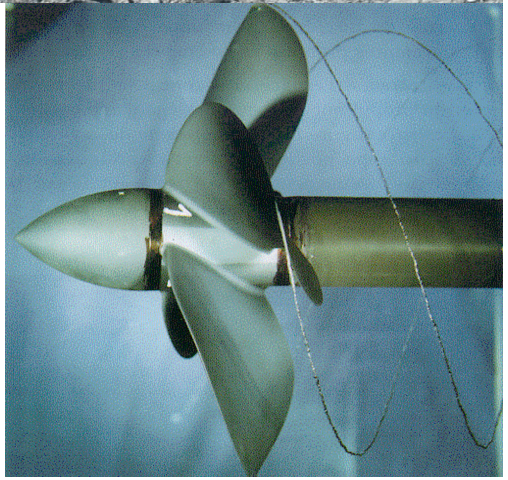
EPFL

Chapter 4: Propellers & Vortex



ME-342 Introduction to turbomachinery

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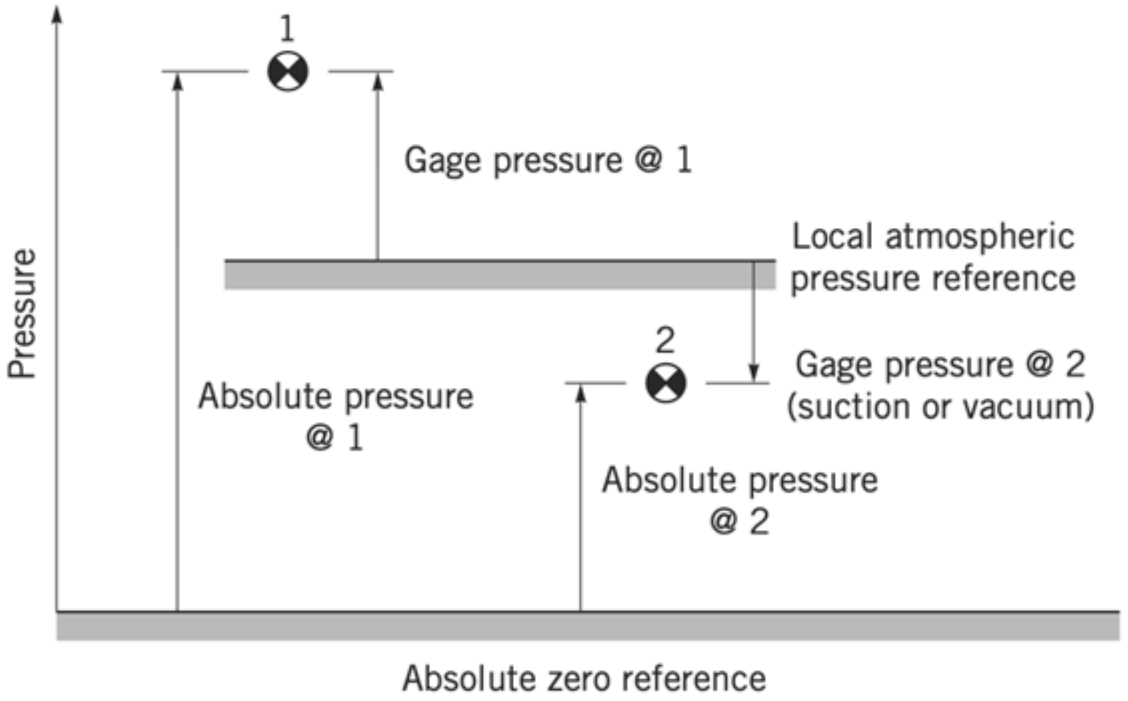
Questions from last exercise session

- Pressure coefficient, C_p

Pressure coefficient	$C_p = \dfrac{p - p_\infty}{\frac{1}{2}\rho U^2}$	$\dfrac{\text{Static pressure}}{\text{Dynamic pressure}}$	Aerodynamics, hydrodynamics
Lift coefficient	$C_L = \dfrac{L}{\frac{1}{2}\rho U^2 A}$	$\dfrac{\text{Lift force}}{\text{Dynamic force}}$	Aerodynamics, hydrodynamics
Drag coefficient	$C_D = \dfrac{D}{\frac{1}{2}\rho U^2 A}$	$\dfrac{\text{Drag force}}{\text{Dynamic force}}$	Aerodynamics, hydrodynamics

The pressure coefficient is a dimensionless form of the pressure

Pressure…



- Ideal gas law → absolute pressure

$$\rho = \frac{p}{RT}$$

The pressure in the ideal gas law must be expressed as an **absolute pressure,** denoted (abs), which means that it is measured relative to absolute zero pressure (a pressure that would only occur in a perfect vacuum).

$$\boxed{P_{abs} = P_{atm} + P_{gage}}$$

The pressure at a point within a fluid mass is designated as either an **absolute pressure** or a **gage pressure**.

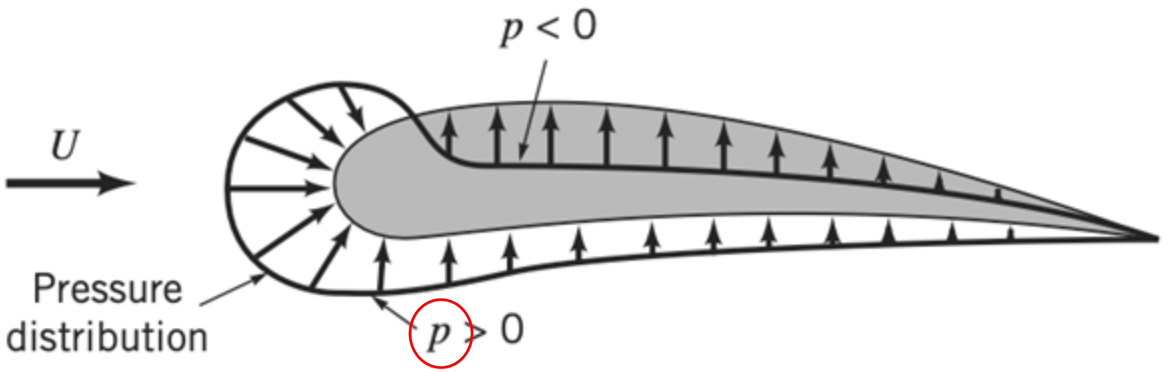
measured relative to a perfect vacuum (absolute zero pressure)

measured relative to the local atmospheric pressure

- Absolute pressures are always positive
- Gage pressure can be positive (above atm) or negative (below atm)
- Zero gage pressure: equal to the local atmospheric pressure
- Negative gage pressure is also referred to as a **suction** or **vacuum pressure**

When flow is involved…

- **Static Pressure**
  - Exerted by a fluid when it is at rest or when there is no directional motion effect at a given point
  - Measured perpendicular to the fluid flow in motion
  - Can be expressed as absolute pressure or gage pressure



Gage pressure

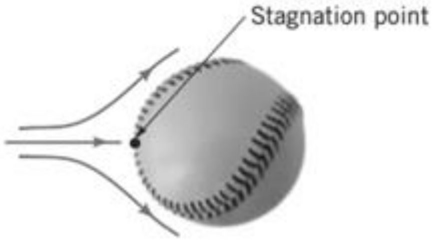
- **Absolute static pressure** = **Static pressure** + **Atmospheric pressure**

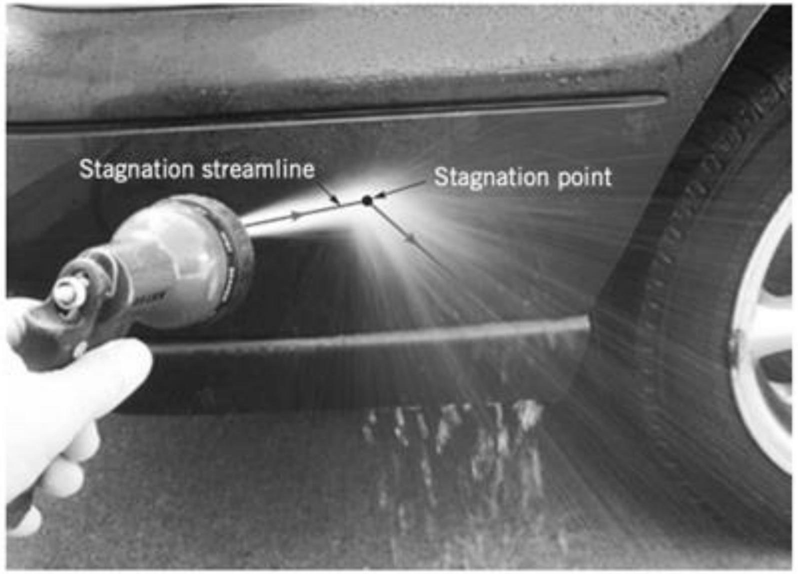
  Total absolute pressure = Static (absolute) pressure + Dynamic pressure

- If static pressure is measured **relative to atmospheric pressure**, then it is the same as gage pressure.
- If static pressure is measured **relative to a vacuum**, then it is absolute pressure.

Stagnation, total pressure

If elevation effects are neglected, the **stagnation pressure**, $p + \rho V^2/2$ is the **largest pressure** obtainable along a given streamline.
→ conversion of all of the kinetic energy into a pressure rise



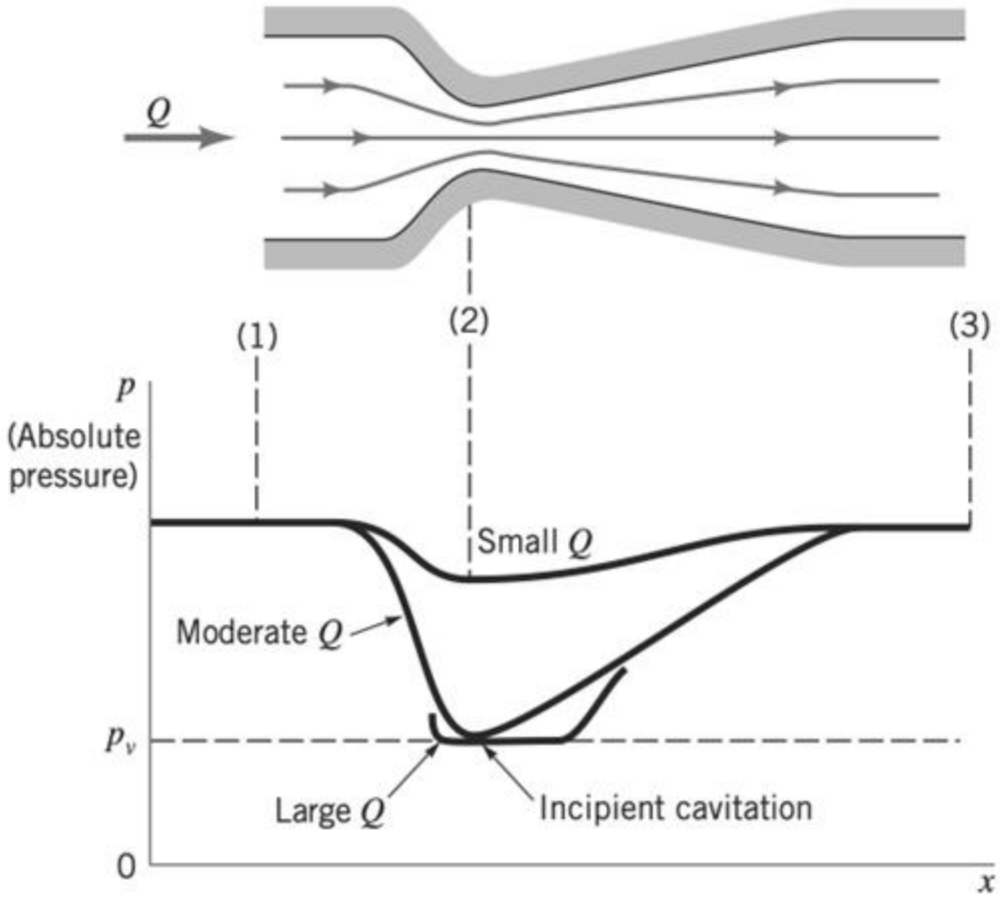


$$p + \frac{1}{2}\rho V^2 + \gamma z = p_T = \text{constant along a streamline}$$

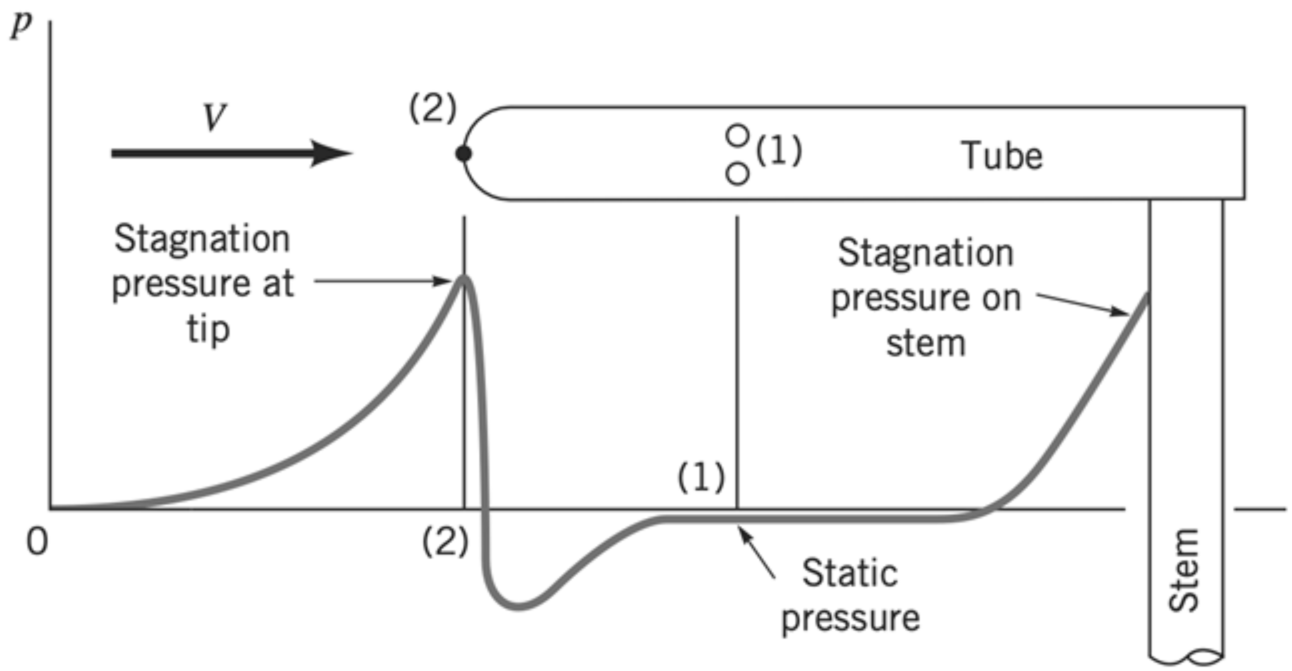
Total pressure, p_T: sum of the static pressure, hydrostatic pressure, and dynamic pressure

Absolute or relative?

- **Venturi tube**

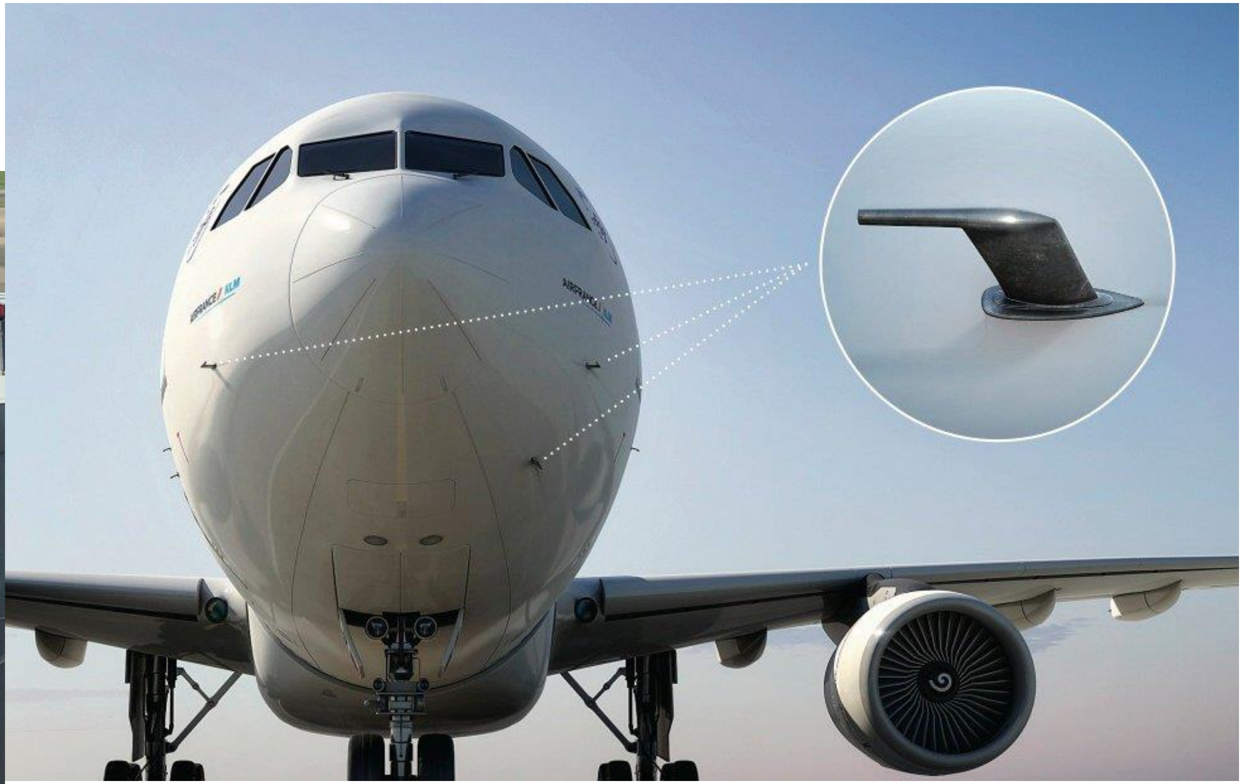


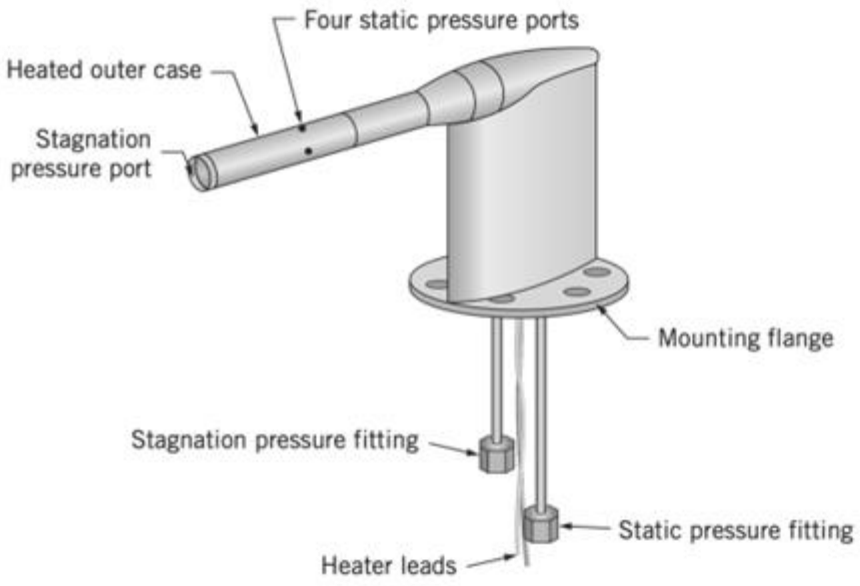
- **Pitot tube**



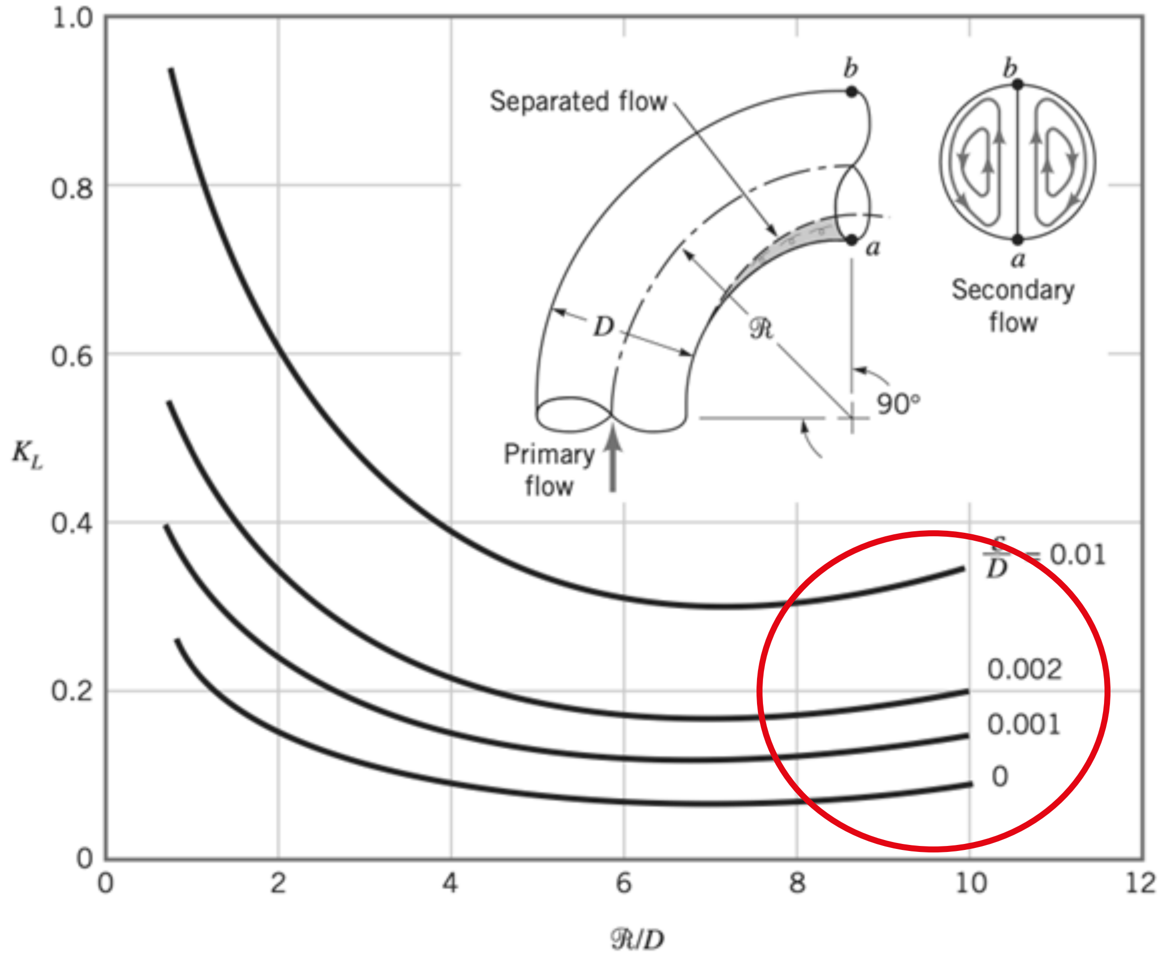
$$p_2 + \frac{1}{2}\rho V_2^2 = p_1$$

Do you remember?





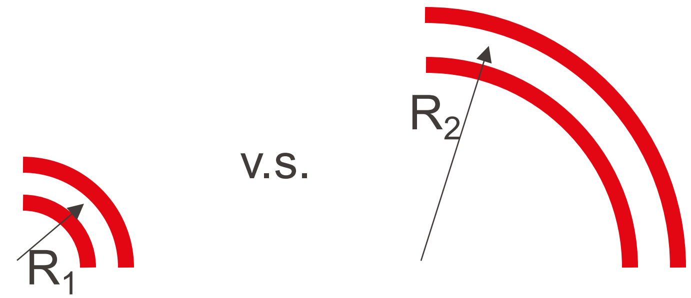
Why does the loss coefficient increase slightly?



$$h_{L\ \mathrm{major}} = f\frac{\ell}{D}\frac{V^2}{2g}$$

$$h_{L\ \mathrm{minor}} = K_L\frac{V^2}{2g}$$

- Increased flow path
  - for a given D, the flow path is increased as R
  - Velocity profile difference between inner and outer bend is larger → secondary flow separation



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One example turbomachinery, which is directly linked to the lift and drag of airfoils?

Propellers (Helicopter)

1-Continuity equation (Mass conservation)

$$B = mb$$

$$\frac{DB_{\text{sys}}}{Dt} = \frac{\partial B_{\text{cv}}}{\delta t} + \dot{B}_{\text{out}} - \dot{B}_{\text{in}}$$

$$\frac{DB_{\text{sys}}}{Dt} = \frac{\partial B_{\text{cv}}}{\partial t} + \rho_2 A_2 V_2 b_2 - \rho_1 A_1 V_1 b_1$$

Time rate of change of the system mass = 0 :

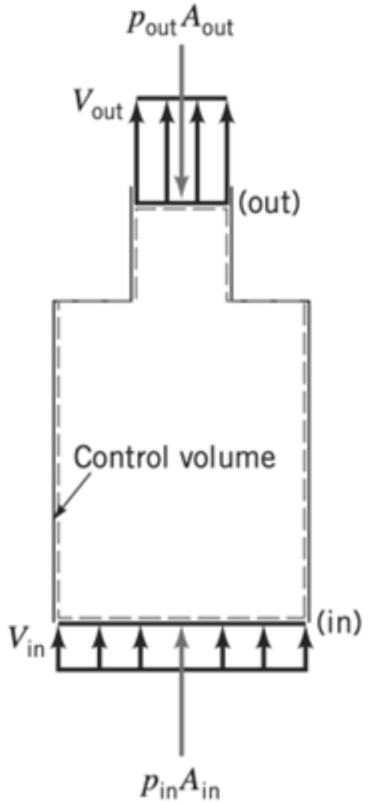
$$\frac{DM_{\text{sys}}}{Dt} = 0$$

Reynolds transport theory

The system mass, M_{sys} is $M_{\text{sys}} = \int_{\text{sys}} \rho d\mathcal{V}$

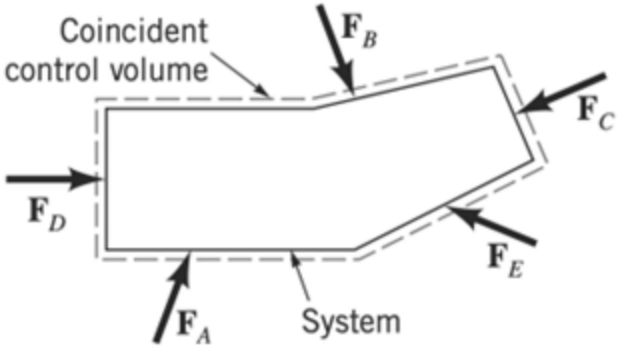
- Continuity equation

- Mass flow rate $\dot{m} = \rho Q = \rho A V$



- cs: control **surface**
- cv: control **volume**

2-Linear momentum equation



Newton's second law of motion

The change of **motion** of an object is **proportional** to the **force** impressed; and is made in the direction of the straight line in which the force is impressed

- Linear momentum equation

$$\boxed{\frac{\partial}{\partial t}\int_{\mathrm{cv}} \mathbf{V}\rho d\mathcal{V} + \int_{\mathrm{cs}} \mathbf{V}\rho \mathbf{V}\cdot\hat{\mathbf{n}}dA = \sum \mathbf{F}_{\substack{\text{contents of the}\\ \text{control volume}}}}$$

3-Energy equation

- cs: control **surface**
- cv: control **volume**

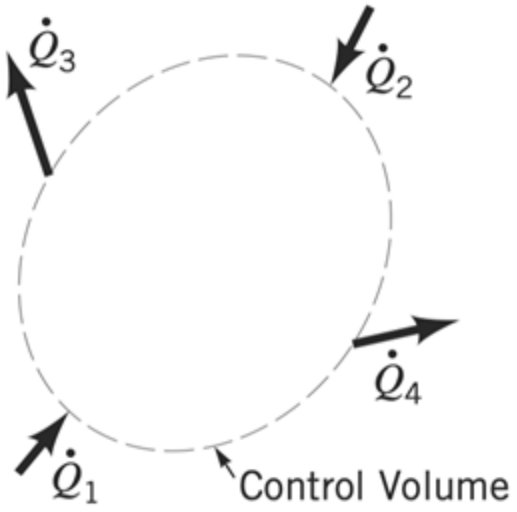
The **first law of thermodynamics**

$$\boxed{\frac{D}{Dt}\int_{\text{sys}} e\rho d\cancel{V} = \frac{\partial}{\partial t}\int_{\text{cv}} e\rho d\cancel{V} + \int_{\text{cs}} e\rho \mathbf{V}\cdot\hat{\mathbf{n}} dA}$$

Time rate of increase of the total stored energy of the system = time rate of increase of the total stored energy of the contents of the control volume + net rate of flow of the total stored energy out of the control volume through the control surface

where $e = \check{u} + \frac{V^2}{2} + gz$ total stored energy per unit mass

internal energy



$$\dot{Q}_{\substack{\text{net}\\ \text{in}}} = \dot{Q}_1 + \dot{Q}_2 - \dot{Q}_3 - \dot{Q}_4$$

$$\boxed{\dot{Q}_{\substack{\text{net}\\ \text{in}}} + \dot{W}_{\substack{\text{shaft}\\ \text{net in}}} = \frac{\partial}{\partial t}\int_{\text{cv}} e\rho d\cancel{V} + \int_{\text{cs}} \left(\check{u} + \frac{p}{\rho} + \frac{V^2}{2} + gz\right)\rho \mathbf{V}\cdot\hat{\mathbf{n}} dA}$$

Heat transfer ratio

Work transfer rate, power

Three governing equations

- Continuity equation

$$\boxed{\frac{\partial}{\partial t}\int_{\mathrm{cv}} \rho d\forall + \int_{\mathrm{cs}} \rho \mathbf{V}\cdot\hat{\mathbf{n}}dA = 0}$$

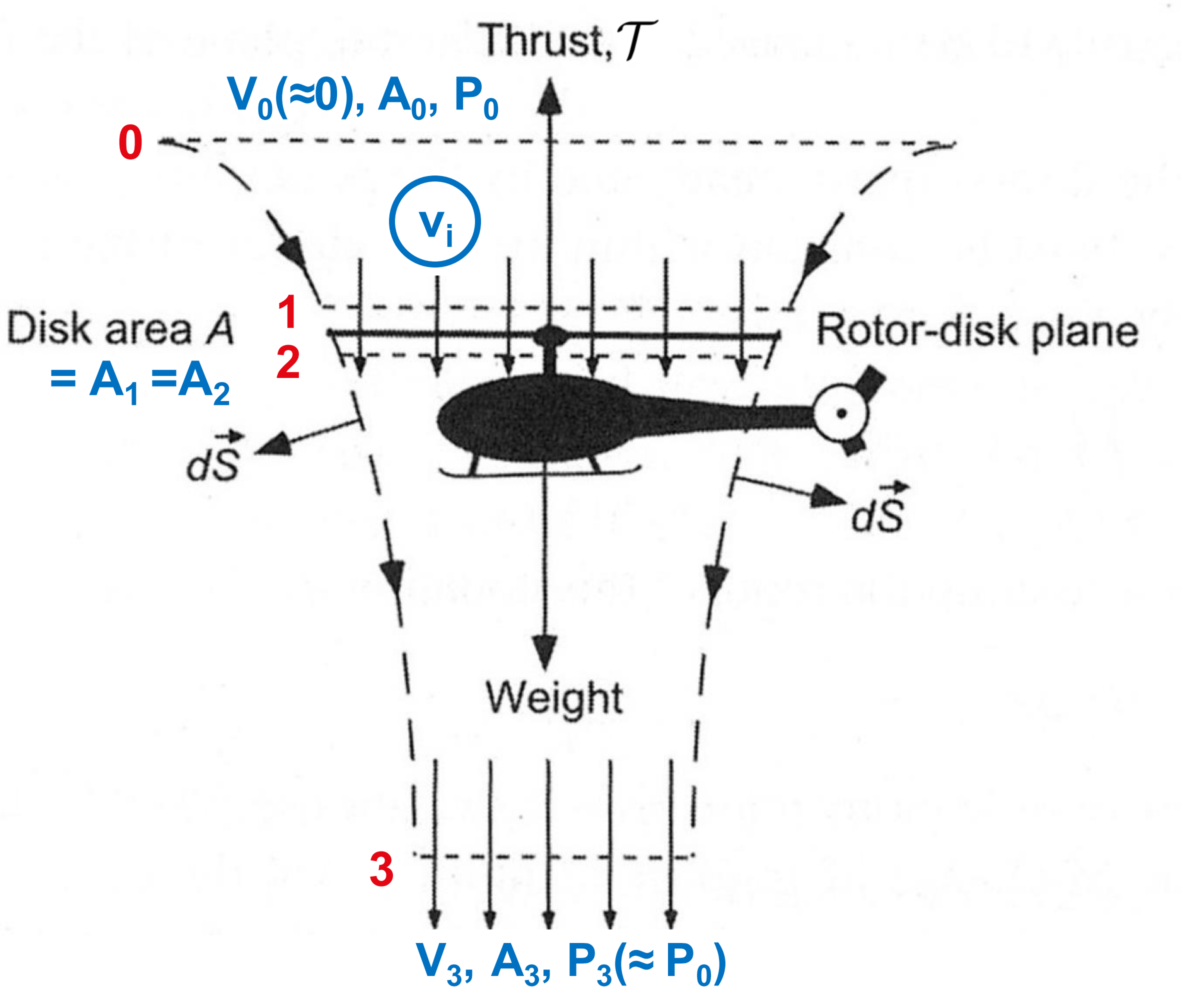
- Linear momentum equation

$$\boxed{\frac{\partial}{\partial t}\int_{\mathrm{cv}} \mathbf{V}\rho d\forall + \int_{\mathrm{cs}} \mathbf{V}\rho\mathbf{V}\cdot\hat{\mathbf{n}}dA = \sum \mathbf{F}_{\substack{\text{contents of the}\\ \text{control volume}}}}$$

- Energy equation

$$\boxed{\dot{Q}_{\substack{\text{net}\\ \text{in}}} + \dot{W}_{\substack{\text{shaft}\\ \text{net in}}} = \frac{\partial}{\partial t}\int_{\mathrm{cv}} e\rho d\forall + \int_{\mathrm{cs}}\left(\check{u} + \frac{p}{\rho} + \frac{V^2}{2} + gz\right)\rho\mathbf{V}\cdot\hat{\mathbf{n}}dA}$$

Helicopters- conservation law for hovering rotor



v_i : induced air velocity
V_3 : far wake velocity

- **Continuity & linear momentum equations:**

$$\boxed{\frac{\partial}{\partial t}\int_{cv}\rho d\cancel{V} + \int_{cs}\rho\mathbf{V}\cdot\hat{\mathbf{n}}dA = 0}$$

$d\vec{\mathbf{S}}$

$$\boxed{\frac{\partial}{\partial t}\int_{cv}\mathbf{V}\rho d\cancel{V} + \int_{cs}\mathbf{V}\rho\mathbf{V}\cdot\hat{\mathbf{n}}dA = \sum\mathbf{F}_{\text{contents of the control volume}}}$$

Steady-state approximation

$$\dot{m} = \int_A \rho\mathbf{V}\cdot\hat{\mathbf{n}}dA$$

Thrust, $\mathcal{T}$ (reaction force)

$$\mathcal{T} = \dot{m}V_3$$

- **Rotor power** (work done by rotor per unit time)

$$P_{\text{rotor}} = \mathcal{T}v_i$$

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Helicopters- conservation law for hovering rotor

- Energy equation (the gain of energy of the fluids per unit time)

adiabatic steady No internal energy change

$$\boxed{\cancel{\dot{Q}_{\substack{\text{net}\\\text{in}}}} + \dot{W}_{\substack{\text{shaft}\\\text{net in}}} = \cancel{\frac{\partial}{\partial t}\int_{\text{cv}} e\rho d\mathcal{V}} + \int_{\text{cs}}\left(\cancel{\check{u}} + \frac{p}{\rho} + \frac{V^2}{2} + gz\right)\rho\mathbf{V}\cdot\hat{\mathbf{n}}dA}$$

$d\vec{\mathbf{S}}$

Mass flow rate

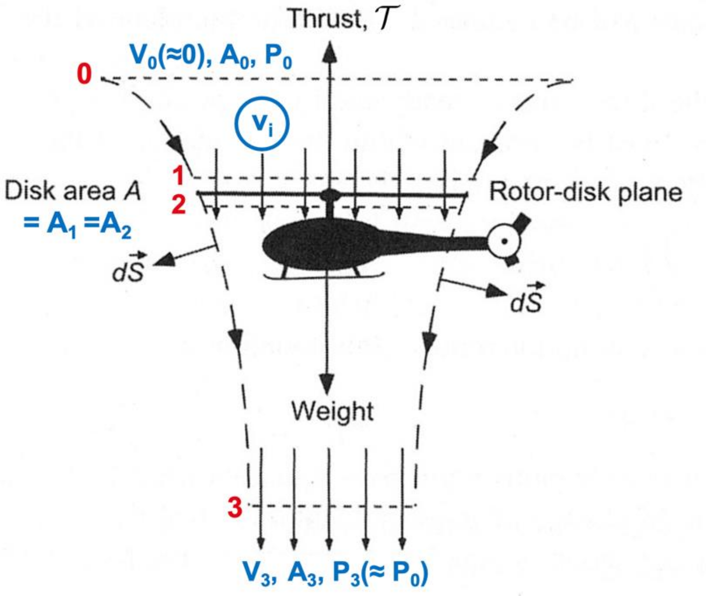
$$\boxed{\dot{m} = \int_A \rho\mathbf{V}\cdot\hat{\mathbf{n}}dA}$$

$$\dot{W}_{\substack{\text{shaft}\\\text{net in}}} = P_{\text{rotor}} =$$

$$=$$

Potential energy change ≪ kinetic energy

$$= \frac{V_3^2}{2}\dot{m}$$



Helicopters- conservation law for hovering rotor

- **Continuity equation:**

$$\mathcal{T} = \dot{m}V_3, \;\; P_{\text{rotor}} = \mathcal{T}v_i$$

- **Energy equation:** $P_{\text{rotor}} = \frac{V_3^2}{2}\dot{m}$

Induced velocity:

Far field velocity is twice of the induced velocity

- **Mass flowrate:** $\dot{m} = \rho A_3 V_3 = \rho A_2 v_i = \rho A v_i$

$$\mathcal{T} = \dot{m}V_3 = \dot{m}(2v_i) = (\rho A v_i)(2vi) = 2\rho A v_i^2$$

The induced velocity: $$\boxed{v_h \equiv v_i = \sqrt{\frac{\mathcal{T}}{2\rho A}} = \sqrt{\left(\frac{\mathcal{T}}{A}\right)\left(\frac{1}{2\rho}\right)}}$$

Disk loading

Helicopters- Figure of merit (FM)

$$P_{\text{rotor}} = \frac{V_3^2}{2}\dot{m} = \dot{m}V_3 v_i = 2\dot{m}v_i^2$$

$$\dot{m} = \rho A v_i$$

$$v_i = \sqrt{\frac{\mathcal{T}}{2\rho A}} = \sqrt{\left(\frac{\mathcal{T}}{A}\right)\left(\frac{1}{2\rho}\right)}$$

$$\boxed{P_{\text{rotor}} = \frac{\mathcal{T}^{3/2}}{\sqrt{2\rho A}}}$$ Ideal rotor power

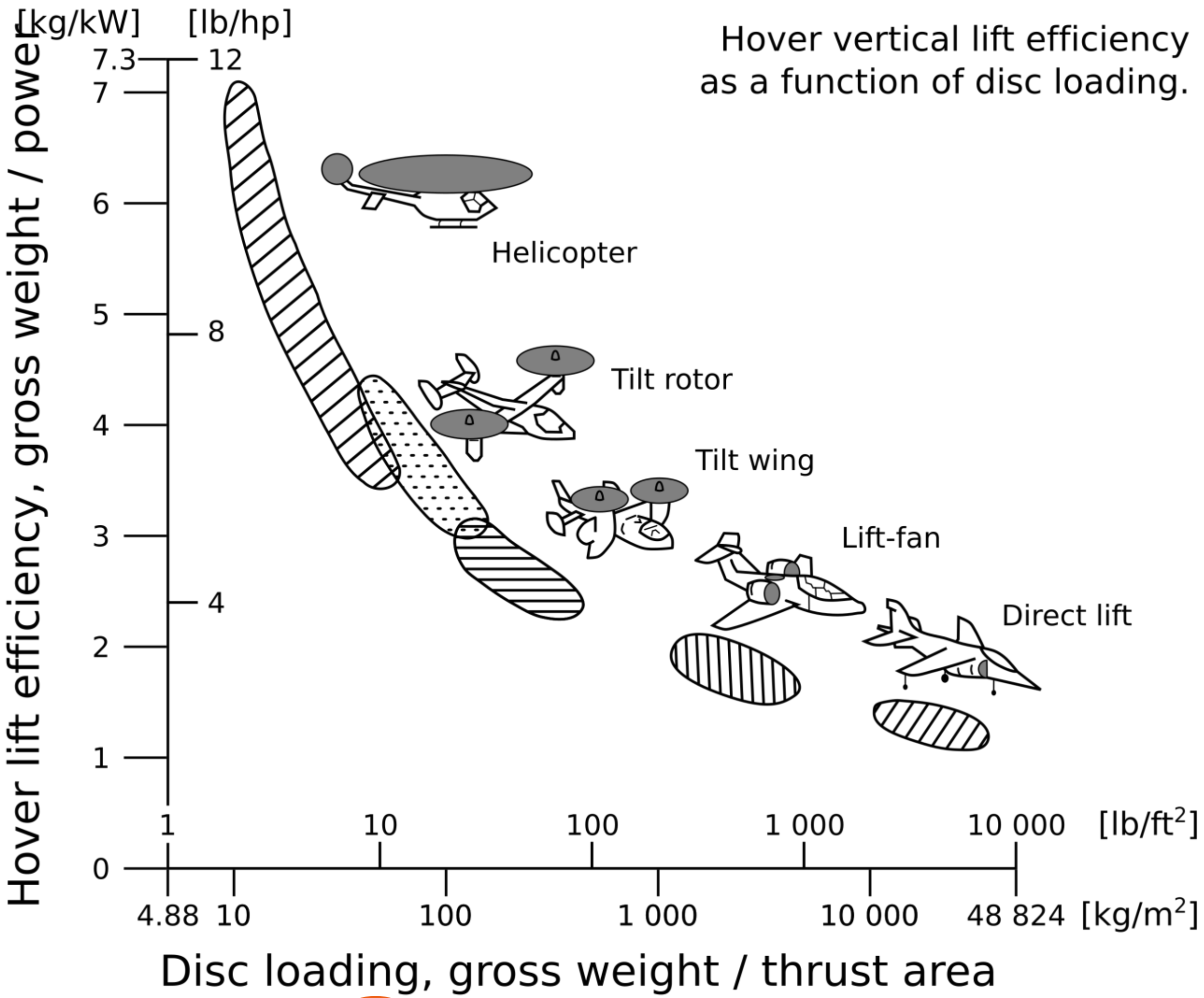
Figure of merit, FM = $\dfrac{\text{Ideal rotor power}}{\text{Actual rotor power}}$

If FM is 0.7 and the computed ideal power is 2000 kW, what is the actual power required?

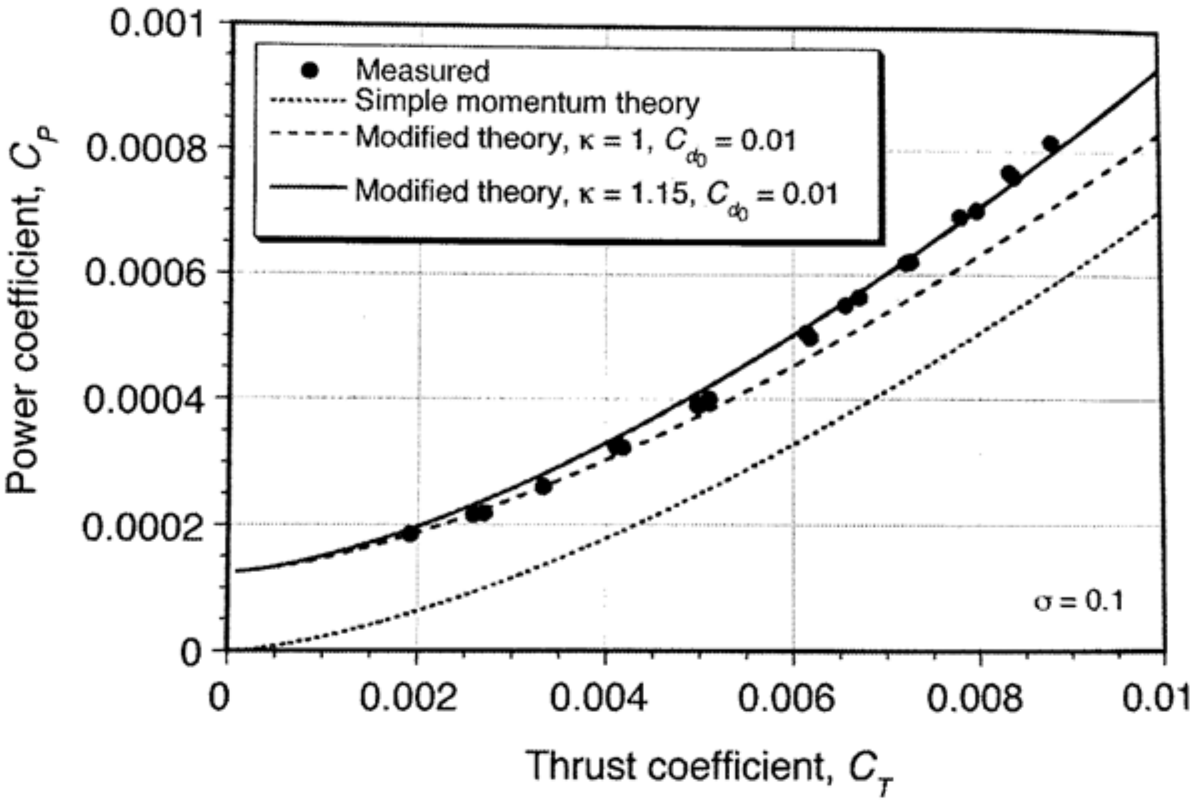
Hovering efficiency vs. disk loading

Ideal power:

$$P_{\text{rotor}} = \frac{\mathcal{T}^{3/2}}{\sqrt{2\rho A}}$$

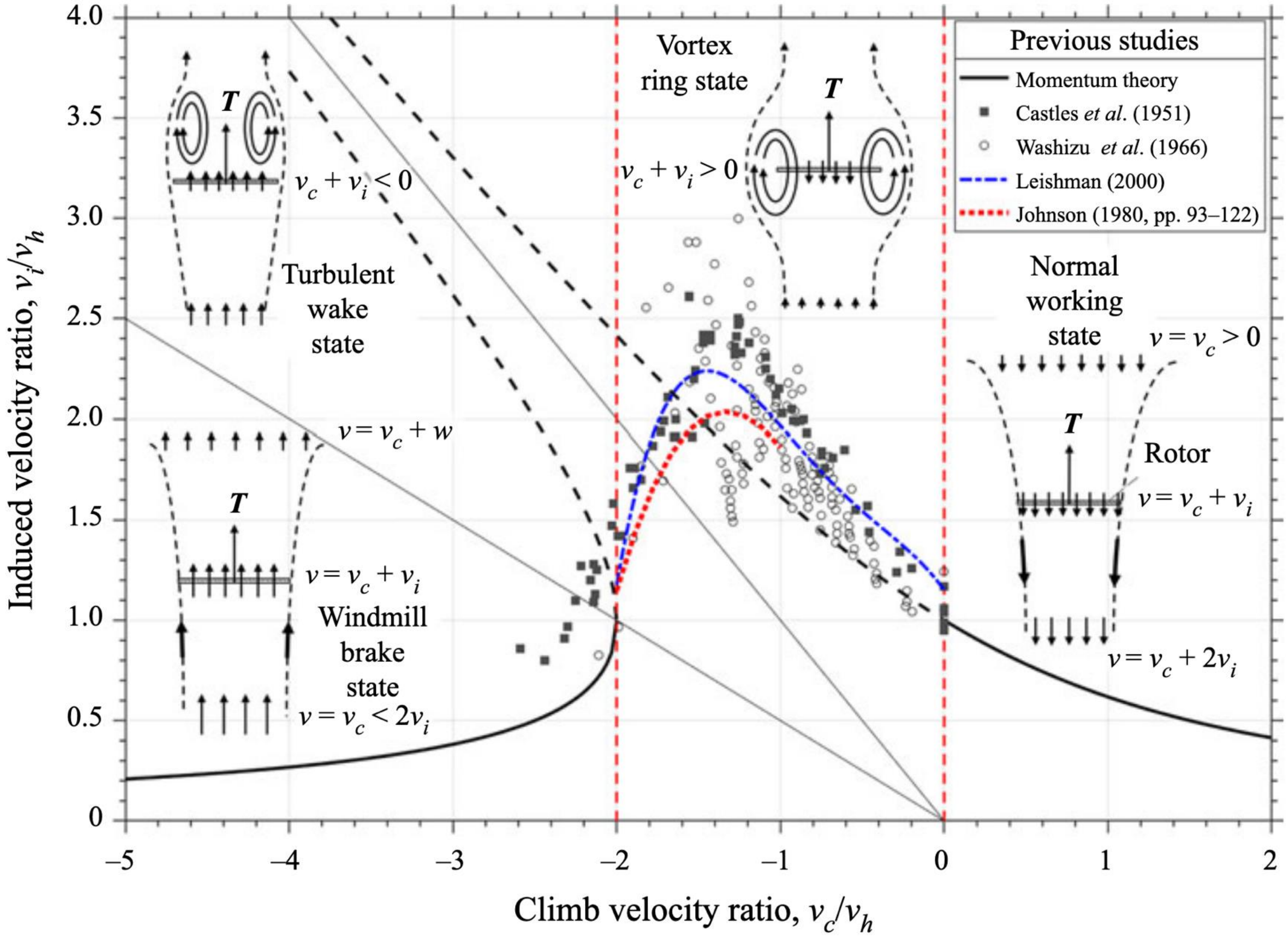


$$\frac{\mathcal{T}}{A}$$



*Note, here C_P is not the pressure coefficient (C_p)

Real helicopter dynamics are much more complex..



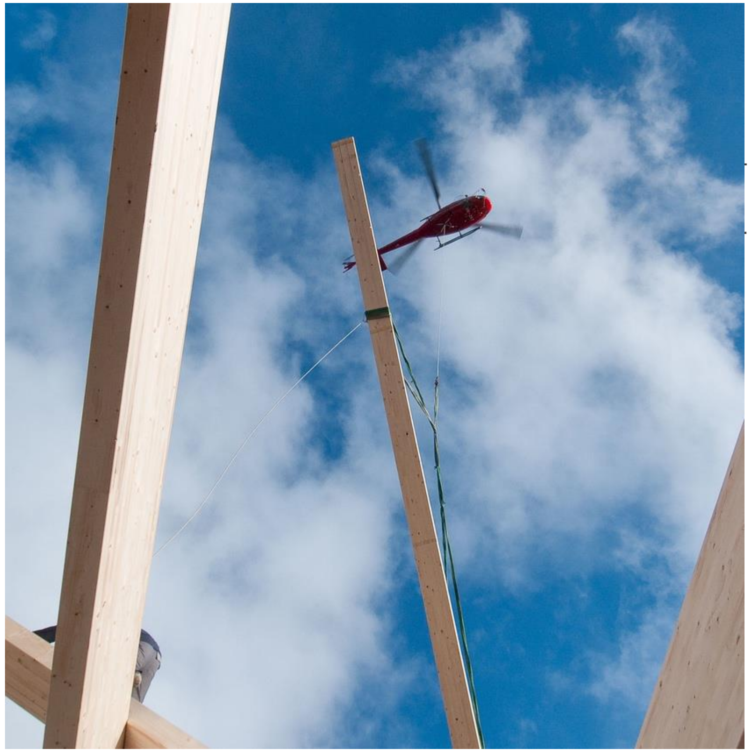
Vortex ring state

Exercise

A helicopter is helping to the construction of a mountain cabane.

(1) Estimate the power required for the helicopter to hover.
(2) What is the amount of fuel required for 10 min hover? (ignore mass change)

- Gross mass of helicopter 20 tonnes
- Rotor diameter 12 m
- Density of air 1.2 kg/m3
- Figure of merit of the rotor 0.75
- Fuel for helicopter (BP Avgas 80), 30 MJ/litre (= 8.3 kWh/litre)



$$P_{\text{rotor}} = \mathcal{T} v_i$$

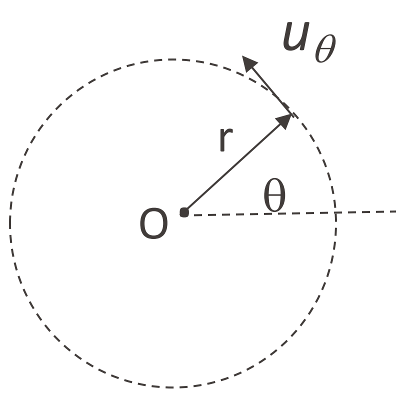
$$v_i = \sqrt{\frac{\mathcal{T}}{2\rho A}}$$

Vortex

Vortex flow

Vortex flow: A vortex may be seen as region of a flow in which the fluid rotates around a straight or curved line

- Free vortex (potential flow):
  - Velocity field:



Vortex intensity

$$\vec{\mathbf{u}} = (0, u_\theta, 0), \quad u_\theta(r) = \frac{\Gamma}{2\pi r}$$

$$\mathbf{\Gamma} = \oint_C \vec{\mathbf{u}} \cdot ds = cst.$$

ds = infinitesimal path element along the closed curve C

  - Irrotational flow

$$\vec{\zeta} = \nabla \times \vec{\mathbf{u}} = \left(0, 0, \frac{1}{r}\frac{\partial\,(r u_\theta)}{\partial r}\right) = (0, 0, 0)\ \forall\ r > 0$$

Vortex flow

- Free vortex (potential flow):
  - Pressure field :
    - Navier-Stokes (radial equilibrium): $\dfrac{\partial p}{\partial r} = \rho \dfrac{u_\theta^2}{r} = \rho \dfrac{\Gamma^2}{4\pi^2 r^3}$

$$u_\theta(r) = \frac{\Gamma}{2\pi r}$$

Integration → $p(r) = p_\infty - \dfrac{\rho\Gamma^2}{8\pi^2 r^2}$

    - Alternate method - Bernoulli equation:

Since the flow is irrotational and steady, Bernoulli equation reads:

$$\forall r > 0, \quad p(r) + \rho\frac{u_\theta^2}{2} = \text{ constant}$$

$$\Rightarrow \forall r > 0, \quad p(r) = p_\infty - \rho\frac{u_\theta^2}{2} = p_\infty - \frac{\rho\Gamma^2}{8\pi^2 r^2}$$

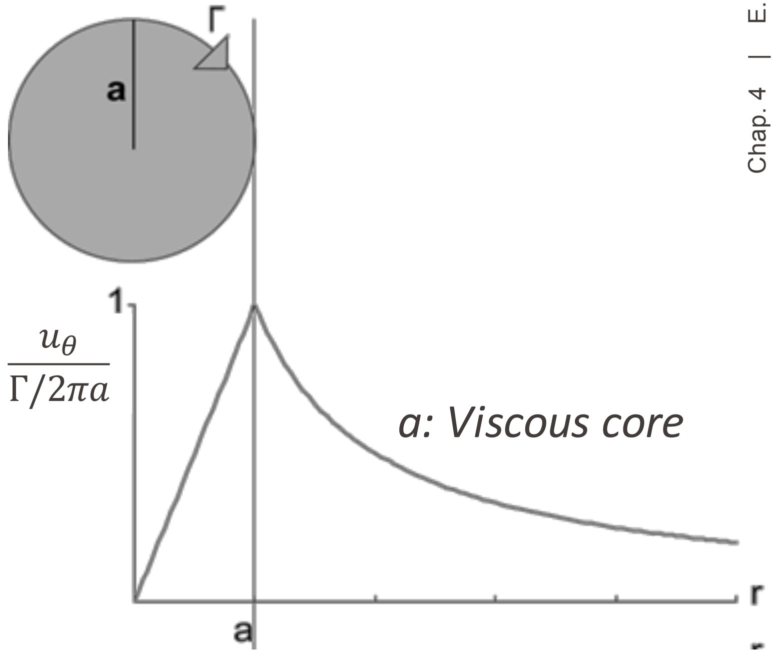
- Limitations:

$$\lim_{r\to 0} u_\theta(r) = +\infty, \quad \lim_{r\to 0} p(r) = -\infty$$

Vortex flow

- Rankine model:
  - Close to the axis, the viscous forces limits the velocity
  - Velocity field:

$$\begin{cases} r \geq a, & u_\theta = \dfrac{\Gamma}{2\pi r} \\ r \leq a, & u_\theta = \omega r \end{cases}$$



where Γ is the circulation:

$$\Gamma = \int_0^{2\pi} u_\theta r d\theta$$

- Continuity of the velocity at $r = a$

$$\omega \mathrm{a} = \frac{\Gamma}{2\pi a} \quad \rightarrow \quad \omega = \frac{\Gamma}{2\pi a^2} \quad \text{or} \quad \Gamma = 2\pi\omega a^2$$

Vortex flow

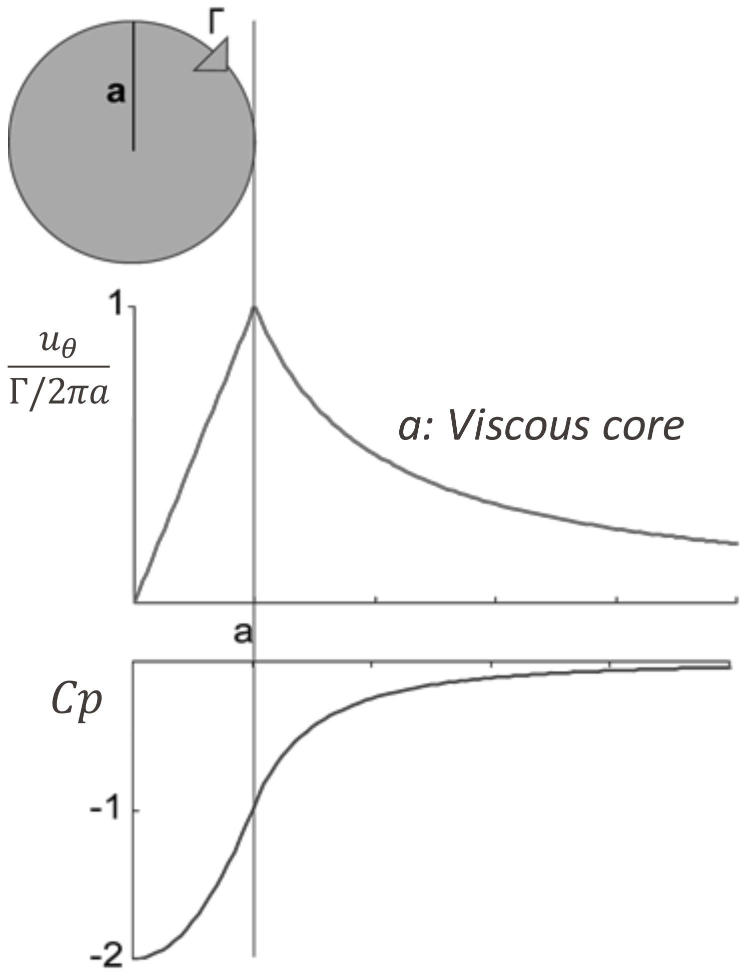
- Rankine model:
  - Pressure field (Navier Stokes)

$$\frac{\partial p}{\partial r} = \rho \frac{u_\theta^2}{r}$$

  - Integration:

$$\begin{cases} r \geq a, \ p(r) = p_\infty - \dfrac{\rho \Gamma^2}{8\pi^2 r^2} \\ r \leq a, \ p_\infty - \dfrac{\rho \Gamma^2}{8\pi^2 a^2}\left(2 - \dfrac{r^2}{a^2}\right) \end{cases}$$

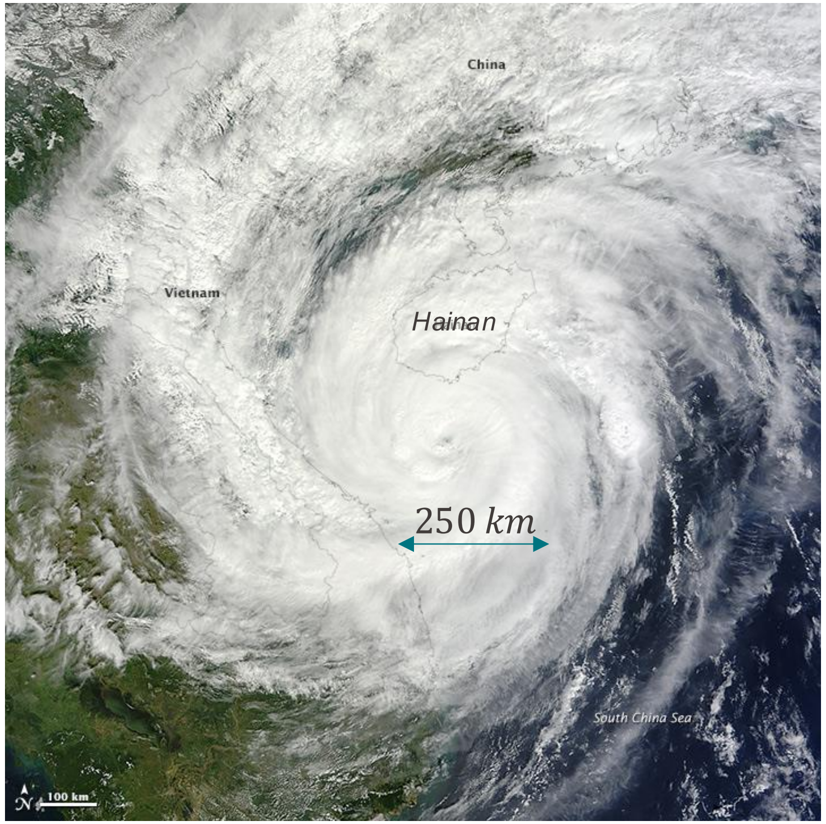
$$C_p(r) = \frac{p(r) - p_\infty}{\frac{1}{2}\rho\left(\frac{\Gamma}{2\pi a}\right)^2} \qquad C_{p,\min} = -2$$



- Discontinuity of the velocity derivative at $r = a$
  - More sophisticated models are available (e.g. Oseen model, Vestas model, …)

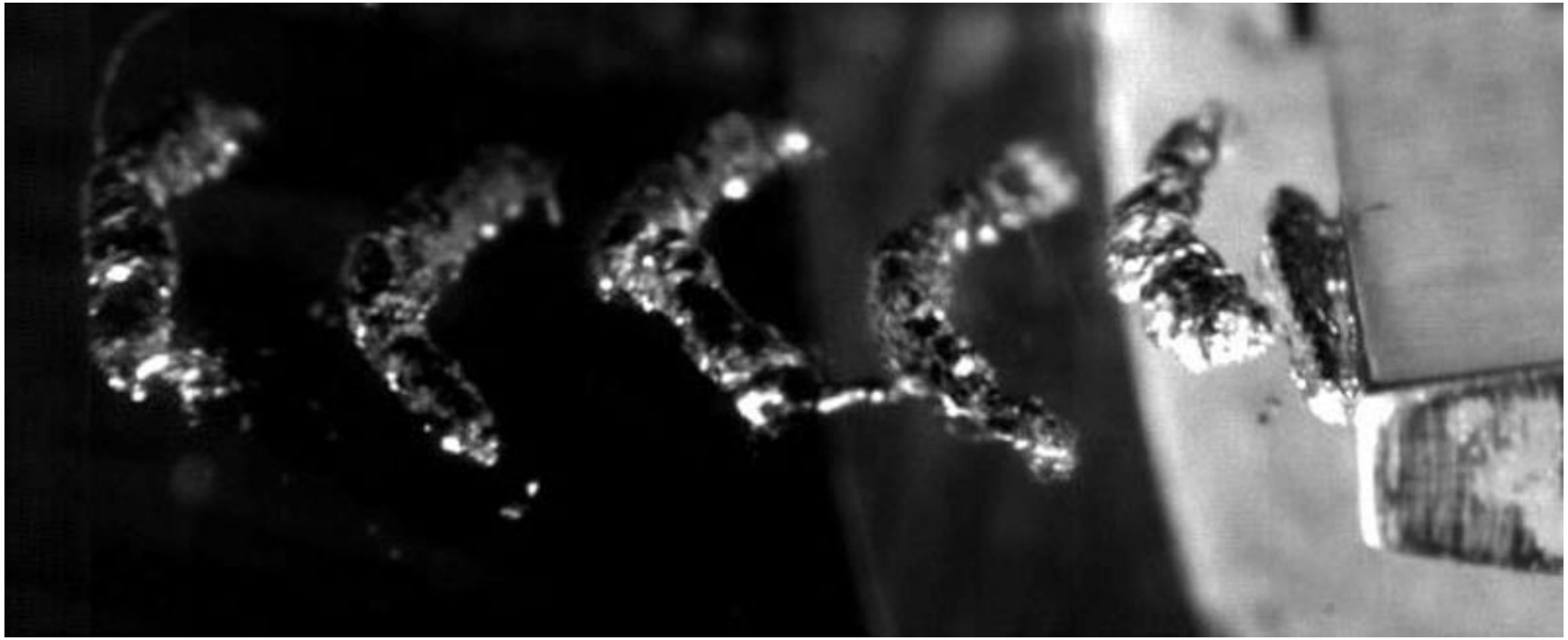
Vortex flow

- Example of vortex flow: Super Typhoon Haiyan, Philippines, November 2013
  - Maximum wind speed ~ 360 km/h
    - Pressure in the vortex axis ?
    - Assumptions:
      - Air density $\rho = 1$ kg/m^3
      - Atm. pressure = $1000\ \mathrm{hPa}$
      - 2D flow



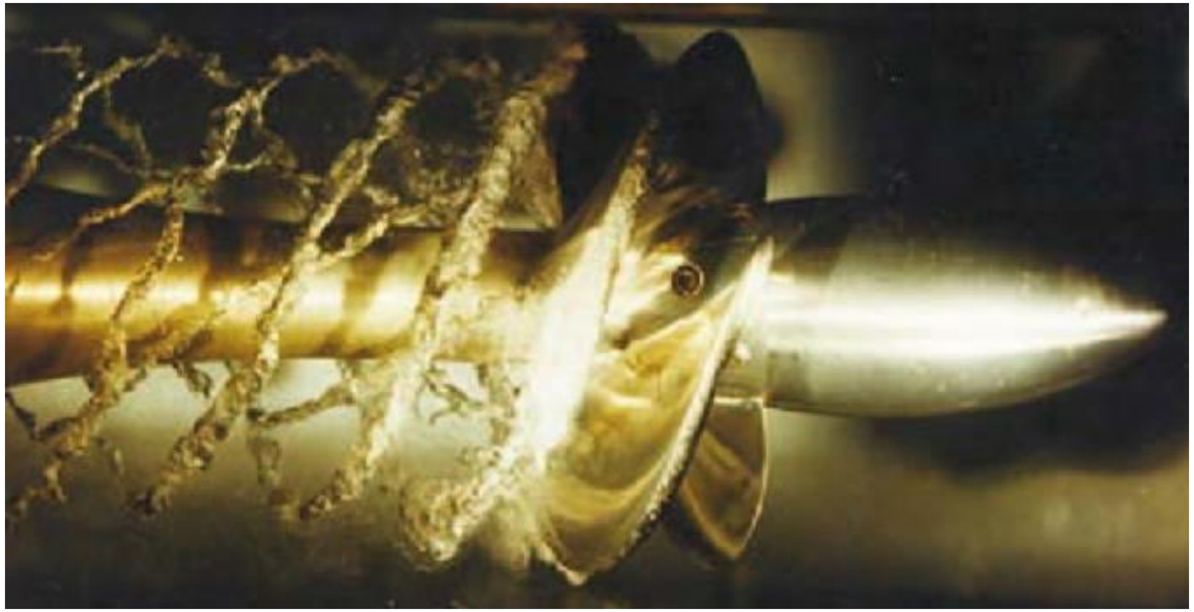
Vortex flow

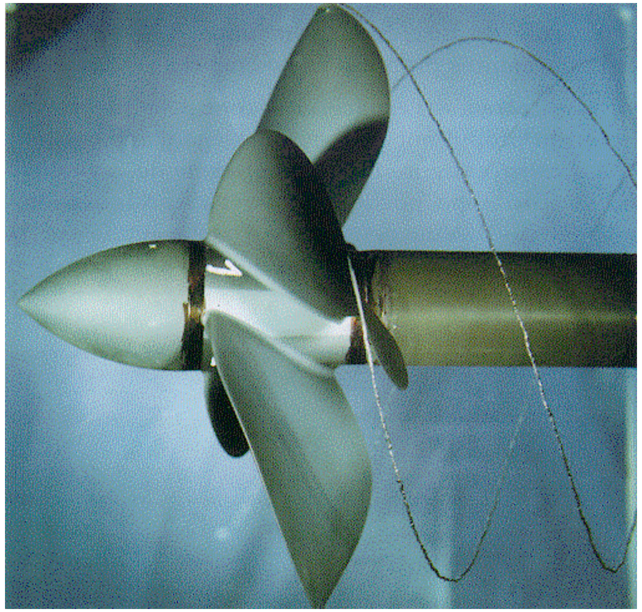
- Example of vortex flow: Karman vortices in the wake of a blunt hydrofoil
  - Flow instability in the wake → Alternate shedding of discrete vortices
  - Low pressure in the core of the vortices → Cavitation
  - Hydro-elastic coupling (Strong induced vibration):
    Vortex shedding frequency ~ resonance frequency of the hydrofoil (torsional mode)



Vortex flow

- Example of vortex flow: Tip vortices in marine propellers
  - Lifting blade with finite span (ducted or non-ducted impeller):
    - A swirl develop at the tip as the flow escapes from high to low pressure sides
    - Risk of cavitation → Source of noise and vibration





Vortex flow

- Example of vortex flow: Rope in Francis turbines
  - The rope is a cavitating vortex that develops at the outlet of a Francis turbine
  - Due to the residual kinetic moment at part load condition
  - Source of strong hydraulic instabilities (noise and vibration)