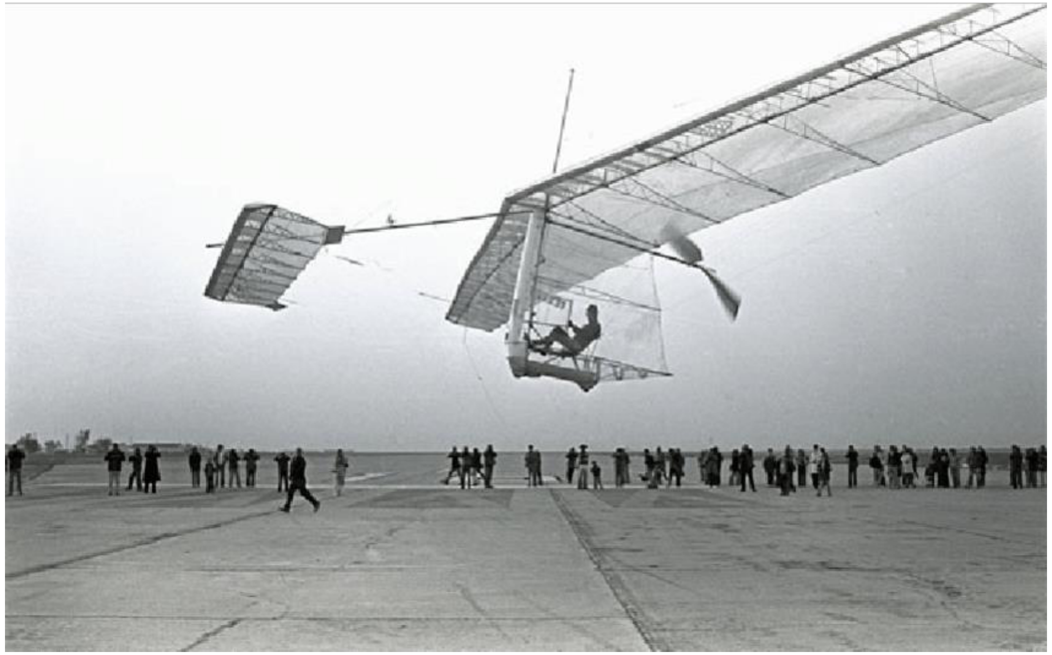
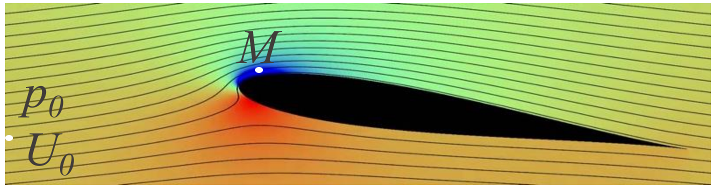
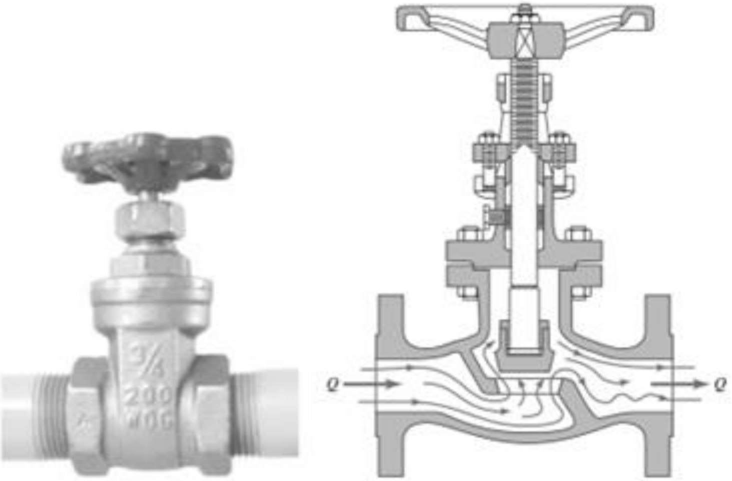






Chapter 3: Basics of Fluid Dynamics – Part2

ME-342 Introduction to turbomachinery

École polytechnique fédérale de Lausanne

Prof. Eunok Yim, HEAD-lab.

What is the number in the circle?

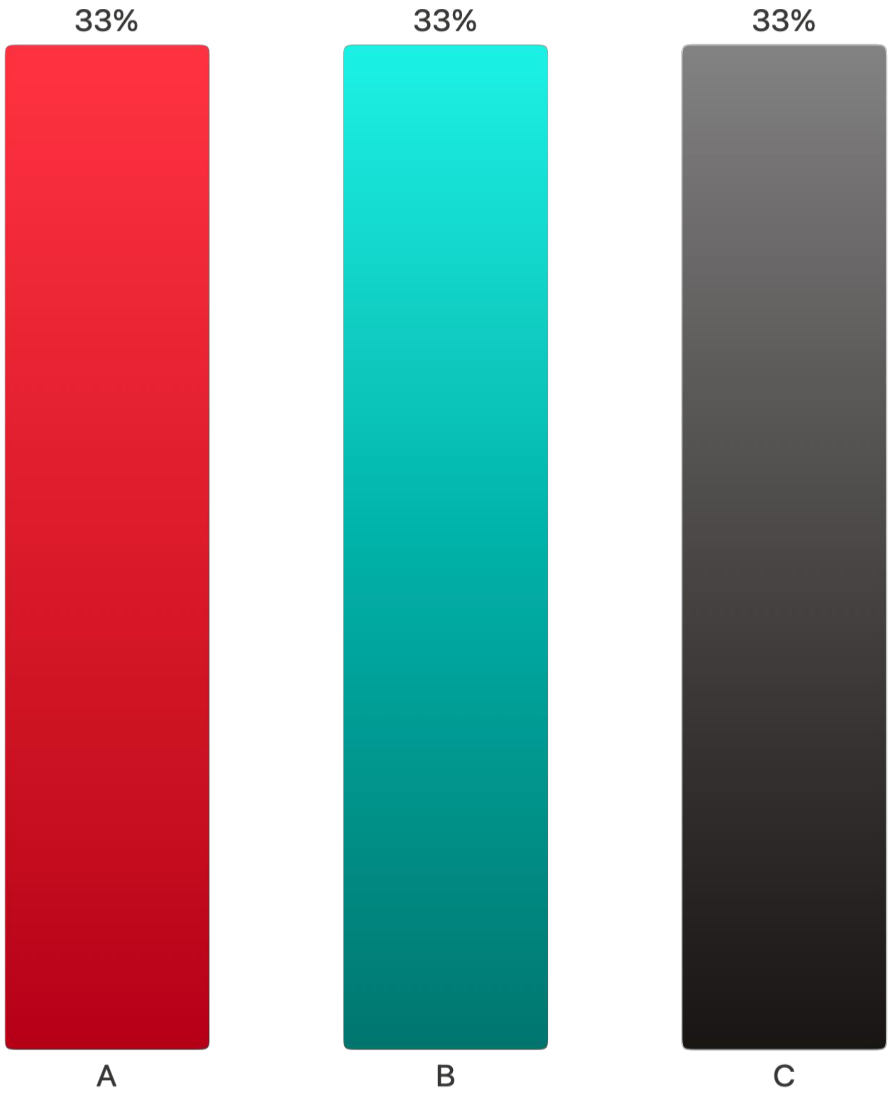
For circular pipe

$$u = V_c\left(1 - \frac{r^{\bigcirc}}{R^{\bigcirc}}\right)$$

A. 2

B. 3

C. 4



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Enter question text here...

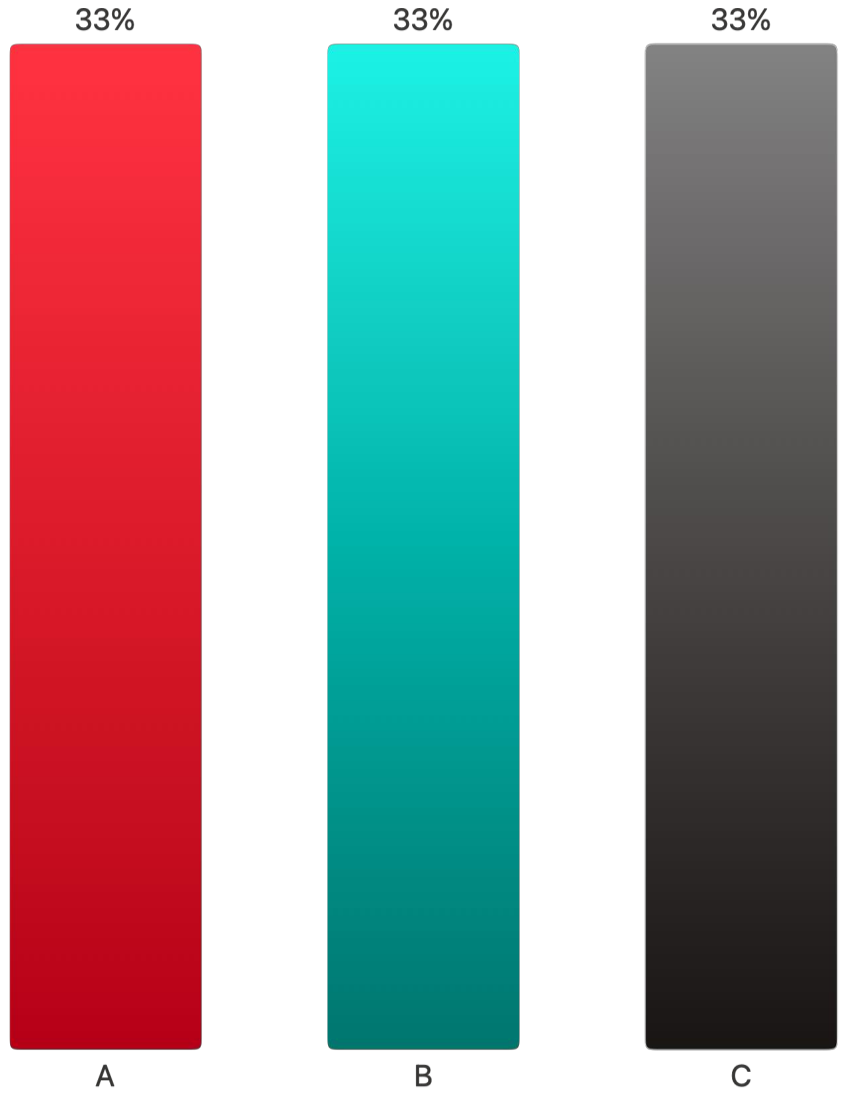
For circular pipe

$$Q = \frac{\pi D^{\bigcirc} \Delta p}{128 \mu \ell}$$

A. 2

B. 3

C. 4



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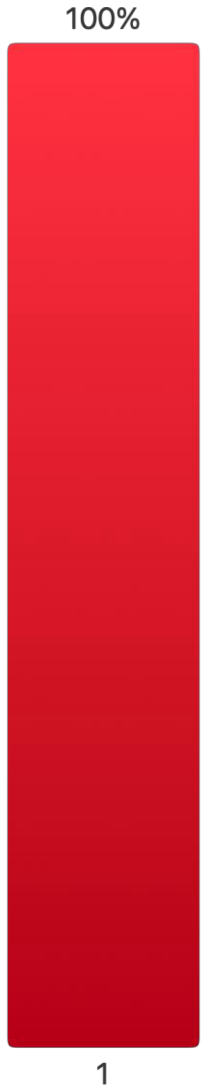
What is this dimensionless number?

[ ] number, $= \frac{\rho U D}{\mu} = \frac{U D}{\nu}$

Rank Responses



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Questions from last lecture..

- -2% error on the pipe diameter (not +2 %) → **±8%**

$$Q_0 = \frac{\pi D_0^4 \Delta p}{128 \mu \ell}$$

→ $D = 1.02 D_0$ → $$Q = \frac{\pi (1.02 D_0)^4 \Delta p}{128 \mu \ell} = 1.02^4 Q_0 = 1.082 Q_0$$ + 8.2%

→ $D = 0.98 D_0$ → $$Q = \frac{\pi (0.98 D_0)^4 \Delta p}{128 \mu \ell} = 0.98^4 Q_0 = 0.922 Q_0$$ - 7.8%

Questions from last lecture..

- Why the energy equation take a form of [m] ?

  - Energy unit :

$$1\ \mathrm{J} = 1\ \mathrm{kg}\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^2 = 1\frac{\mathrm{kg}\cdot\ \mathrm{m}^2}{\mathrm{s}^2}$$

  - Mechanical energy equation or extended Bernoulli equation

$$\frac{p_{\text{in}}}{\rho} + \frac{\alpha_{\text{in}}\ \bar{V}_{\text{in}}^2}{2} + gz_{\text{in}} + w_{\text{shaft}} - \text{loss} = \frac{p_{\text{out}}}{\rho} + \frac{\alpha_{\text{out}}\ \bar{V}_{\text{out}}^2}{2} + gz_{\text{out}}$$

$$1\ \mathrm{Pa} = 1\ \mathrm{N/m^2} = 1\ \mathrm{kg/(m\cdot s^2)}$$

$$1\ \mathrm{W} = 1\ \mathrm{J/s} = 1\ \mathrm{N}\cdot\ \mathrm{m/s} = 1\ \mathrm{kg}\cdot\ \mathrm{m}^2\cdot\ \mathrm{s}^{-3}$$

$$w_{\text{shaft}} = \frac{\dot{W}_{\text{shaft}}}{\dot{m}} = \frac{[\mathrm{W}]}{[\mathrm{kg/s}]}$$

Unit = m^2/s^2

Energy per unit mass

Divide by gravitational acceleration, g [m/s^2]

$$\frac{p_1}{\gamma} + \alpha_1\frac{\bar{V}_1^2}{2g} + z_1 + h_s - h_L = \frac{p_2}{\gamma} + \alpha_2\frac{\bar{V}_2^2}{2g} + z_2$$

$\gamma = \rho g$ Specific weight

Head loss

$$h_s = \frac{w_{\text{shaft net in}}}{g} = \frac{\dot{W}_{\text{shaft net in}}}{\dot{m}g}$$

Energy per unit weight

Unit = m

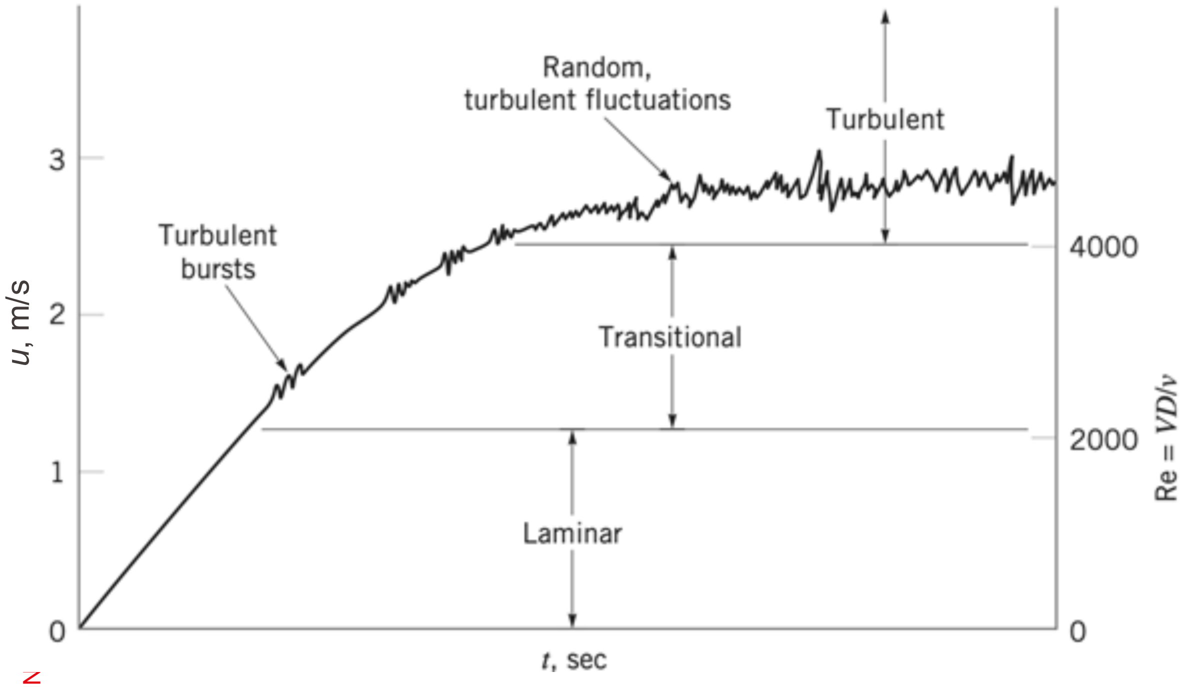
Questions from last lecture..

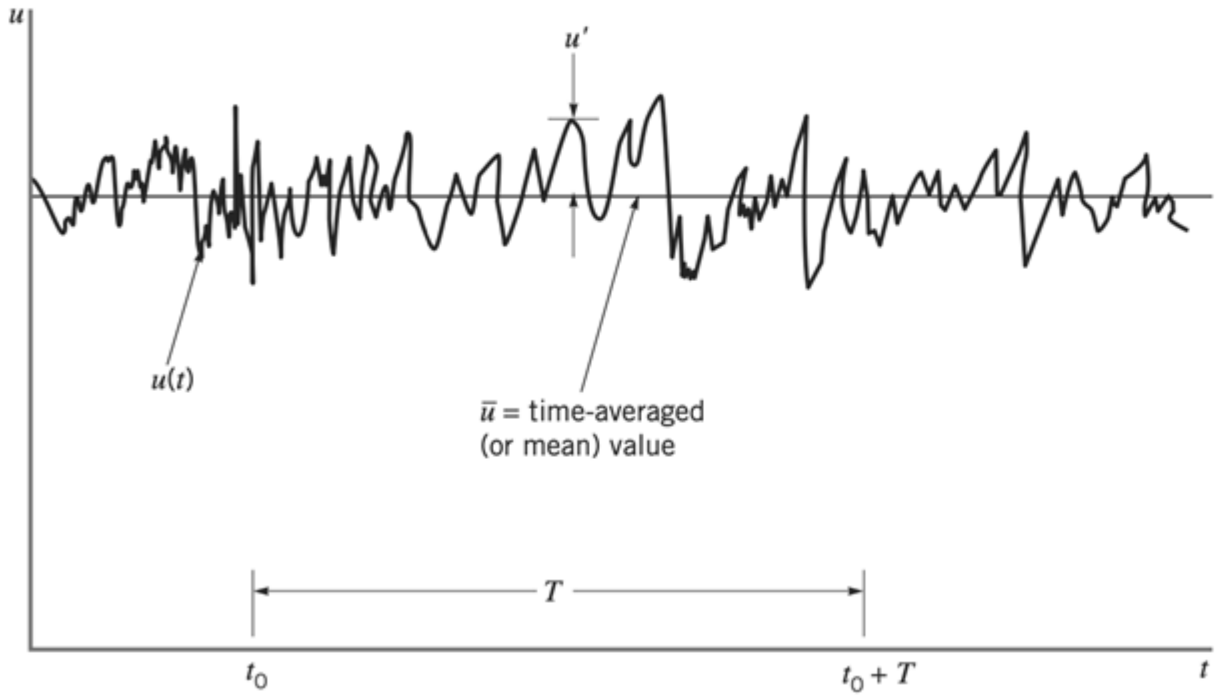
- What are the assumptions of using the energy equation?
  - Along the streamline flow along a streamline between two sections
  - Incompressible
  - Steady
  - Irrotational flow (fluid domain)
  - Inviscid

In practice, none of these assumptions is exactly true

Velocity approximation in turbulent pipe

- **Some basics of Turbulent flow**





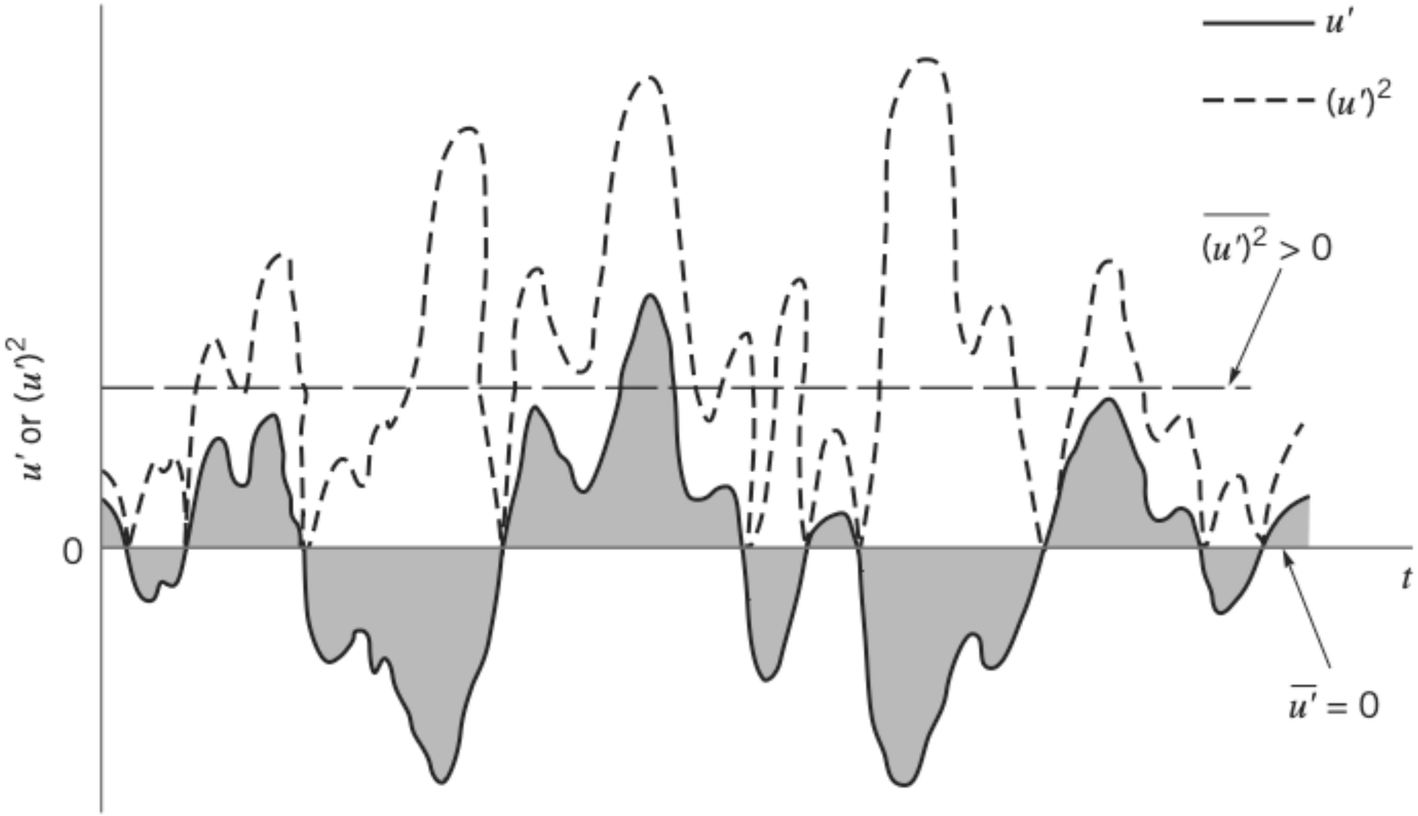
$$\mathbf{u}(\mathbf{x},t) = \overline{\mathbf{u}(\mathbf{x})} + \mathbf{u}'(\mathbf{x},t)$$

Time-averaged meanflow Fluctuations

$$\bar{\mathbf{u}} = \frac{1}{T}\int_{t_0}^{t_0+T} \mathbf{u}(\mathbf{x},t)dt \qquad \mathbf{u}' = \frac{1}{T}\int_{t_0}^{t_0+T} \mathbf{u}'(\mathbf{x},t)dt = 0$$

Stochastic or random fluctuations

- Squared average is larger than zero



$$\overline{\mathbf{u}'^2} = \frac{1}{T}\int_{t_0}^{t_0+T} \mathbf{u}'^2(\mathbf{x},t)dt > 0$$

Shear stress in presence of turbulence

- **Turbulent shear stress**

$$\tau = \mu \frac{d\bar{u}}{dy} - \rho \overline{u'v'} = \tau_{\mathrm{lam}} + \tau_{\mathrm{turb}}$$

turbulent shear stress > 0

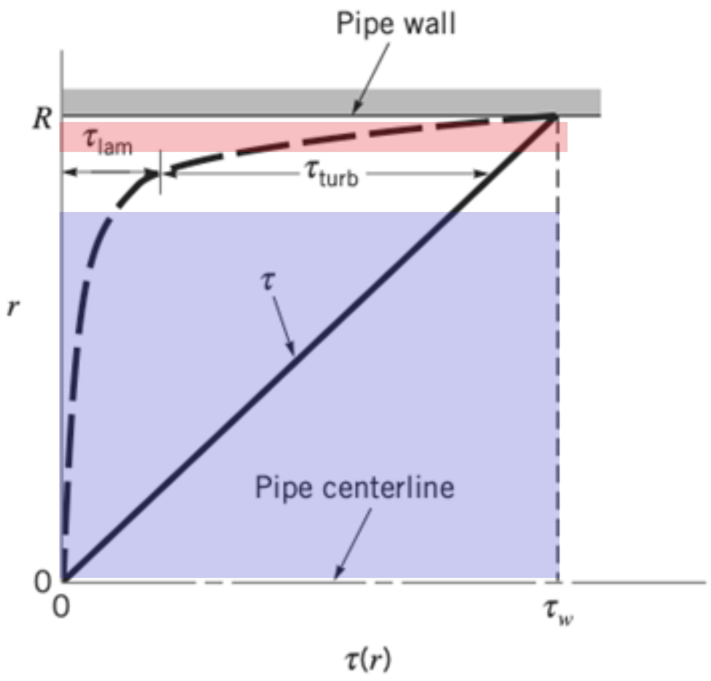
- Shear stress is greater in turbulent flow than in laminar flow
- The forms $-\rho\overline{u'v'},\ -\rho\overline{u'w'},\ -\rho\overline{v'w'}$ are called **Reynolds stress**

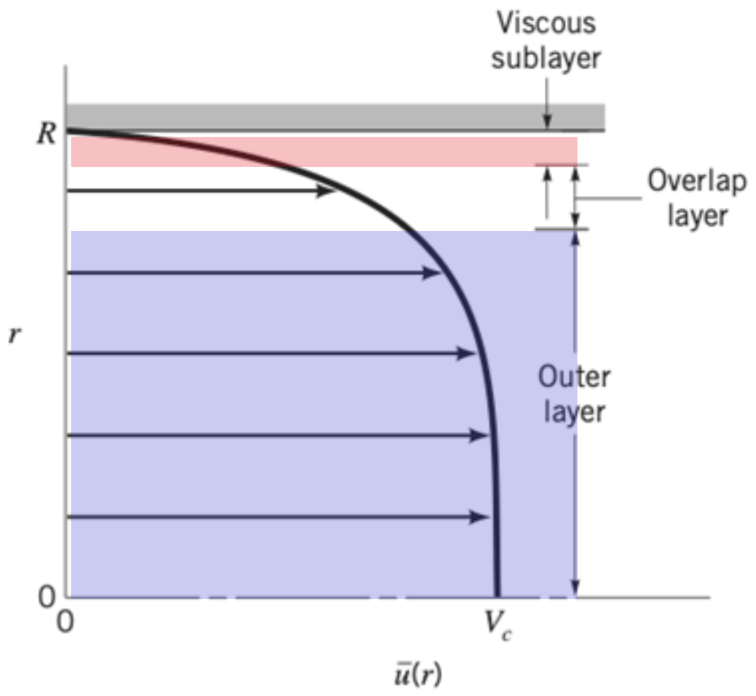
Shear stress composition

- **From various observations...**

$$\tau = \mu \frac{d\bar{u}}{dy} - \rho\overline{u'v'} = \tau_{\mathrm{lam}} + \tau_{\mathrm{turb}}$$

- **Viscous sublayer:** In a very narrow region near the wall, the laminar shear stress is dominant.
- **Outer layer:** Away from the wall, the turbulent portion of the shear stress is dominant.
- **Overlap layer**: The transition between these two regions occurs in the overlap layer.





Velocity profiles in layers

- **In the viscous sublayer**

$$\frac{\bar{u}}{u^*} = \frac{yu^*}{\nu}$$ Valid very near the smooth wall

$y = R - r$ Distance from the wall

$u^* = \sqrt{\frac{\tau_w}{\rho}}$ Friction velocity (not an actual velocity)

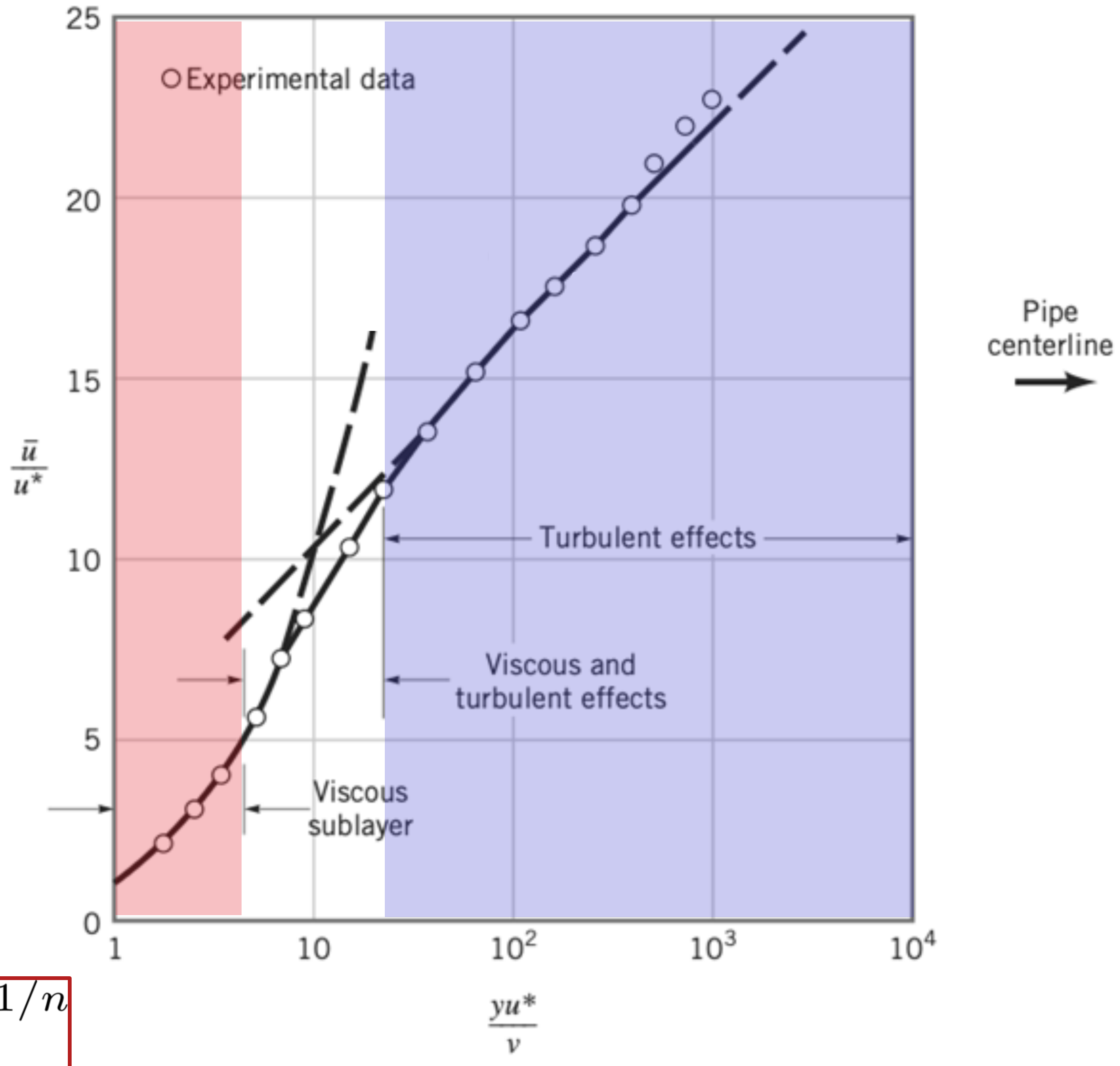
- **In the overlap region**

$$\frac{\bar{u}}{u^*} = 2.5 \ln\left(\frac{yu^*}{\nu}\right) + 5.0$$

Constants 2.5 and 5.0 are determined experimentally

- **In the outer turbulent layer**

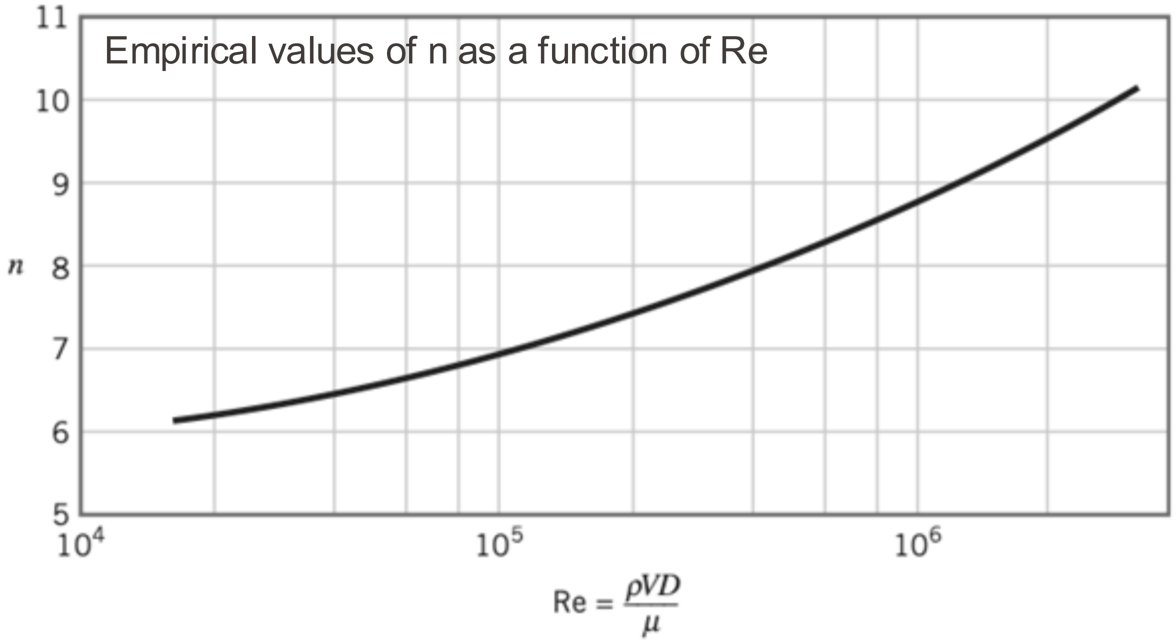
$$\frac{(V_c - \bar{u})}{u^*} = 2.5 \ln\left(\frac{R}{y}\right)$$ or $$\frac{\bar{u}}{V_c} = \left(1 - \frac{r}{R}\right)^{1/n}$$

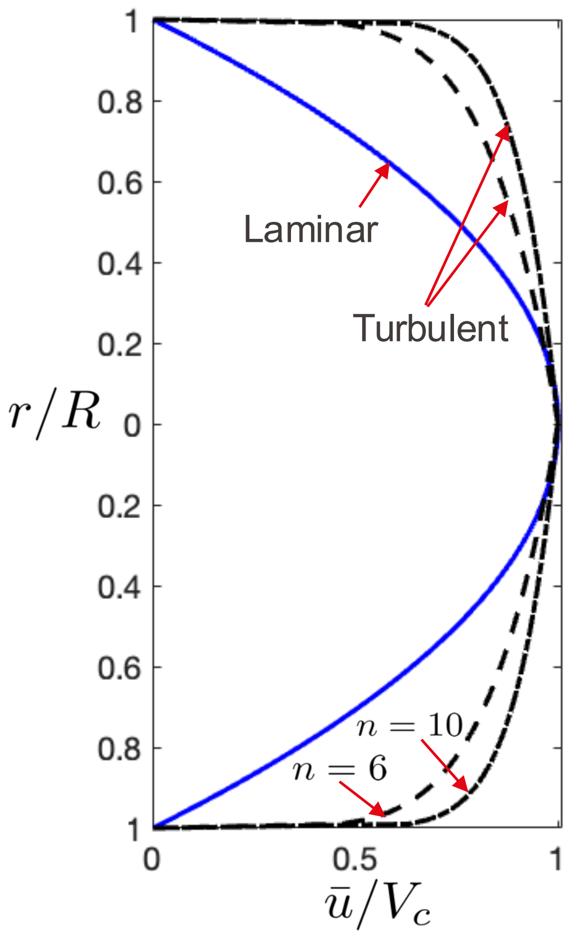


Velocity profile in turbulent pipe

- **Power-law velocity profile**

$$\frac{\bar{u}}{V_c} = \left(1 - \frac{r}{R}\right)^{1/n}$$



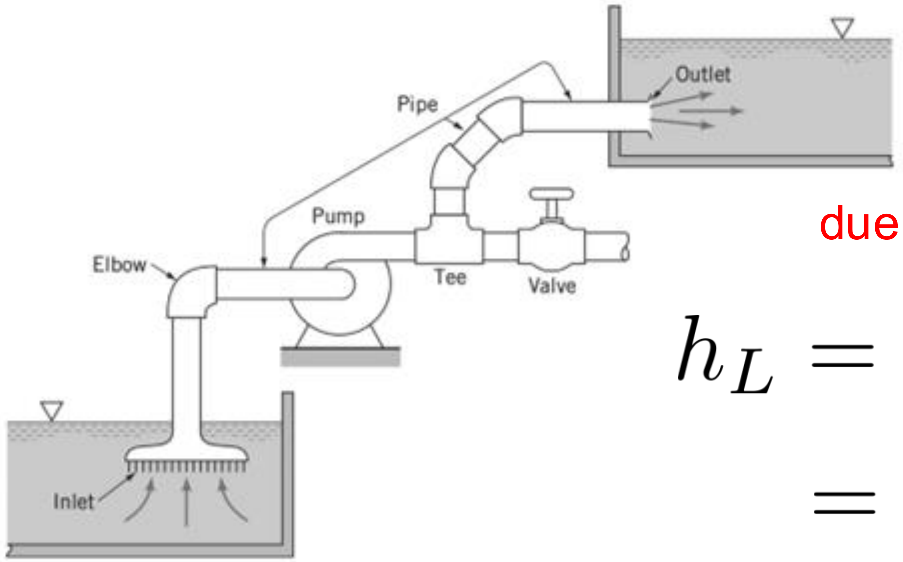


Examine the power-law velocity profile and discuss if it makes sense at :

- Validity r=R
- Validity r=0

Dimensional Analysis of Pipe Flow

- **Turbulent flow** (the chaotic, irregular movement of fluids) is an extremely complicated subject that is hard to fully understand or predict.
- Despite extensive research, scientists and engineers have not yet developed a complete, mathematically rigorous theory that can fully describe or predict turbulence in all situations. It remains one of the major unsolved problems in fluid dynamics.
- Most turbulent pipe flow analyses are based on experimental data and semi-empirical formulas.



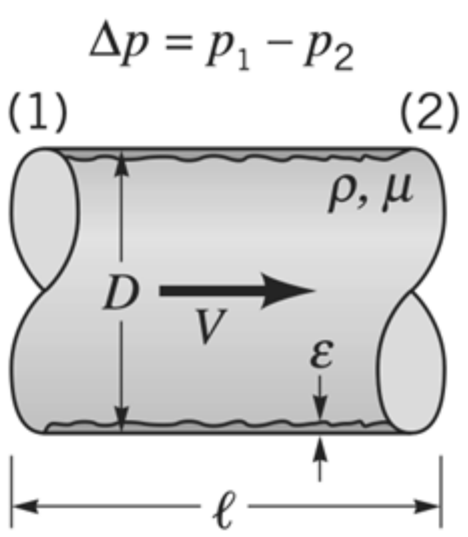
The head loss in turbulent and non-idealized pipe

due to viscous effects — due to various pipe components

$$h_L = h_{L,\text{major}} + h_{L,\text{minor}}$$
$$= h_{L,\text{regular}} + h_{L,\text{singular}}$$

- do not necessarily reflect the relative importance of each type of loss

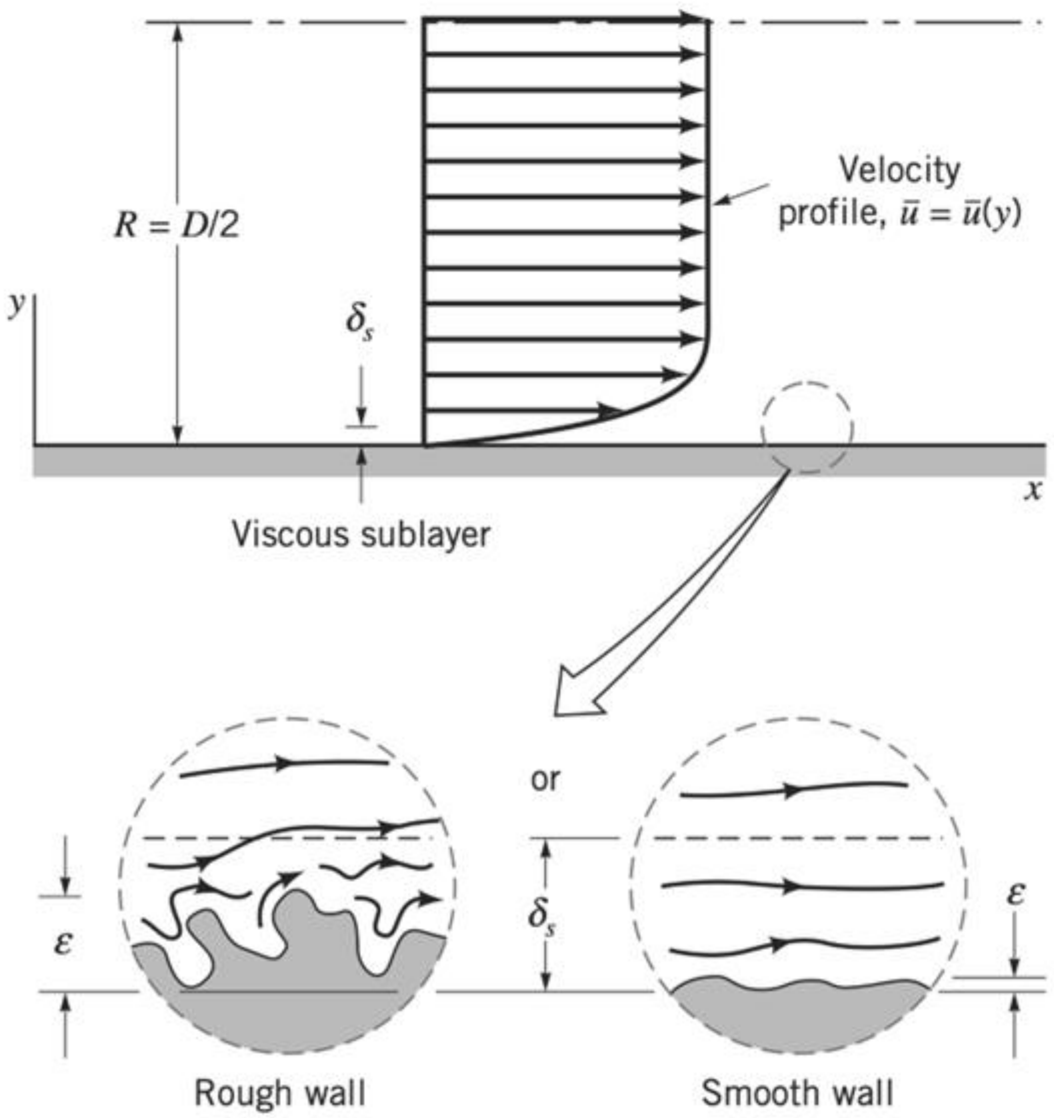
Major Losses (Regular losses)



Head loss in a pipe are dependent on the wall shear stress, τ_w

$$\Delta p = F(V, D, \ell, \varepsilon, \mu, \rho)$$

We consider $0 \leq \varepsilon/D \leq 0.05$ (larger roughness should be considered as a pipe geometry)



Other parameters (surface tension, vapor pressure, etc) do not affect the pressure drop for the conditions stated (steady, incompressible flow; round, horizontal pipe)

Major Losses

$$\Delta p = F(V, D, \ell, \varepsilon, \mu, \rho) \longrightarrow \frac{\Delta p}{\frac{1}{2}\rho V^2} = \tilde{\phi}\left(\frac{\rho V D}{\mu}, \frac{\ell}{D}, \frac{\varepsilon}{D}\right)$$

From experimental observations, the pressure drop is proportional to the length,

$$\frac{\Delta p}{\frac{1}{2}\rho V^2} = \frac{\ell}{D}\phi\left(Re, \frac{\varepsilon}{D}\right)$$

$$\Delta p = f \frac{\ell}{D}\frac{\rho V^2}{2}$$

$$f = \phi\left(Re, \frac{\varepsilon}{D}\right)$$

Fully developed laminar flow, f=64/Re

- Darcy–Weisbach equation

$$h_{L\ \text{major}} = f \frac{\ell}{D}\frac{V^2}{2g}$$

EPFL

Moody diagram

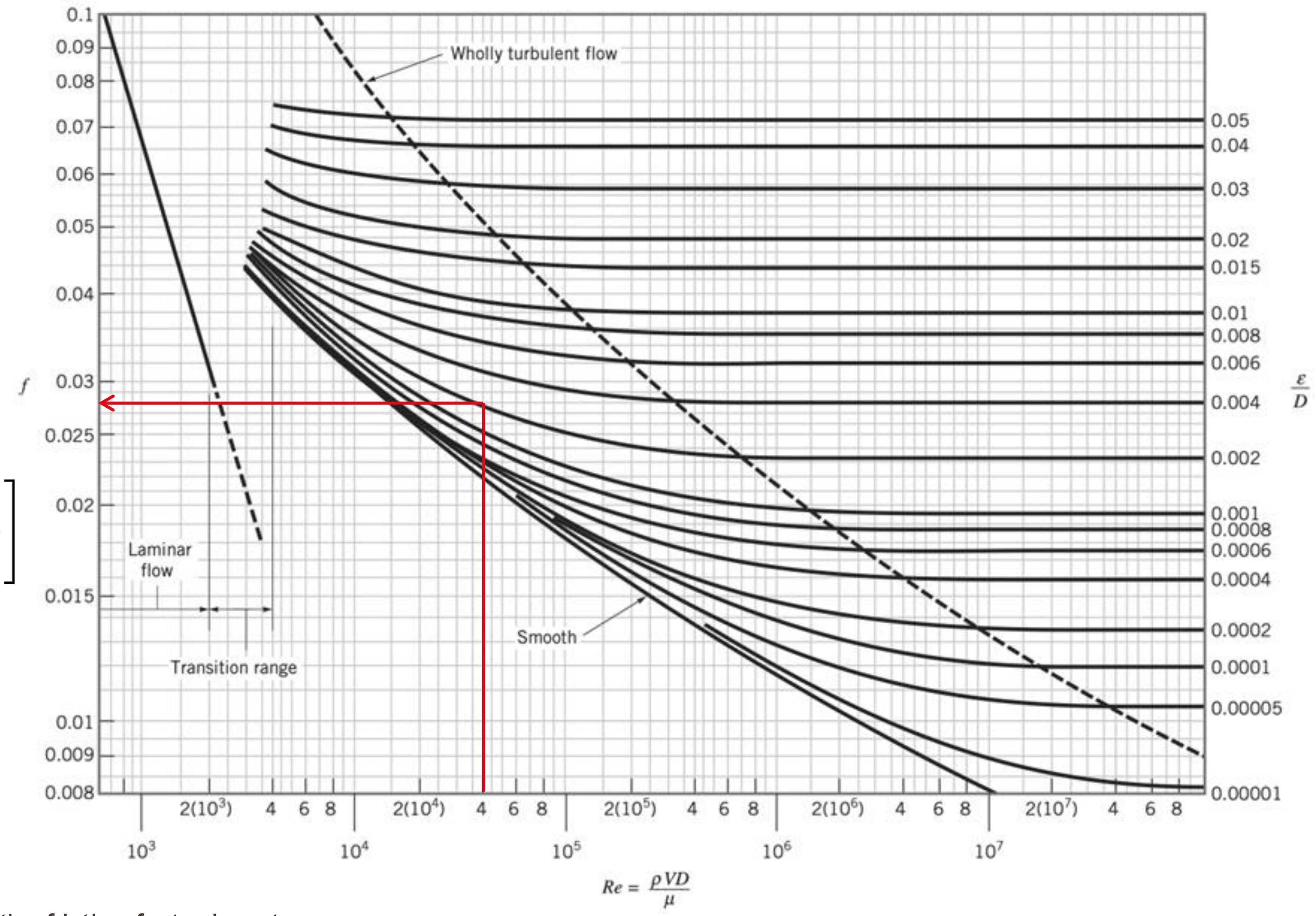
- Colebrook formula

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right)$$

- Haaland equation

$$\frac{1}{\sqrt{f}} = -1.8 \log\left[\left(\frac{\varepsilon/D}{3.7}\right)^{1.11} + \frac{6.9}{\mathrm{Re}}\right]$$

e.g., Re = 4×10^4,
e/D = 0.002
f = 0.028



Note that even for smooth pipes (ε=0) the friction factor is not zero:

- There is a head loss in any pipe, no matter how smooth the surface is made. This is a result of the no-slip boundary condition that requires any fluid to stick to any solid surface it flows over.
- Even when the roughness is considerably less than the viscous sublayer thickness, such pipes are called *hydraulically smooth*.

Minor Losses (Singular losses)

Due to valves, bends, tees, expansion, contraction…

- **Loss coefficient, K_L :**

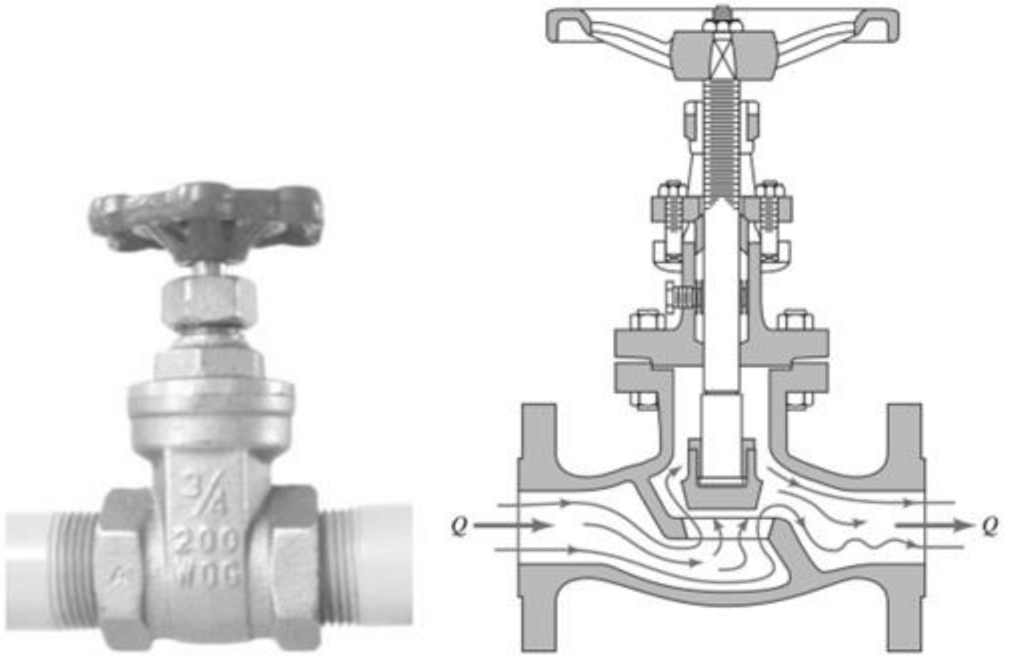
$$K_L = \frac{h_{L\ \text{minor}}}{(V^2/2g)} = \frac{\Delta p}{\frac{1}{2}\rho V^2}$$

$$\Delta p = K_L \frac{1}{2}\rho V^2$$

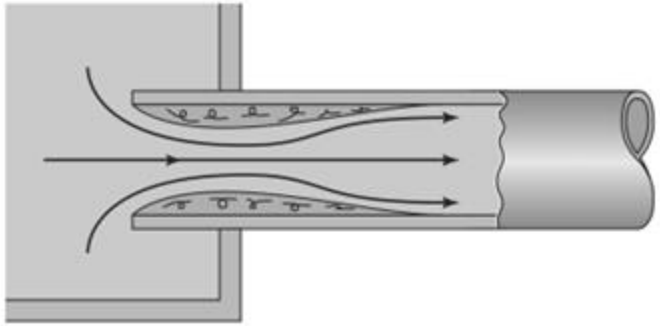
$$h_{L\ \text{minor}} = K_L \frac{V^2}{2g}$$

$$K_L = \phi(\text{geometry}, Re)$$

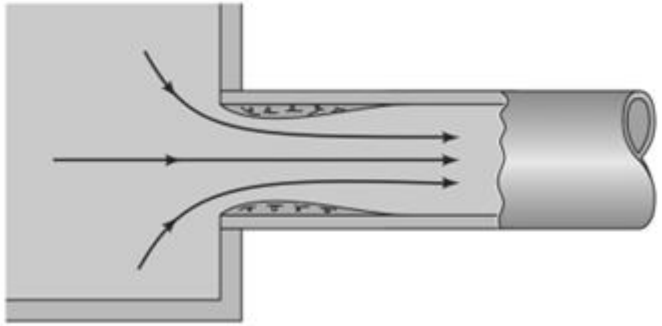
Determined mostly empirically



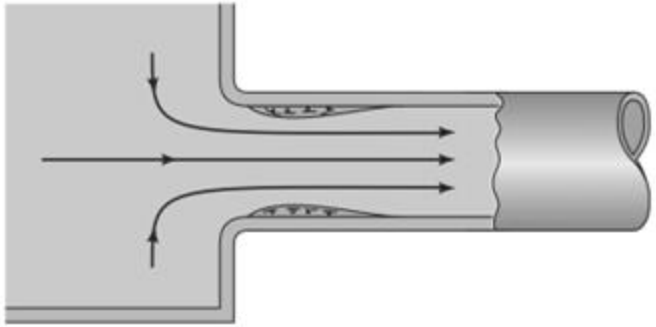
Minor Losses



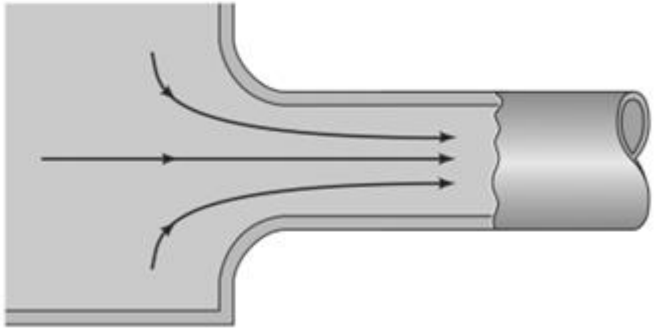
Reentrant, K_L = 0.8



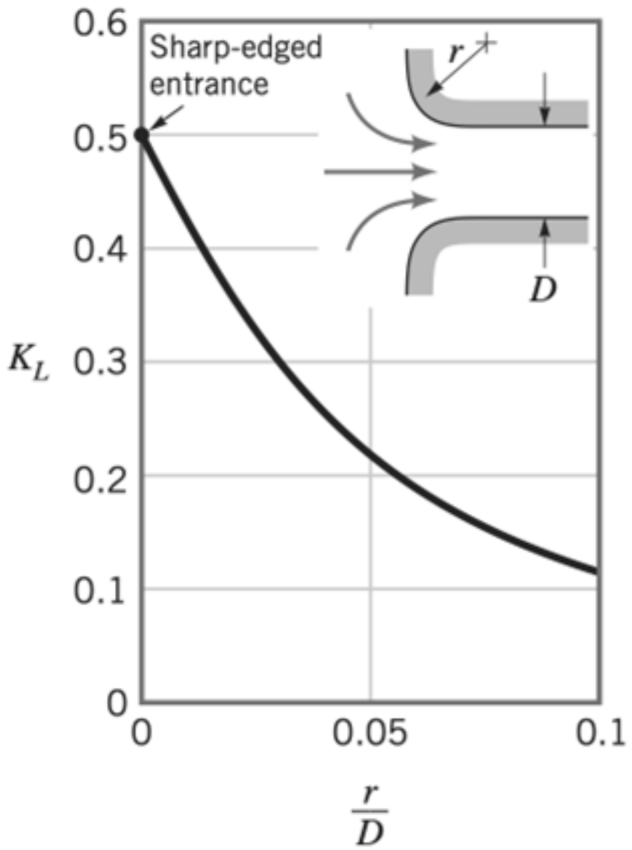
Sharp-edged, K_L = 0.5

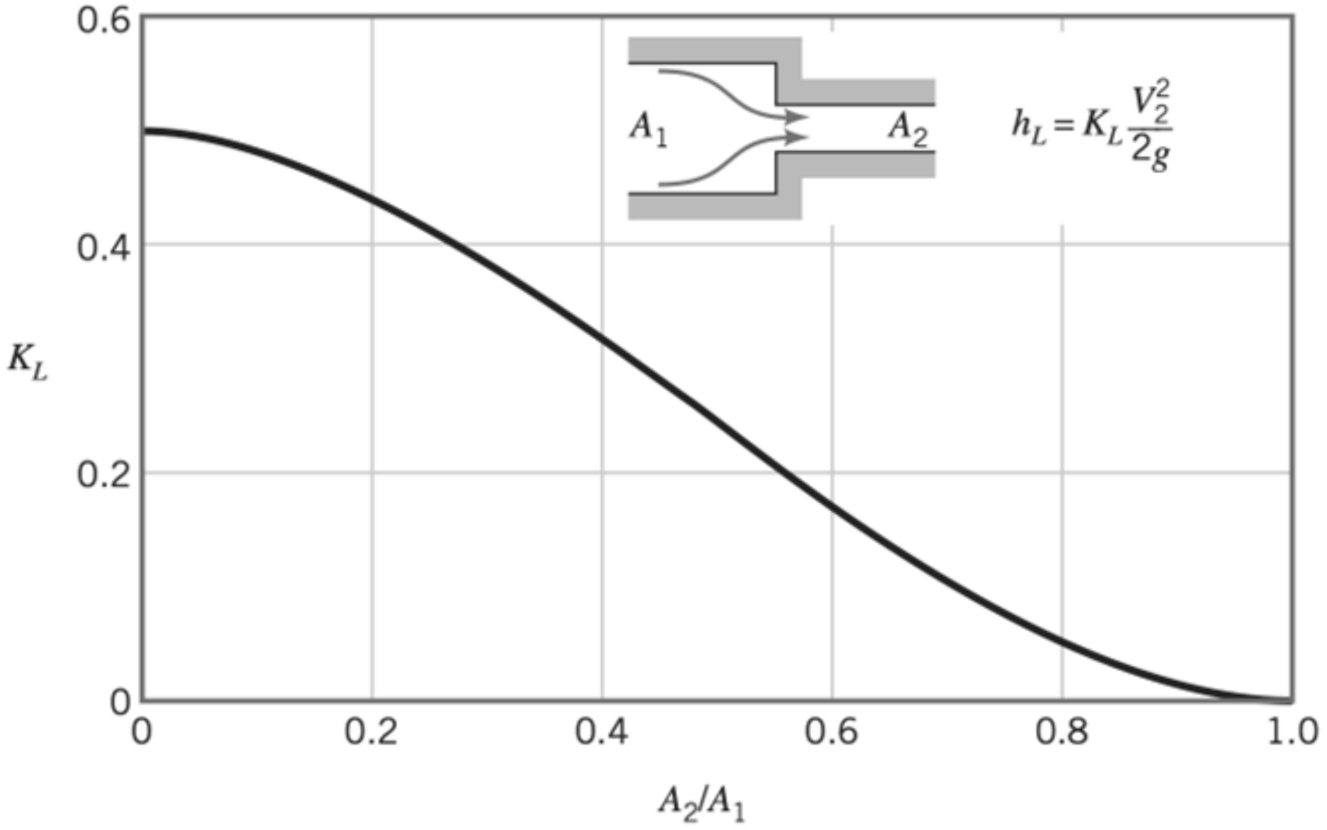


Slightly rounded, K_L = 0.2

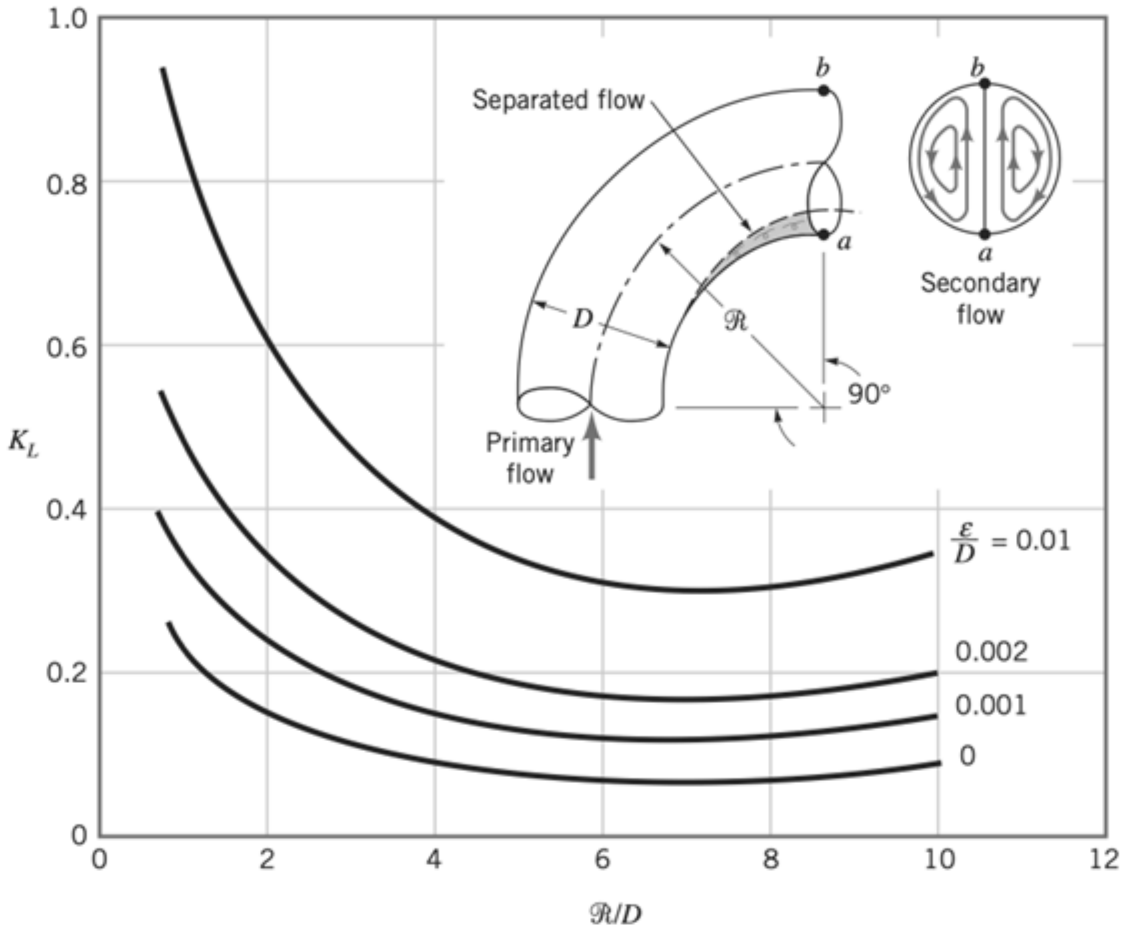


Well-rounded, K_L = 0.04

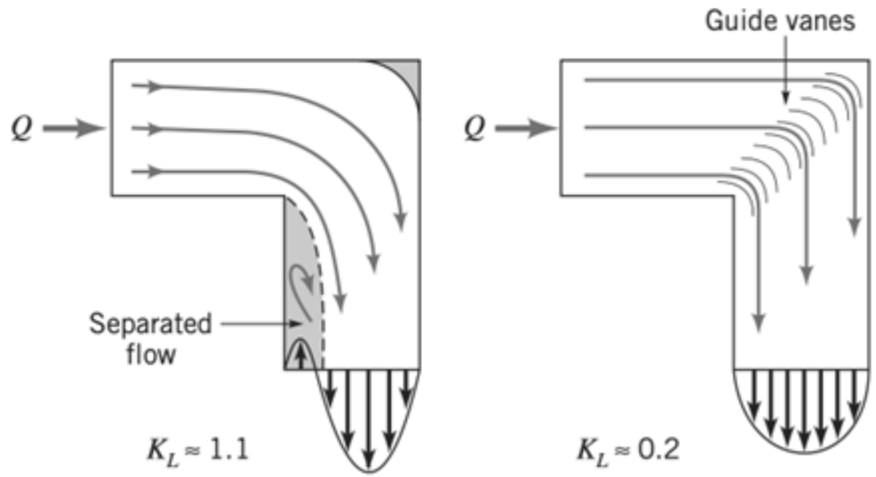




$h_L = K_L \frac{V_2^2}{2g}$
A_1
A_2
K_L
A_2/A_1
0
0.2
0.4
0.6
0.8
1.0



Separated flow
Secondary flow
Primary flow
b
a
D
ℛ
90°
K_L
ℛ/D
$\frac{\varepsilon}{D}$ = 0.01
0.002
0.001
0

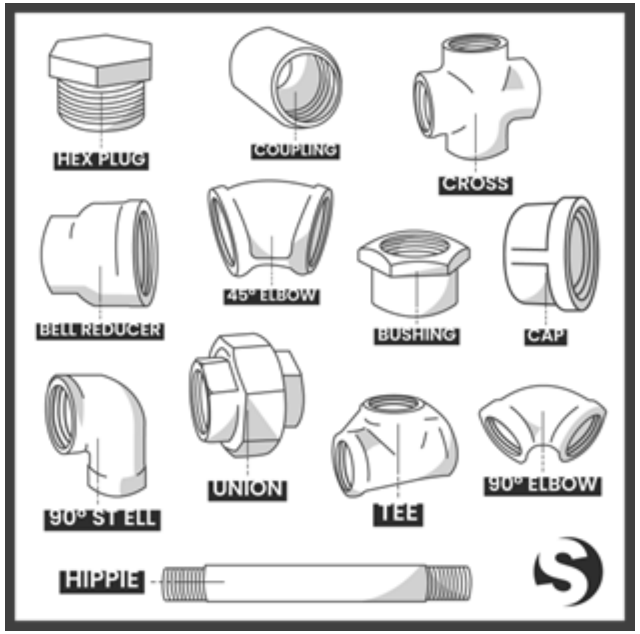


Q
Separated flow
Guide vanes
$K_L \approx 1.1$
$K_L \approx 0.2$

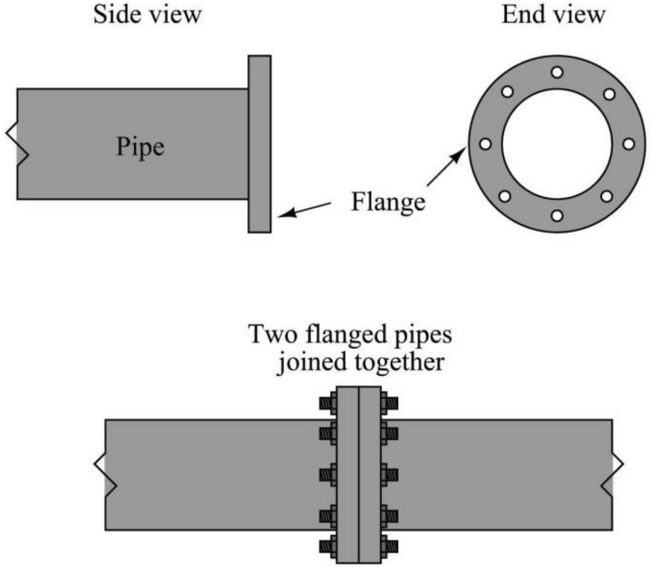
EPFL

For your information

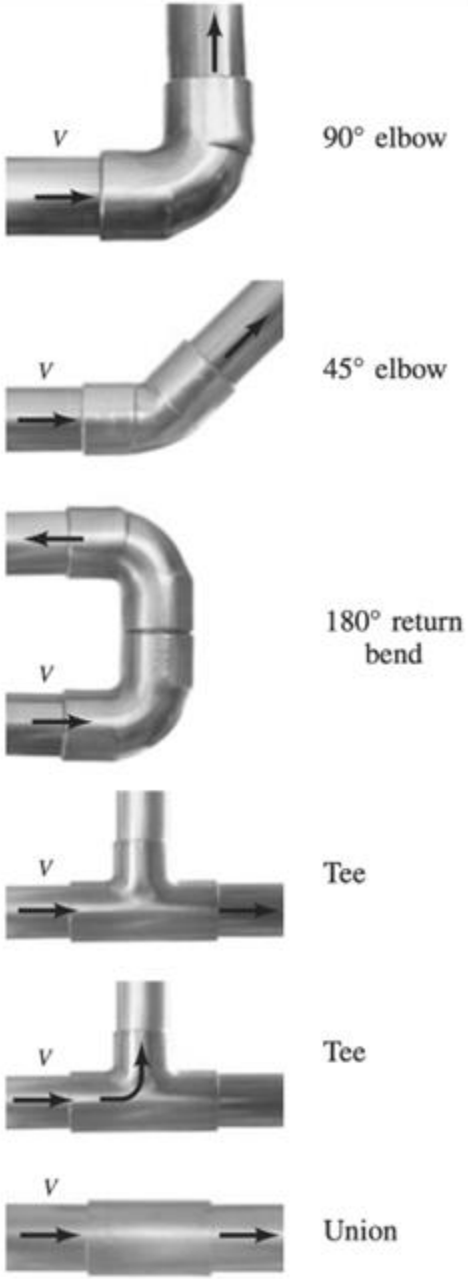
threaded



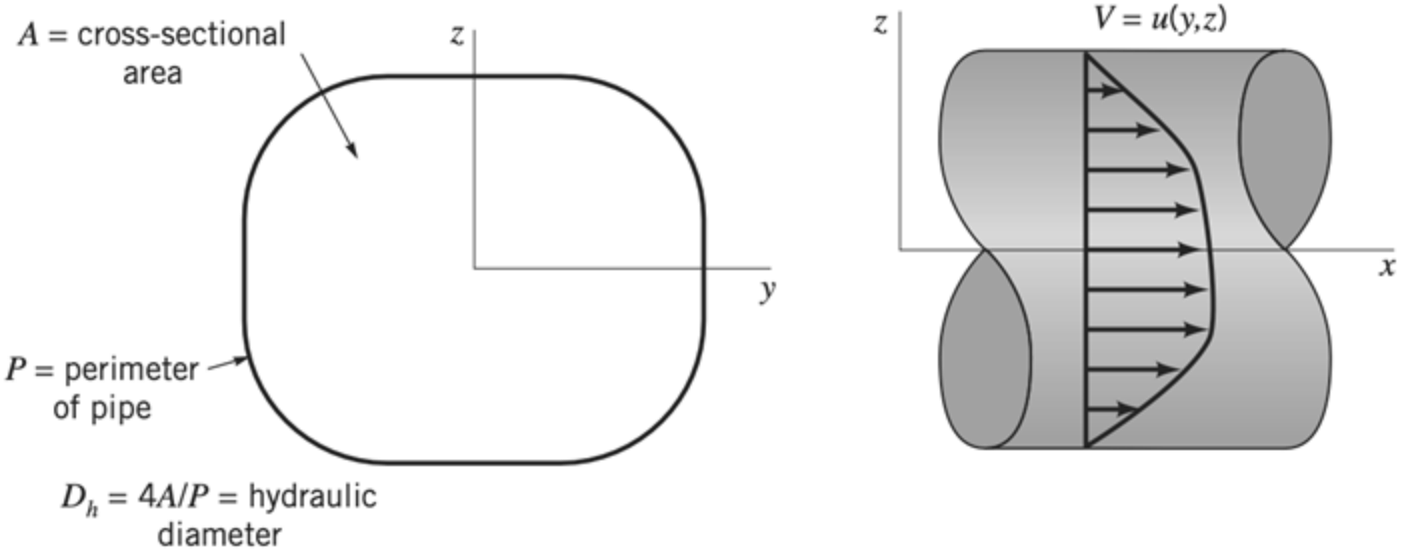
flanged



Component	K_L
a. Elbows	
Regular 90°, flanged	0.3
Regular 90°, threaded	1.5
Long radius 90°, flanged	0.2
Long radius 90°, threaded	0.7
Long radius 45°, flanged	0.2
Regular 45°, threaded	0.4
b. 180° return bends	
180° return bend, flanged	0.2
180° return bend, threaded	1.5
c. Tees	
Line flow, flanged	0.2
Line flow, threaded	0.9
Branch flow, flanged	1.0
Branch flow, threaded	2.0
d. Union, threaded	0.08
e. Valves	
Globe, fully open	10
Angle, fully open	2
Gate, fully open	0.15
Gate, $\frac{1}{4}$ closed	0.26
Gate, $\frac{1}{2}$ closed	2.1
Gate, $\frac{3}{4}$ closed	17
Swing check, forward flow	2
Swing check, backward flow	∞
Ball valve, fully open	0.05
Ball valve, $\frac{1}{3}$ closed	5.5
Ball valve, $\frac{2}{3}$ closed	210



Non-circular duct → use hydraulic diameter



Hydraulic diameter : $D_h = \frac{4A}{P}$

$$f = \frac{C}{Re_h} \qquad Re_h = \frac{V D_h}{\nu} \qquad h_L = f \frac{l}{D_h} \frac{V^2}{2g} \qquad \frac{\varepsilon}{D_h}$$

Dimensional Analysis of Pipe Flow

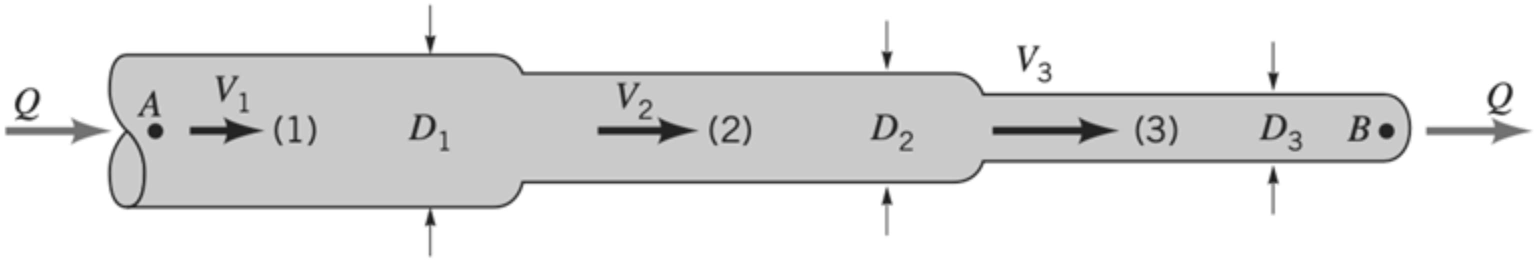
$$h_L = h_{L,\text{major}} + h_{L,\text{minor}}$$

$$= f \frac{l}{D} \frac{V^2}{2g} + K_L \frac{V^2}{2g}$$

Types of problems

- Type 1
  - Flowrate/average velocity: given
  - Pressure difference/head loss: determine
- Type 2
  - Flowrate/average velocity: determine
  - Pressure difference/head loss: given
- Type 3
  - Flowrate/average velocity: given
  - Pressure difference/head loss: given
  - Diameter of pipe: determine

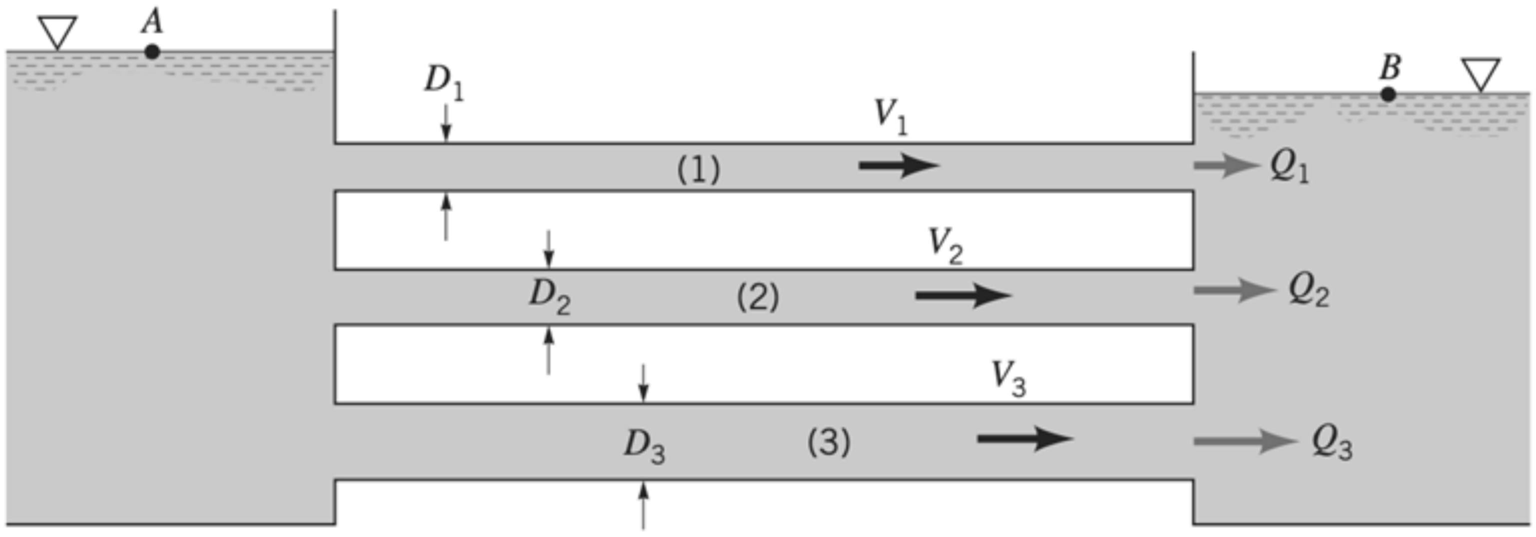
Multiple pipe systems



(a)

$$Q_1 = Q_2 = Q_3$$

$$h_{L_{A-B}} = h_{L_1} + h_{L_2} + h_{L_3}$$



(b)

Parallel pipe system

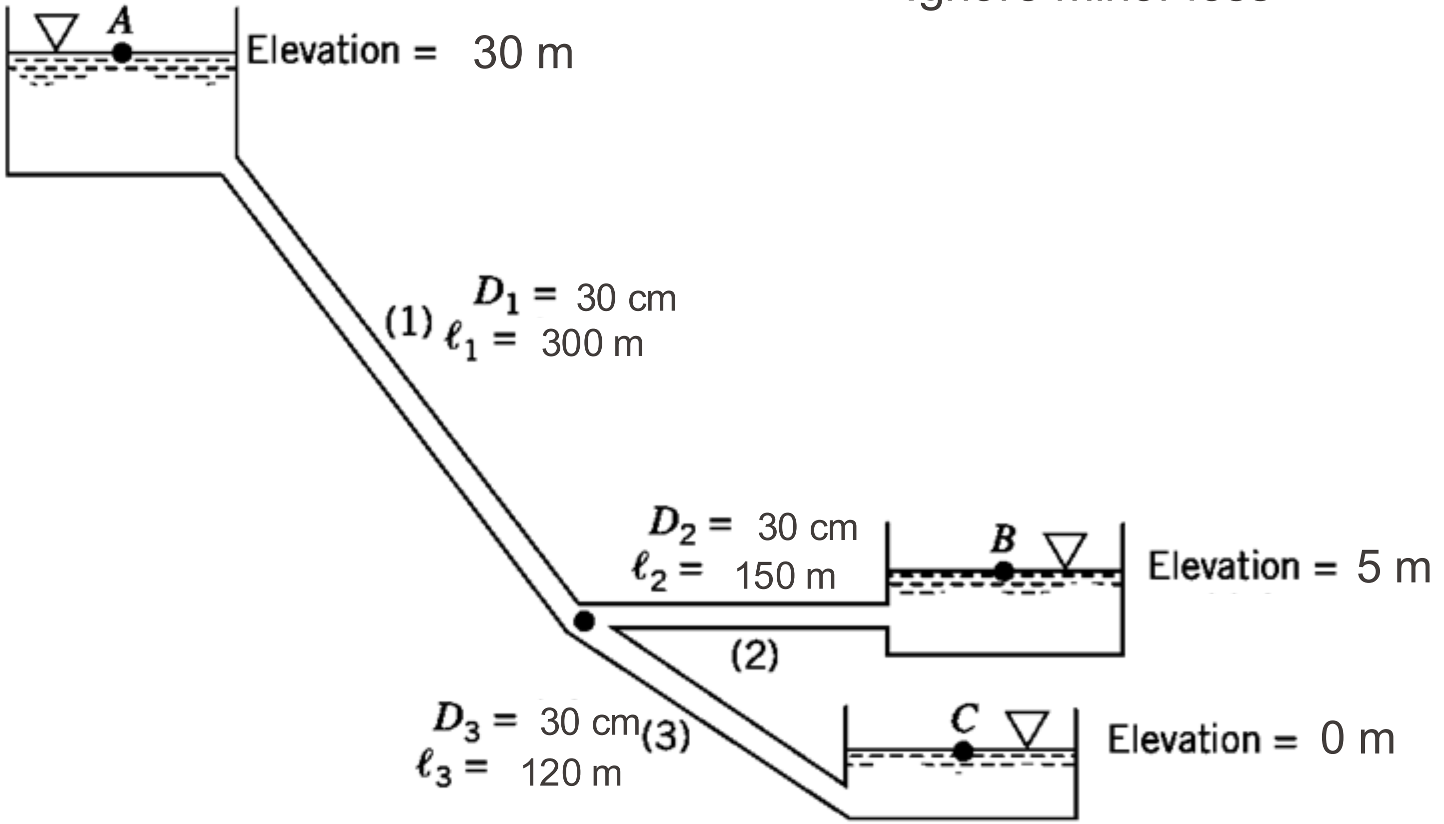
$$Q = Q_1 + Q_2 + Q_3$$

$$h_{L_1} = h_{L_2} = h_{L_3}$$

$$\frac{p_1}{\gamma} + \frac{\bar{V}_1^2}{2g} + z_1 + h_s - h_L = \frac{p_2}{\gamma} + \frac{\bar{V}_2^2}{2g} + z_2$$

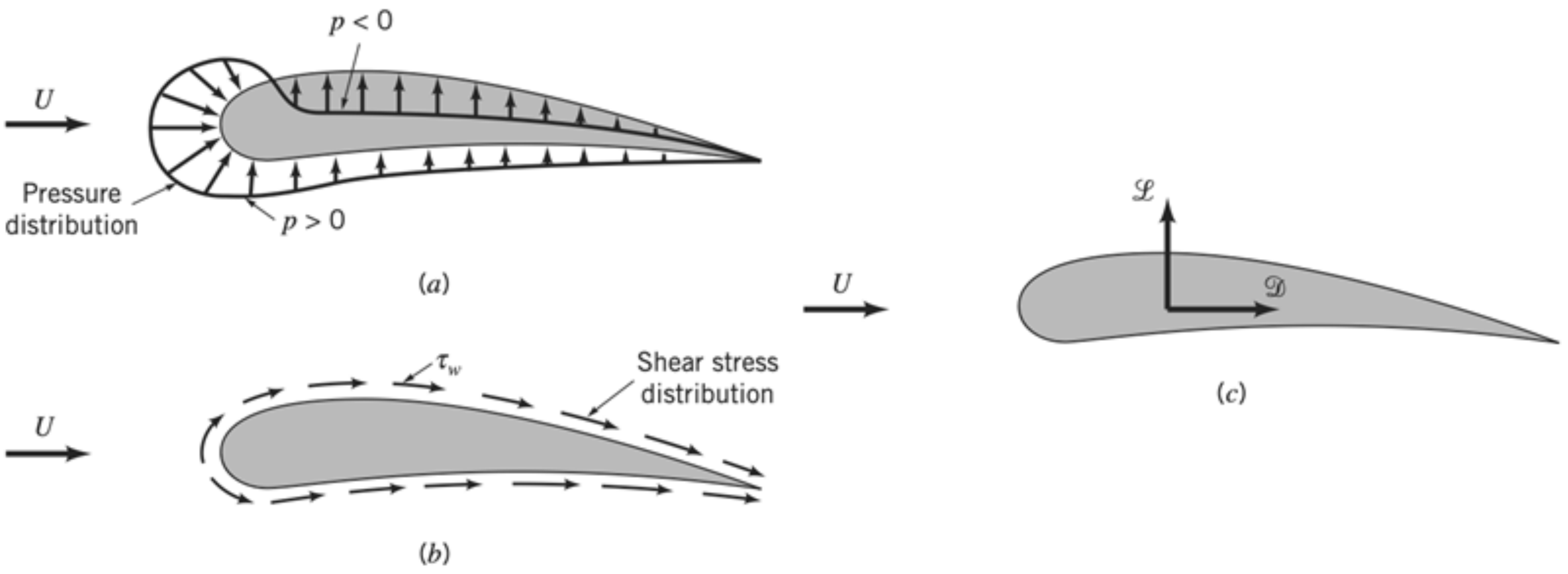
Determine the flowrate into or out of each reservoir

- Friction factor 0.02
- Ignore minor loss



Flow around immersed body

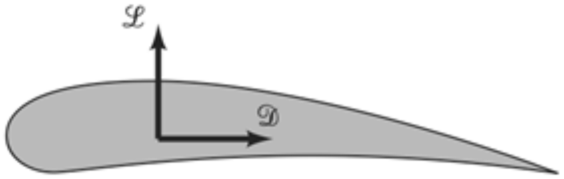
Flow around immersed body

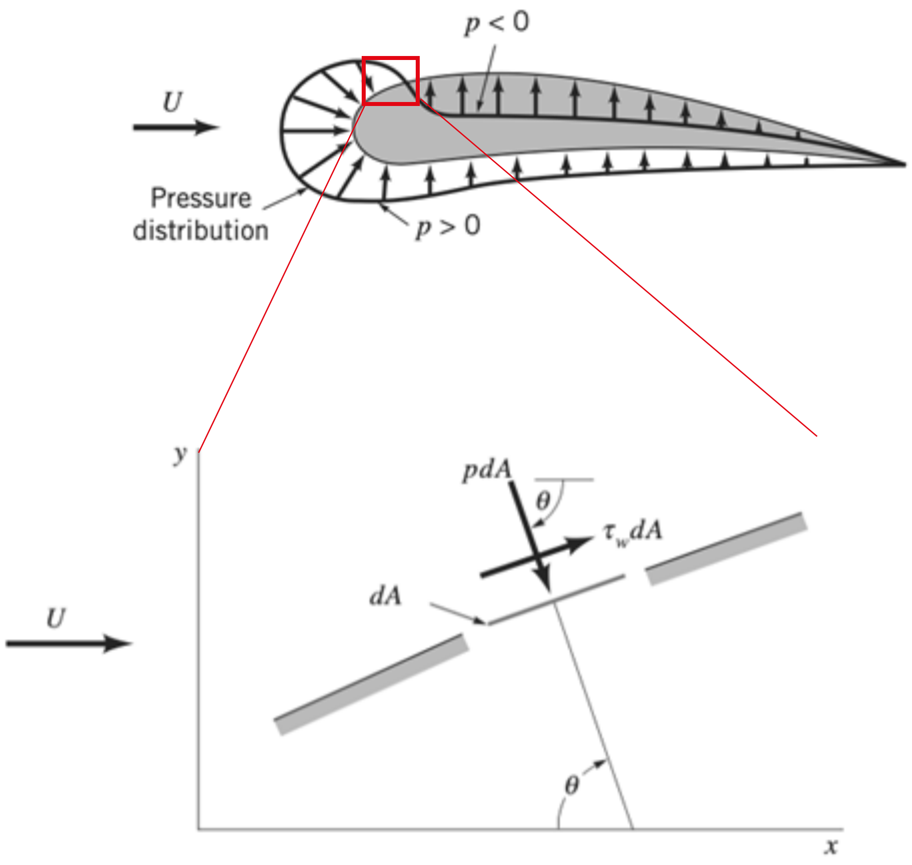


It is useful to know the detailed distribution of **shear stress** and **pressure** over the surface of the body (such information is difficult to obtain).
Only the integrated or resultant effects of these distributions are needed. The resultant force in the direction of the upstream velocity is termed the **drag, D**, and the resultant force normal to the upstream velocity is termed the **lift, L**

- Drag, D: a net **force** in the direction of flow due to the pressure and shear forces on the surface parallel to upstream flow direction
- Lift, L: Component of the **force** perpendicular to upstream flow direction

Flow around immersed body





- Element force in x

$$dF_x = (pdA)\cos\theta + (\tau_w dA)\sin\theta$$

- Element force in y

$$dF_y = -(pdA)\sin\theta + (\tau_w dA)\cos\theta$$

- Integration of element force in x, Drag

$$D = \int dF_x = \int p\cos\theta dA + \int \tau_w \sin\theta dA$$

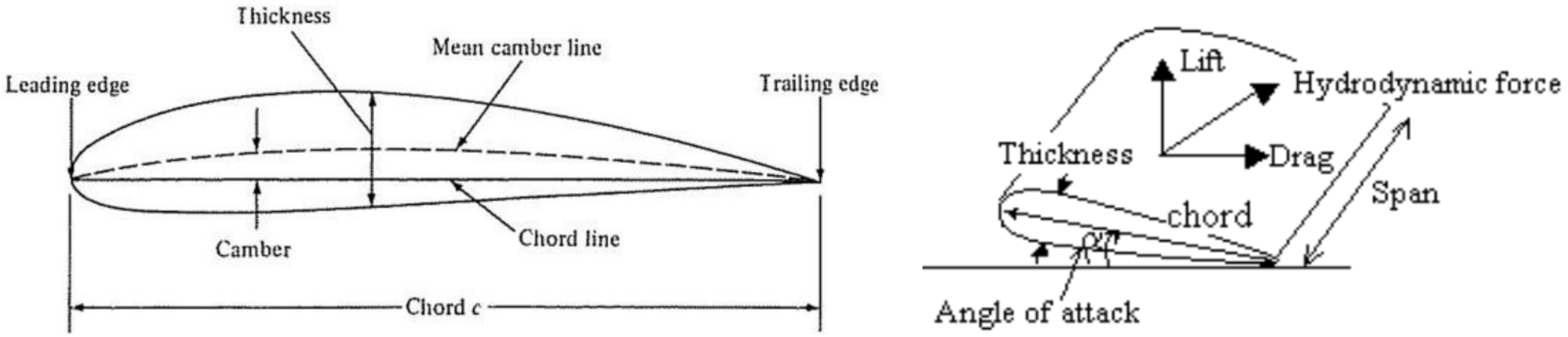
- Integration of element force in y, lift

$$L = \int dF_y = -\int p\sin\theta dA + \int \tau_w \cos\theta dA$$

To obtain D, L, not only the distribution of pressure and shear stress are known, but also ______ should be known

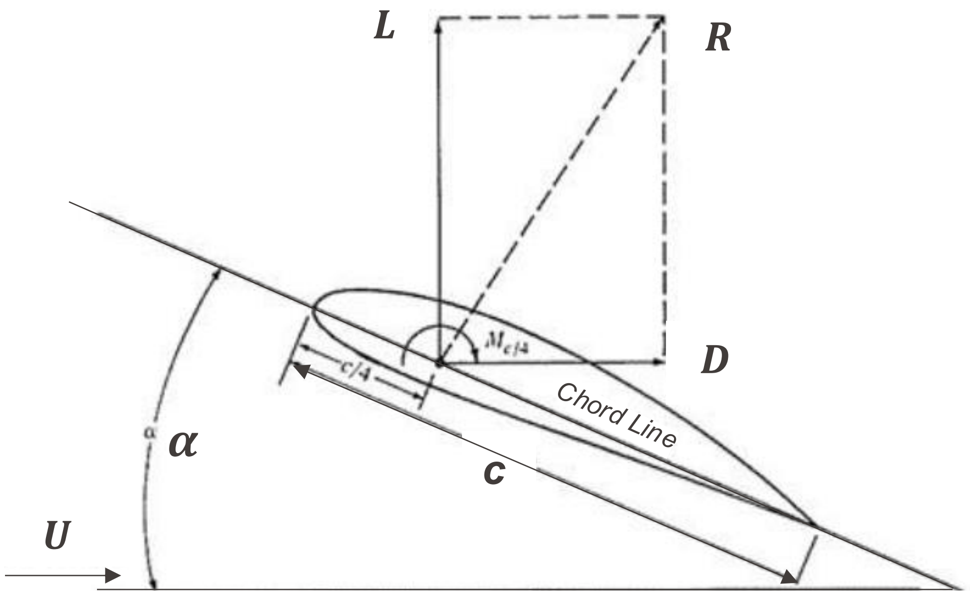
Flow around a profiled body

- **Nomenclature airfoil :**
  - **Chord line**: Straight line connecting the leading and trailing edges
  - **Chord length**: Distance between leading and trailing edges (along the chord line)
  - **Mean camber line**: located halfway between upper and lower surfaces
  - **Camber**: Maximum distance between mean camber line and chord line
    Measured perpendicularly to the chord line. (Zero camber → Symmetric foil)
  - **Span**: width of the foil

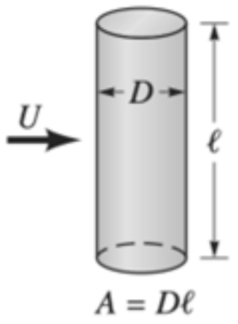


Lift and Drag

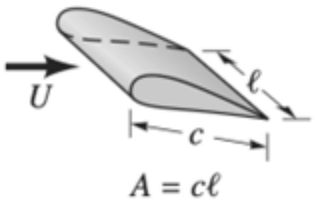
- Without detailed information concerning the shear stress and pressure distributions on a body the previous equations for D, L are useless
- The widely used alternative is to define dimensionless lift and drag coefficients and determine their approximate values by means of either a simplified analysis, some numerical technique, or an appropriate experiment: **lift coefficient,** C_L, and **drag coefficient** C_D
- Lift and drag forces vary with the flow velocity and the angle of attack, α (Angle between the upstream flow direction and chord line)



Lift coefficient: $$C_L = \frac{L}{\frac{1}{2}\rho U^2 A} = \frac{L}{q_\infty A}$$



Drag coefficient: $$C_D = \frac{D}{\frac{1}{2}\rho U^2 A} = \frac{D}{q_\infty A}$$

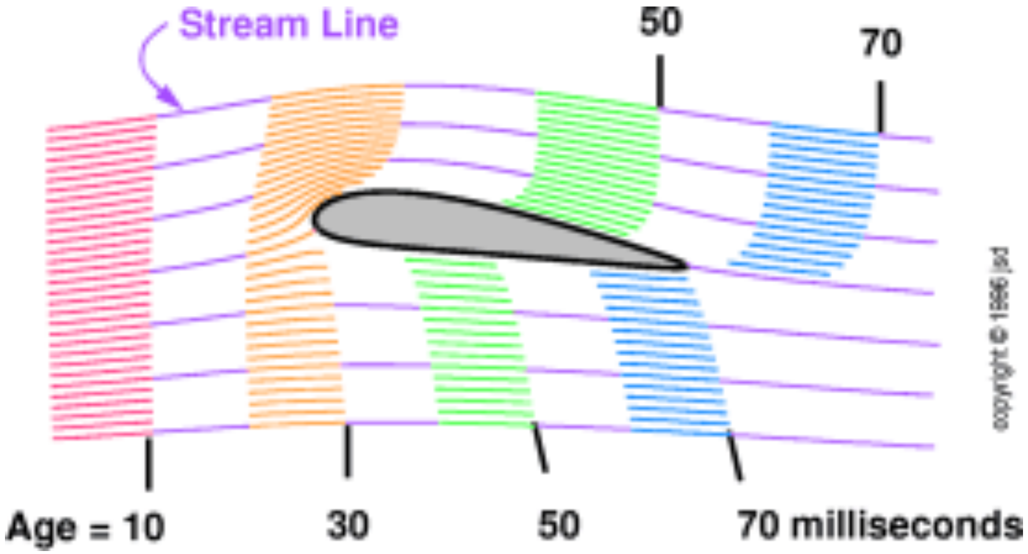


With $$q_\infty = \frac{1}{2}\rho U^2$$

and A is a reference surface (chord × span)

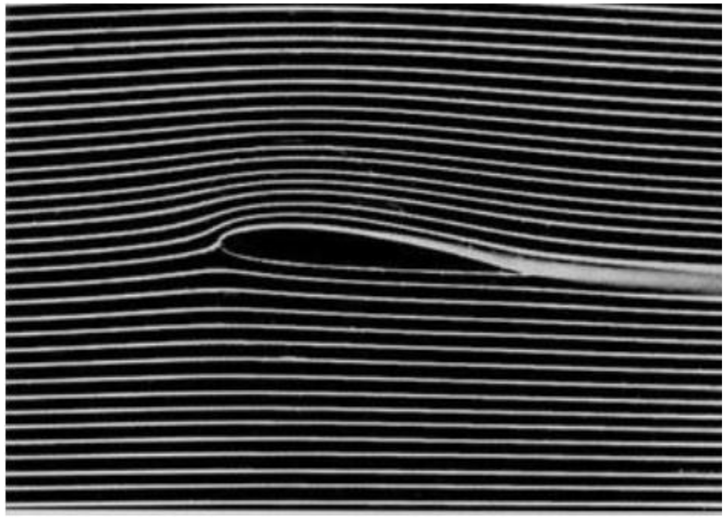
Flow around a profiled body

- **Mechanism of lift generation**:
  - Illustration of the fluid displacement around a foil using transient injections of colored smokes (during 10 sec)

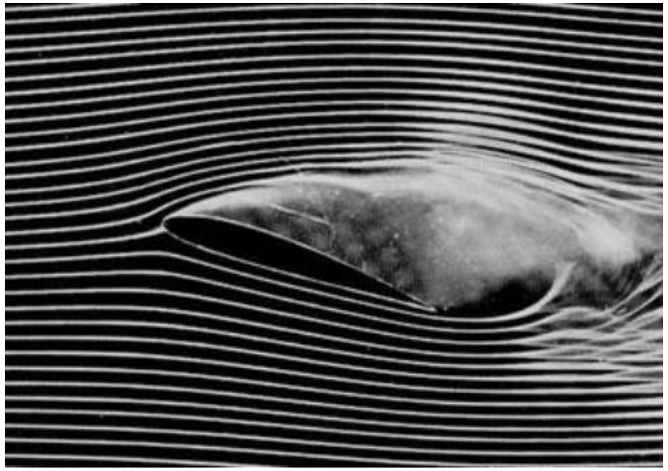


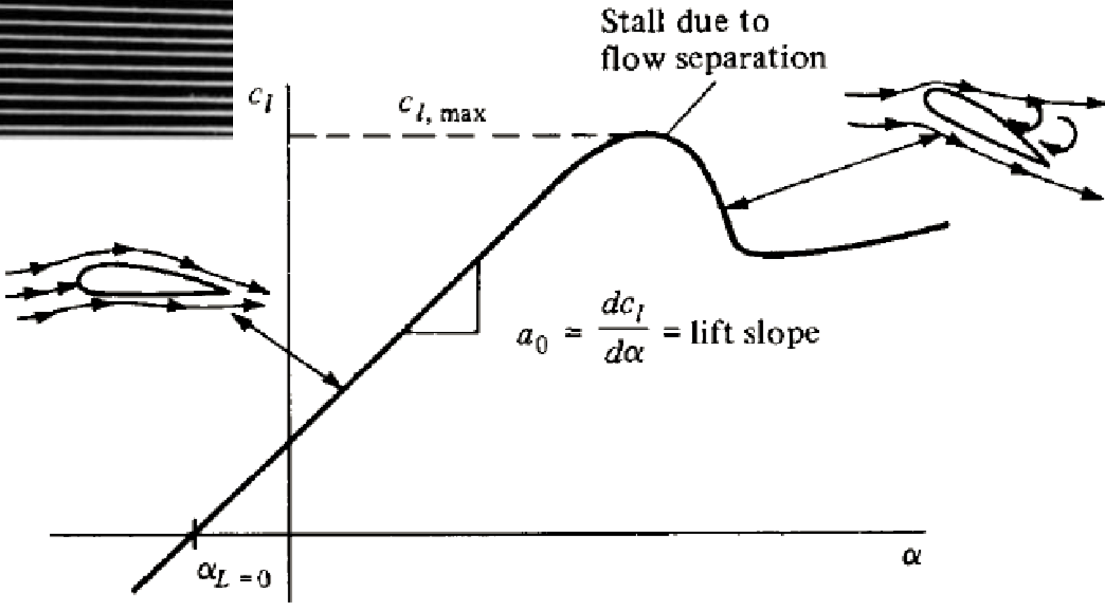
  - The flow is accelerated on the upper surface (suction side) with respect to the lower surface (pressure side) → pressure difference → lift generation

Flow around a profiled body



Stall

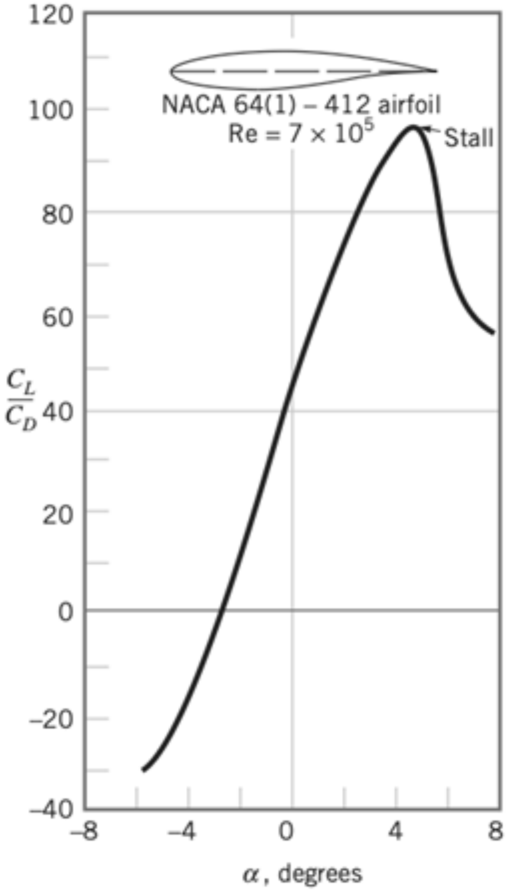


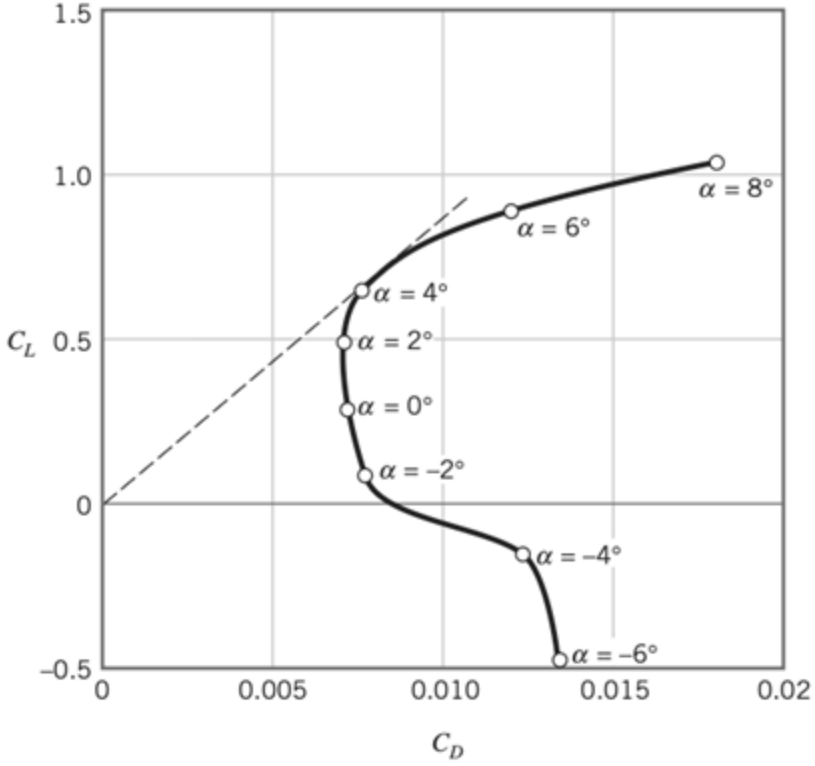


Stall : If α is too large, the boundary layer on the upper surface separates, the flow over the wing develops a wide, turbulent wake region, the lift decreases, and the drag increases.

Flow around a profiled body – C_L/C_D ratio

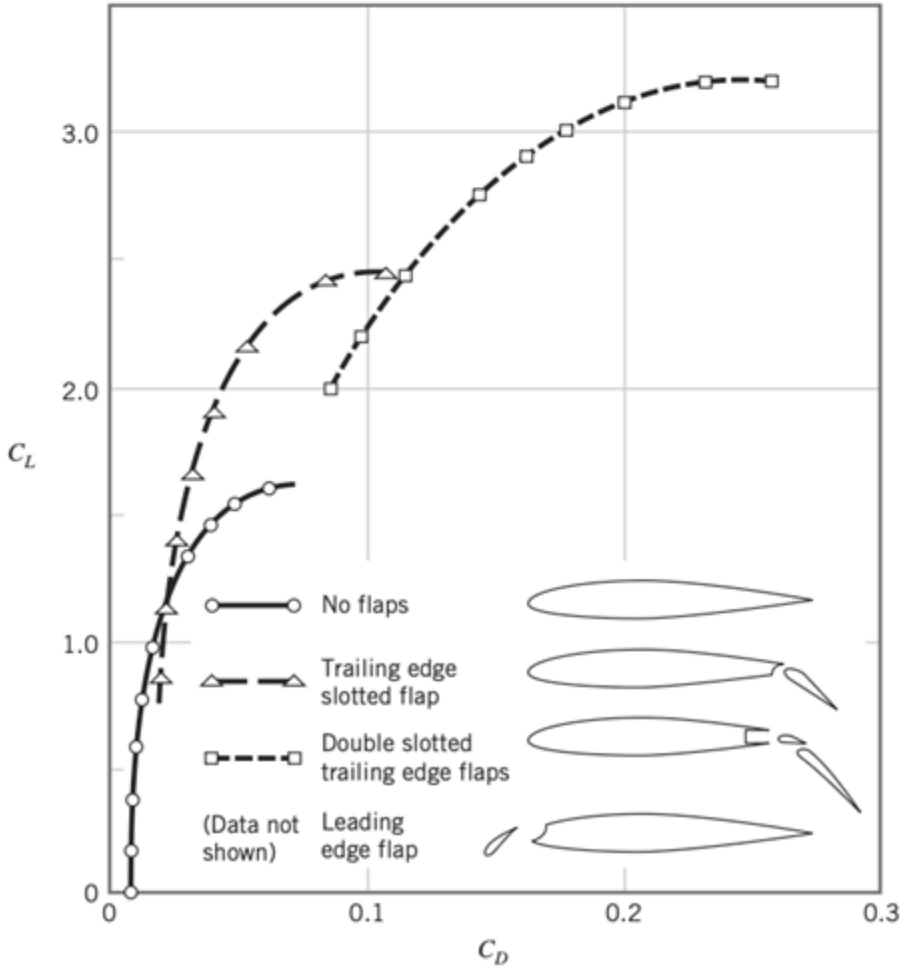
- **Aerodynamic efficiency** C_L/C_D is important : more lift with less drag
- High performance airfoils generate lift about 100 times more than their drag
- In still air, glide horizontal distance of 100 m for each 1 m drop in altitude
- The lift and drag can be altered by changing angle of attack → represents a change in shape
- Modern airplanes it is common to use leading edge & trailing edge flaps





Flow around a profiled body – C_L/C_D ratio

- Modern aircraft uses the flap to generate the necessary lift during the relatively low-speed (landing and takeoff)
- Flaps enhance the lift although the price to pay is an increase in drag and 'dirty' configuration
- During the normal flight, the flaps are retracted ('clean' configuration) flying in a relatively small drag



Flow around a profiled body

- Pressure and velocity distributions on the surface of a profiled body as well as lift and drag forces are crucial for the design of turbomachines

- In the lack of analytical solutions, engineers always use numerical simulations and experiments

- A significant progress was made in the field of numerical flow simulations with different degree of sophistication, such as:
    - RANS: Reynolds-averaged Navier Stokes solvers
    - LES: Large eddy simulation solvers
    - DNS: Direct numerical simulation solvers

- JAVAFOIL solver: is a relatively simple solver, which uses several traditional methods for the analysis of a subsonic flow around a profiled body and provides a quick and fair estimation of the velocity and pressure fields along with the lift and drag forces.

  → See Exercise

Exercise

Gossamer Condor

Flight speed = 5 m/s
Wing side = b = 30 m, c = 2.3 m (average)
Mass flight = W = 30 kg
Mass pilot = W_p = 65 kg
Drage coefficient C_D = 0.046
Power train efficiency η = 0.8
Density of air ρ = 1.2 kg/m^3

- For a steady flight condition, find the lift coefficient C_L

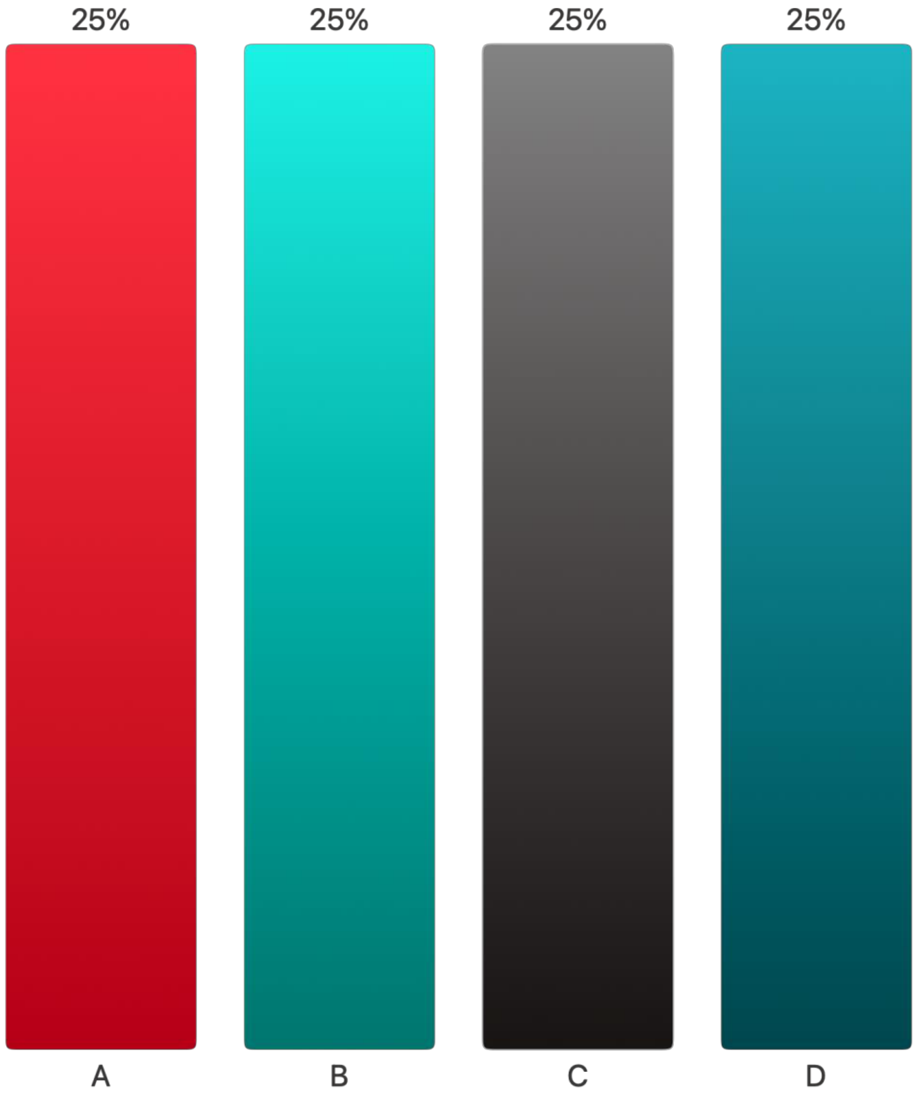
- The power required from the pilot (P=DU)

With the obtained required power, how long a fit man can fly?

A. 1 min – 10 min
B. 15 min – 30 min
C. 45 min – 60 min
D. > 60 min

Enter Code
me342

echo360poll.eu



How fit to fly Gossamer Condor?

- How much power humans can generate and for how long varies with physical form.

- The specific power may be expressed in watts per kilogram of body mass.

- Active cyclists can produce from
  - 2.2 W/kg (average untrained)
  - 3.0 W/kg (good [fitness])
  - 6.6 W/kg (top-class male athletes) at their functional threshold power (about one hour)
  - 5 W/kg is about the level reachable by excellent male or exceptional female amateurs.

- Maximum sustained power levels for one hour are recorded from about 200 W (NASA experimental group of "healthy men") to 500 W (Eddy Merckx on ergometer 1975).