Chapter 6: Centrifugal Pump

ME-342 Introduction to turbomachinery

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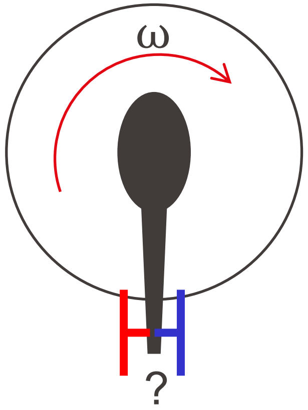
Recall-About torques in helicopter

Two main rotors in a helicopter
- horizontal lift-producing rotor
- vertical tail rotor



Without the **tail rotor** to **cancel** the **torque** from the main rotor, the helicopter body would spin out of control in the direction opposite to that of the main rotor.

Tailless jet-rotor
Jet-rotor concept removes the need for a tail rotor in helicopters.
Gas (hot exhaust gases or compressed air) is directed through the blades/Exhausted from nozzles at blade tips, perpendicular to blade axis.
Uses the **angular momentum principle** for rotation. Similar to a rotating lawn sprinkler or dishwasher arm. No drive shaft needed to turn the main rotor, eliminating torque issues

Recall-Hiller YH-32 Hornet

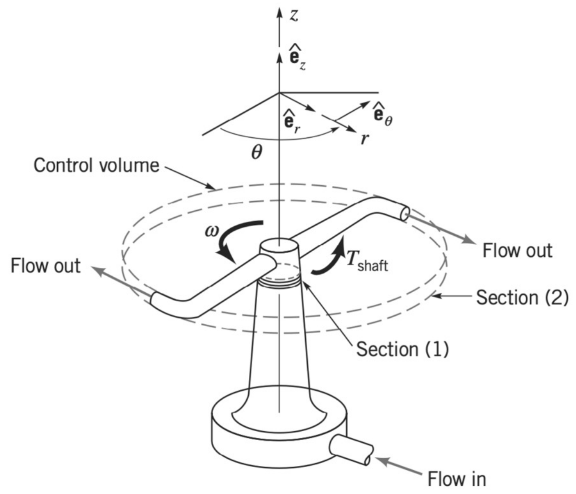
Used **peroxide-fueled tip jets** to rotate the blades

A catalyst (silver or platinum) decomposed the **hydrogen peroxide** into **high-temperature steam and oxygen**.

Not efficient compare d to conventional helicopter, fuel consumption, too noisy

SNCASO SO.1221 Djinn

First commercialised tip-jet helicopter
Cold jet – a Turbomeca gas turbine compresses air and sends it to the rotor tip



Recall - Basic governing equations for turbomachine

- **Shaft torque**

$$T_{\text{shaft}} = -\dot{m}_1 \left(r_1 V_{\theta 1}\right) + \dot{m}_2 \left(r_2 V_{\theta 2}\right)$$

- **Shaft power**

$$\dot{W}_{\text{shaft}} = T_{\text{shaft}}\,\omega = -\dot{m} r_1 V_{\theta 1} \omega + \dot{m} r_2 V_{\theta 2} \omega$$

U_1 U_2

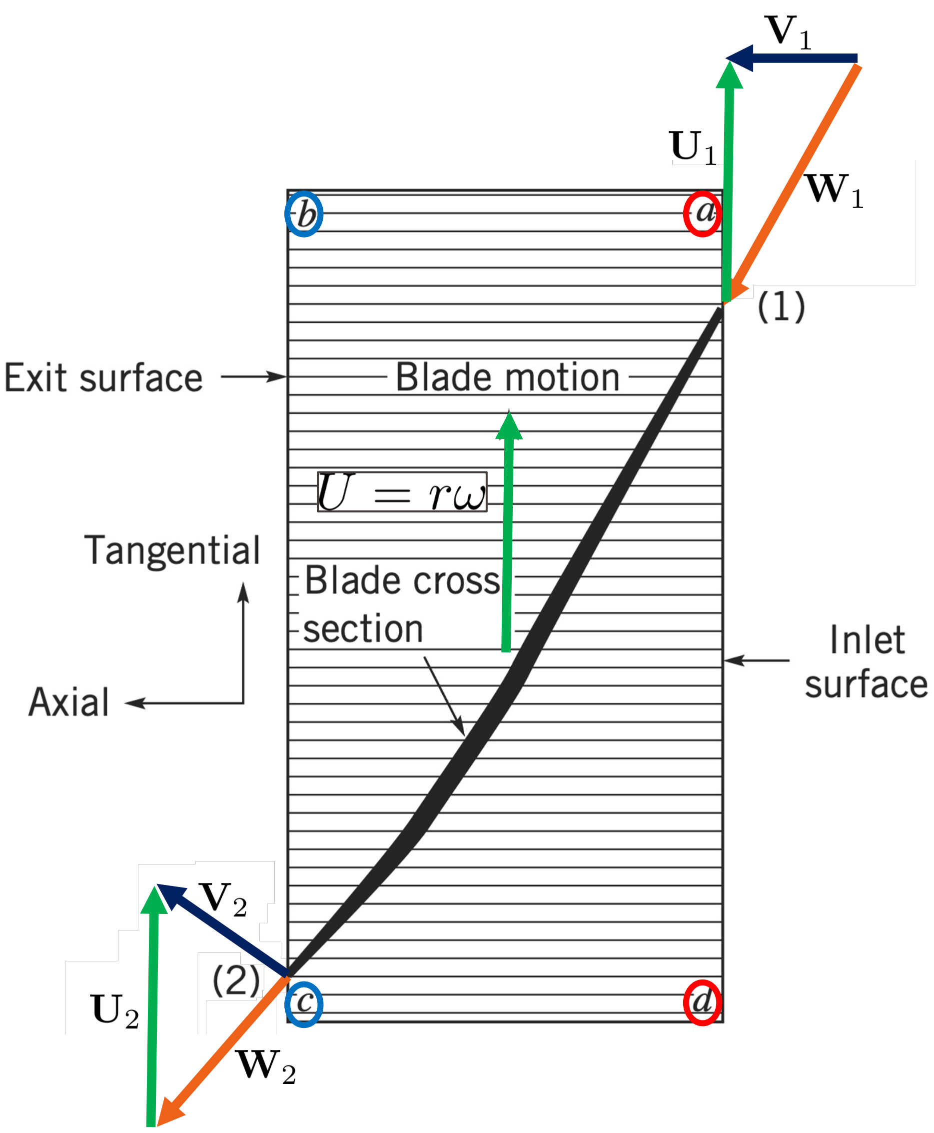
$$\dot{W}_{\text{shaft}} = \left(-\dot{m}_1\right)\left(U_1 V_{\theta 1}\right) + \dot{m}_2 \left(U_2 V_{\theta 2}\right)$$ [W] = [kg·m²/s³]

- **Shaft work per unit mass (shaft power per unit mass flow rate),** $\dot{m}_1 = \dot{m}_2$

$$w_{\text{shaft}} = -\left(U_1 V_{\theta 1}\right) + \left(U_2 V_{\theta 2}\right)$$ [m²/s²]

- Basic governing equations for pumps or turbines whether the machines are radial-, mixed-, or axial-flow devices and for compressible and incompressible flows
- Note it is only the function of tangential component of velocity, no V_r, V_x

Recall - Basic Energy Considerations



- **Velocity diagram:**
  The actual (**absolute**) velocity is the vector sum of the **relative** and **blade** velocities

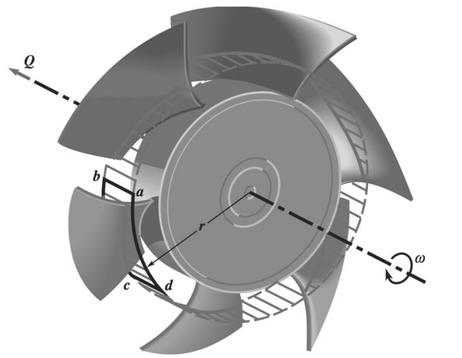
$$\mathbf{V} = \mathbf{W} + \mathbf{U}$$

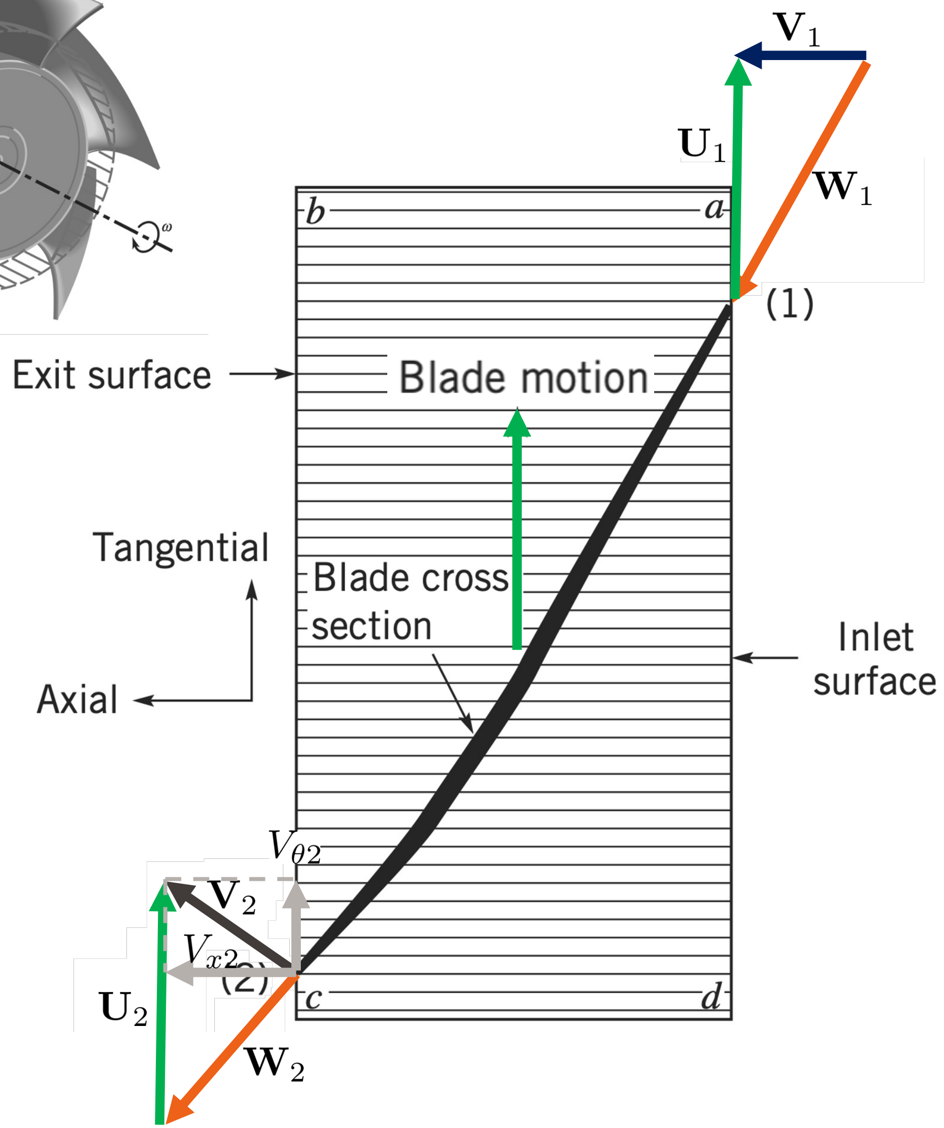
$\mathbf{V}$ = Absolute fluid velocity

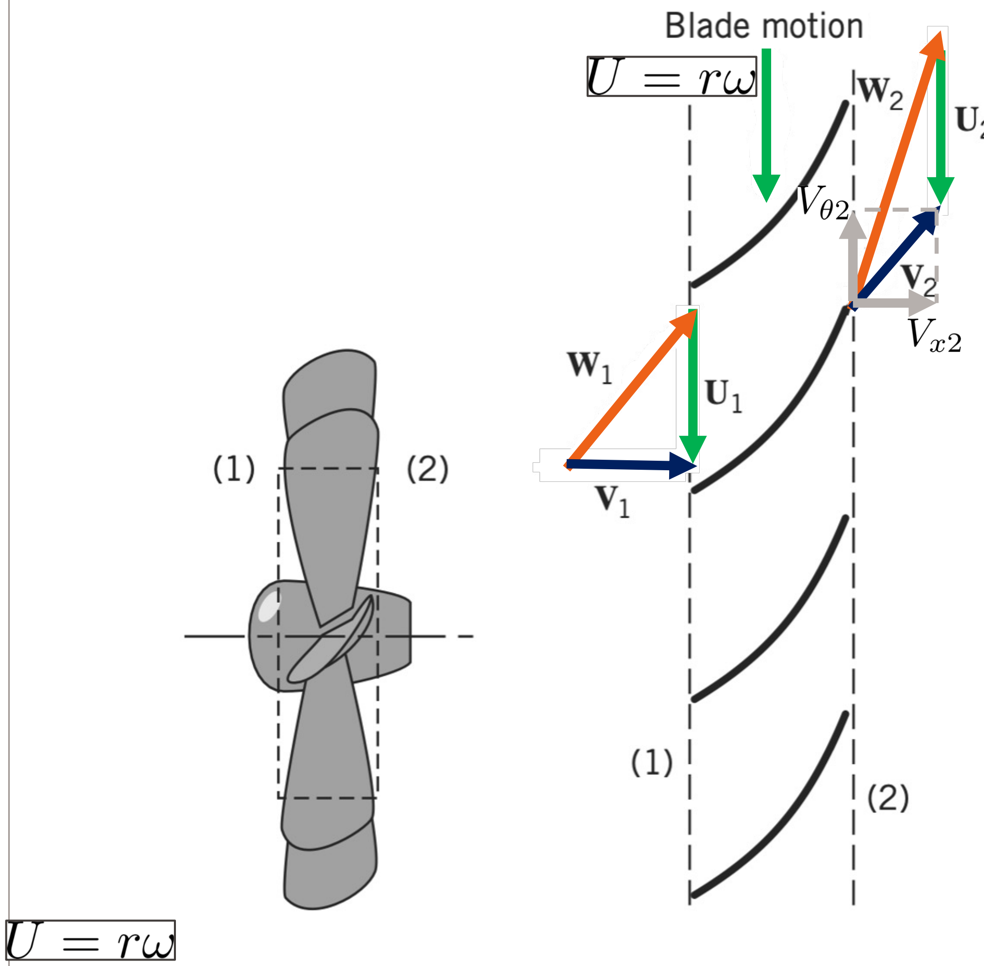
$\mathbf{W}$ = Relative velocity

$\mathbf{U}$ = Blade speed, ωr

Recall - Pump vs. turbine

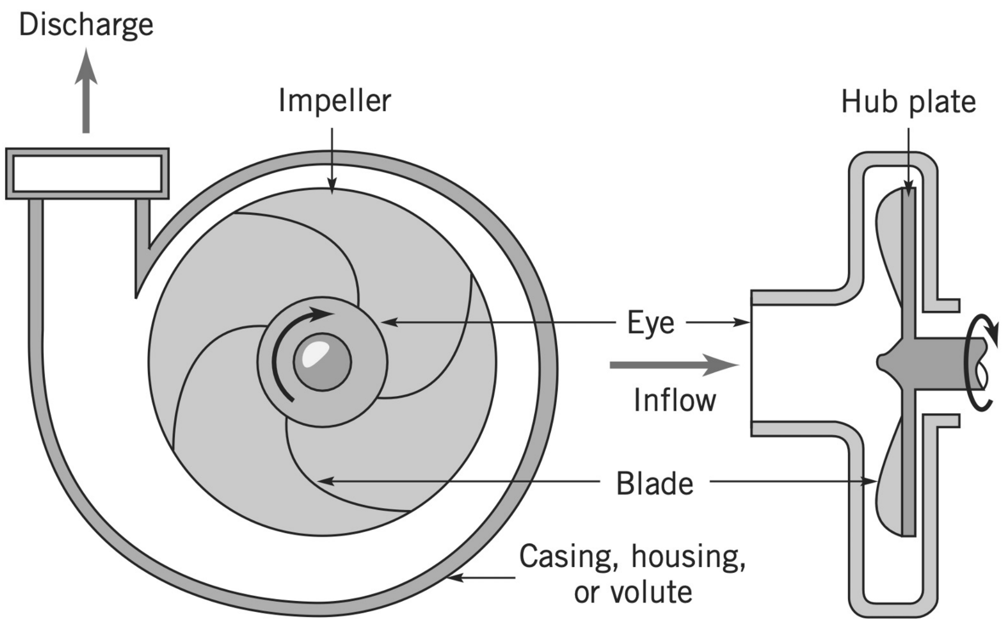


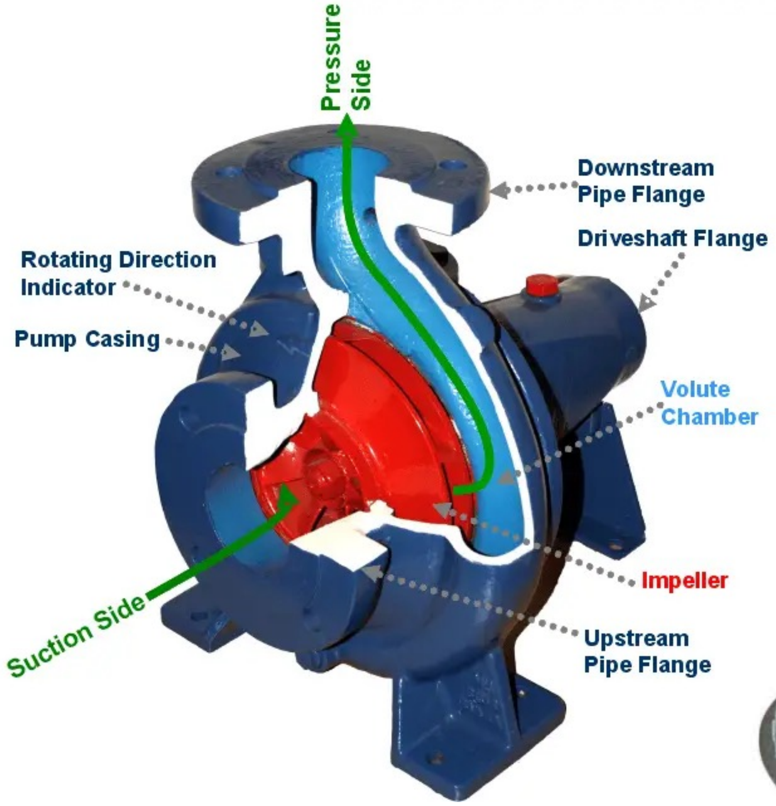


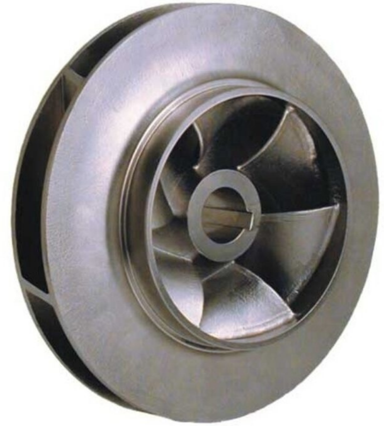


Centrifugal Pump

Centrifugal pump

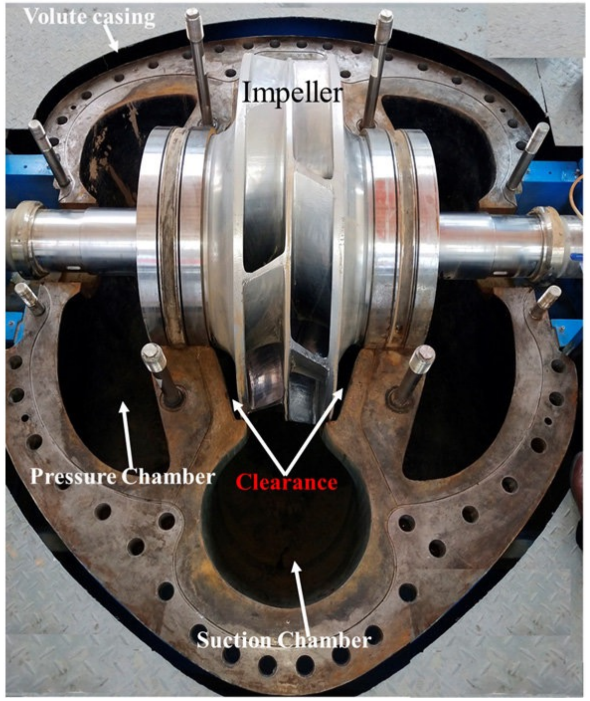




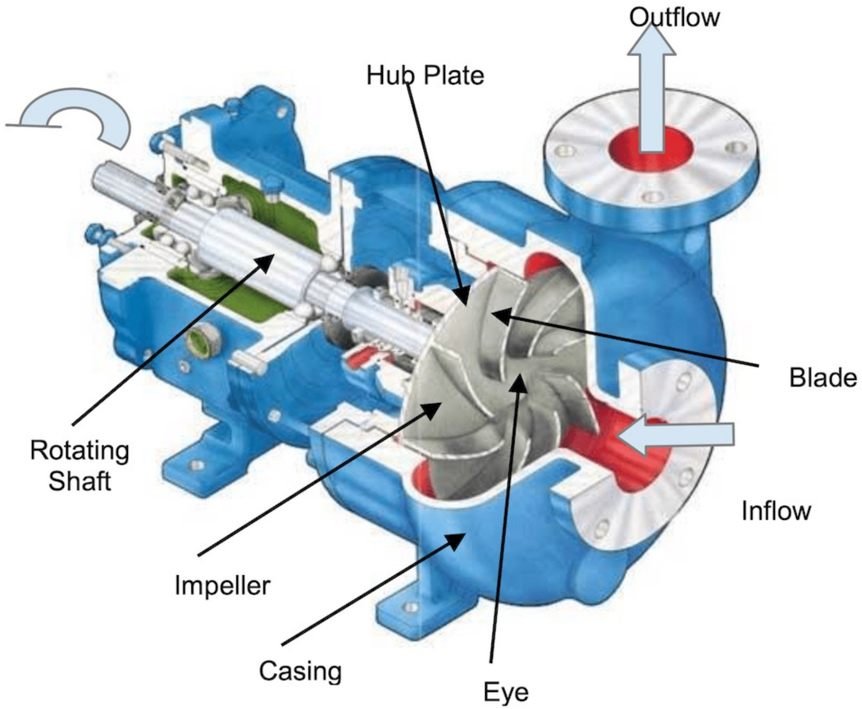


Open or shrouded impellers

Centrifugal pump



Double suction

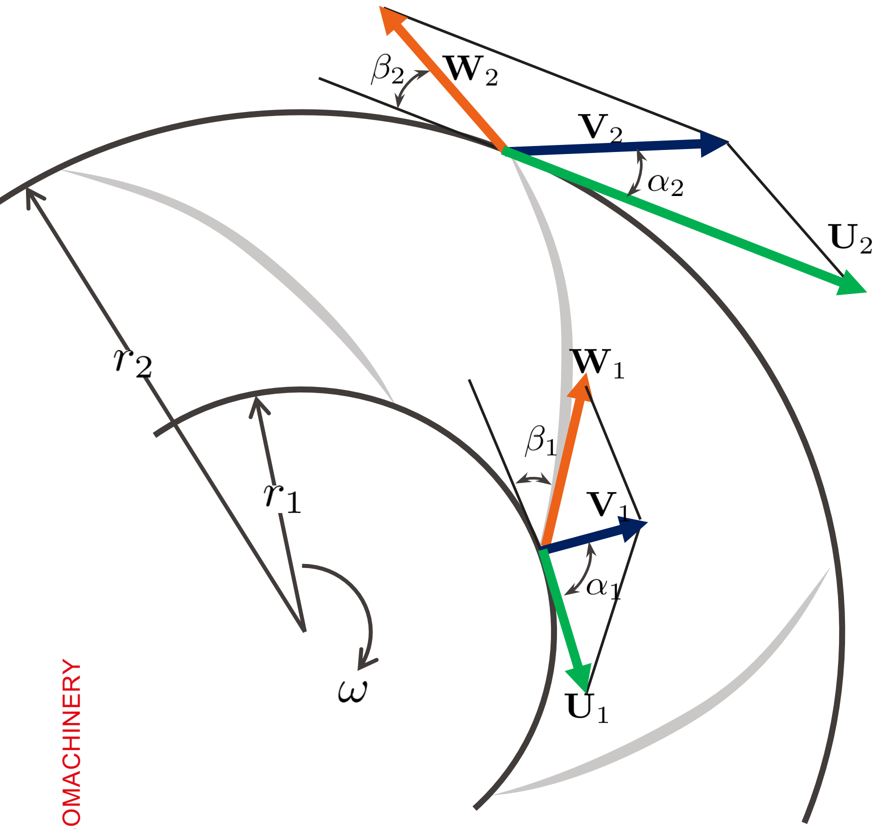


Single suction

Multi-stage
- Operation in series
- Very large discharge pressure, or head, can be developed by a multistage pump

Creating a large increase in kinetic energy of the fluid flowing through the impeller.
→ converted into an increase in pressure as the fluid flows from the impeller into the casing enclosing the impeller.

Theoretical consideration



Euler turbomachine equation:

$$T_{\text{shaft}} = \dot{m}(r_2 V_{\theta 2} - r_1 V_{\theta 1}) \qquad \dot{m} = \rho Q$$

$$\dot{W}_{\text{shaft}} = T_{\text{shaft}}\,\omega$$

$$w_{\text{shaft}} = \frac{\dot{W}_{\text{shaft}}}{\rho Q} = U_2 V_{\theta 2} - U_1 V_{\theta 1}$$

For incompressible pump flow, the energy equation

$$w_{\text{shaft}} = \left(\frac{p_{\text{out}}}{\rho} + \frac{V_{\text{out}}^2}{2} + g z_{\text{out}}\right) - \left(\frac{p_{\text{in}}}{\rho} + \frac{V_{\text{in}}^2}{2} + g z_{\text{in}}\right) + \text{ loss}$$

$$U_2 V_{\theta 2} - U_1 V_{\theta 1} = \left(\frac{p_{\text{out}}}{\rho} + \frac{V_{\text{out}}^2}{2} + g z_{\text{out}}\right) - \left(\frac{p_{\text{in}}}{\rho} + \frac{V_{\text{in}}^2}{2} + g z_{\text{in}}\right) + \text{ loss}$$

Divide by gravity, g

$$\frac{U_2 V_{\theta 2} - U_1 V_{\theta 1}}{g} = \left(\frac{p_{\text{out}}}{\rho g} + \frac{V_{\text{out}}^2}{2g} + z_{\text{out}}\right) - \left(\frac{p_{\text{in}}}{\rho g} + \frac{V_{\text{in}}^2}{2g} + z_{\text{in}}\right) + h_L$$

Total head, H

$$h_L = \text{loss}/\text{g}$$

$$\frac{U_2 V_{\theta 2} - U_1 V_{\theta 1}}{g} = H_{\text{out}} - H_{\text{in}} + h_L$$

EPFL

Theoretical consideration

$$\frac{U_2 V_{\theta 2} - U_1 V_{\theta 1}}{g} = H_{out} - H_{in} + h_L$$

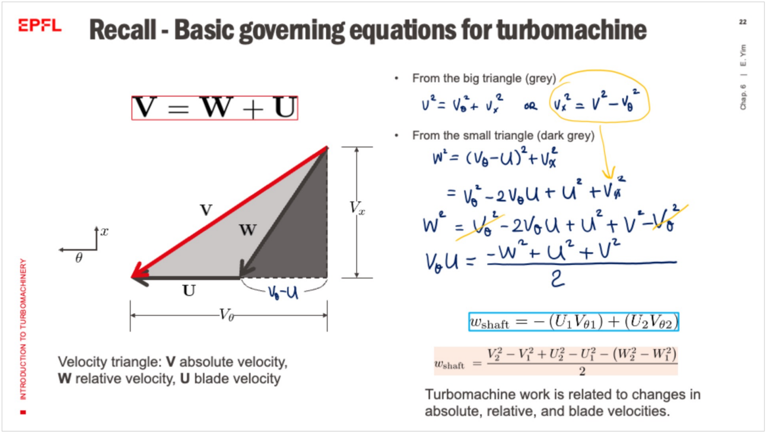
Ideal head rise, **h_i**

Actual head rise, **h_a**

$$\frac{U_2 V_{\theta 2} - U_1 V_{\theta 1}}{g} = \left(\frac{p_{out}}{\rho g} + \frac{V_{out}^2}{2g} + z_{out}\right) - \left(\frac{p_{in}}{\rho g} + \frac{V_{in}^2}{2g} + z_{in}\right) + h_L$$

Ideal head rise $h_i = \frac{1}{2g}\left[(V_2^2 - V_1^2) + (U_2^2 - U_1^2) + (W_1^2 - W_2^2)\right]$

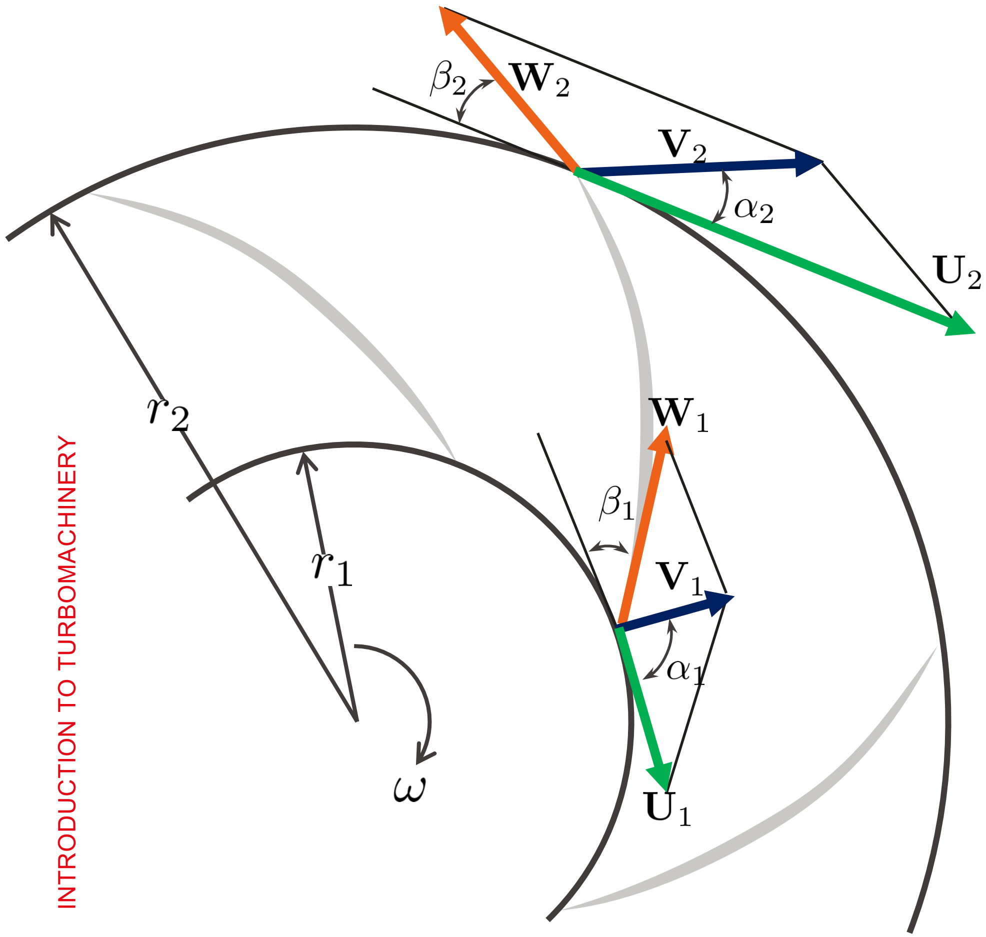
- $(V_2^2 - V_1^2)$: increase in the kinetic energy of the fluid
- $(U_2^2 - U_1^2) + (W_1^2 - W_2^2)$: pressure head rise that develops across the impeller
  - $(U_2^2 - U_1^2)$: Centrifugal effect
  - $(W_1^2 - W_2^2)$: Diffusion of relative flow in the blade passage



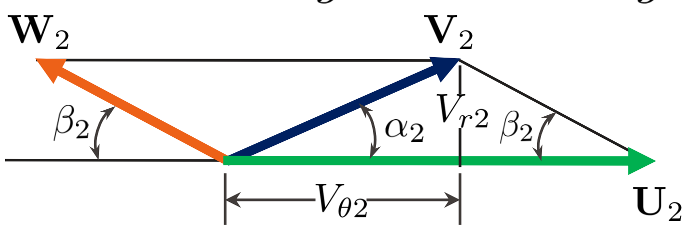
Theoretical consideration

For centrifugal pump inlet with purely radial, (no $V_{\theta 1}$ component)

$$\alpha_1 = 90^\circ \quad \mathbf{V}_1 = [V_{r1}, 0, 0]$$



$$h_i = \frac{U_2 V_{\theta 2} - U_1 V_{\theta 1}}{g} = \frac{U_2 V_{\theta 2}}{g}$$



$$\tan\beta_2 = \frac{V_{r2}}{U_2 - V_{\theta 2}} \rightarrow \cot\beta_2 = \frac{U_2 - V_{\theta 2}}{V_{r2}}$$

$$h_i = \frac{U_2^2}{g} - \frac{U_2 V_{r2} \cot\beta_2}{g} \qquad h_i = \frac{U_2^2}{g} - \frac{U_2 \cot\beta_2}{2\pi r_2 b_2 g} Q$$

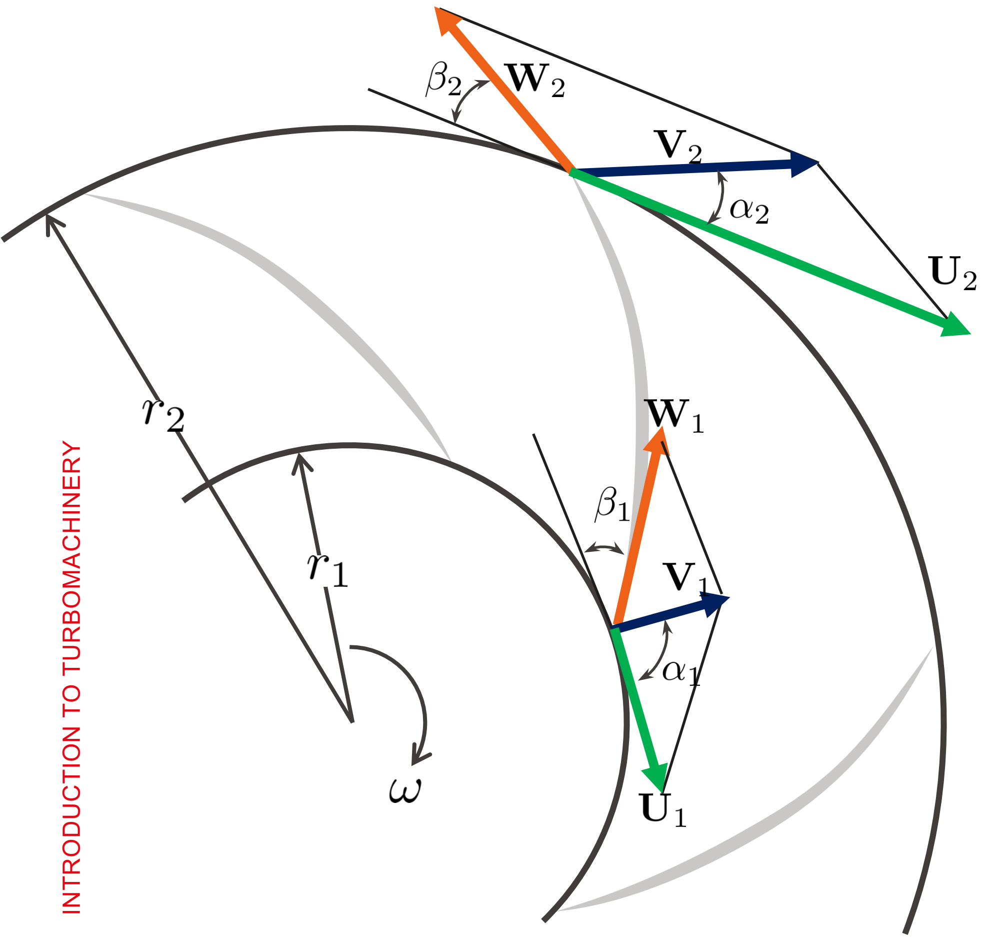
Flowrate, Q, is related to the **radial** component of the **absolute** velocity:

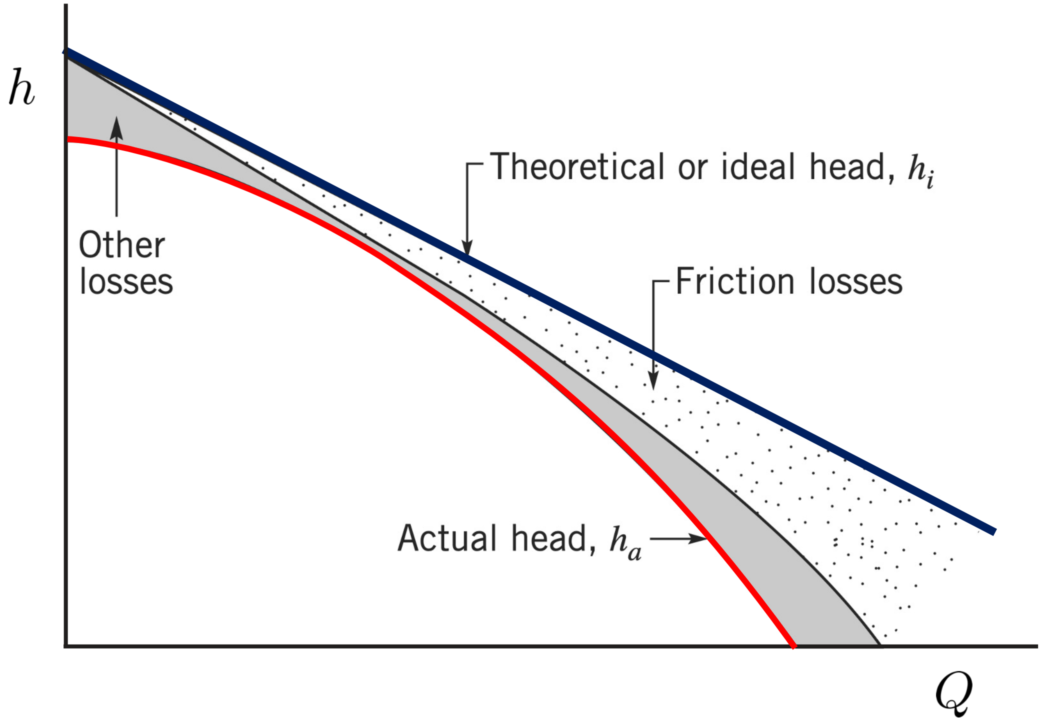
$$Q = 2\pi r_2 b_2 V_{r2}$$

Theoretical consideration

Ideal head rise can be represented as a function of flow rate and geometrical parameters of impeller only

$$h_i = \frac{U_2^2}{g} - \frac{U_2 \cot \beta_2}{2\pi r_2 b_2 g} Q$$





Losses: skin friction in the blade passages, which vary as Q^2, and other losses due to such factors as flow separation, impeller blade-casing clearance flows, and other three-dimensional flow effects

Pump performance characteristics

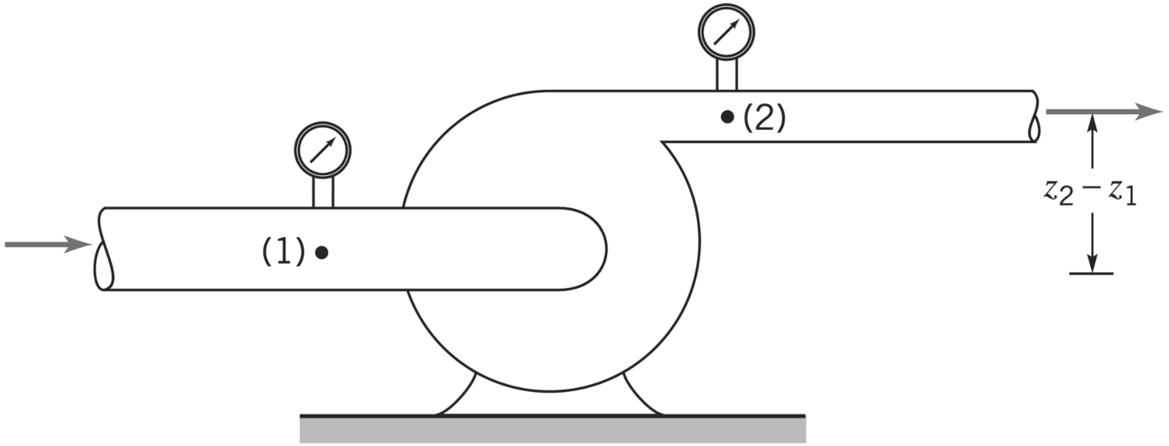
$$h_a = H_2 - H_1$$

$$H = \frac{p}{\rho g} + \frac{V^2}{2g} + z$$

$$h_a = \frac{p_2 - p_1}{\gamma} + z_2 - z_1 + \frac{V_2^2 - V_1^2}{2g}$$

Typically, the differences in elevations (Δz) and velocities (ΔV^2) are small

$$h_a \approx \frac{p_2 - p_1}{\gamma}$$



Power gained by the fluids, $P_f = \gamma Q h_a$

Overall efficiency

$$\eta = \frac{\text{power gained by the fluid}}{\text{shaft power driving the pump}} = \frac{P_f}{\dot{W}_{\text{shaft}}}$$

Pump performance characteristics

Overall efficiency $$\eta = \frac{\text{power gained by the fluid}}{\text{shaft power driving the pump}} = \frac{P_f}{\dot{W}_{\text{shaft}}}$$

= power actually gained by the fluid to the shaft power supplied

Hydraulic losses: Skin friction, flow separation, 3D and unsteady effects
→ Hydraulic efficiency η_h

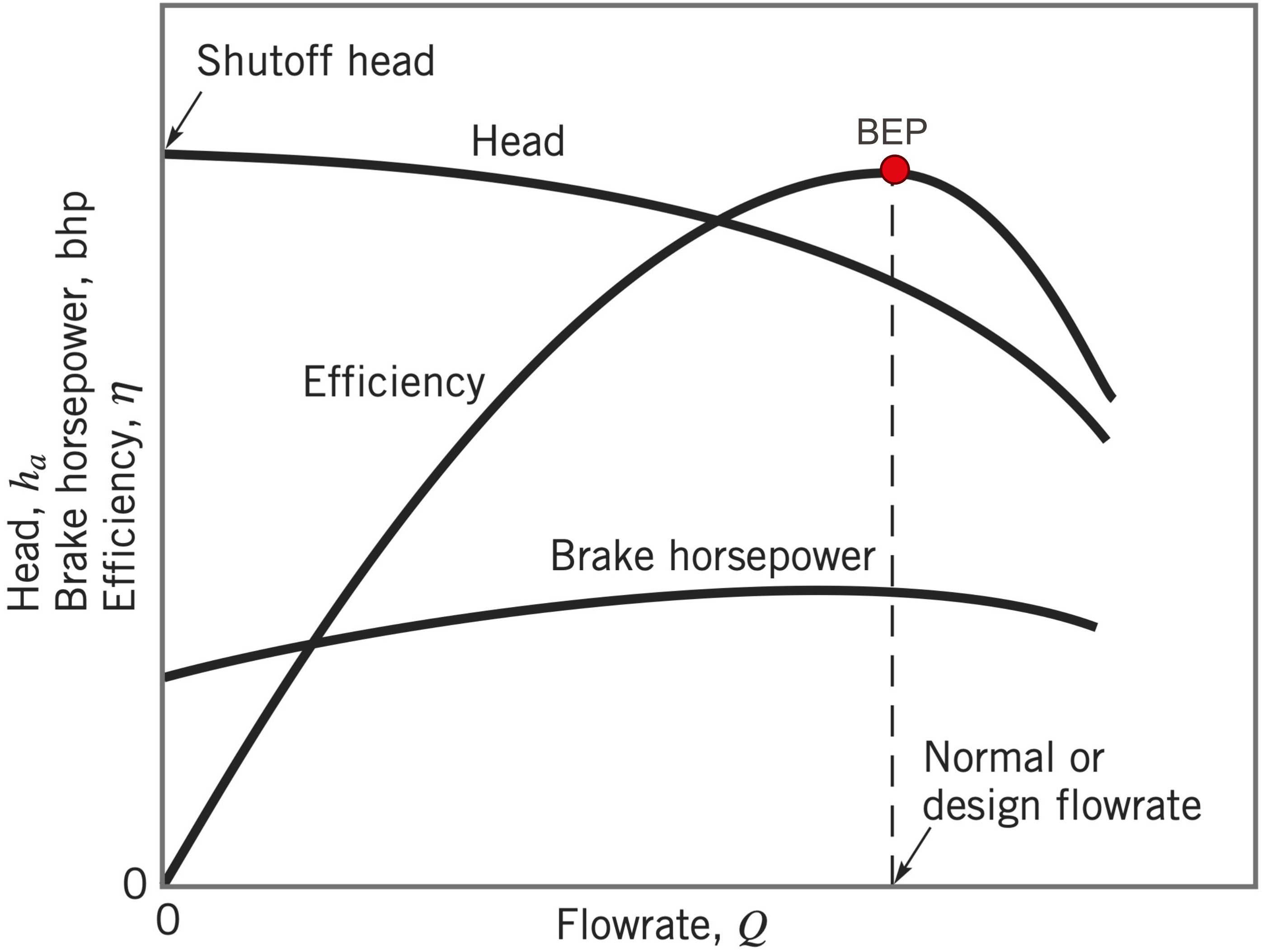
Mechanical losses: bearing and sealing losses
→ Mechanical efficiency η_m

Volumetric losses: flow leakage components
→ Volumetric efficiency η_v

$$\eta = \eta_h \eta_m \eta_v$$

Pump performance characteristics

Typical performance characteristics for a centrifugal pump of a given size operating at a constant impeller speed



- Shutoff head: pressure head rise across the pump with the discharge valve closed. Since there is no flow with the valve closed, the related efficiency is zero, and the power supplied by the pump is simply dissipated as heat
- Best efficiency point (BEP): maximum efficiency at designed flowrate
- Brake horsepower (power as horsepower, hp unit)

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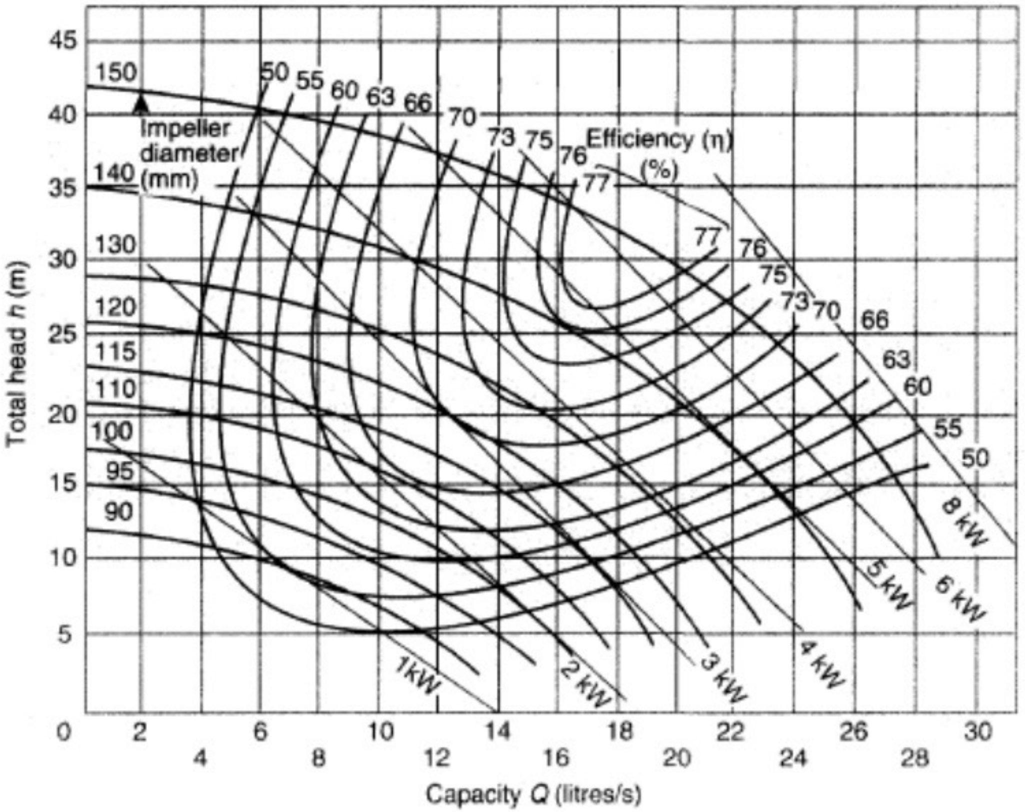
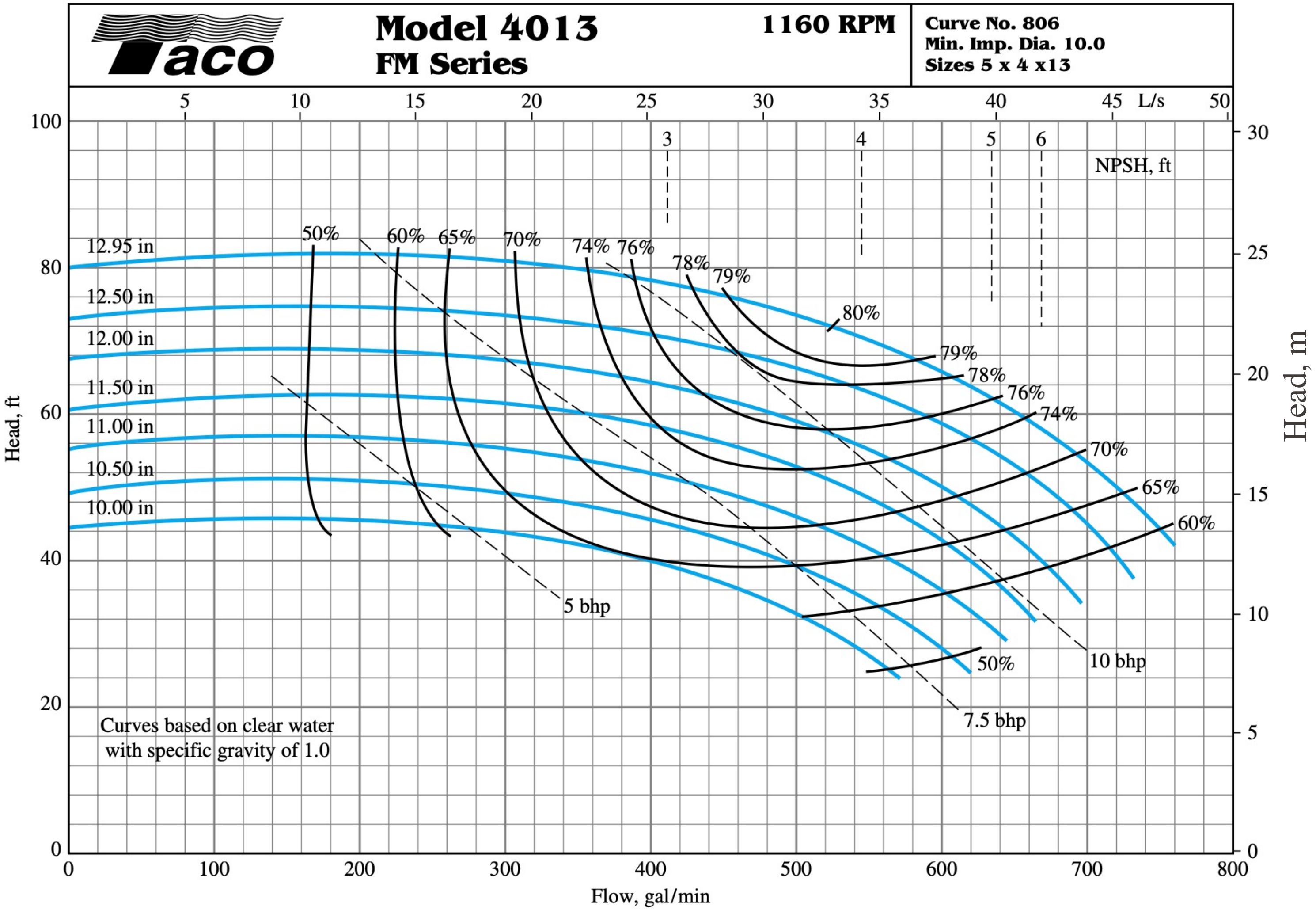
Performance curve for centrifugal pump





Net Positive Suction Head (NPSH)

- Cavitation: when liquid pressure at a given location is reduced to the vapor pressure, vapor form (the liquid starts to boil) → loss of efficiency and structural damage

characterize the potential for cavitation, the difference between the total head on the suction side, near the pump impeller inlet $p_s/\gamma + V_s^2/2g$ and the liquid vapor pressure head, p_v/γ is used.

Net positive suction head $$\text{NPSH} = \frac{p_s}{\gamma} + \frac{V_s^2}{2g} - \frac{p_v}{\gamma}$$

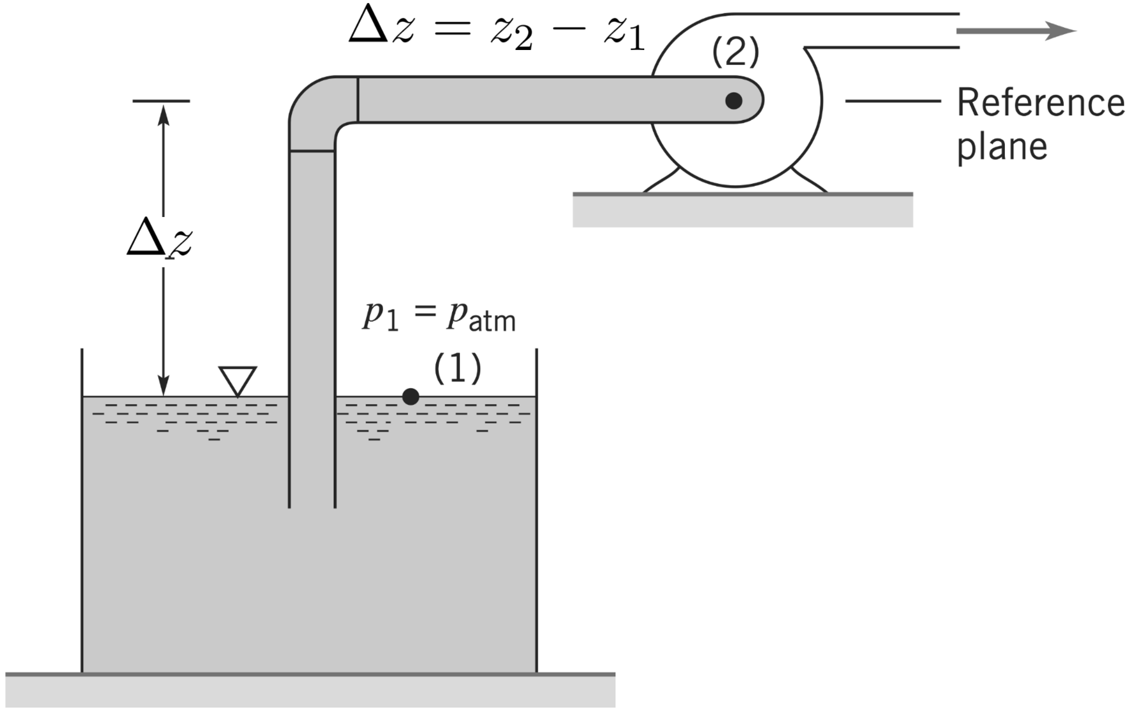
Required NPSH, NPSH_R must be maintained, or exceeded, so that cavitation will not occur
detecting cavitation or observing a change in the head–flowrate curve

Available NPSH, NPSH_A represents the head that actually occurs for the particular flow system.

NPSH

$$\frac{p_{atm}}{\gamma} + z_1 = \frac{p_s}{\gamma} + \frac{V_s^2}{2g} + z_2 + \sum h_L$$

$$\frac{p_s}{\gamma} + \frac{V_s^2}{2g} = \frac{p_{atm}}{\gamma} - \Delta z - \sum h_L$$



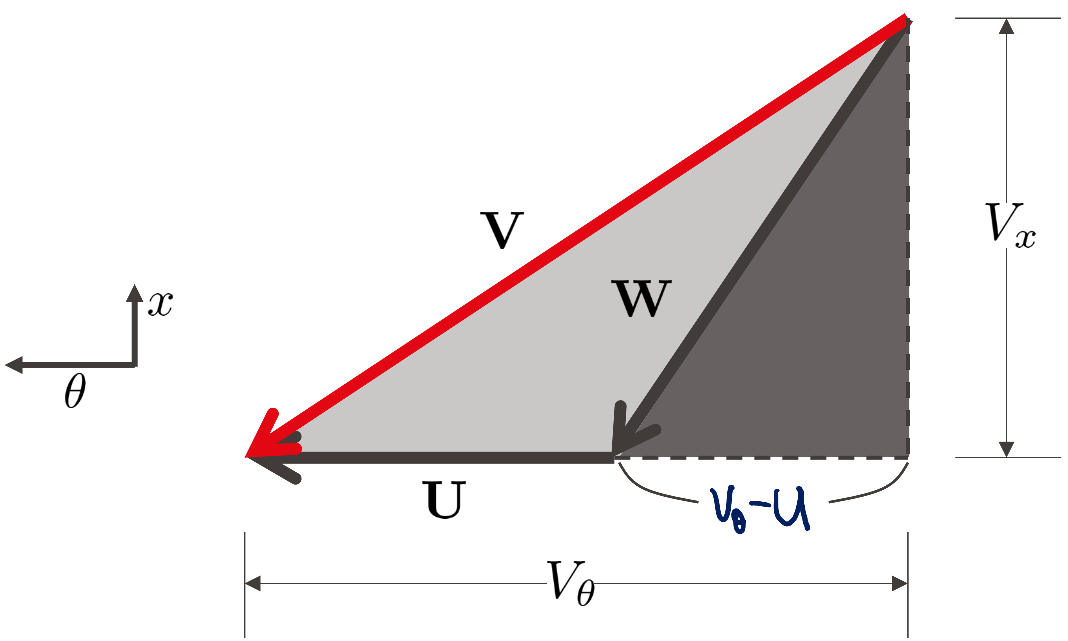
$$\mathrm{NPSH_A} = \frac{p_{atm}}{\gamma} - \Delta z - \sum h_L - \frac{p_v}{\gamma}$$

For proper pump operation, it is necessary $\mathrm{NPSH}_A \geq \mathrm{NPSH_R}$

If z_1 increases, $\mathrm{NPSH_A}$ decreases
→ matching $\mathrm{NPSH_A} = \mathrm{NPSH_R}$
Pump operates with cavitation

Recall - Basic governing equations for turbomachine

$$\mathbf{V} = \mathbf{W} + \mathbf{U}$$



Velocity triangle: **V** absolute velocity, **W** relative velocity, **U** blade velocity

- From the big triangle (grey)

$$V^2 = V_\theta^2 + V_x^2 \quad \text{OR} \quad V_x^2 = V^2 - V_\theta^2$$

- From the small triangle (dark grey)

$$W^2 = (V_\theta - U)^2 + V_x^2$$

$$= V_\theta^2 - 2V_\theta U + U^2 + V_x^2$$

$$W^2 = V_\theta^2 - 2V_\theta U + U^2 + V^2 - V_\theta^2$$

$$V_\theta U = \frac{-W^2 + U^2 + V^2}{2}$$

$$w_{\text{shaft}} = -(U_1 V_{\theta 1}) + (U_2 V_{\theta 2})$$

$$w_{\text{shaft}} = \frac{V_2^2 - V_1^2 + U_2^2 - U_1^2 - (W_2^2 - W_1^2)}{2}$$

Turbomachine work is related to changes in absolute, relative, and blade velocities.