{1}{4}, \frac{1}{5}$$

The similarity laws is not very accurate between tests on a model pump in water and pump in viscous oil

Exercise – similarity



Liquid water with density p = 998 kg/m^3 flows through an axial flow pump, with a rotor diameter of 30 cm at a rate of 200 m^3/h. The pump operates at 1600 rpm, and its efficiency is 0.78. The pump power is 10 kW. If a second pump in the same series has a diameter of 20 cm and operates at 3200 rpm, at the condition of the same efficiency, find (a) mass flow rate, (b) the total pressure increase across it, and the input power.

(a) Since the pumps are geometrically similar and their efficiencies are the same, dynamic similarity may be assumed. Thus

(b) With equal power coefficient and efficiency, it follows that

The total pressure rise is given by

$$P_f = \gamma Q h_a$$
$$h_a = \Delta P / \gamma$$

Specific speed

- Flow coefficient

$$C_Q = \frac{Q}{\omega D^3}$$

- Head rise coefficient

$$C_H = \frac{g h_a}{\omega^2 D^2}$$

$$N_s = \frac{C_Q^{1/2}}{C_H^{3/4}} =$$

Dimensionless !

Specific speed

- Flow coefficient

$$C_Q = \frac{Q}{\omega D^3}$$

- Head rise coefficient

$$C_H = \frac{g h_a}{\omega^2 D^2}$$

$$N_s = \frac{\omega\sqrt{Q}}{(g h_a)^{3/4}}$$

Dimensionless !

It is customary to specify a value of specific speed at the peak efficiency flow coefficient only

If required head h_a, flowrate Q and speed ω are specified it is possible to select the most efficient/appropriate type of pump

Both N_s and N_{sd} have the same physical meaning, but their magnitudes differ by a constant conversion factor (N_{sd} =2733 N_s) when ω is in rad/s

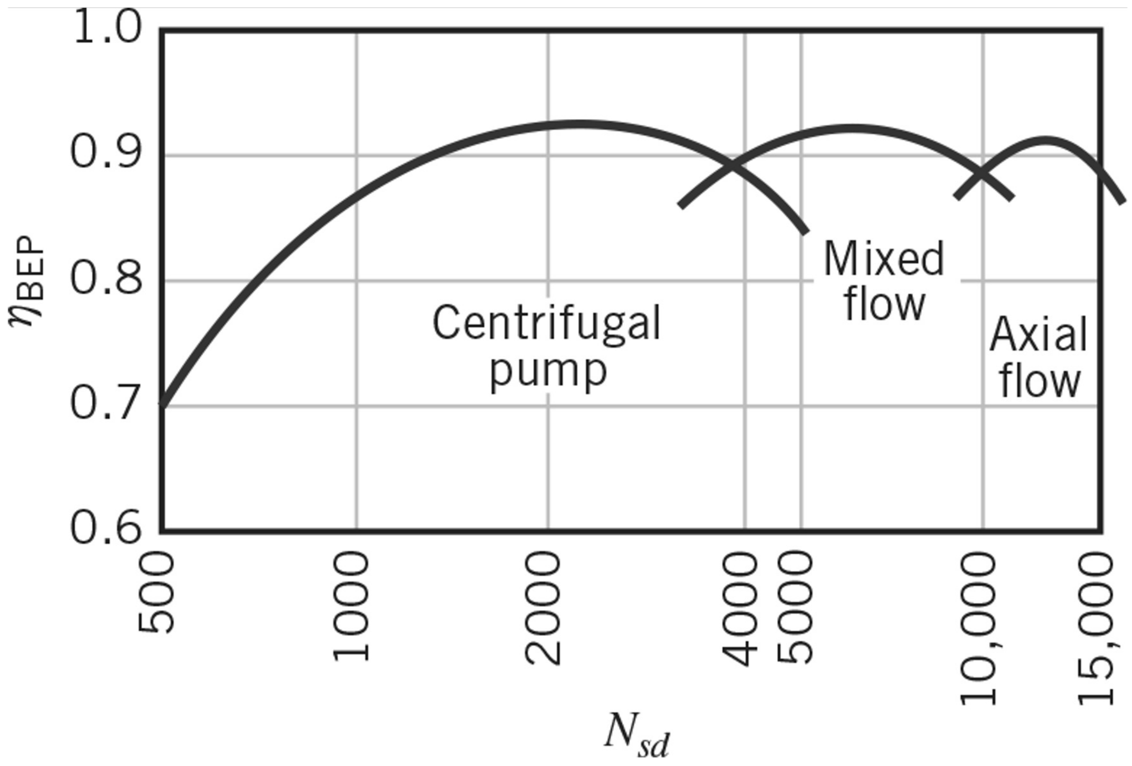
However… in the United States a modified, dimensional form of specific speed

U.S. customary units

$$N_{sd} = \frac{\omega(\mathrm{rpm})\sqrt{Q(\mathrm{gpm})}}{[h_a(\mathrm{ft})]^{3/4}}$$

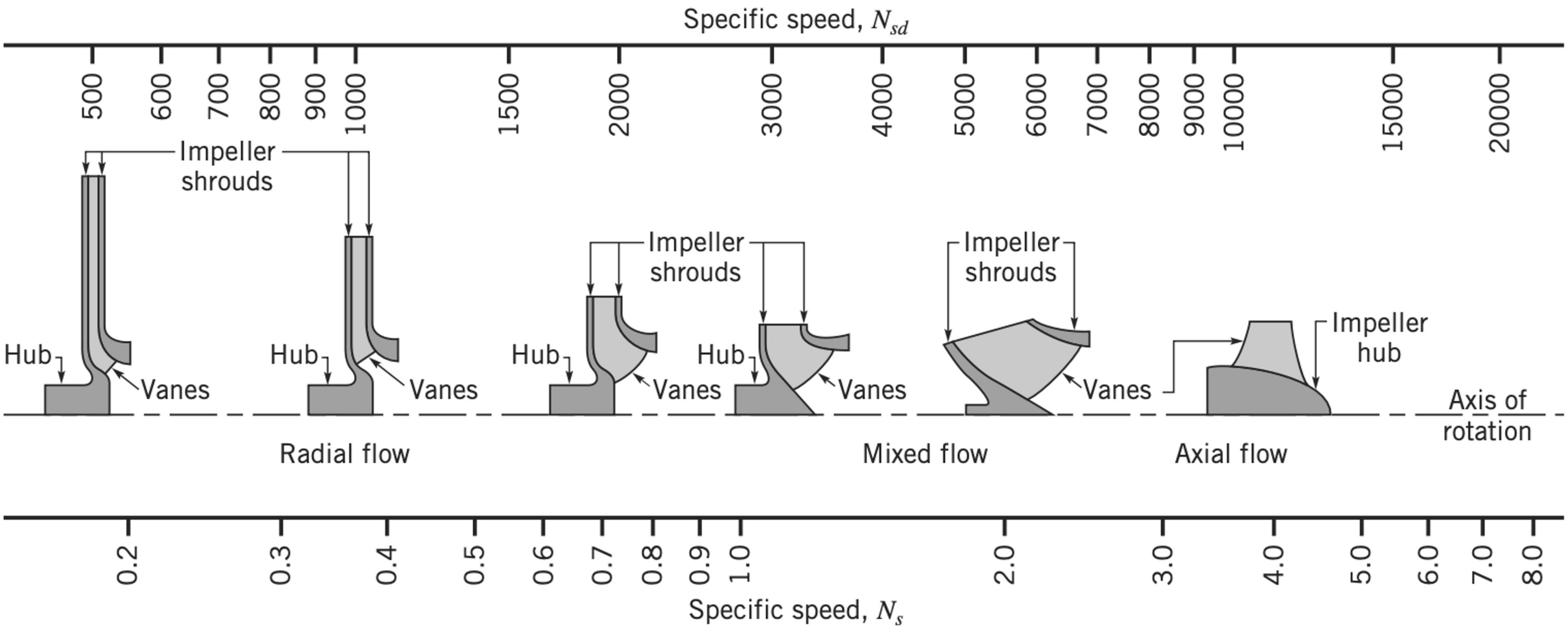
Specific speed

Variation in specific speed at maximum efficiency with types of pump



Specific speed

Variation in specific speed at maximum efficiency with types of pump



Cordier diagram

The **Cordier diagram** is based on an intensive empirical analysis of proven turbomachinery using extensive experimental data.

A pump is to be selected to pump water at the rate of **50 L/s**. The increase in total head across the pump is to be **35 m**. An electric motor, connected with a direct drive and a rotational speed of **3450 rpm**, provides the power to the pump. Water is drawn from a pool at atmospheric temperature and pressure. Its density is ρ = 998 kg/m³.

(a) Determine the type of pump for this application

$$N_s = \frac{\omega\sqrt{Q}}{(gh_a)^{3/4}} = \frac{3450\cdot\pi}{30}\,\frac{\sqrt{0.05}}{(9.81\cdot 35)^{3/4}} = 1.013$$

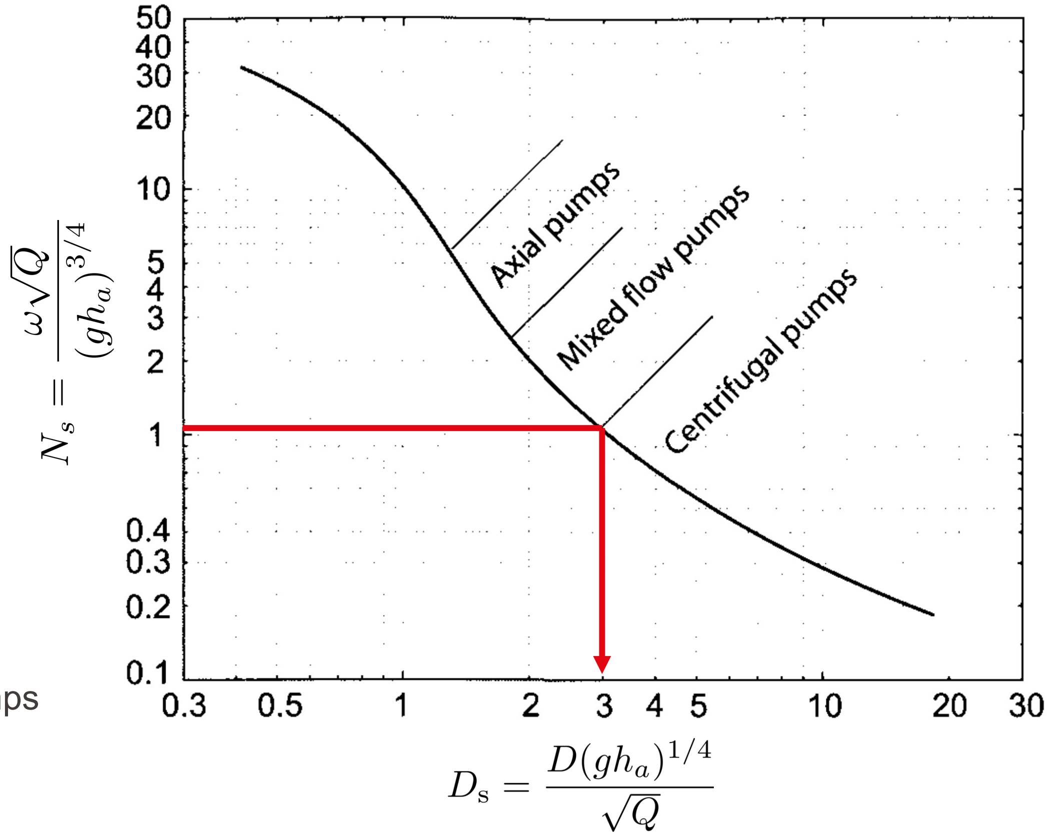
Mixed or centrifugal pumps

(b) Calculate the pump diameter

$$D = D_s\frac{Q}{(gh_a)^{1/4}} = \frac{3\sqrt{0.05}}{(9.81\cdot 35)^{1/4}} = 0.156\ \text{m}$$

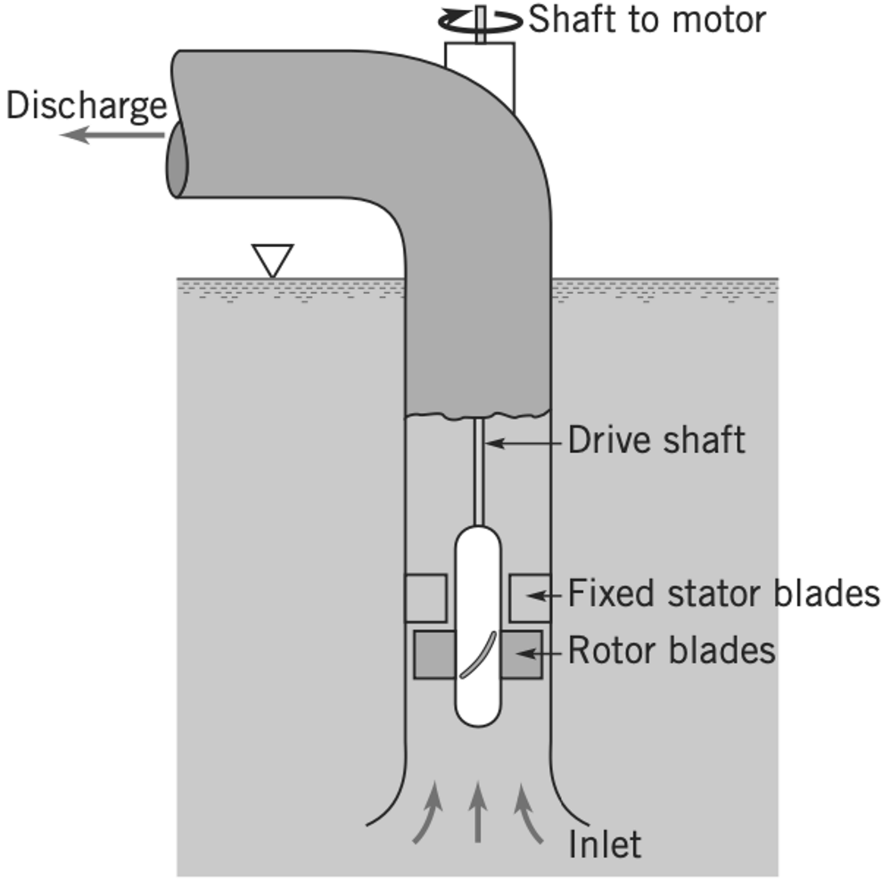
(c) Calculate the power needed (pump efficiency of 80%)

$$\dot{W} = \frac{\gamma Q h_a}{\eta} = \frac{(998\cdot 9.81)\cdot 0.05\cdot 35}{0.8} = 21.0\ \text{kW}$$



Axial-flow pumps

For high-specific speed $N_s = \frac{\omega\sqrt{Q}}{(gh_a)^{3/4}}$, low head rise, high flowrate usually $N_s > 3$

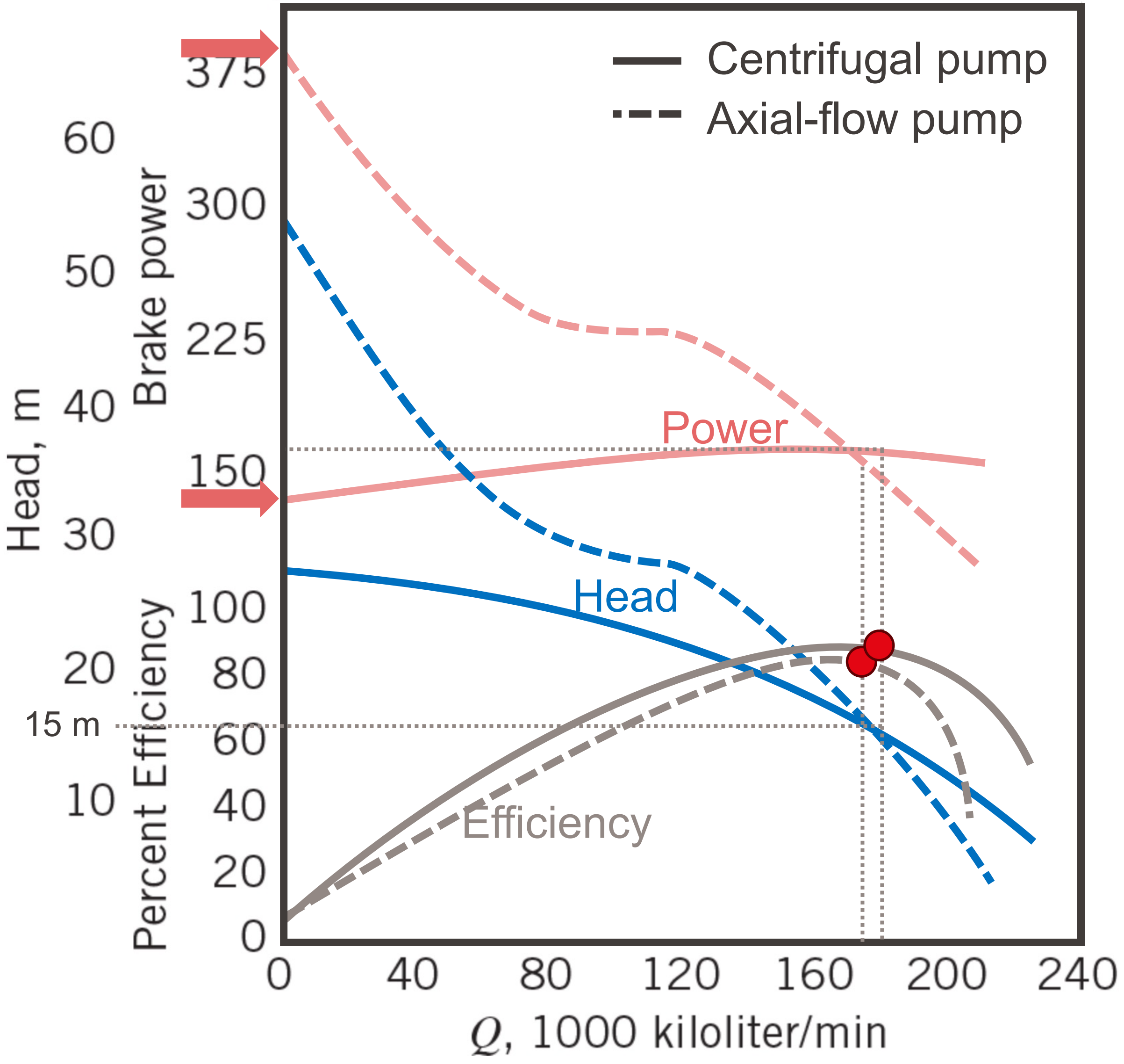


Propeller pump

EPFL

Axial-flow pumps

Typical characteristics



At a similar design capacity, the head and brake power

- Shutoff power:
  130 kW (Centrifugal), 380 kW (Axial)
  Overload when Q ↓
- Head curve:
  Axial pump has steeper head curve
  Large change in *h* with a small change in Q
- Efficiency
  Axial pump has lower efficiency
  → adjustable blades

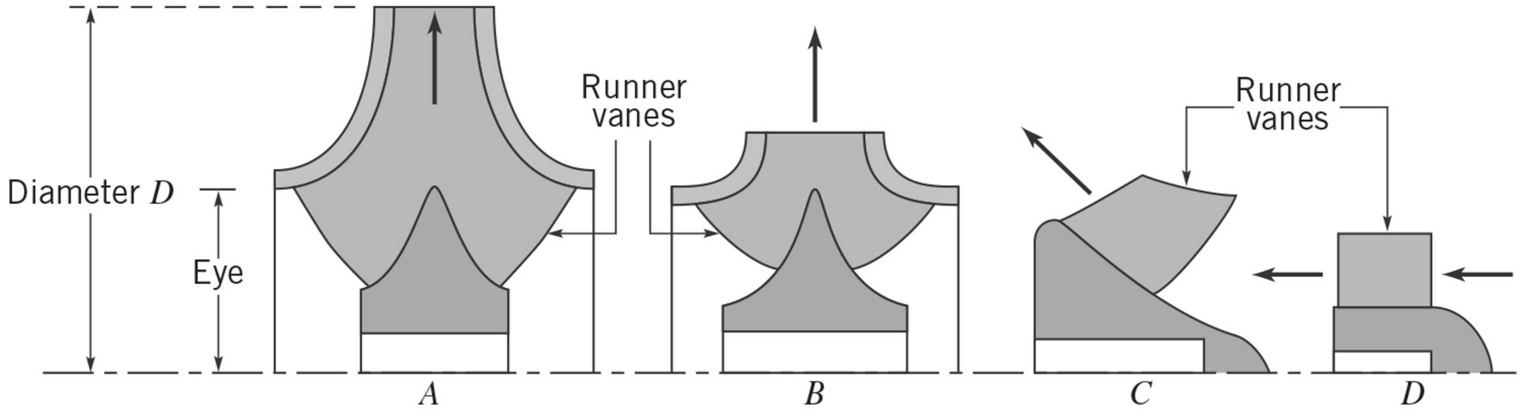
A centrifugal pump: be started with the system closed (pump requires less power with no flow)

An axial-flow pump: be started wide open (to avoid the power spike at zero flow)

Intermediate $1.5 < N_s < 3.3$ → mixed-flow pumps

Some examples different type of pumps

For the same flow rate :



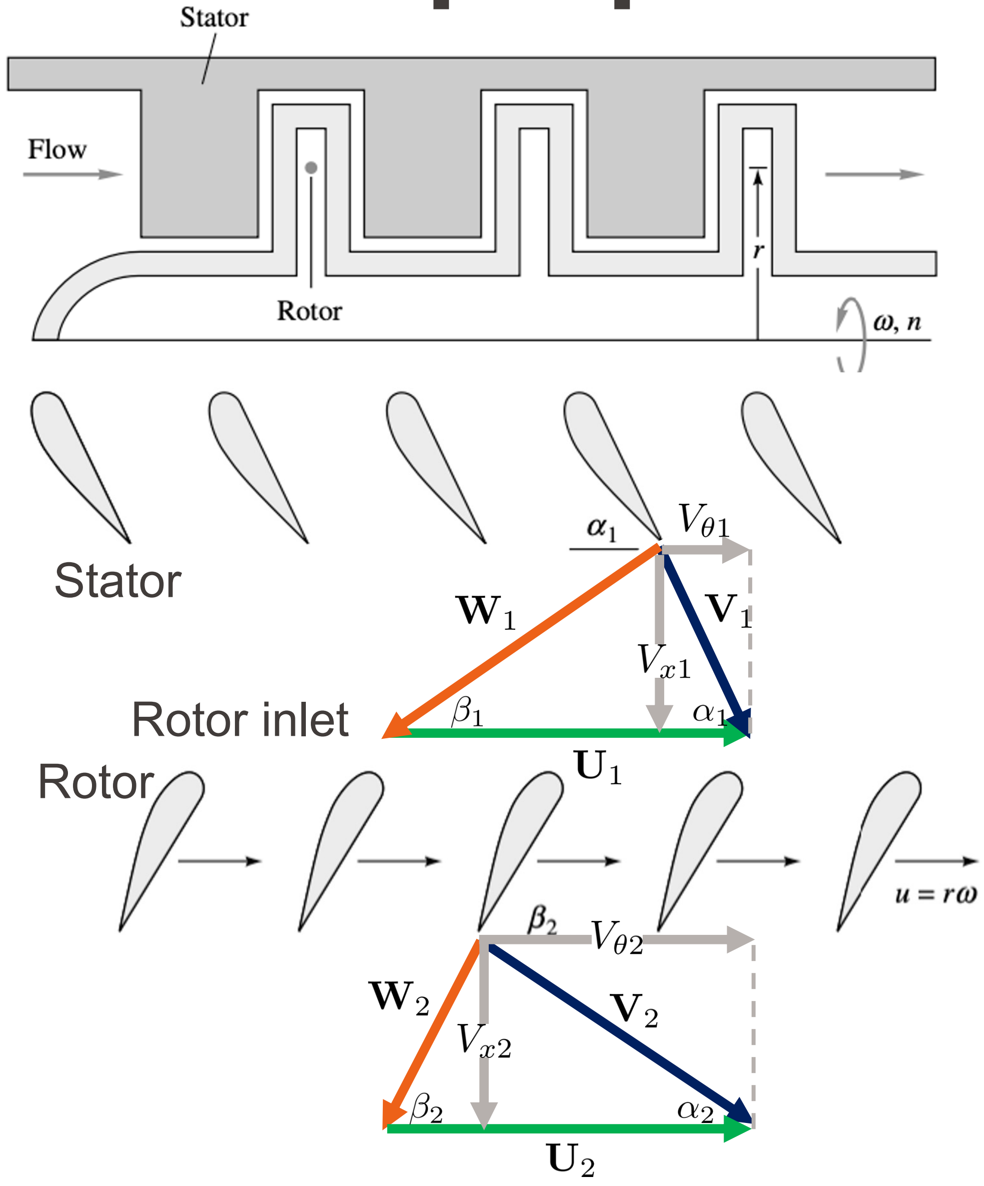
Type	Centrifugal	Centrifugal	Mixed flow	Axial flow	
N_s	0.5	0.8	2.3	5	↑
kiloliter/min	9	9	9	9	
Head, km	21	15	10	6	↓
Rpm	870	1160	1750	2600	↑
D, cm	48	30	25	18	↓
D_{eye}/D	0.5	0.7	0.9	1.0	↑

Centrifugal pump → mixed-flow pump → axial-flow pump:

specific speed increases, head decreases, speed increases, impeller diameter decreases, and eye diameter increases

The dimensionless parameters and scaling relationships apply to all three types of pumps.

Axial pump



Since the stator is fixed, ideally the absolute velocity V_1 is parallel to the trailing edge of the blade

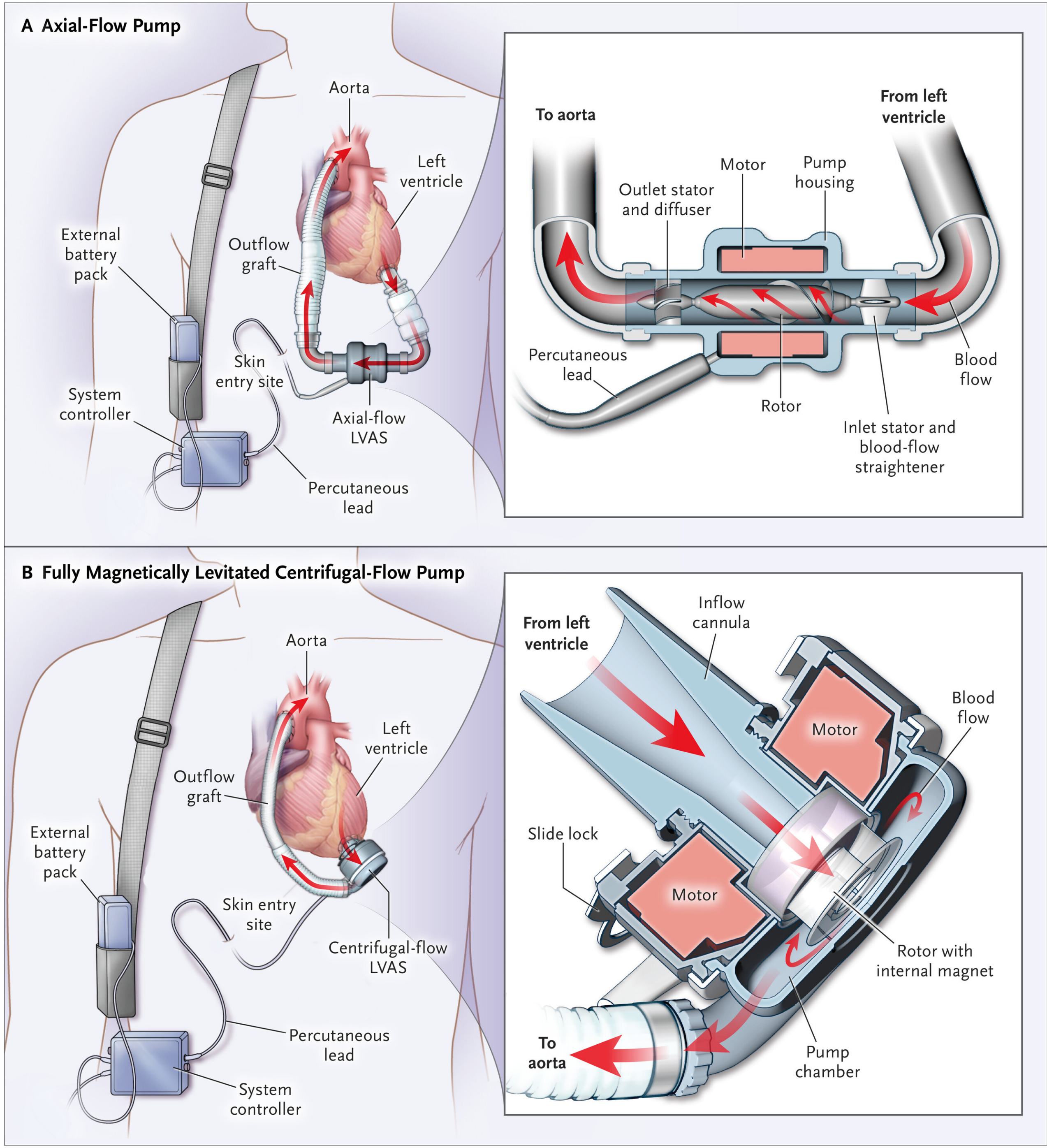
$$V_{x1} = V_{x2} = V_x = \frac{Q}{A} = \text{const}$$

The ideal head expressed stator angle α_1 and rotor angle β_2

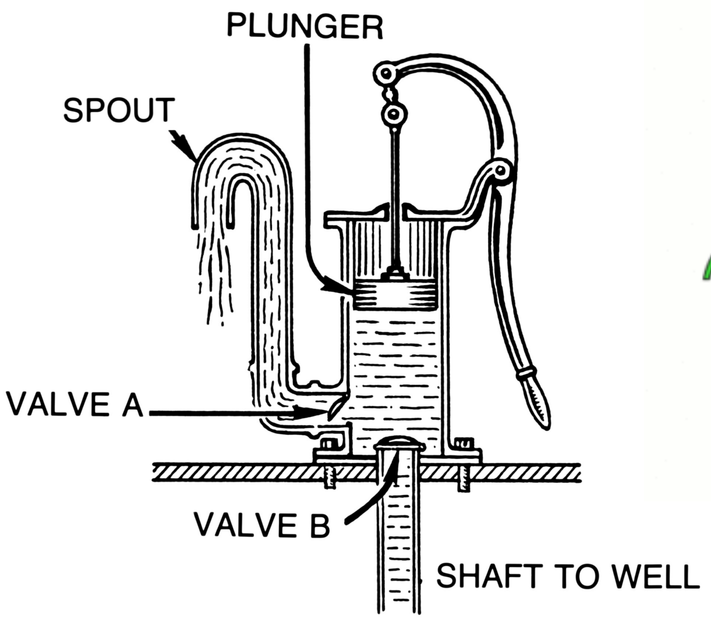
Strictly speaking, this applies only to a single streamtube of radius r, but it is a good approximation for very short blades if r denotes the average radius.

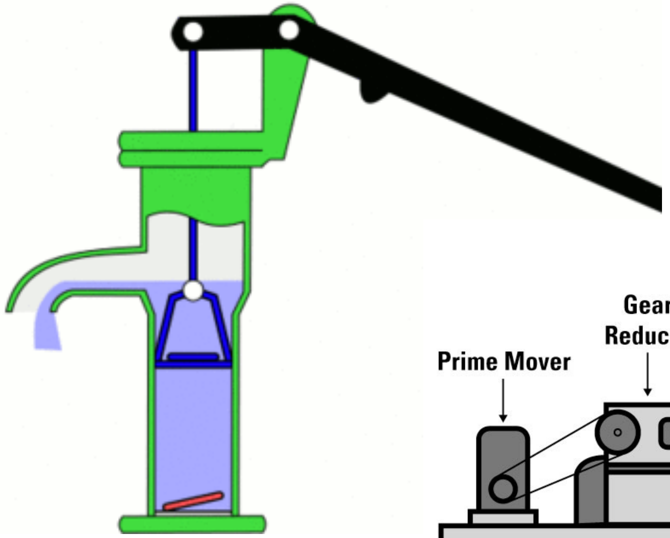
Mechanical heart assist devices, LVAD

- Our heart is also a pump, it can also suffer from malfunctions.
- One of the more promising techniques is use of a left-ventricular assist device (LVAD), which supplements a diseased heart.
- LVAD pump is implanted alongside the heart and works in parallel with the cardiovascular system to assist the pumping function of the heart's left ventricle (The left ventricle supplies oxygenated blood to the entire body and performs about 80% of the heart's work).
- Risks of infection, bleeding, and device-related complications like pump thrombosis or stroke.
- Patients must manage external components like batteries and power cords, which can restrict activities such as swimming or showering.

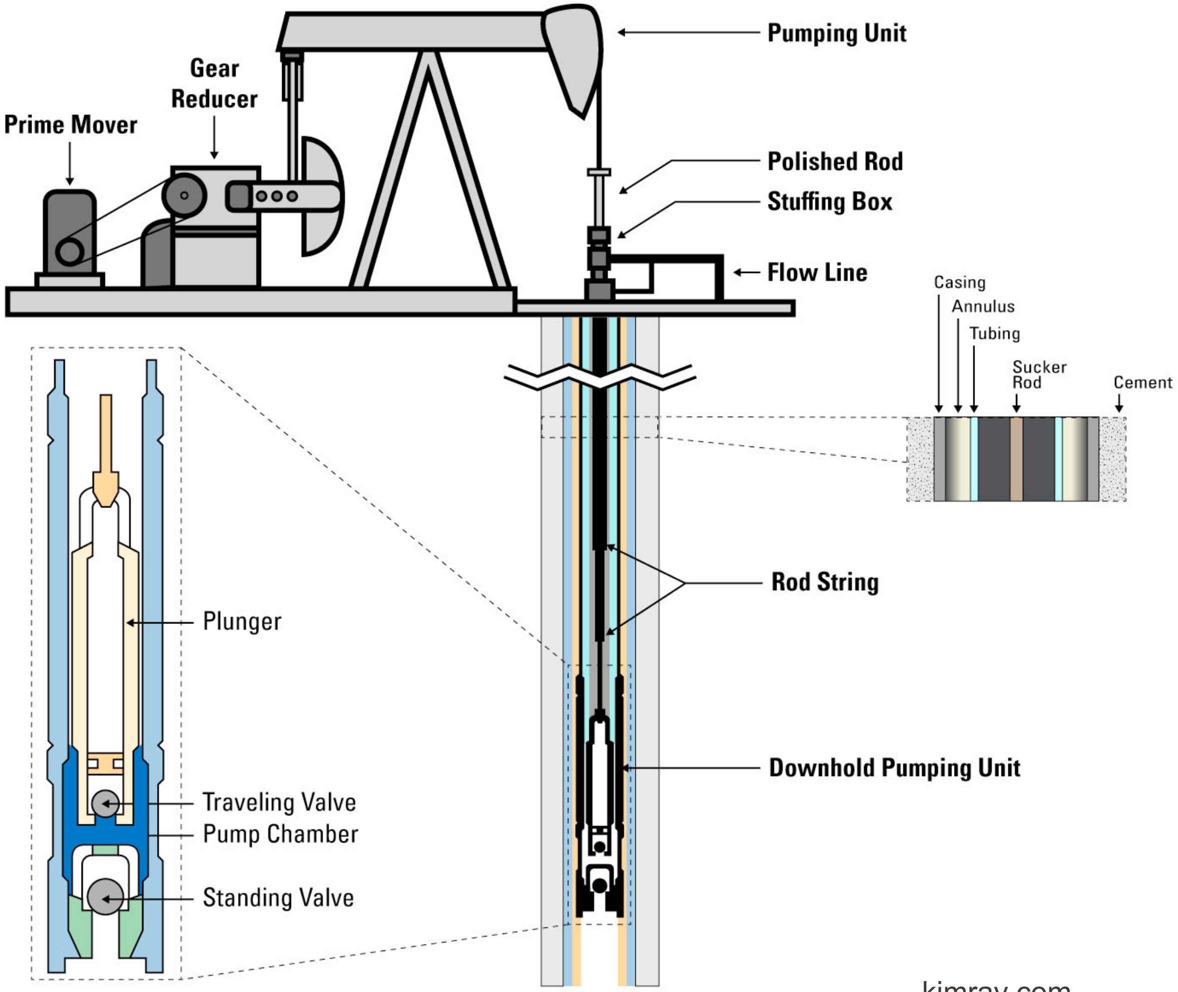


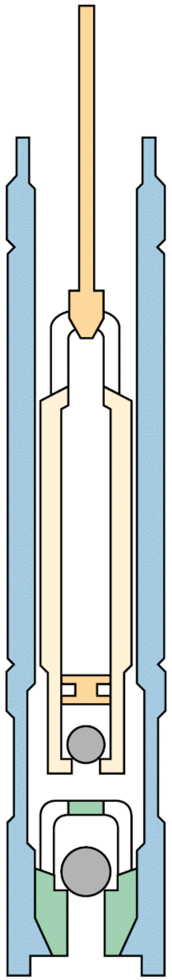
Other types of pump – piston pump





Hand pump
(Piston pump)





kimray.com