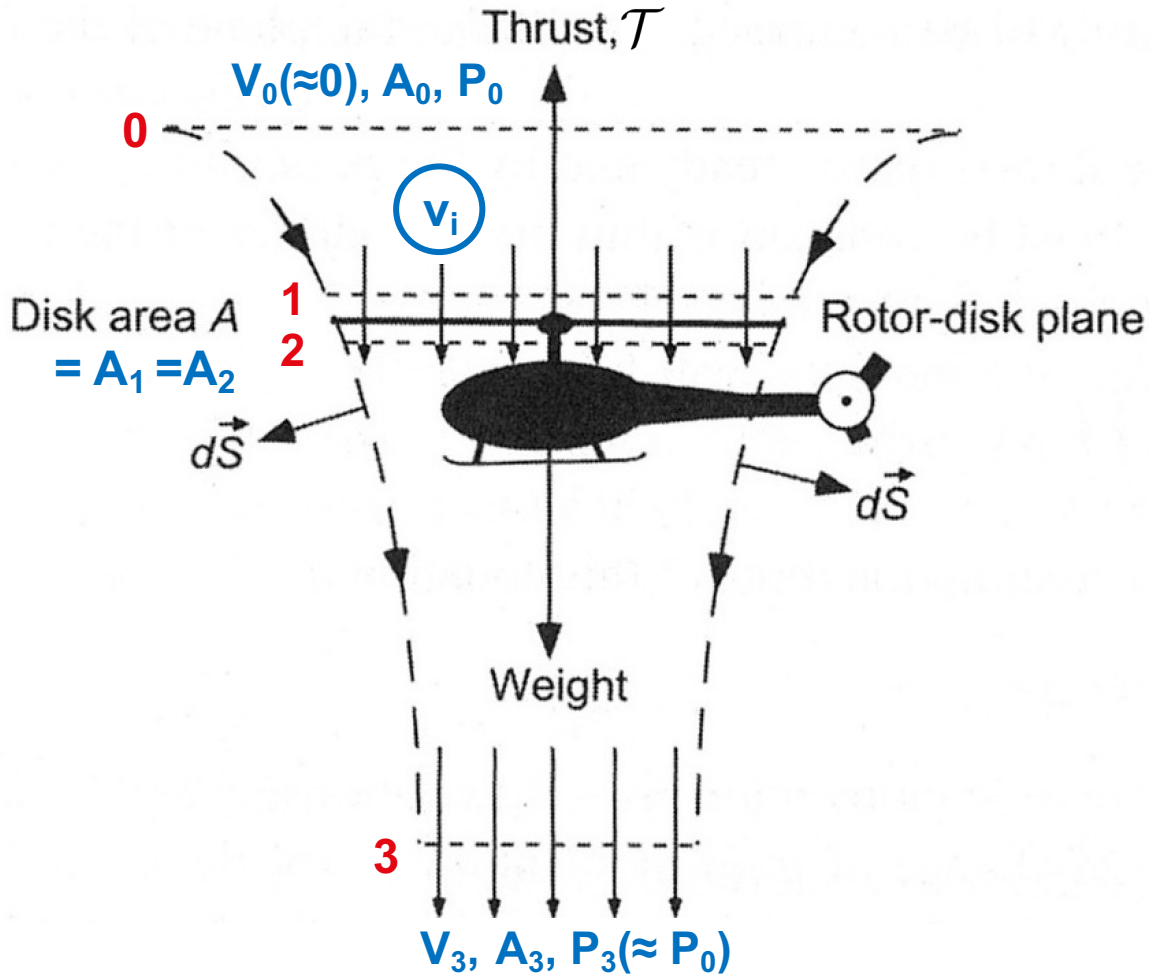


Chapter 5: Euler turbomachine equation

ME-342 Introduction to
turbomachinery

Prof. Eunok Yim, HEAD-lab.

Helicopters- conservation law for hovering rotor



v_i : induced air velocity

V_3 : far wake velocity

- **Continuity & linear momentum equations:**

$$\frac{\partial}{\partial t} \int_{\text{CV}} \rho dV + \int_{\text{CS}} \rho \mathbf{V} \cdot \hat{\mathbf{n}} dA = 0$$

$$\frac{\partial}{\partial t} \int_{\text{cv}} \mathbf{V} \rho dV + \int_{\text{cs}} \mathbf{V} \rho \mathbf{V} \cdot \underline{\hat{\mathbf{n}}} dA = \sum \mathbf{F} \quad \text{contents of the control volume}$$

Steady-state approximation

$$\int_{cs} \rho \vec{V} \cdot d\vec{S} = 0$$

$$\int_{CS} \vec{v} \cdot \rho \cdot \vec{v} d\vec{s} = \Sigma F$$

Thrust, T (reaction force)

$$-F = T = \int_3 \underbrace{\vec{v} \rho \vec{v}}_{\text{in}} d\vec{S} - \int_0 \vec{v} \rho \vec{v} d\vec{S}$$

$$\mathcal{T} = \dot{m}V_3$$

$$\dot{m} = \int_A \rho \mathbf{V} \cdot \hat{\mathbf{n}} dA$$

- **Rotor power** (work done by rotor per unit time)

$$P_{\text{rotor}} = \mathcal{T} v_i$$

Helicopters- conservation law for hovering rotor

- Continuity equation:

$$\mathcal{T} = \dot{m} V_3, \quad P_{\text{rotor}} = \mathcal{T} v_i$$

$$P_{\text{rotor}} = \frac{V_3^2}{2} \dot{m} = \dot{m} V_3 v_i$$

- Energy equation: $P_{\text{rotor}} = \frac{V_3^2}{2} \dot{m}$

Induced velocity:

$$v_i = \frac{1}{2} V_3$$

Far field velocity is twice of the induced velocity

- Mass flowrate: $\dot{m} = \rho A_3 V_3 = \rho A_2 v_i = \rho A v_i$

$$\mathcal{T} = \dot{m} V_3 = \dot{m} (2v_i) = (\rho A v_i) (2v_i) = 2\rho A v_i^2$$

$$\dot{m} = \rho A_3 V_3 = \rho A v_i \rightarrow A_3 = \frac{1}{2} A$$

Remember! The energy equation is valid along the streamlines.

The induced velocity:

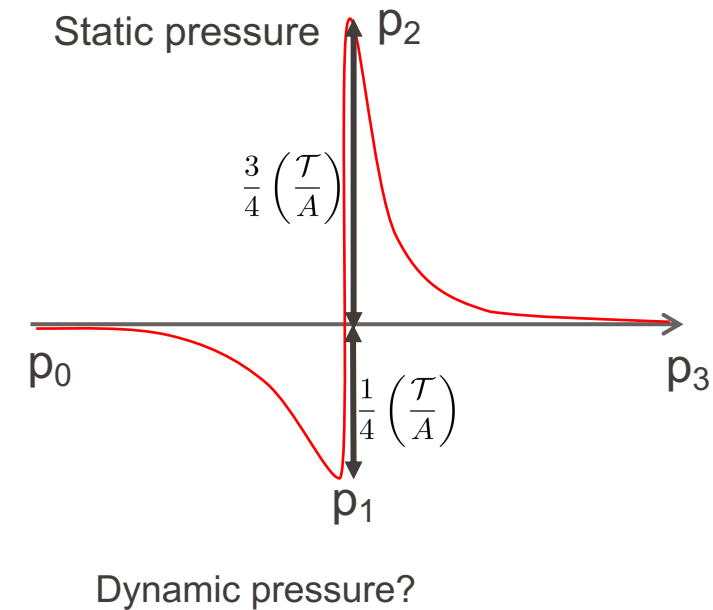
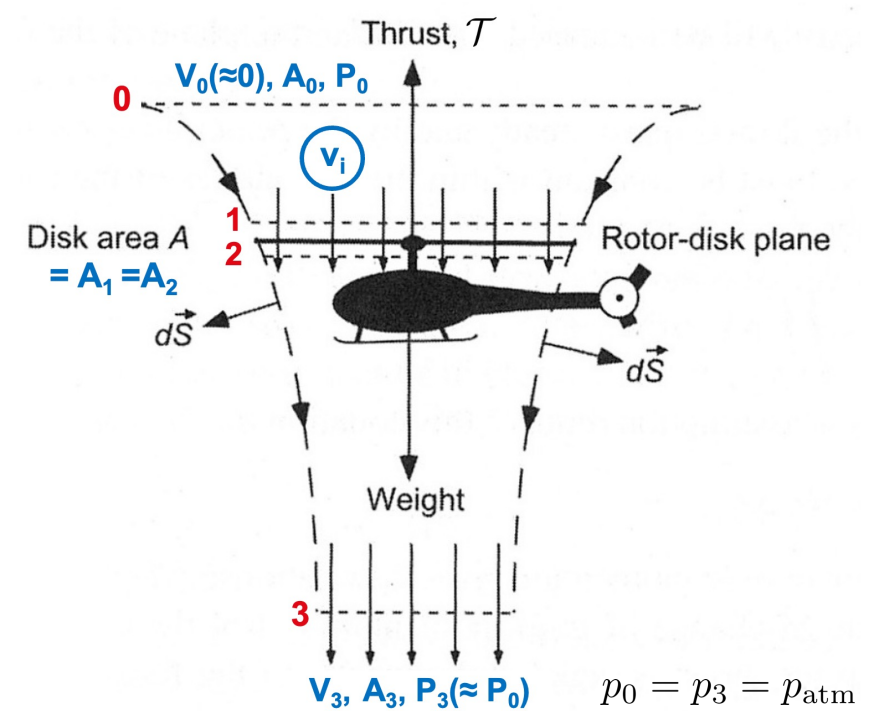
$$v_h \equiv v_i = \sqrt{\frac{\mathcal{T}}{2\rho A}} = \sqrt{\left(\frac{\mathcal{T}}{A}\right) \left(\frac{1}{2\rho}\right)}$$

Disk loading

Pressure around the helicopter

$$\Delta p = p_2 - p_1 = \frac{\mathcal{T}}{A}$$

Ignore the hydrostatic pressure : $\rho \Delta z \sim \epsilon$



$$p_1 = p_{\text{atm}} - \frac{1}{4} \left(\frac{\mathcal{T}}{A} \right), \quad p_2 = p_{\text{atm}} + \frac{3}{4} \left(\frac{\mathcal{T}}{A} \right)$$

The ideal and actual powers

- Last week, we learned ideal power
 - It can be estimated the thrust needed for hover
- Actual power

$$P_{\text{actual}} = P_{\text{induced}} + P_{\text{profile}} + P_{\text{parasitic}} + P_{\text{climb}}$$

Induced power =
modified ideal power
with factor

Viscous effect

Climb power = Weight x V_c

P_{profile} : power required to overcome the rotor **drag** → Torque

$P_{\text{parasitic}}$: power required to overcome the helicopter body (fuselage) **drag** and **tail rotor power**

EPFL Helicopters- Figure of merit (FM)

$$P_{\text{rotor}} = \frac{V_3^2}{2} \dot{m} = \dot{m} V_3 v_i = 2 \dot{m} v_i^2$$

$$\dot{m} = \rho A v_i$$

$$v_i = \sqrt{\frac{T}{2\rho A}} = \sqrt{\left(\frac{T}{A}\right) \left(\frac{1}{2\rho}\right)}$$

$$P_{\text{rotor}} = \frac{T^{3/2}}{\sqrt{2\rho A}} \quad \text{Ideal rotor power}$$

$$\text{Figure of merit, FM} = \frac{\text{Ideal rotor power}}{\text{Actual rotor power}}$$

If FM is 0.7 and the computed ideal power is 2000 kW, what is the actual power required?

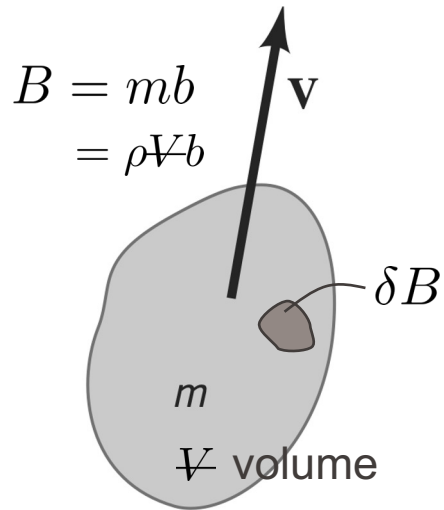
Reynolds transport theorem

Reynolds transport theorem

System: a collection of matter of fixed identity (always the same atoms or fluid particles), which may move, flow, and interact with its surroundings.

Control volume: a volume in space (a geometric entity, independent of mass) through which fluid may flow.

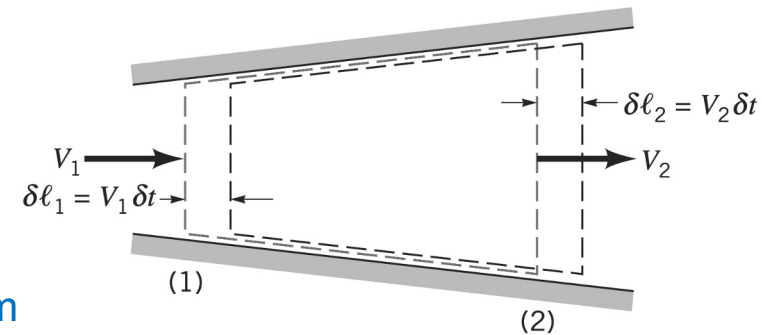
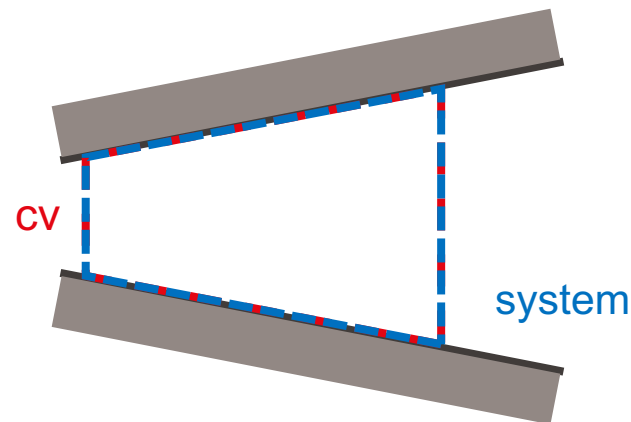
System vs. control volume (Lagrangian vs. Eulerian)



$$\delta B = b \rho \delta V \longrightarrow B_{\text{sys}} = \lim_{\delta V \rightarrow 0} \sum_i b_i (\rho_i \delta V) = \int_{\text{sys}} \rho b dV$$

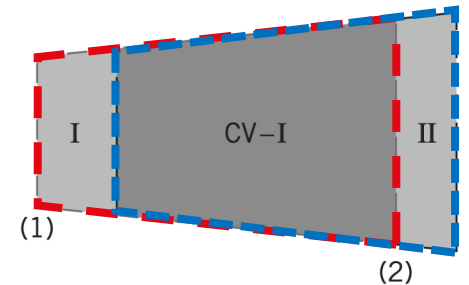
$$B_{\text{cv}} = \int_{\text{cv}} \rho b dV$$

B	$b = B/m$
m	1
$m\mathbf{V}$	\mathbf{V}
$\frac{1}{2}mV^2$	$\frac{1}{2}V^2$



--- Fixed control surface and system boundary at time t

--- System boundary at time $t + \delta t$

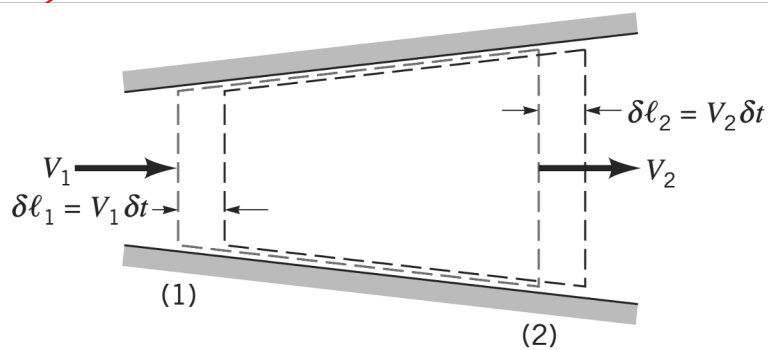


Reynolds transport theorem

$$t = t_0, \quad B_{\text{sys}}(t_0) = B_{\text{cv}}(t_0)$$

$$t = t_0 + \delta t, \quad B_{\text{sys}}(t_0 + \delta t) = B_{\text{cv}}(t_0 + \delta t) - B_{\text{I}}(t_0 + \delta t) + B_{\text{II}}(t_0 + \delta t)$$

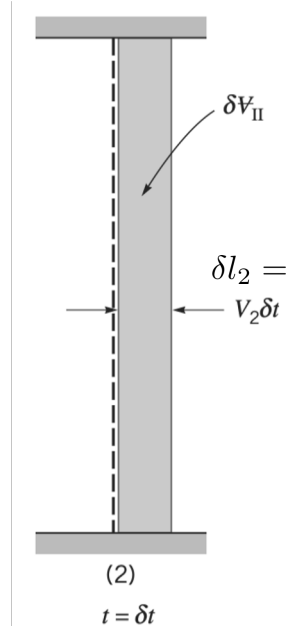
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--- Fixed control surface and system boundary at time t

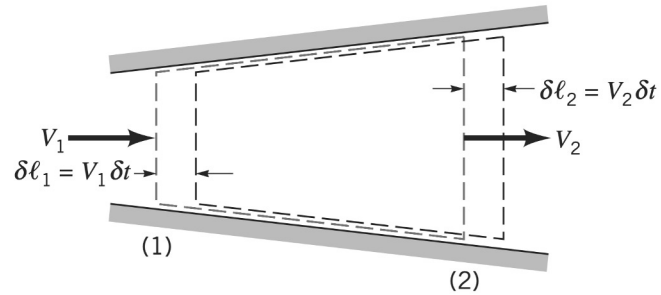
--- System boundary at time $t + \delta t$

$$\frac{DB_{\text{sys}}}{Dt} = \frac{\partial B_{\text{cv}}}{\partial t} + \rho_2 A_2 V_2 b_2 - \rho_1 A_1 V_1 b_1$$



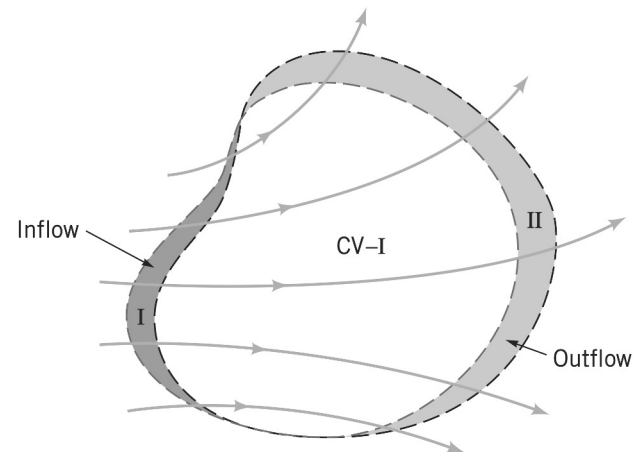
Reynolds transport theorem

• General case



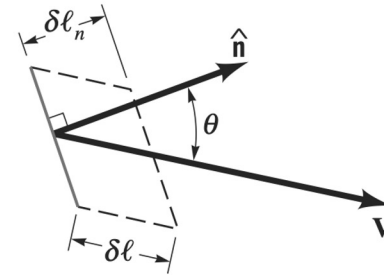
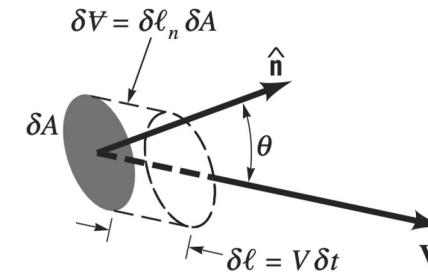
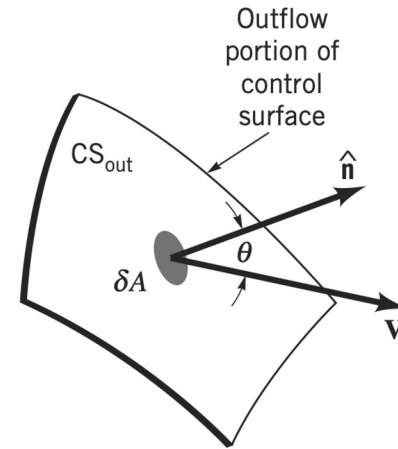
--- Fixed control surface and system boundary at time t

--- System boundary at time $t + \delta t$



--- Fixed control surface and system boundary at time t

--- System boundary at time $t + \delta t$



$$\dot{B}_{out} = \int_{CS_{out}} d\dot{B}_{out} = \int_{CS_{out}} \rho b V \cos \theta dA$$

$$V \cos \theta = \mathbf{V} \cdot \hat{n}$$

$$\begin{aligned} \dot{B}_{out} - \dot{B}_{in} &= \int_{CS_{out}} \rho b \mathbf{V} \cdot \hat{n} dA - \left(- \int_{CS_{in}} \rho b \mathbf{V} \cdot \hat{n} dA \right) \\ &= \int_{CS} \rho b \mathbf{V} \cdot \hat{n} dA \end{aligned}$$

$$\frac{DB_{sys}}{Dt} = \frac{\partial}{\partial t} \int_{CV} \rho b dV + \int_{CS} \rho b \mathbf{V} \cdot \hat{n} dA$$

B	$b = B/m$
m	1
$m\mathbf{V}$	\mathbf{V}
$\frac{1}{2}mV^2$	$\frac{1}{2}V^2$

Recall Last lecture: linear momentum equation

Newton's second law of motion

The change of **motion** of an object is **proportional** to the **force** impressed; and is made in the direction of the straight line in which the force is impressed

$$\frac{D}{Dt}$$

$$\text{momentum} = \text{mass} \times \text{(its) velocity} = \int_{\text{sys}} \rho \vec{V} dV$$

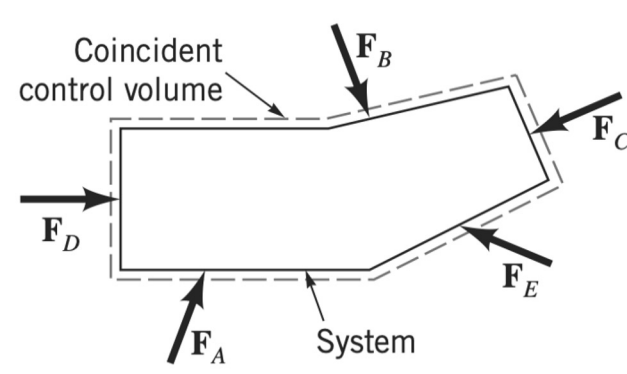
$$= \int_{\text{sys}} \rho dV \quad \vec{V}$$

$$\frac{D}{Dt} \left(\int_{\text{sys}} \rho \vec{V} dV \right) = \frac{\partial}{\partial t} \int_{\text{cv}} \rho \vec{V} dV + \int_{\text{cs}} \vec{V} \cdot \rho \vec{V} \hat{n} dA$$

Reynolds transport theorem

- Linear momentum equation

$$\frac{\partial}{\partial t} \int_{\text{cv}} \rho \vec{V} dV + \int_{\text{cs}} \rho \vec{V} \cdot \hat{n} dA = \sum \mathbf{F}_{\text{contents of the control volume}}$$



Moment-of Momentum (angular momentum)

- Linear momentum equation

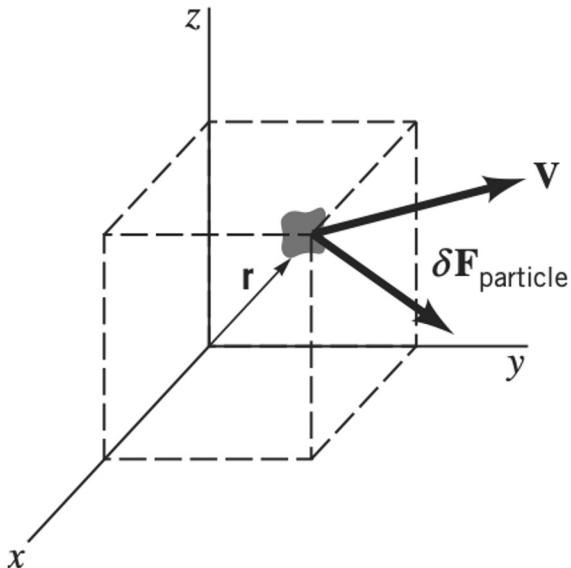
$$\frac{D}{Dt} \int_{sys} \mathbf{V} \rho dV = \frac{\partial}{\partial t} \int_{cv} \mathbf{V} \rho dV + \int_{cs} \mathbf{V} \rho \mathbf{V} \cdot \hat{\mathbf{n}} dA = \sum \mathbf{F}_{\text{contents of the control volume}}$$

- Linear momentum for a particle volume of δV

$$\frac{D}{Dt} (\mathbf{V} \rho \delta V) = \delta \mathbf{F}_{\text{particle}}$$

- Form the moment*** w.r.t. origin of an inertial coordinate system

$$\mathbf{r} \times \frac{D}{Dt} (\mathbf{V} \rho \delta V) = \mathbf{r} \times \delta \mathbf{F}_{\text{particle}}$$



*Apply force at a distance

Every particle of a system $\frac{D}{Dt}[(\mathbf{r} \times \mathbf{V})\rho\delta V] = \mathbf{r} \times \delta \mathbf{F}_{\text{particle}}$

For a system (collection of fluid particles)

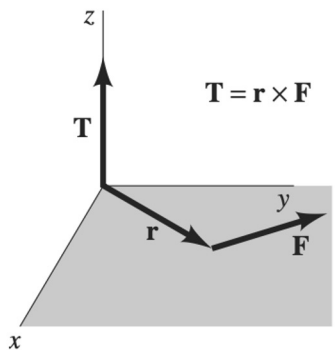
$$\int_{\text{sys}} \frac{D}{Dt}[(\mathbf{r} \times \mathbf{V})\rho dV] = \sum (\mathbf{r} \times \mathbf{F})_{\text{sys}}$$

For a control volume that is **instantaneously coincident** with the system

$$\sum (\mathbf{r} \times \mathbf{F})_{\text{sys}} = \sum (\mathbf{r} \times \mathbf{F})_{\text{cv}}$$

- Moment-of-momentum equation

$$\frac{\partial}{\partial t} \int_{\text{cv}} (\mathbf{r} \times \mathbf{V})\rho dV + \int_{\text{cs}} (\mathbf{r} \times \mathbf{V})\rho \mathbf{V} \cdot \hat{\mathbf{n}} dA = \sum (\mathbf{r} \times \mathbf{F})_{\text{contents of the control volume}}$$



$$\mathbf{T} = \mathbf{r} \times \mathbf{F}$$

Moment-of-momentum equation is Torque

Linear momentum equation

$$\frac{D}{Dt} \int_{\text{sys}} \mathbf{V}\rho dV = \frac{\partial}{\partial t} \int_{\text{cv}} \mathbf{V}\rho dV + \int_{\text{cs}} \mathbf{V}\rho \mathbf{V} \cdot \hat{\mathbf{n}} dA = \sum \mathbf{F}_{\text{contents of the control volume}}$$

- Moment-of-momentum equation

$$\frac{\partial}{\partial t} \int_{\text{cv}} (\mathbf{r} \times \mathbf{V}) \rho dV + \int_{\text{cs}} (\mathbf{r} \times \mathbf{V}) \rho \mathbf{V} \cdot \hat{\mathbf{n}} dA = \sum (\mathbf{r} \times \mathbf{F})_{\text{contents of the control volume}}$$

In turbomachine, a series of particles (a continuum) passes through the rotor.
For steady flow (or for turbomachine rotors with steady-in-the-mean or steady-on-average cyclical flow)

$$\sum (\mathbf{r} \times \mathbf{F}) = \int_{\text{cs}} (\mathbf{r} \times \mathbf{V}) \rho \mathbf{V} \cdot \hat{\mathbf{n}} dA$$

external torques (moments) acting
on the contents of the control volume

net rate of flow of moment-of-momentum
(**angular momentum**) through the control surface

$$\int_{\text{cs}} (\mathbf{r} \times \mathbf{V}) \rho \mathbf{V} \cdot \hat{\mathbf{n}} dA = \int_{\text{Outlet } A_2} (\mathbf{r} \times \mathbf{V}) \rho \mathbf{V} \cdot \hat{\mathbf{n}} dA - \int_{\text{Inlet } A_1} (\mathbf{r} \times \mathbf{V}) \rho \mathbf{V} \cdot \hat{\mathbf{n}} dA$$

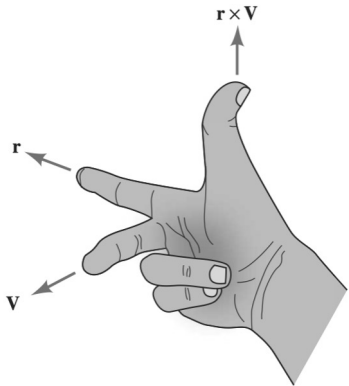
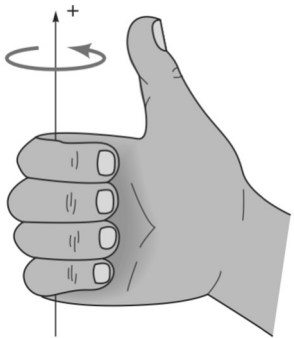
Assume:

- The only external torque is the shaft torque
- The flow is steady and uniform at the inlet and outlet, meaning $(\mathbf{r} \times \mathbf{V})$ and $\rho \mathbf{V} \cdot \hat{\mathbf{n}}$ are constant over the areas A_1 & A_2
- The mass flow rate $\dot{m} = \rho \mathbf{V} \cdot \hat{\mathbf{n}} A$ is the same at inlet and outlet (steady flow)

- Derivation of Euler turbomachine equation

$$\sum (\mathbf{r} \times \mathbf{F}) = \int_{cs} (\mathbf{r} \times \mathbf{V}) \rho \mathbf{V} \cdot \hat{\mathbf{n}} dA$$

$\mathbf{r} \times \mathbf{V} =$	\hat{e}_r	\hat{e}_θ	\hat{e}_x
	r	0	x
	V_r	V_θ	V_x



The axial component is interesting for turbomachinery as it rotates along the **shaft** axis

$$\sum \left[(\mathbf{r} \times \mathbf{F})_{\text{contents of the control volume}} \right]_{\text{axial}} = T_{\text{shaft}}$$

$$[(\mathbf{r}_2 \times \mathbf{V}_2) \dot{m}_2 - (\mathbf{r}_1 \times \mathbf{V}_1) \dot{m}_1]_{\text{axial}} = \dot{m}_2 (r_2 V_{\theta 2}) - \dot{m}_1 (r_1 V_{\theta 1})$$

$$T_{\text{shaft}} = -\dot{m}_1 (r_1 V_{\theta 1}) + \dot{m}_2 (r_2 V_{\theta 2})$$

Euler turbomachine equation

= axial component of moment-of-momentum equation

Positive/negative according to the right-hand rule

Application of moment-of-momentum equation

Select a fixed and nondeforming control volume that includes the rotating parts and the fluid in an instant.

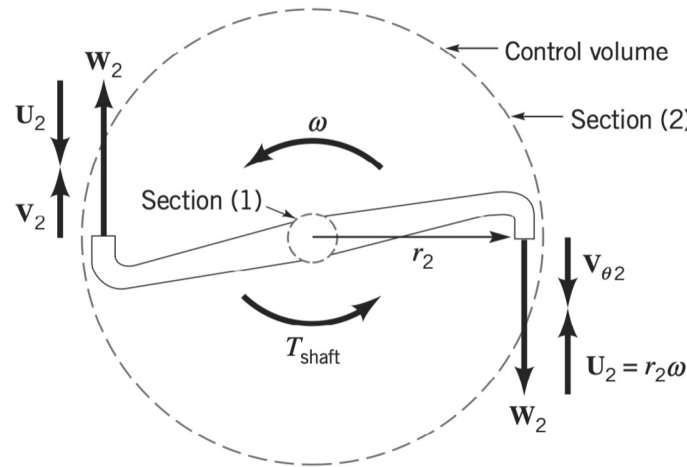
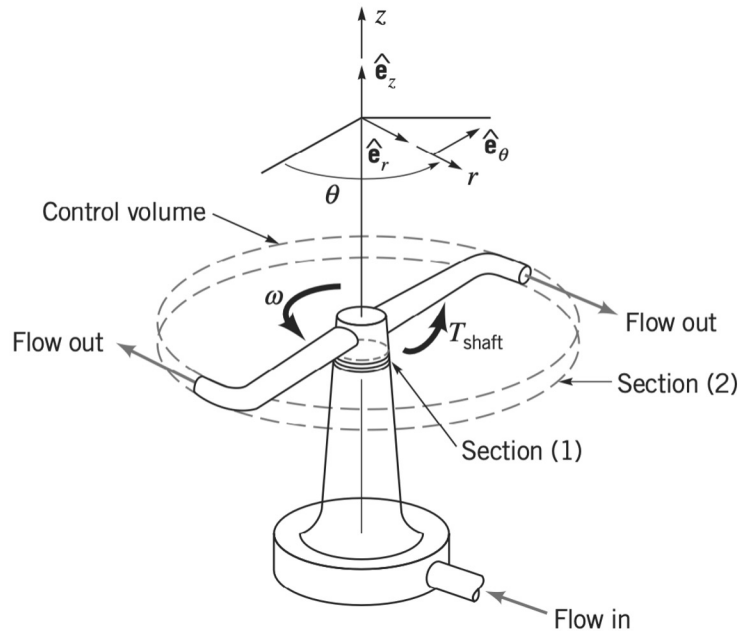
The flow within this control volume is cyclical, but steady in the mean.

The only torque we consider is the driving shaft torque, T_{shaft} .

$$\int_{\text{CS}} (\mathbf{r} \times \mathbf{V}) \rho \mathbf{V} \cdot \hat{\mathbf{n}} dA = \sum (\mathbf{r} \times \mathbf{F})_{\text{contents of the control volume}}$$

$$\sum \left[(\mathbf{r} \times \mathbf{F})_{\text{contents of the control volume}} \right]_{\text{axial}} = T_{\text{shaft}}$$

$$\left[\int_{\text{CS}} (\mathbf{r} \times \mathbf{V}) \rho \mathbf{V} \cdot \hat{\mathbf{n}} dA \right]_{\text{axial}} = (-r_2 V_{\theta 2}) (+\dot{m})$$



$$\mathbf{V} = \mathbf{W} + \mathbf{U}$$

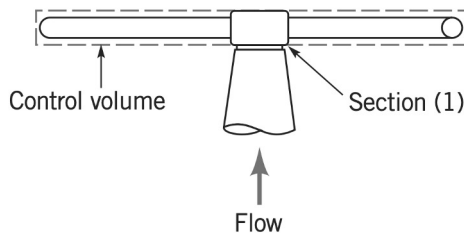
\mathbf{W} : Relative velocity

\mathbf{V} : Absolute velocity

\mathbf{U} : Moving nozzle velocity

Sign of the axial component of $\mathbf{r} \times \mathbf{V}$
 +: If \mathbf{V}_θ and \mathbf{U} are same direction
 -: If \mathbf{V}_θ and \mathbf{U} are opposite direction

$$-r_2 V_{\theta 2} \dot{m} = T_{\text{shaft}}$$



- Shaft power

ω : Angular velocity

$$\dot{W}_{\text{shaft}} = T_{\text{shaft}} \omega = -\frac{r_2 V_{\theta 2} \dot{m}}{U_2} \omega$$

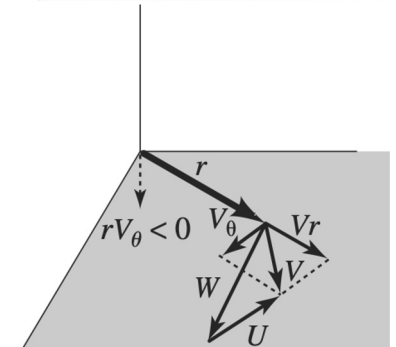
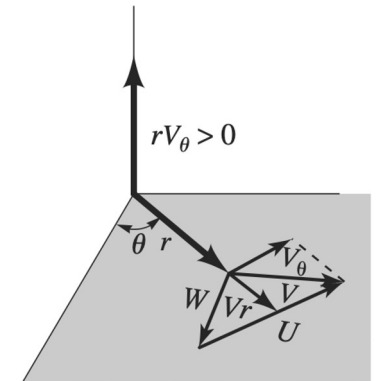
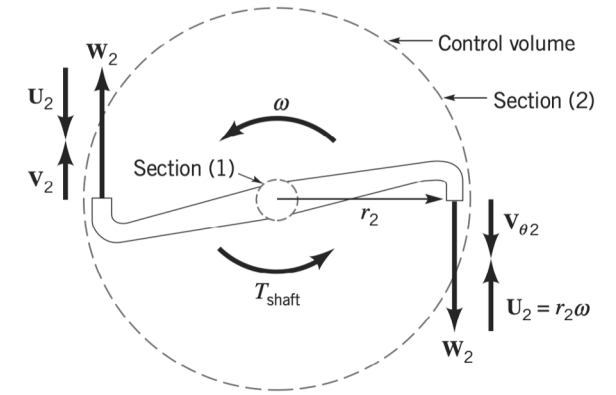
$$\dot{W}_{\text{shaft}} = -U_2 V_{\theta 2} \dot{m}$$

- Shaft work per unit mass (shaft power per unit mass flow rate)

$$w_{\text{shaft}} = -U_2 V_{\theta 2}$$

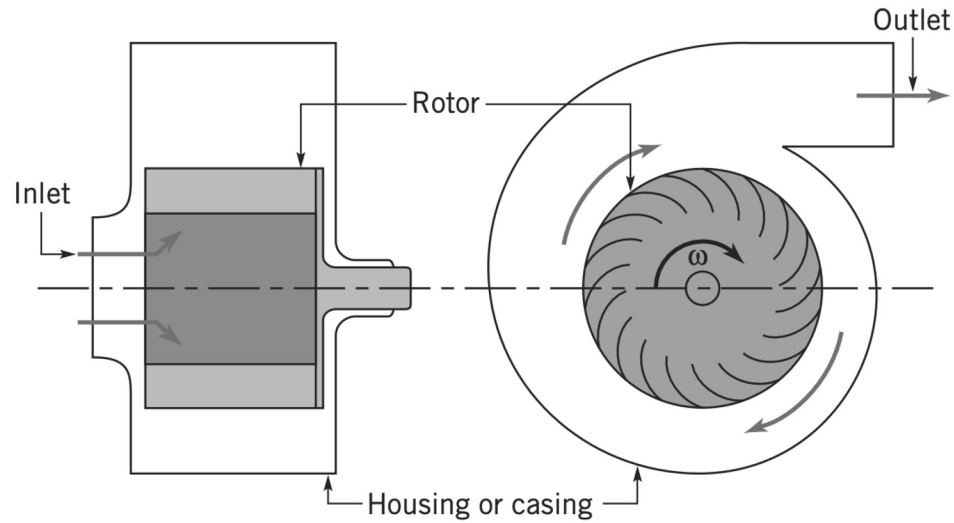
When the shaft **torque** and shaft **rotation**

- are in the **same** direction, power is *transferred* into the fluid (pump)
- are in **opposite** directions, power is *extracted* from the fluid (turbine)

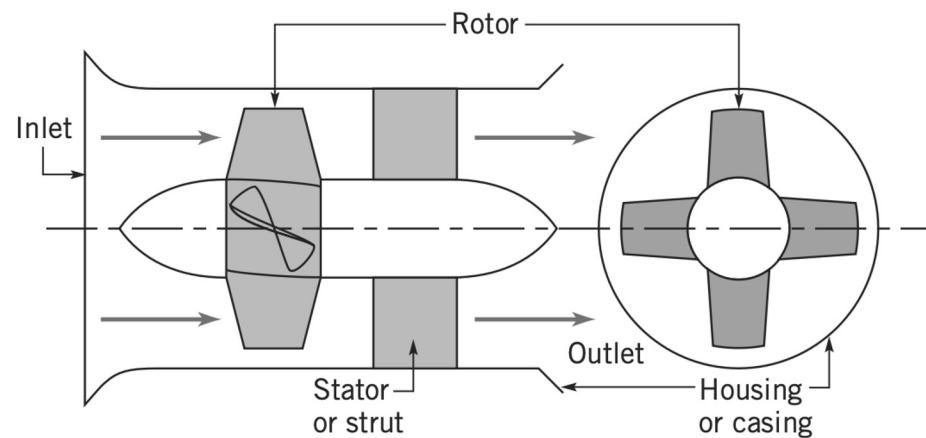


Basic energy conversion

- Depending on the predominant direction of the fluid motion relative to the rotor axis as the fluid passes the blades



Radial-flow fan

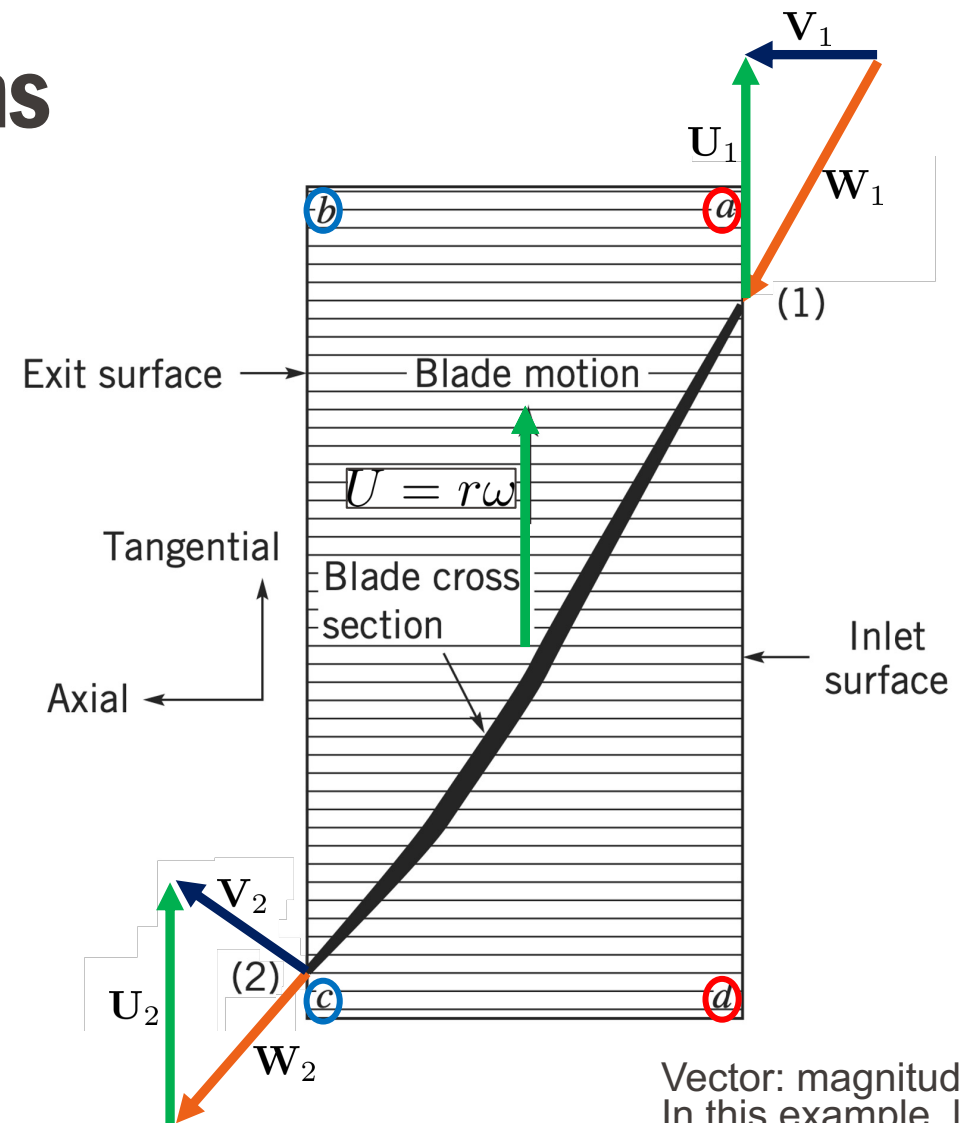
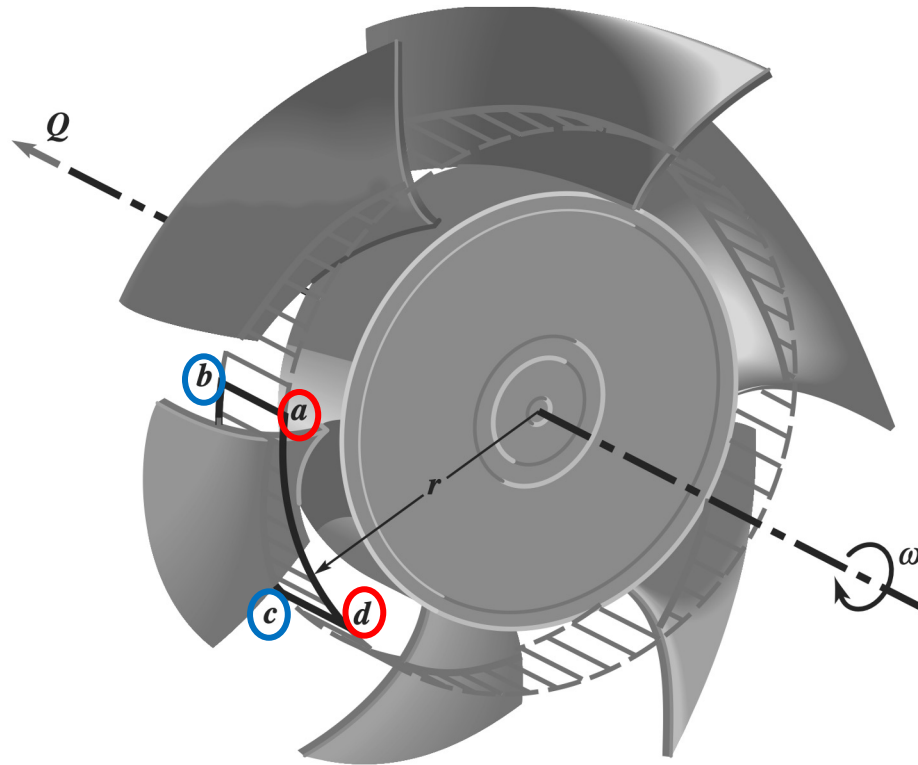


Axial-flow fan



Basic Energy Considerations

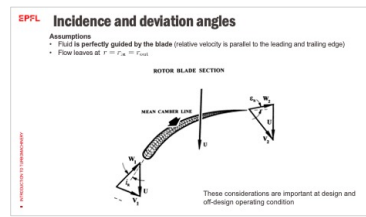
- Velocity diagram (fan)

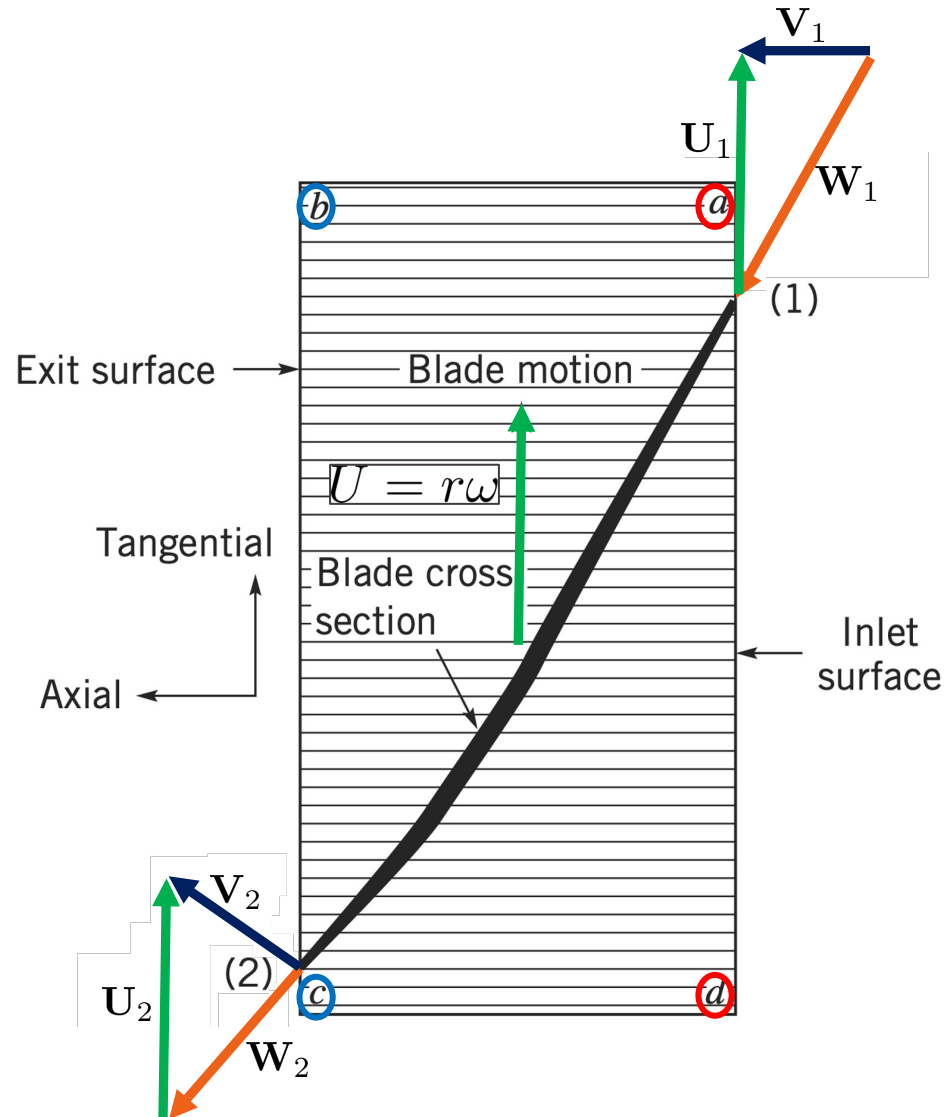


Assumptions

- Fluid is **perfectly guided by the blade** (relative velocity is parallel to the leading and trailing edge)*
- Flow leaves at $r = r_{in} = r_{out}$

*sometimes leading edge (inlet) \mathbf{V} is prescribed





- Velocity diagram:**

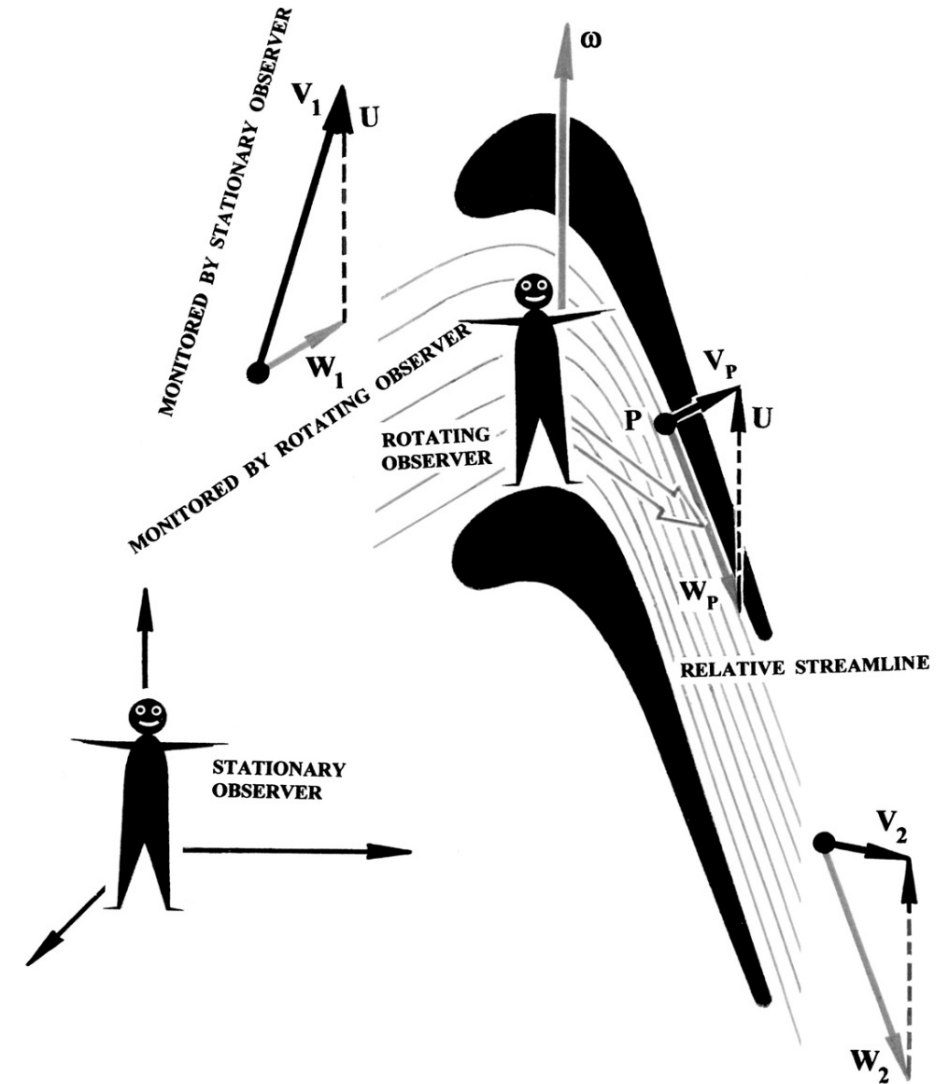
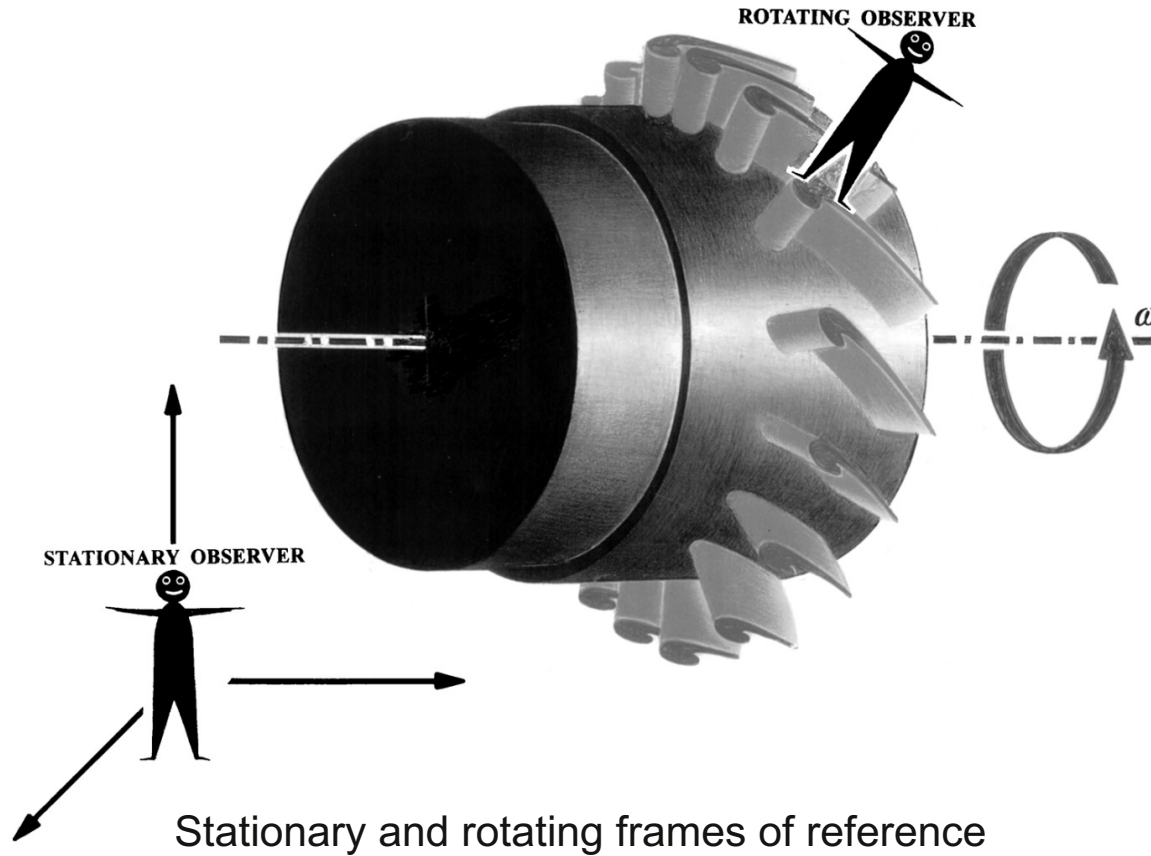
The actual (**absolute**) velocity is the vector sum of the **relative** and **blade** velocities

$$\mathbf{V} = \mathbf{W} + \mathbf{U}$$

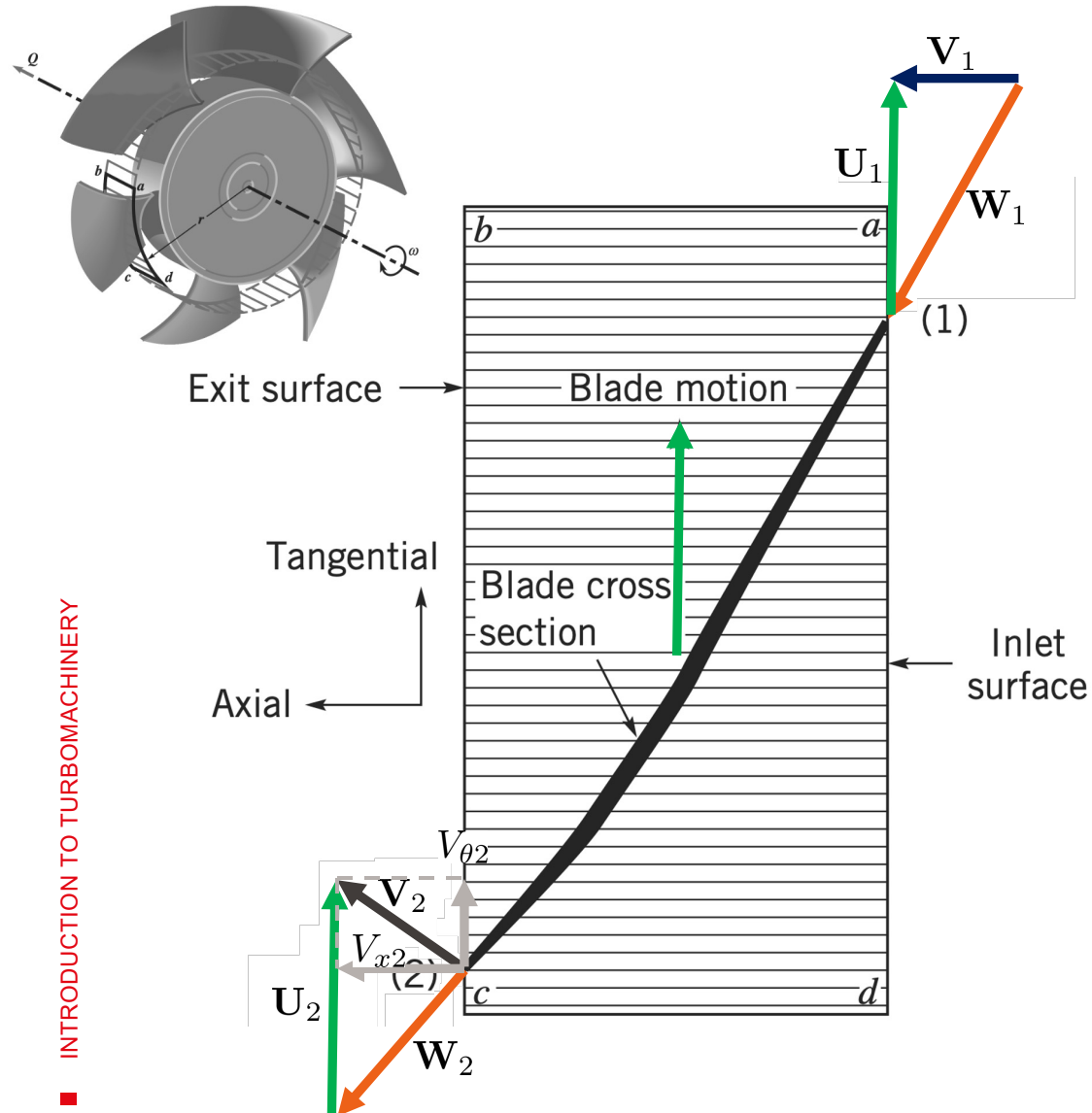
\mathbf{V} = Absolute fluid velocity

\mathbf{W} = Relative velocity

\mathbf{U} = Blade speed, ωr

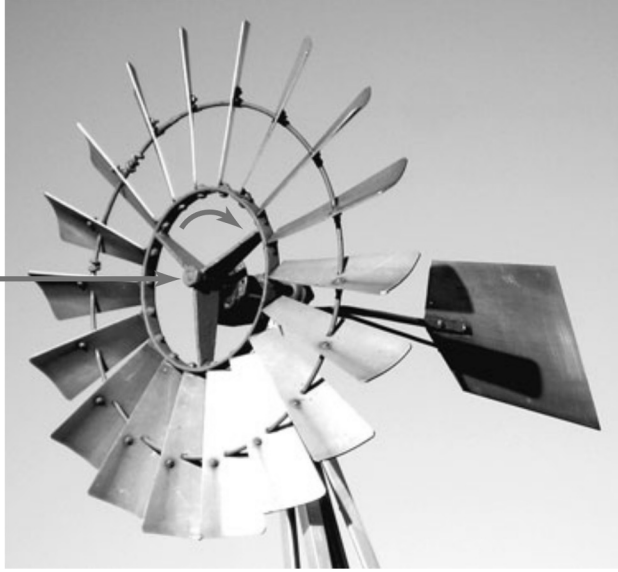


- Velocity triangle/diagram (fan)

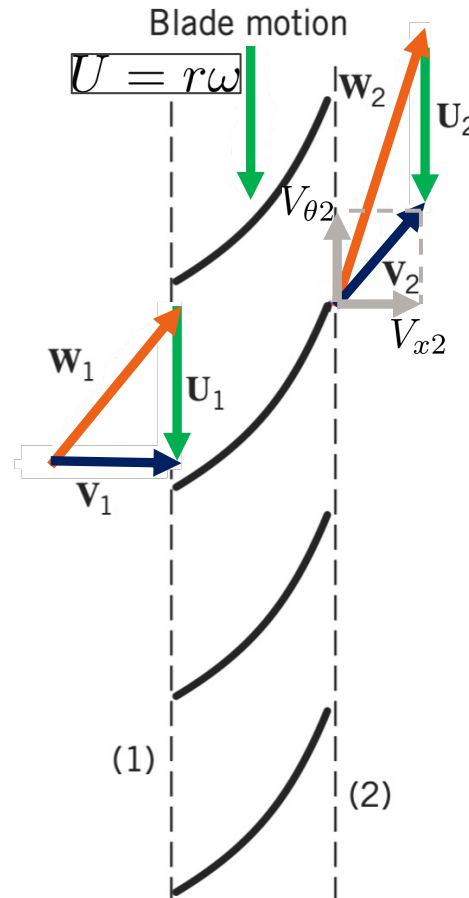
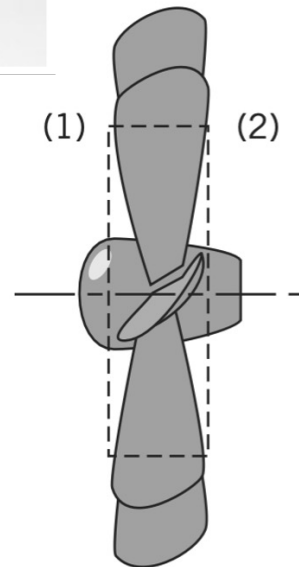
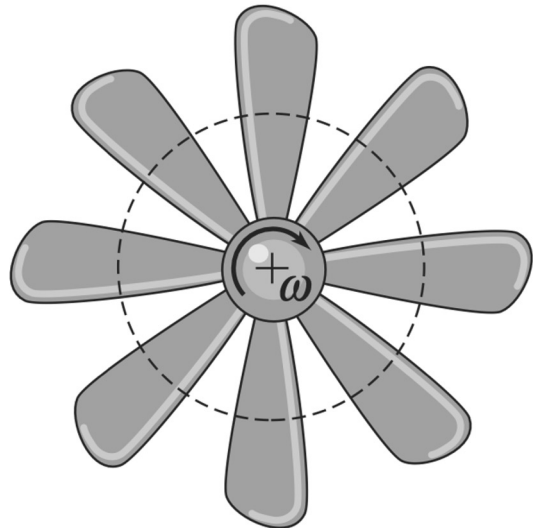


- The fan blade (because of its **shape** and **motion**) “pushes” the fluid, causing it to change direction. The absolute velocity vector, \mathbf{V} , is turned during its flow across the blade from section (1) to section (2) :
 - Inlet (1): the fluid had **no tangential** component of **absolute** velocity (in the direction of the motion of the blade, θ)
 - Outlet (2): the **tangential** component of absolute velocity is **nonzero**. For this to occur, the blade must push on the fluid in the tangential direction. That is, the blade exerts a tangential force component on the fluid in the direction of the motion of the blade.
- This tangential force component and the blade motion are in the same direction—the blade does work on the fluid. **This device is a pump**

- Velocity diagram (turbine)



Blades move in the direction of the lift force exerted on each blade by the wind blowing through the rotor

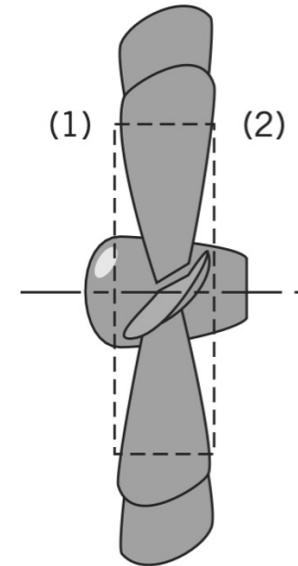
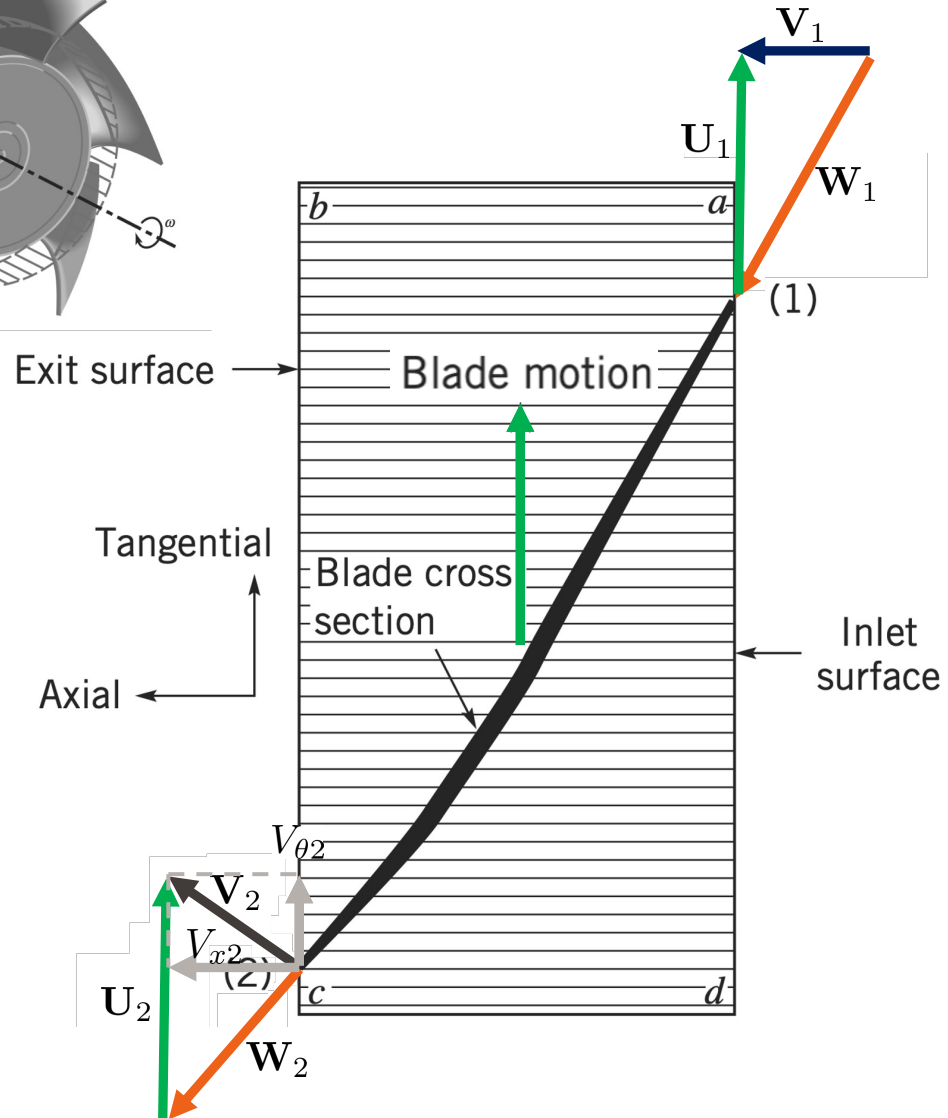
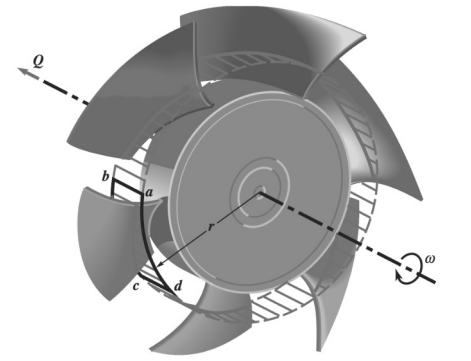


Due to the blade **shape** and **motion**:

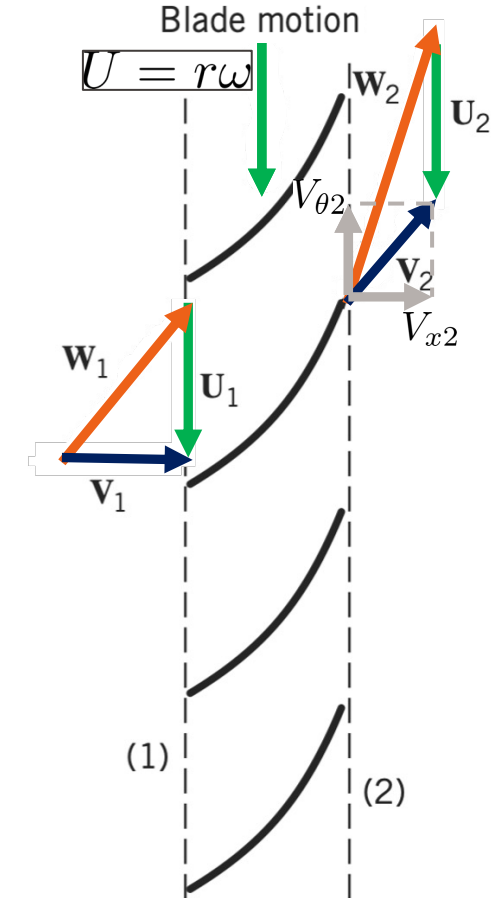
- Absolute velocity vectors at (1) and (2), \mathbf{V}_1 and \mathbf{V}_2 , have different directions
- The blades must have pushed up on the fluid → opposite to the direction of blade motion.
- Because of equal and opposite forces action-reaction, the fluid must have pushed on the blades in the direction of their motion—the **fluid does work on the blade**

→ Extraction of energy from the fluid, purpose of a **turbine**

Pump and turbine



$$U = r\omega$$



Euler turbomachine equation

$$T_{\text{shaft}} = -\dot{m}_1 (r_1 V_{\theta 1}) + \dot{m}_2 (r_2 V_{\theta 2})$$

mass flowrate
into CV

Outflow from
CV

$$\mathbf{V} = [V_r, V_\theta, V_x]$$

Sign of the axial component of $\mathbf{r} \times \mathbf{V}$ or tangential component of $\mathbf{V}_{\text{tangential}} = \mathbf{V}_\theta = V_\theta$
 +: If \mathbf{V}_θ and \mathbf{U} are same direction
 -: If \mathbf{V}_θ and \mathbf{U} are opposite direction

T_{shaft} Positive same direction as rotation

T_{shaft} Negative opposite direction of rotation

- Shaft torque is directly proportional to the mass flowrate $\dot{m} = \rho Q$
 → for the same volume flowrate, ~1000 more torque is required to pump water than air
- **Tangential** component of the **absolute velocity** is important

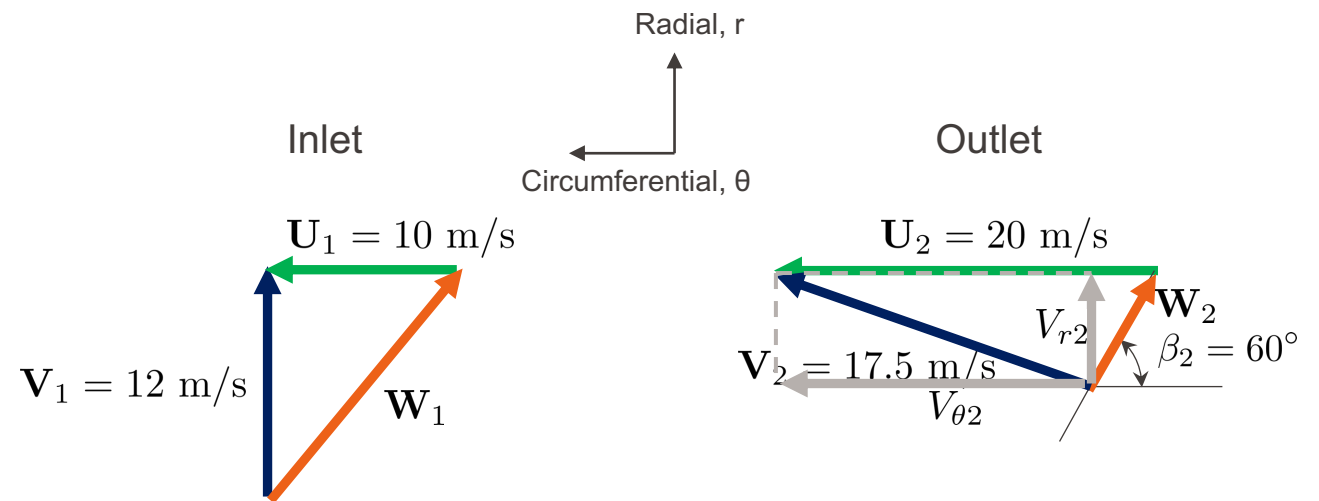
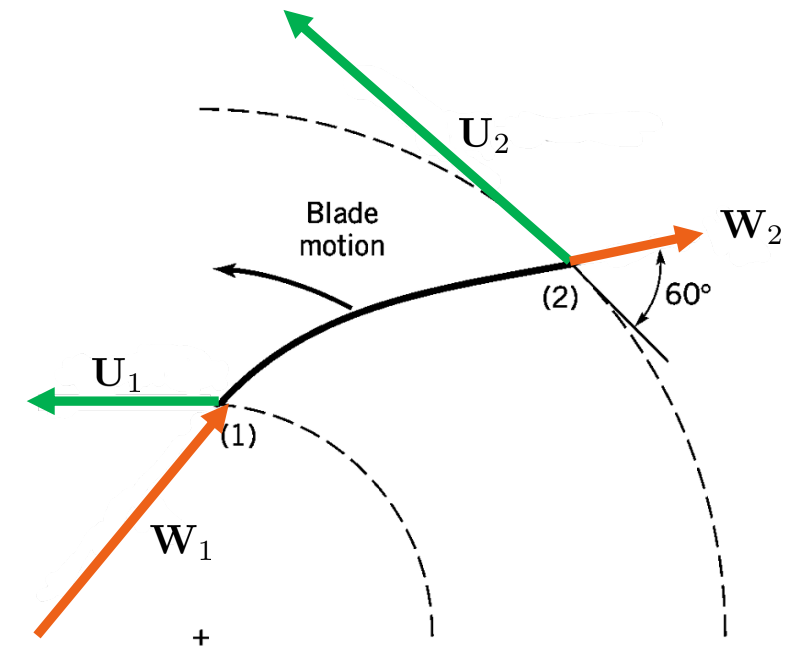
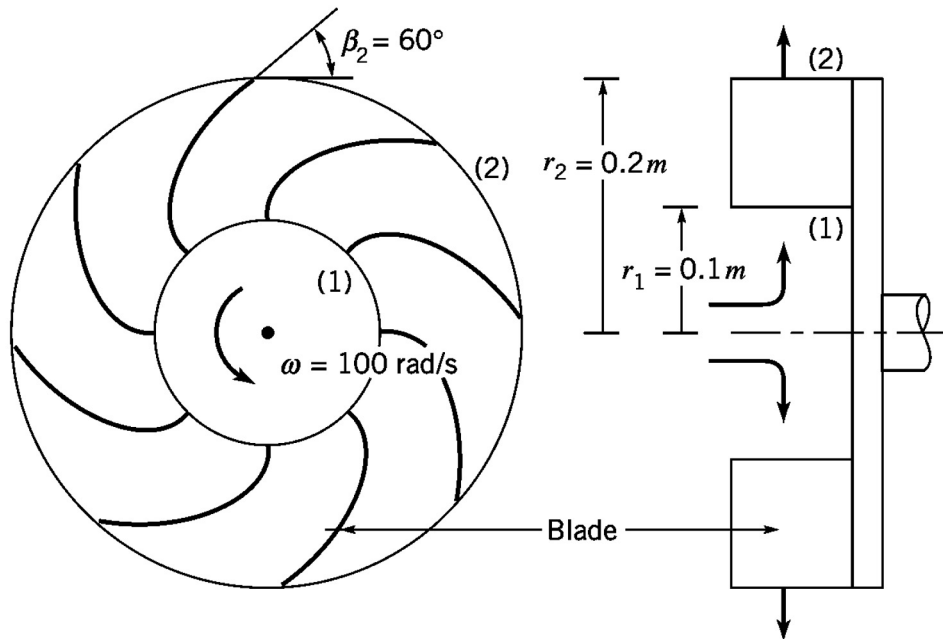
Basic Energy Considerations

• Exercise: Turbine or pump?

Known:

- Rotation speed $\omega = 100 \text{ rad/s}$
- $r_1 = 0.1 \text{ m}$, $r_2 = 0.2 \text{ m}$
- Absolute velocity at the inlet is purely radial
- Absolute velocities are measured $V_1 = 12 \text{ m/s}$, $V_2 = 17.5 \text{ m/s}$

Tell if it is a pump or turbine



Basic governing equations for turbomachine

- **Shaft torque**

$$T_{\text{shaft}} = -\dot{m}_1 (r_1 V_{\theta 1}) + \dot{m}_2 (r_2 V_{\theta 2})$$

- **Shaft power**

$$\dot{W}_{\text{shaft}} = T_{\text{shaft}} \omega = -\dot{m} \underbrace{r_1 V_{\theta 1}}_{U_1} \omega + \dot{m} \underbrace{r_2 V_{\theta 2}}_{U_2} \omega$$

$$\dot{W}_{\text{shaft}} = (-\dot{m}_1) (U_1 V_{\theta 1}) + \dot{m}_2 (U_2 V_{\theta 2}) \quad [W] = [\text{kg} \cdot \text{m}^2/\text{s}^3]$$

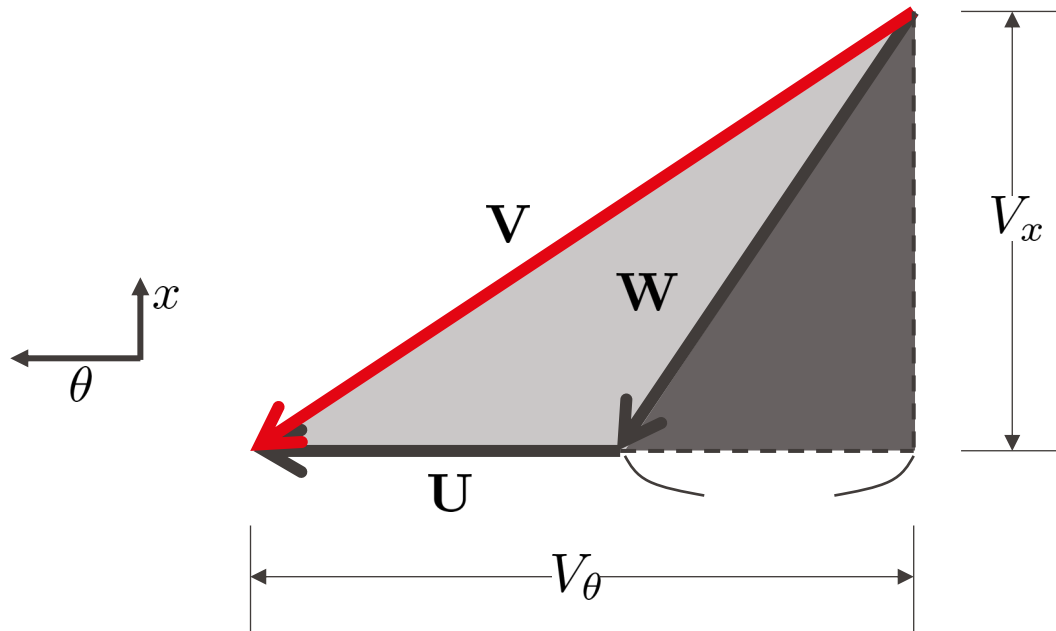
- **Shaft work per unit mass (shaft power per unit mass flow rate), $\dot{m}_1 = \dot{m}_2$**

$$w_{\text{shaft}} = - (U_1 V_{\theta 1}) + (U_2 V_{\theta 2}) \quad [\text{m}^2/\text{s}^2]$$

- Basic governing equations for pumps or turbines whether the machines are radial-, mixed-, or axial-flow devices and for compressible and incompressible flows
- Note it is only the function of tangential component of velocity, no V_r , V_x

$$\mathbf{V} = \mathbf{W} + \mathbf{U}$$

- From the big triangle (grey)
- From the small triangle (dark grey)



Velocity triangle: **V** absolute velocity,
W relative velocity, **U** blade velocity

$$w_{\text{shaft}} = -(U_1 V_{\theta 1}) + (U_2 V_{\theta 2})$$

$$w_{\text{shaft}} = \frac{V_2^2 - V_1^2 + U_2^2 - U_1^2 - (W_2^2 - W_1^2)}{2}$$

Turbomachine work is related to changes in absolute, relative, and blade velocities.

About torques in helicopter



Two main rotors in a helicopter

- horizontal lift-producing rotor
- vertical tail rotor

Without the **tail rotor** to **cancel** the **torque** from the main rotor, the helicopter body would spin out of control in the direction opposite to that of the main rotor.

Tailless jet-rotor

Jet-rotor concept removes the need for a tail rotor in helicopters.

Gas (hot exhaust gases or compressed air) is directed through the blades/Exhausted from nozzles at blade tips, perpendicular to blade axis.

Uses the **angular momentum principle** for rotation. Similar to a rotating lawn sprinkler or dishwasher arm. No drive shaft needed to turn the main rotor, eliminating torque issues

Hiller YH-32 Hornet

Used **peroxide-fueled tip jets** to rotate the blades

A catalyst (silver or platinum) decomposed the **hydrogen peroxide** into **high-temperature steam and oxygen**.

Not efficient compared to conventional helicopter, fuel consumption, too noisy

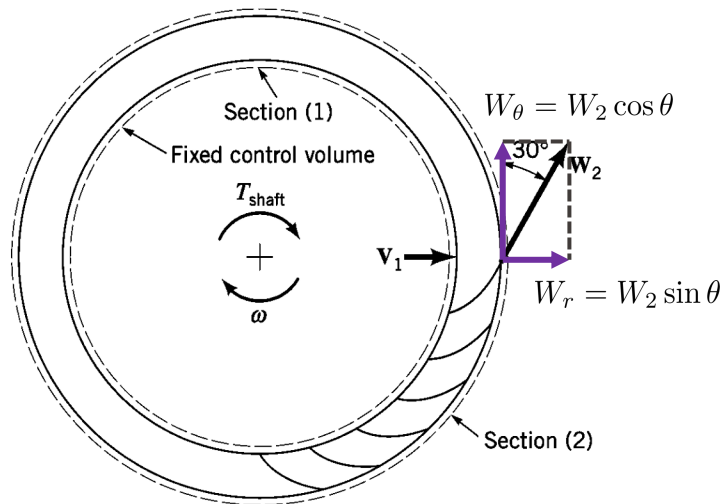


An air fan has a bladed rotor of 30 cm. outside diameter and 25 cm. inside diameter. The height of each rotor blade is constant at 2.5 cm from blade inlet to outlet. The flowrate is steady, on a time-average basis, at $0.1 \text{ m}^3/\text{s}$ and the absolute velocity of the air at blade inlet, \mathbf{V}_1 , is radial. The blade discharge angle is 30° from the tangential direction. The rotor rotates at a constant speed of 1725 rpm. Air density is $\rho = 1.2 \text{ kg/m}^3$, the only torque we consider is the driving shaft torque.

- Estimate the power required to run the fan

$$\begin{aligned}\dot{W}_{\text{shaft}} &= (-\dot{m}_{\text{in}}) (U_{\text{in}} V_{\theta \text{ in}}) + \dot{m}_{\text{out}} (U_{\text{out}} V_{\theta \text{ out}}) \\ &= (-\dot{m}_1) (U_1 V_{\theta 1}) + \dot{m}_2 (U_2 V_{\theta 2})\end{aligned}$$

$$\mathbf{V} = \mathbf{W} + \mathbf{U}$$



Power that needs to be delivered through the fan shaft. Ideally, all of this power would go into the flowing air.

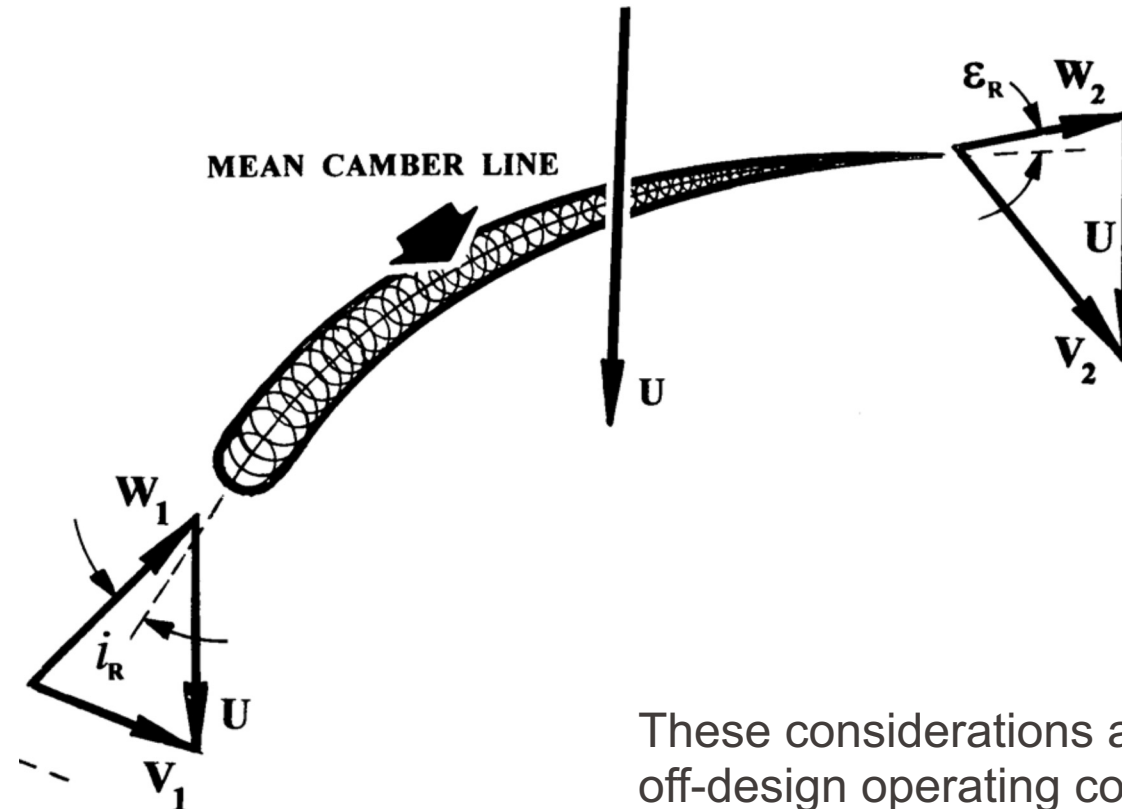
appendix

Incidence and deviation angles

Assumptions

- Fluid is perfectly guided by the blade (relative velocity is parallel to the leading and trailing edge)
- Flow leaves at $r = r_{\text{in}} = r_{\text{out}}$

ROTOR BLADE SECTION



These considerations are important at design and off-design operating condition

Helicopters- Figure of merit (FM)

$$P_{\text{rotor}} = \frac{V_3^2}{2} \dot{m} = \dot{m} V_3 v_i = 2 \dot{m} v_i^2$$

$$\dot{m} = \rho A v_i$$

$$v_i = \sqrt{\frac{\mathcal{T}}{2\rho A}} = \sqrt{\left(\frac{\mathcal{T}}{A}\right) \left(\frac{1}{2\rho}\right)}$$

$$P_{\text{rotor}} = \frac{\mathcal{T}^{3/2}}{\sqrt{2\rho A}} \quad \text{Ideal rotor power}$$

$$\text{Figure of merit, FM} = \frac{\text{Ideal rotor power}}{\text{Actual rotor power}}$$

If FM is 0.7 and the computed ideal power is 2000 kW, what is the actual power required?