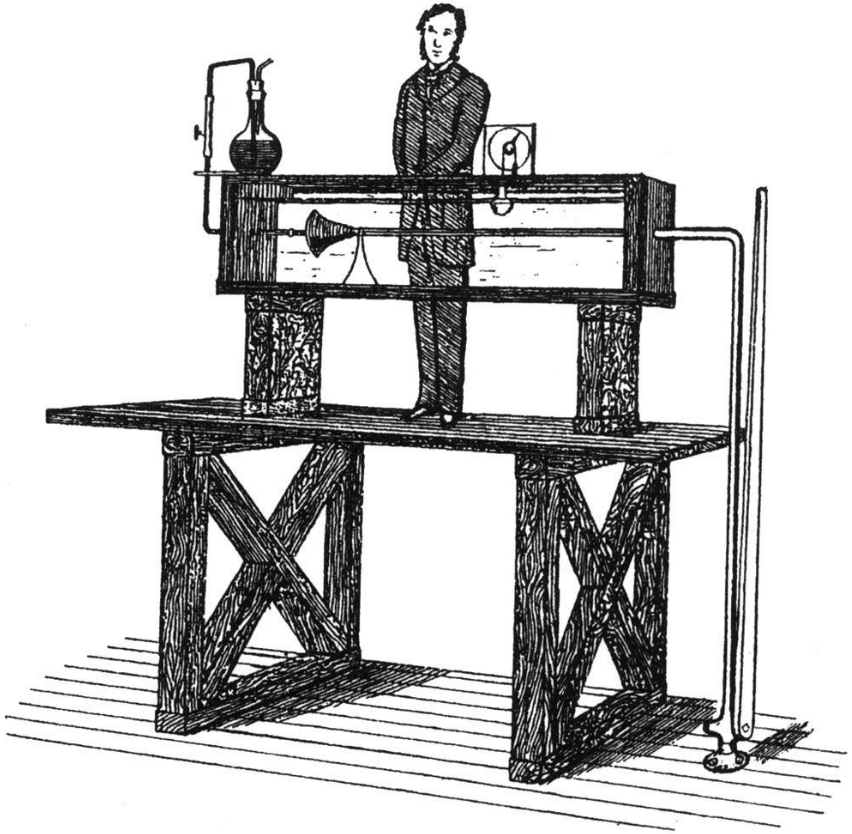
EPFL



Chapter 2: Basics of Fluid Dynamics - Part1

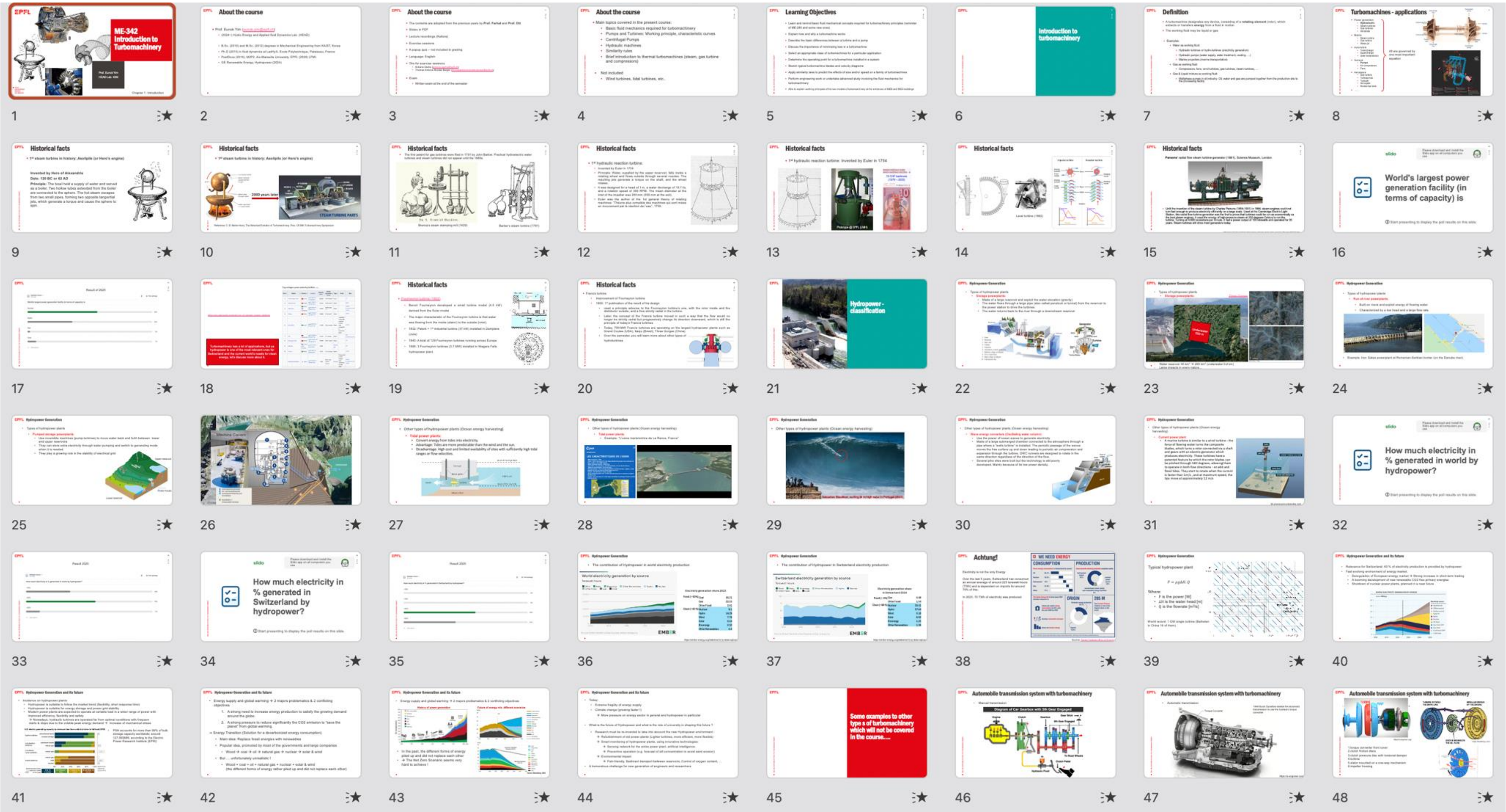
ME-342 Introduction to turbomachinery

École polytechnique fédérale de Lausanne

Prof. Eunok Yim, HEAD-lab.

Review the last lecture

- Sorry for the sound issue of the recording
- Some answers to the repeated questions about the exam:
  - Written exam (100%) at the end of the semester
  - Closed book



What stayed in your mind from the last lecture?

Word cloud

Enter Code

_me342_

echo360poll.eu

Do you recall....

Last year, Fluids Mechanics course

- Energy equation (extended Bernoulli equation)

(Do you recall) Energy consideration

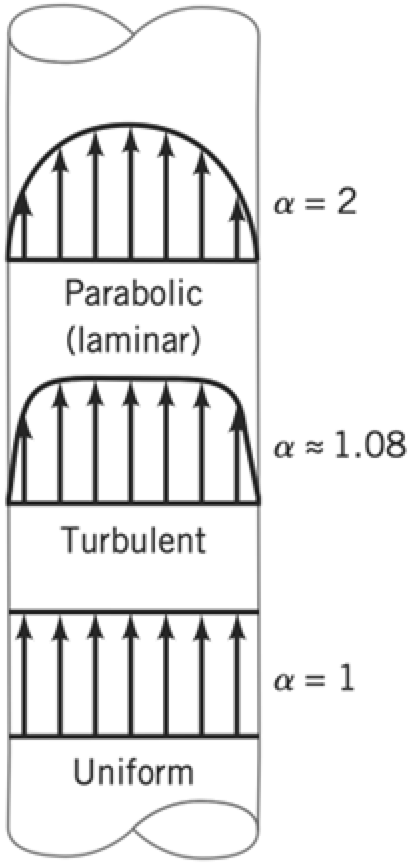
- **Energy equation for incompressible, nonuniform flow**

  ….. your last year's course

  - Mechanical energy equation or extended Bernoulli equation

Turbine
$\pm h_T$?

$$\frac{p_{\text{in}}}{\rho} + \frac{\alpha_{\text{in}}\,\bar{V}_{\text{in}}^2}{2} + g z_{\text{in}} + w_{\text{shaft}} - \text{loss} = \frac{p_{\text{out}}}{\rho} + \frac{\alpha_{\text{out}}\,\bar{V}_{\text{out}}^2}{2} + g z_{\text{out}}$$



$$\boxed{\frac{p_1}{\gamma} + \alpha_1 \frac{\bar{V}_1^2}{2g} + z_1 + h_s - h_L = \frac{p_2}{\gamma} + \alpha_2 \frac{\bar{V}_2^2}{2g} + z_2}$$

$\gamma = \rho g$ Specific weight

$\alpha =$ Kinetic energy coefficient

$h_s =$ Shaft work head

$$h_s = \frac{w_{\text{shaft net in}}}{g} = \frac{\dot{W}_{\text{shaft net in}}}{\dot{m} g}$$

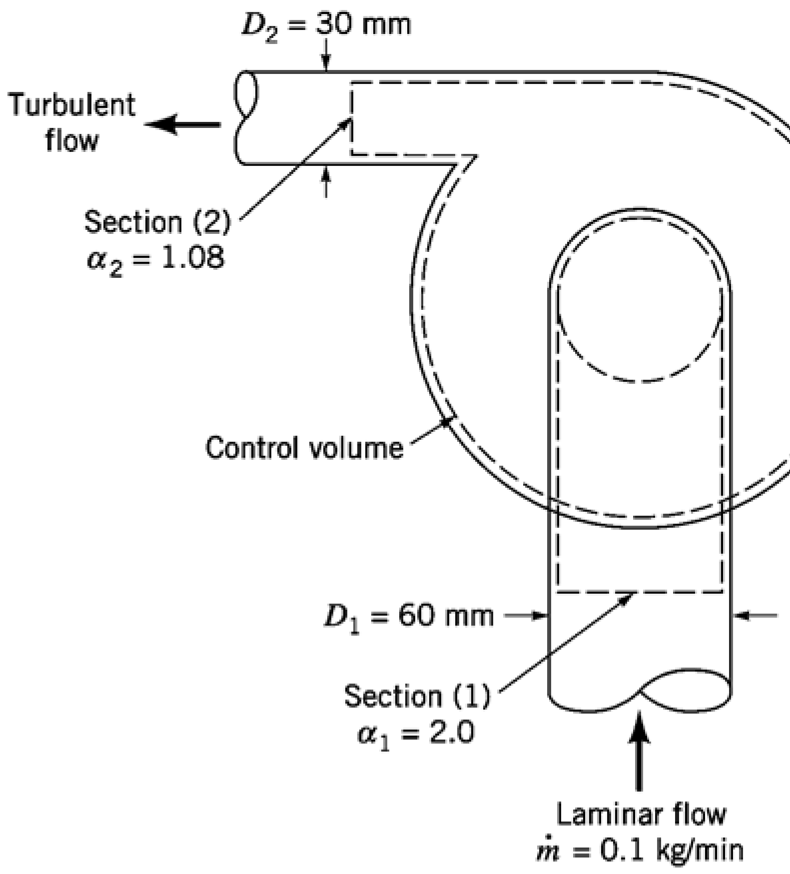
$h_L =$ Head loss

Head = energy per unit weight

Total head : $H = \dfrac{p}{\gamma} + \dfrac{V^2}{2g} + z$

(Do you recall) Energy consideration

- **Exercise, a fan**



$$\boxed{\frac{p_1}{\gamma} + \alpha_1 \frac{\bar{V}_1^2}{2g} + z_1 + h_s - h_L = \frac{p_2}{\gamma} + \alpha_2 \frac{\bar{V}_2^2}{2g} + z_2}$$

Static pressure raise = 0.1 kPa
Fan motor power = 0.14 W

ρ = 1.23 kg/m^3
g = 9.8 m/s^2

$$h_s = \frac{w_{\text{shaft net in}}}{g} = \frac{\dot{W}_{\text{shaft net in}}}{\dot{m} g}$$

$$\dot{W} = \gamma Q h$$

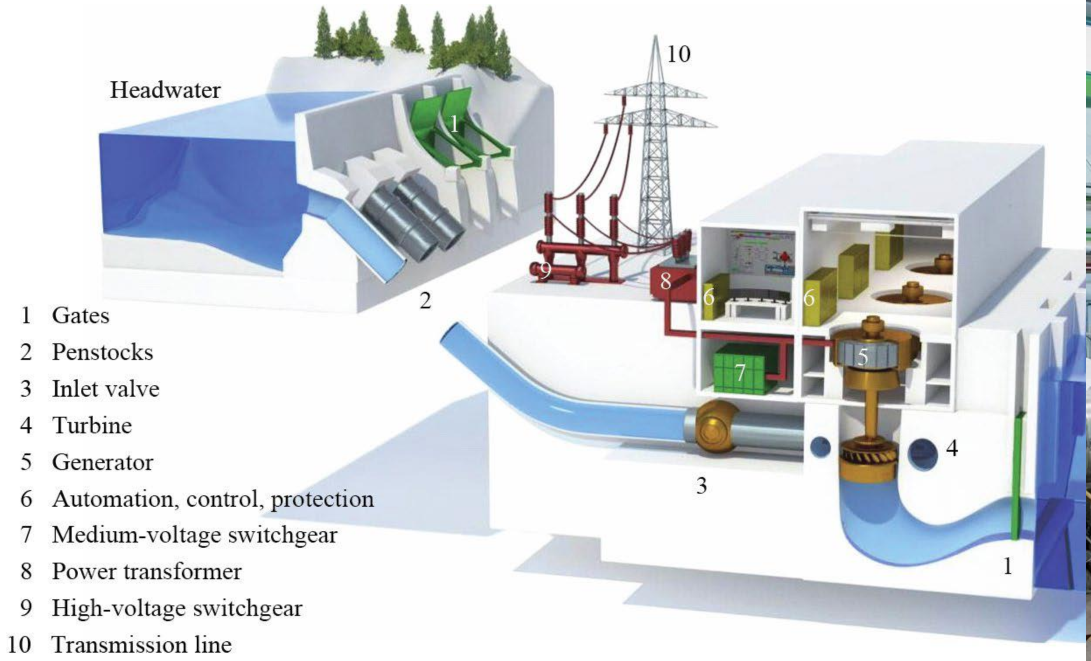
What is head loss h_L?

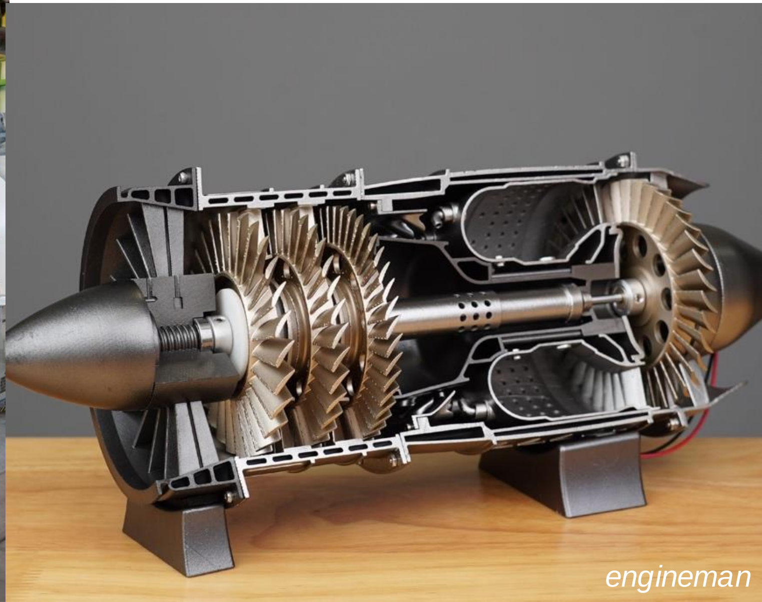
Turbomachinery linked Basic Fluid Mechanics

- Flow in pipes
- Laminar turbulent transition
- Vortex flow
- Boundary layer
- Flow around a profiled body

Relevant Fluid Mechanic basics

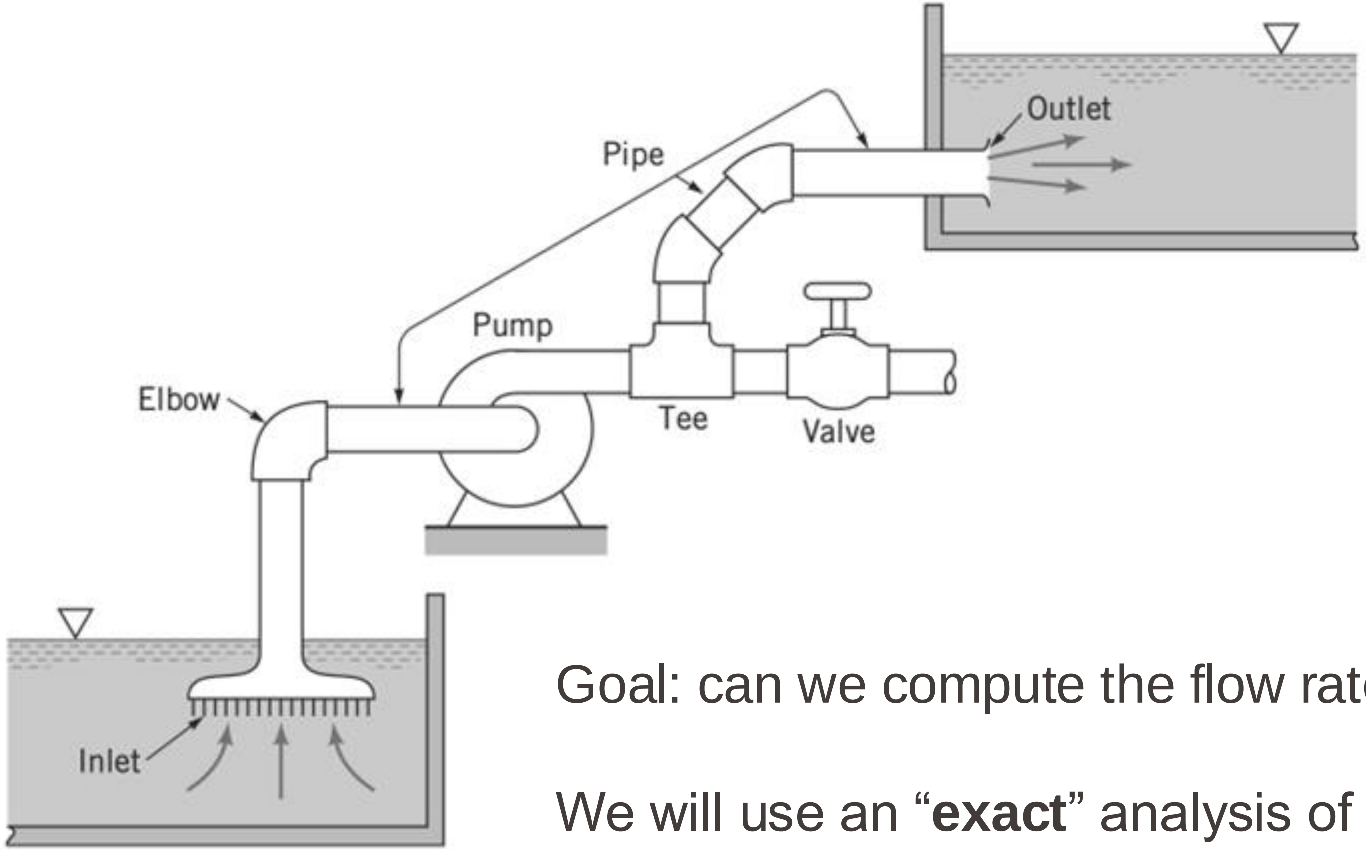
- Turbomachines involve large variety of flow phenomena:
    - Flows in pipes and channels (Laminar, turbulent)
    - Flows over a rotating set of blades
    - Gap flows (tip leakage vortices)
    - Free surface flows (jets)
    - Two-phase flows (gas-liquid mixture, cavitation, …)
    - …





Pipe flow

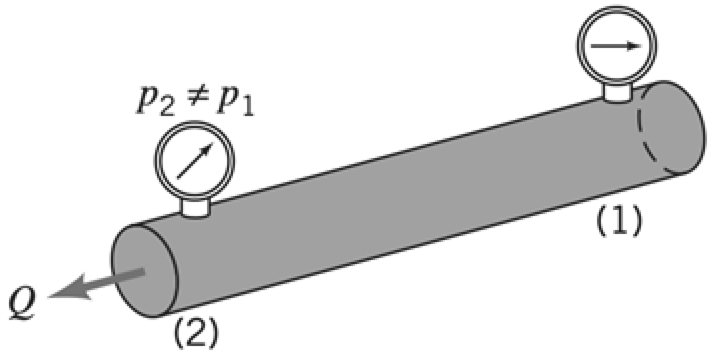
Viscous flow in pipe



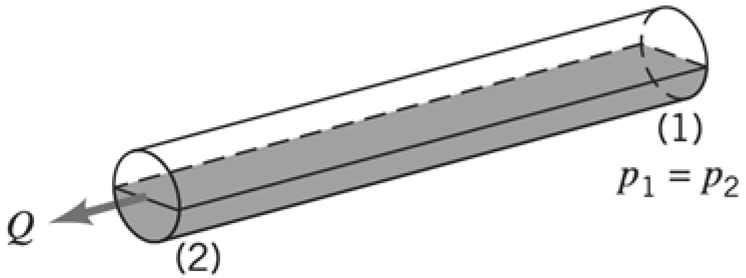
Goal: can we compute the flow rate, power required to the pump, etc…?

We will use an “**exact**” analysis of the simplest pipe flow topics (such as laminar flow in long, straight, constant diameter pipes) and “**dimensional analysis**” considerations combined with experimental results for the other pipe flow topics.

Assumption for the exact analysis



Pipe flow

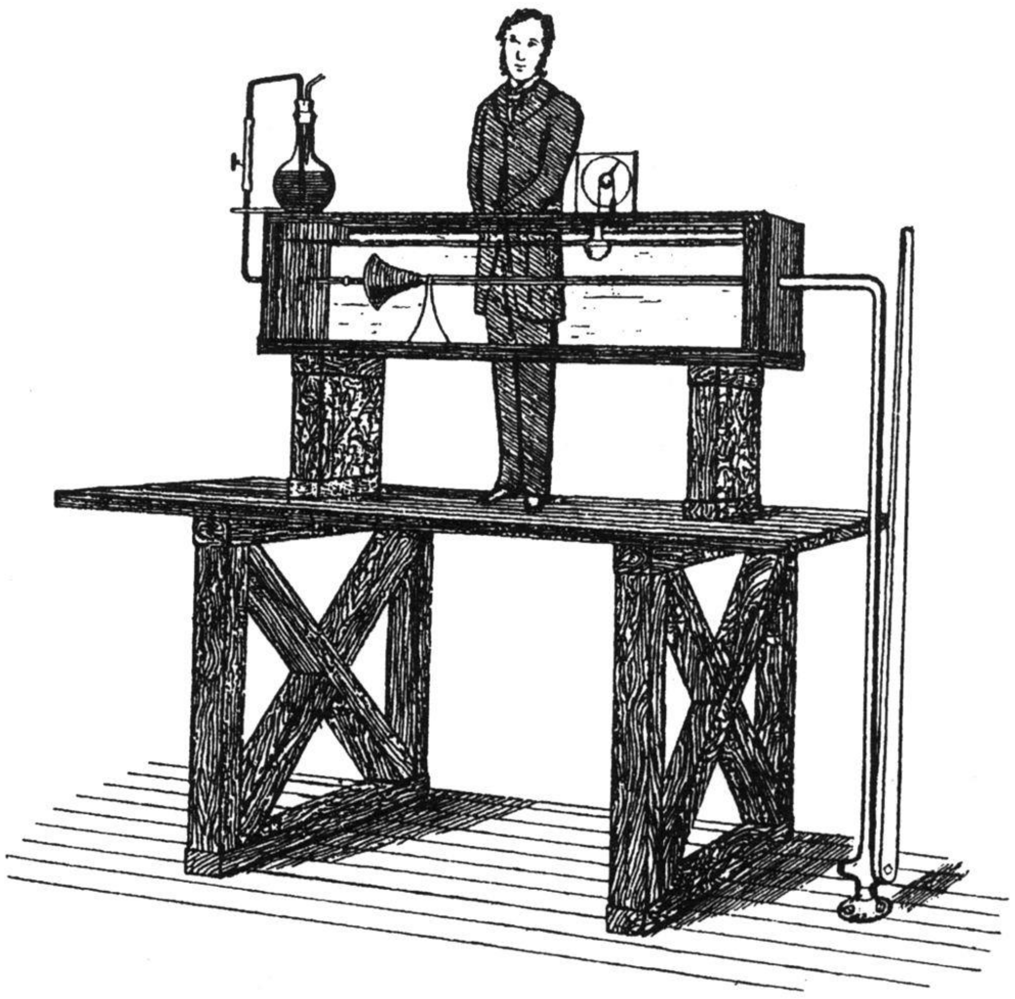


Open-channel flow

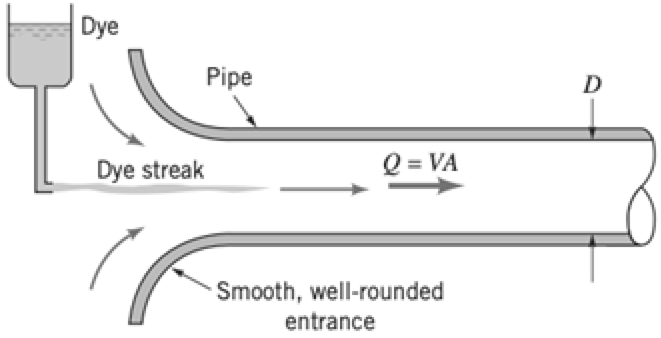
- Pipe is completely filled with the flow
- The fundamental mechanisms between pipe flow and open-channel flow are different (gravity vs. pressure difference)
  - If the pipe is not full, the pressure difference cannot be maintained
- Let’s consider first circular pipe with diameter, D
  (later you can adopt hydraulic diameter)

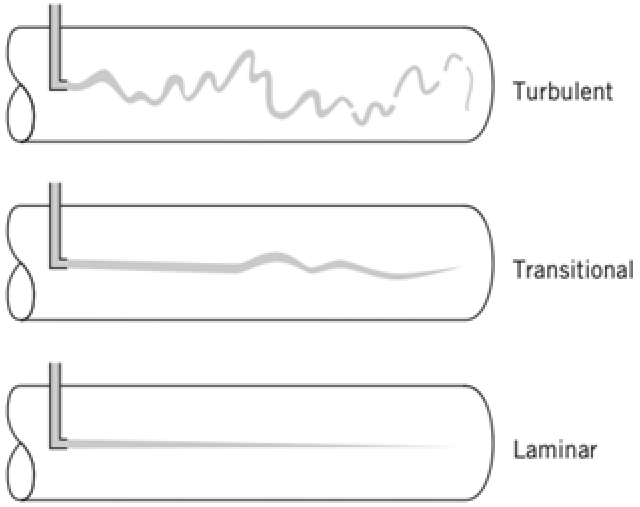
Reynolds number

- Laminar and turbulent transition



Osborne Reynolds's apparatus of 1883





- Reynolds number:

ratio of inertia over viscous forces

$$\boxed{Re = \frac{\rho U D}{\mu} = \frac{U D}{\nu}}$$

μ: dynamic viscosity, ν: kinematic viscosity
U: characteristic velocity
D: characteristic length, diameter

Reynolds number

- Reynolds number:

  ratio of inertia over viscous forces

$$\boxed{Re = \frac{\rho U D}{\mu} = \frac{U D}{\nu}}$$

μ: dynamic viscosity, ν: kinematic viscosity
U : characteristic velocity
D : characteristic length, diameter

For pipe (circular cross-section) flows :

- $Re < 2100$: laminar flow
- $2100 < Re < 4000$: Transitional
- $Re > 4000$: Turbulent flow

Example

Kinematic viscosity of water at 20 ℃,
$\nu = 1.002 \cdot 10^{-6}$ m²⁄s

Case of a water flow in a pipe:

D = 1 mm, Transition to turbulence for U = m/s

D = 1 m, Transition to turbulence for U = mm/s

Most of pipe flows are turbulent

Fully developed flow

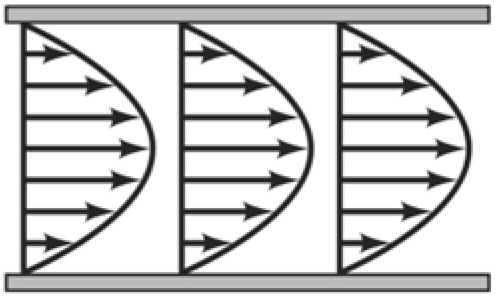
Although most flows are turbulent rather than laminar, and many pipes are not long enough to allow the attainment of **fully developed flow***(what is it?), a theoretical treatment and full understanding of fully developed laminar flow is important..

- It represents one of **the few theoretical viscous analyses** that can be carried out **exactly** (within the framework of quite general assumptions).

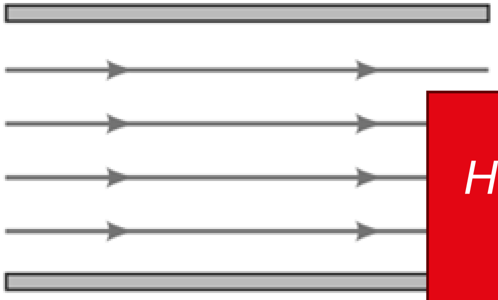
- The knowledge of the velocity profile can lead directly to other useful information such as pressure drop, head loss, and flowrate.

- There are many practical situations involving the use of fully developed laminar pipe flow

***Fully developed flow:** the velocity profile is the same at any cross-section of the pipe



Velocity profiles

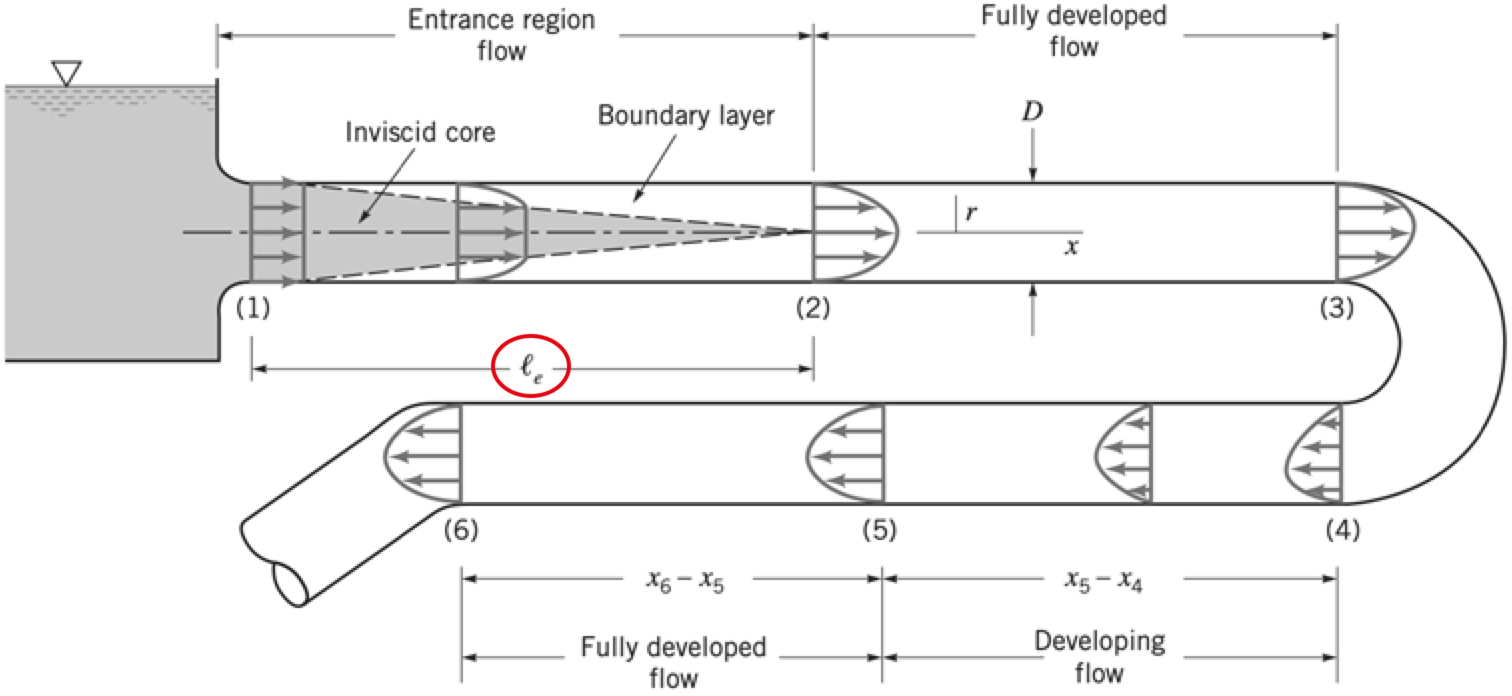


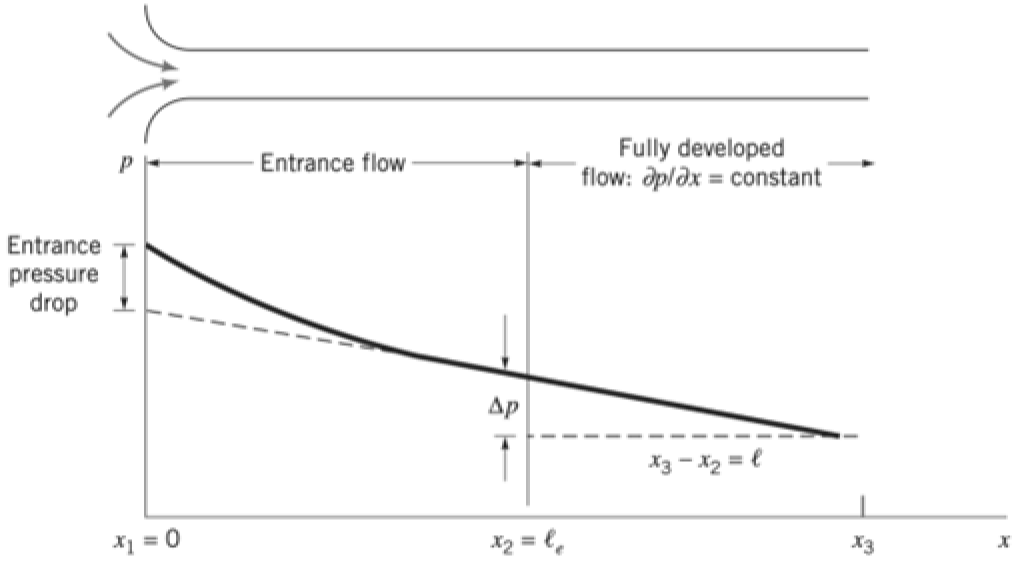
Streamlines

How long does it take to be fully developed?

Entrance and fully developed flow

- **Entrance length**





- Entrance length

$$\frac{\ell_e}{D} = 0.06Re$$: laminar flow

$$\frac{\ell_e}{D} = 4.4(Re)^{1/6}$$: turbulent flow

Re = 5 000, pipe with diameter D= 0.1 m what is the entrance length?

- Entrance length

$$\frac{\ell_e}{D} = 0.06 Re \quad \text{: laminar flow}$$

$$\frac{\ell_e}{D} = 4.4 (Re)^{1/6} \quad \text{: turbulent flow}$$

A. 30 m

B. 18 m

C. 1.8 m

D. 0.2 m

Enter Code

me342

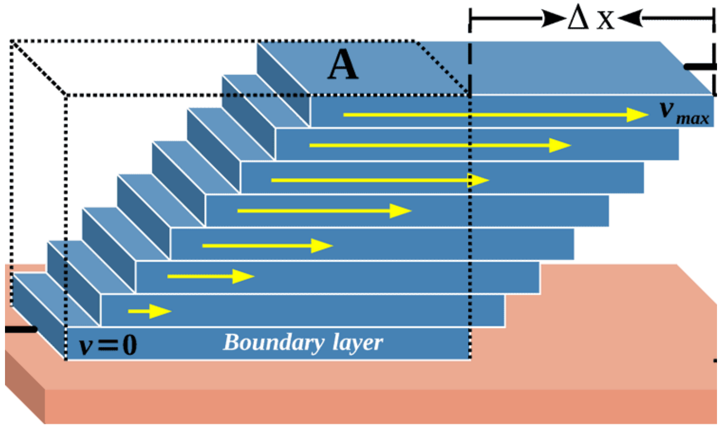
echo360poll.eu

0%	0%	0%	0%
A	B	C	D

Do you recall.. Viscous shear stress

- Physical properties of liquids and gases:
  - Viscosity
    - – Velocity gradient in a flowing fluid → shear force
    - – Newtonian fluids:
      the shear stress (tangential force/surface unit) proportional to velocity gradient:

$$\tau = \mu \frac{\partial u}{\partial y} = \rho \nu \frac{\partial u}{\partial y}$$



μ: dynamic viscosity, ν: kinematic viscosity

- – Shear forces always small in comparison with pressure
- – For ideal fluids ($\mu = 0$), with no internal friction → inviscid flows

If we make 2% error in diameter for laminar pipe flow, how much is the consequent error for the flowrate?

(answer with your intuition)

A. 2%

B. 4%

C. 8%

D. 16%

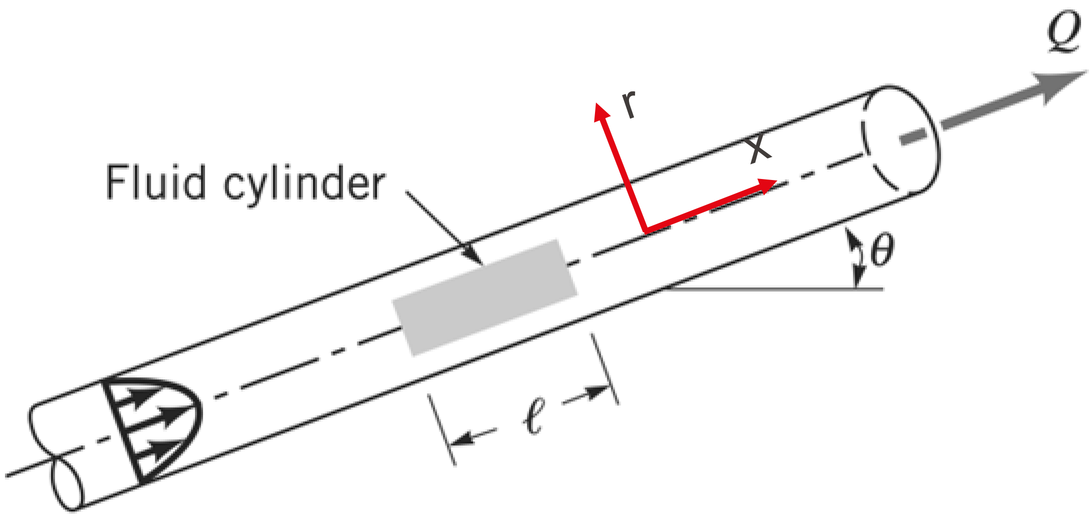
Enter Code

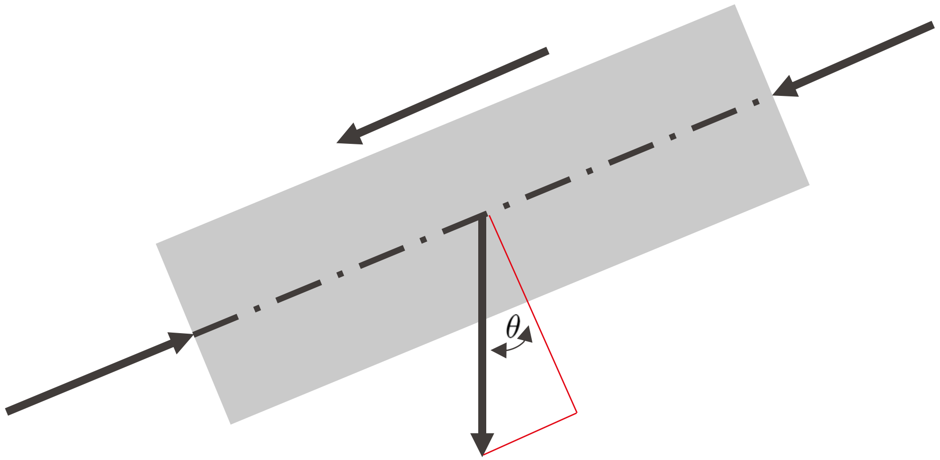
me342

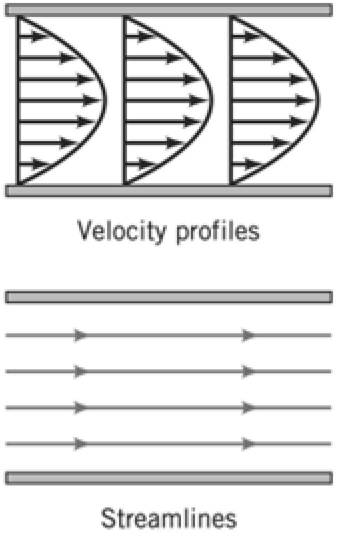
echo360poll.eu

0%	0%	0%	0%
A	B	C	D

Fully developed laminar flow







- Pressure difference

$$\Delta p = \frac{2\ell}{r}\tau$$

Fully developed laminar flow

- Wall shear stress

$$\Delta p = \frac{2\ell}{r}\tau$$

- Velocity $\boxed{u = V_c\left(1 - \frac{r^2}{R^2}\right)}$

Newtonian fluids shear stress :

$$\tau = \mu\frac{\partial u}{\partial y}$$

Cylindrical-coordinate
decrease of *u* in *r* (τ >0)

$$\tau = -\mu\frac{\partial u}{\partial r}$$

- Flowrate $\boxed{Q = \frac{\pi D^4 \Delta p}{128\mu\ell}}$

Discuss the results

If we make 2% error in diameter for laminar pipe flow, how much is the consequent error for the flowrate?

A. 2%
B. 4%
C. 8%
D. 16%

Enter Code
me342

echo360poll.eu

0%	0%	0%	0%
A	B	C	D

Let’s go back to the quiz

If we make 2% error in diameter for laminar pipe flow, how much is the consequent error for the flow rate?

Derivation from Navier-Stokes equation

- **Incompressible Navier-Stokes equation**

$$\nabla \cdot \mathbf{u} = 0,$$

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u}\right) = -\nabla p + \underset{\rho g}{\cancel{\mathbf{g}}} + \mu\nabla^2\mathbf{u}$$

- Fully developed pipe flow

$$\mathbf{u} = (u_r, u_\theta, u_x) = (0, 0, u_x(r))$$

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z\frac{\partial v_r}{\partial z}\right)$$
$$= -\frac{\partial p}{\partial r} + \rho g_r + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_r}{\partial r}\right) - \frac{v_r}{r^2} + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2}\right]$$

(θ direction)

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z\frac{\partial v_\theta}{\partial z}\right)$$
$$= -\frac{1}{r}\frac{\partial p}{\partial \theta} + \rho g_\theta + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_\theta}{\partial r}\right) - \frac{v_\theta}{r^2} + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_\theta}{\partial z^2}\right]$$

(z direction)

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right)$$
$$= -\frac{\partial p}{\partial z} + \rho g_z + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right]$$

→ exercise session

Friction factor

- **Flow rate**

$$Q = \frac{\pi D^4 \Delta p}{128 \mu \ell} = VA$$

$$\Delta p = \frac{32 \mu \ell V}{D^2}$$

- Divide by dynamic pressure to obtain a dimensionless form

$$\frac{\Delta p}{\frac{1}{2}\rho V^2} = f \frac{\ell}{D}$$

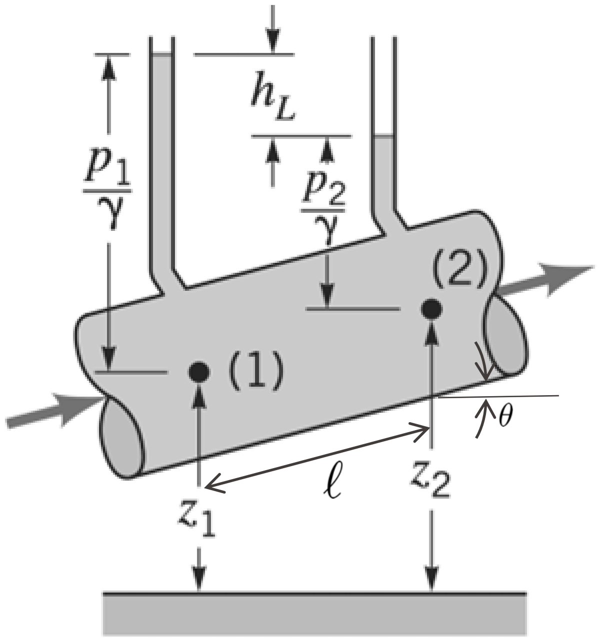
- **Friction factor** or Darcy friction factor (fully developed laminar flow)

$$\boxed{f = \frac{64}{Re}}$$

- Express f with wall shear stress (τ_w)

$$f = \frac{8\tau_w}{\rho V^2}$$

Let's go back to the energy consideration



- Absence of shaft work & turbine loss

$$\boxed{\frac{p_1}{\gamma} + \alpha_1 \frac{\bar{V}_1^2}{2g} + z_1 - h_L = \frac{p_2}{\gamma} + \alpha_2 \frac{\bar{V}_2^2}{2g} + z_2}$$

- In fully developed flow, the kinetic energy is the same

$$\frac{p_1}{\gamma} + z_1 - h_L = \frac{p_2}{\gamma} + z_2$$

$$h_L = \frac{p_1}{\gamma} - \frac{p_2}{\gamma} + z_1 - z_2$$

$$= \frac{\Delta p}{\gamma} - \Delta z = \frac{2l\tau}{r\gamma} = \frac{4l\tau_w}{\gamma D}$$

The head loss in a pipe is a result of the viscous shear stress on the wall.

Discussion on the pressure drop-shear stress relation

- The nonzero pressure gradient along the horizontal pipe results from ________ effects.

$$\Delta p = \frac{2\ell}{r}\tau = -\frac{2\ell}{r}\mu\frac{\partial u}{\partial r}$$

- From a ***force balance perspective***, the pressure force is required to counteract the _______ ______ acting on the fluid.

- From an ***energy balance perspective***, the work done by the pressure force compensates for the energy lost due to ________________ throughout the fluid.

- The ***head loss in a pipe*** is a result of the __________________ on the wall.

$$h_L = \frac{4l\tau_w}{\gamma D}$$

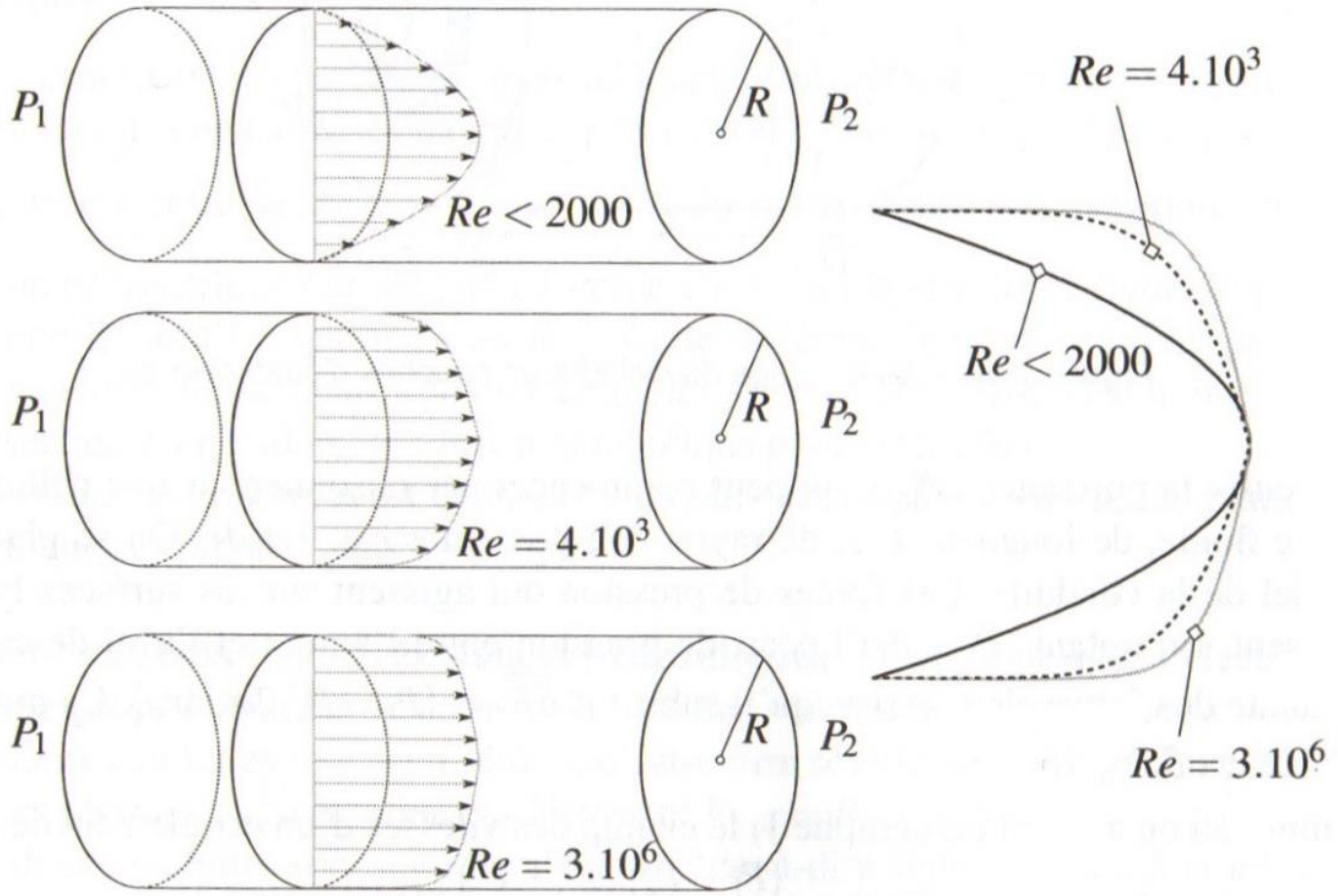
Exercise

An oil with a viscosity μ = 0.4 N·s/m^2 and density ρ = 900 kg/m^3 flows in a pipe of diameter D = 0.02 m.

(1) What pressure drop dP is needed to produce a flowrate of Q = 2.0x10^-5 m^3/s if the pipe is horizontal with length ℓ = 10 m ?

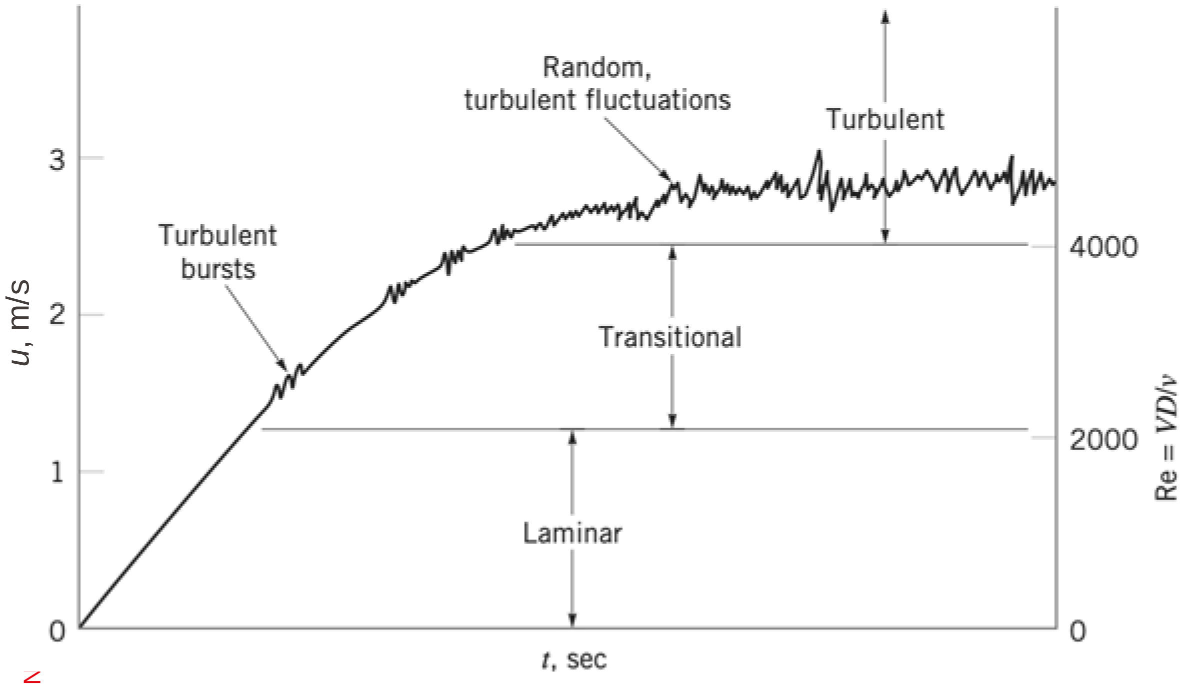
(2) How steep a hill θ, must the pipe be on if the oil is to flow through the pipe at the same rate as in (1) but with dP = 0?

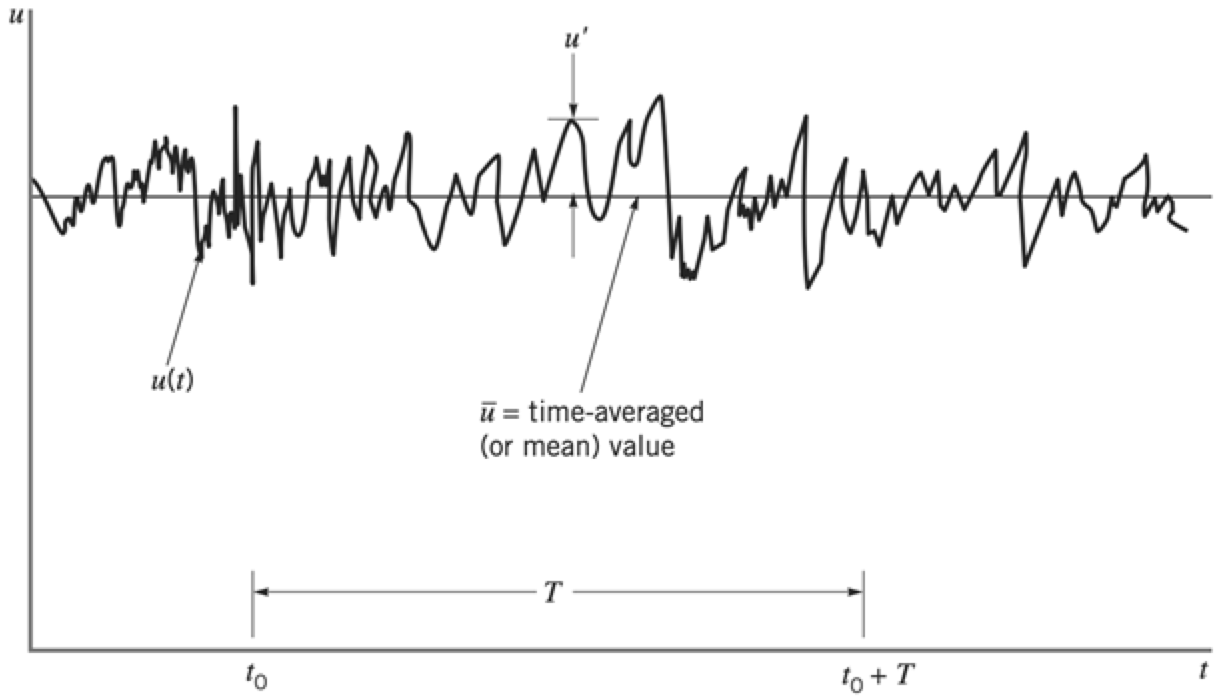
Pipe flow in turbulent regime



Velocity approximation in turbulent pipe

- Some basics of Turbulent flow





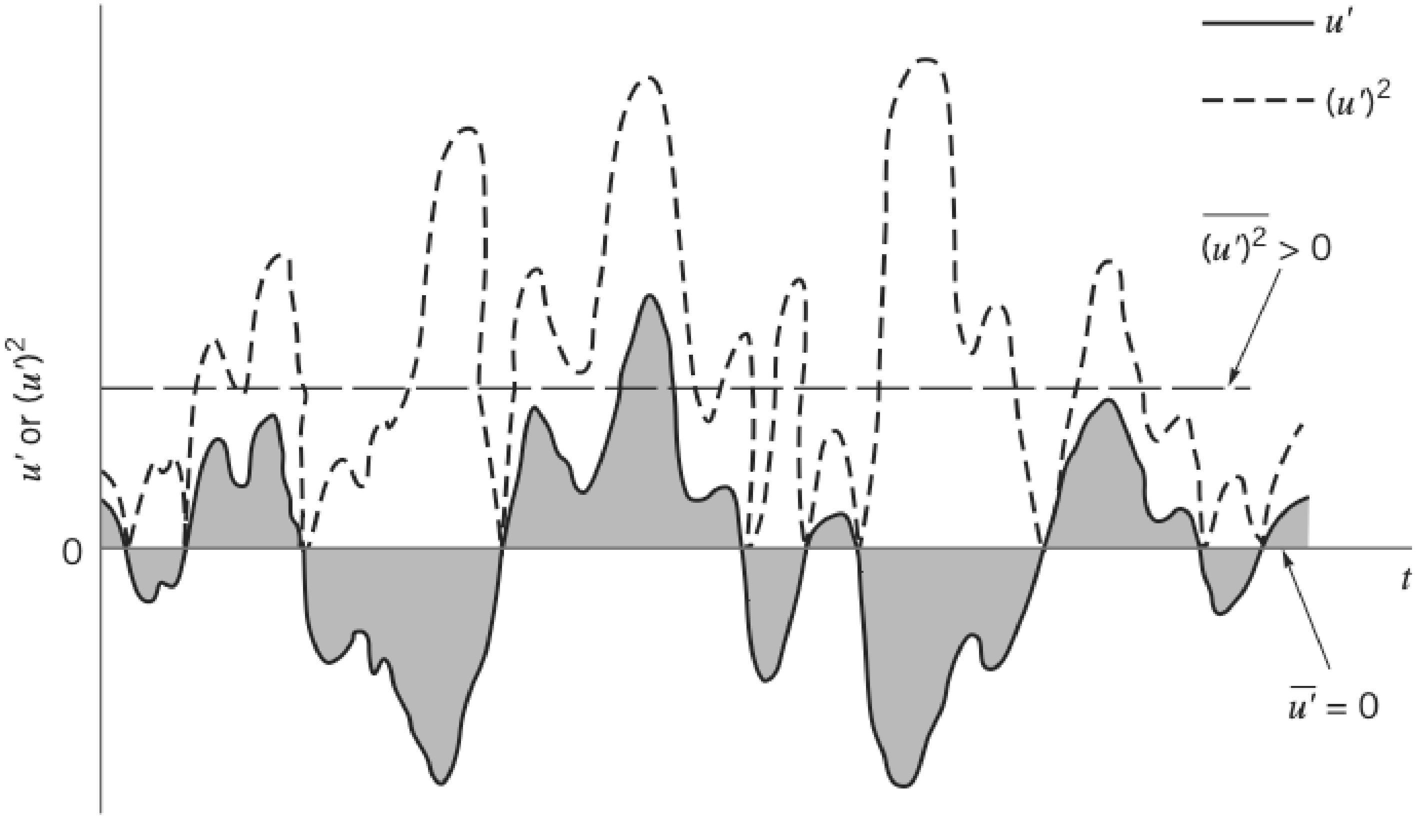
$$\boxed{\mathbf{u}(\mathbf{x},t) = \overline{\mathbf{u}(\mathbf{x})} + \mathbf{u}'(\mathbf{x},t)}$$

Time-averaged meanflow Fluctuations

$$\bar{\mathbf{u}} = \frac{1}{T}\int_{t_0}^{t_0+T} \mathbf{u}(\mathbf{x},t)dt \qquad \mathbf{u}' = \frac{1}{T}\int_{t_0}^{t_0+T} \mathbf{u}'(\mathbf{x},t)dt = 0$$

Stochastic or random fluctuations

- **Squared average is larger than zero**



$$\boxed{\overline{\mathbf{u}'^2} = \frac{1}{T}\int_{t_0}^{t_0+T} \mathbf{u}'^2(\mathbf{x}, t)dt > 0}$$

Shear stress in presence of turbulence

- **Turbulent shear stress**

$$\tau = \mu \frac{d\bar{u}}{dy} - \rho \overline{u'v'} = \tau_{\mathrm{lam}} + \tau_{\mathrm{turb}}$$

turbulent shear stress > 0

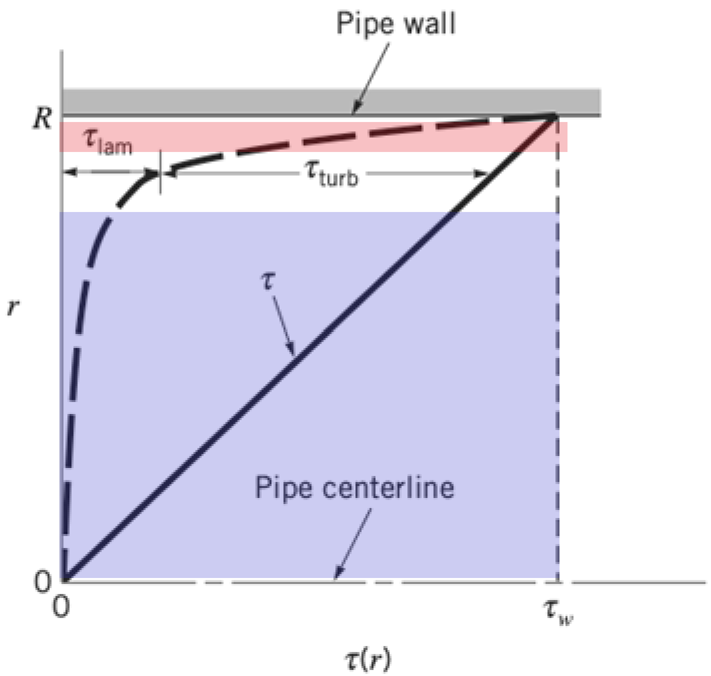
- Shear stress is greater in turbulent flow than in laminar flow
- The forms $-\rho\overline{u'v'},\ -\rho\overline{u'w'},\ -\rho\overline{v'w'}$ are called **Reynolds stress**

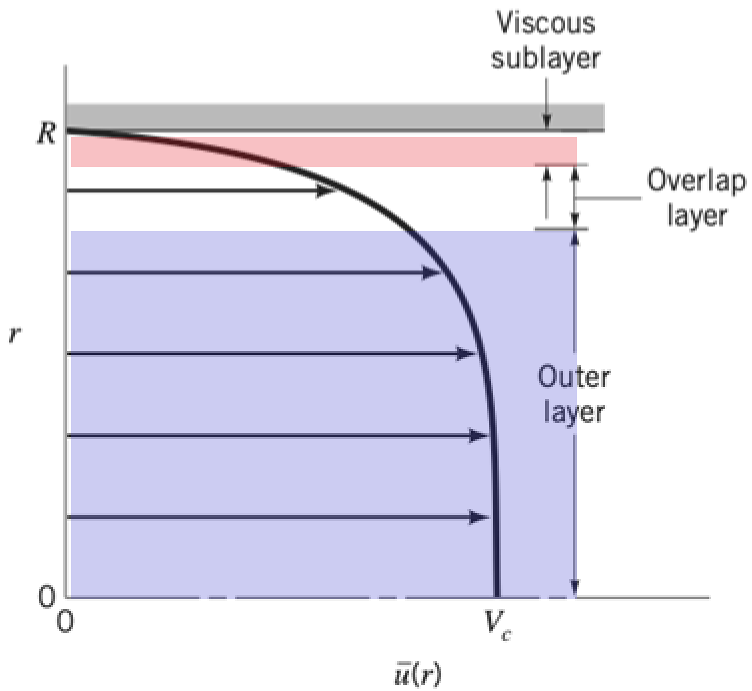
Shear stress composition

- From various observations...

$$\tau = \mu \frac{d\bar{u}}{dy} - \rho \overline{u'v'} = \tau_{\text{lam}} + \tau_{\text{turb}}$$

- **Viscous sublayer:** In a very narrow region near the wall, the laminar shear stress is dominant.
- **Outer layer:** Away from the wall, the turbulent portion of the shear stress is dominant.
- **Overlap layer**: The transition between these two regions occurs in the overlap layer.





Velocity profiles in layers

- **In the viscous sublayer**

$$\boxed{\frac{\bar{u}}{u^*} = \frac{yu^*}{\nu}}$$ Valid very near the smooth wall

$y = R - r$ Distance from the wall

$u^* = \sqrt{\frac{\tau_w}{\rho}}$ Friction velocity (not an actual velocity)

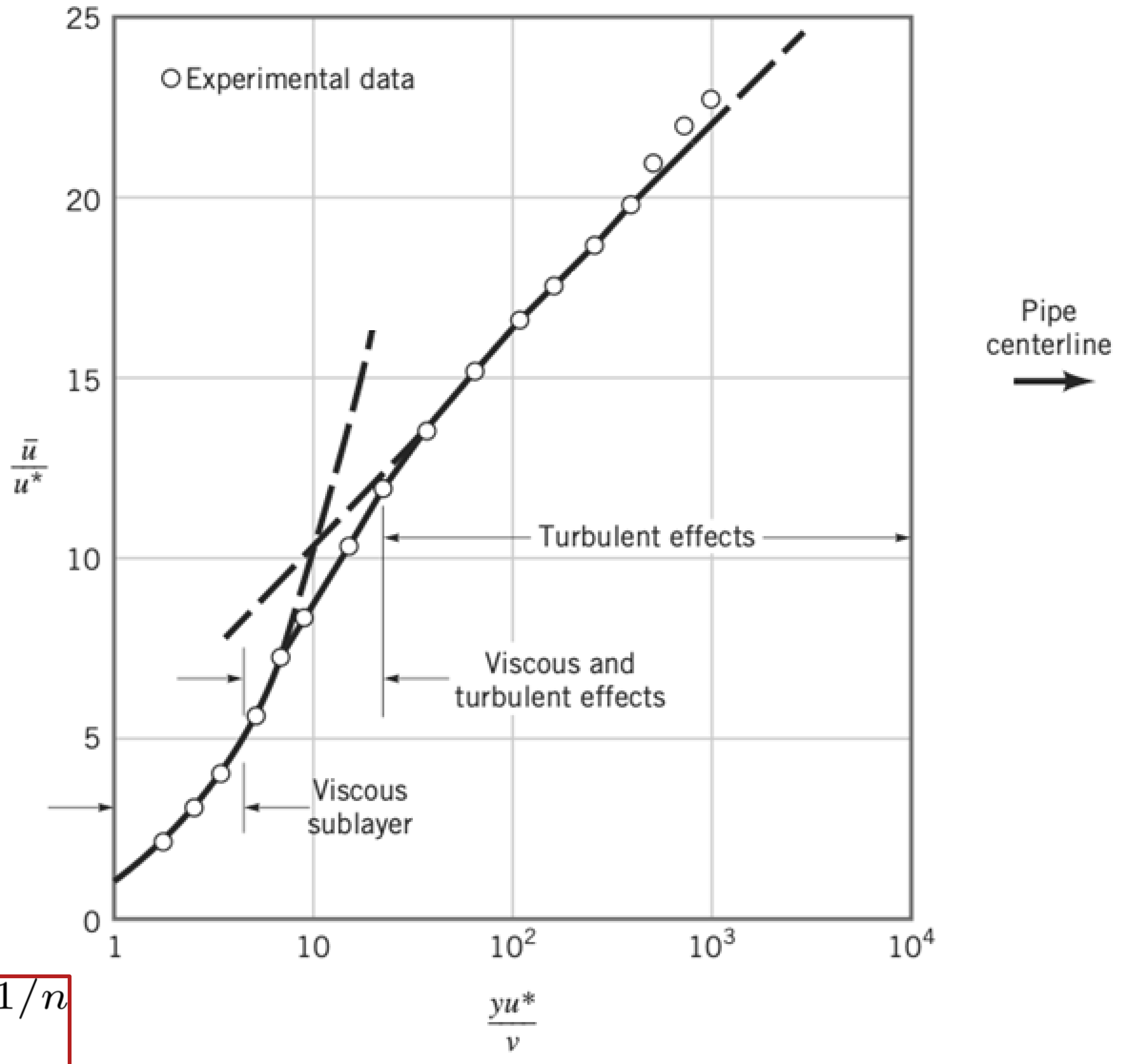
- **In the overlap region**

$$\boxed{\frac{\bar{u}}{u^*} = 2.5 \ln\left(\frac{yu^*}{\nu}\right) + 5.0}$$

Constants 2.5 and 5.0 are determined experimentally

- **In the outer turbulent layer**

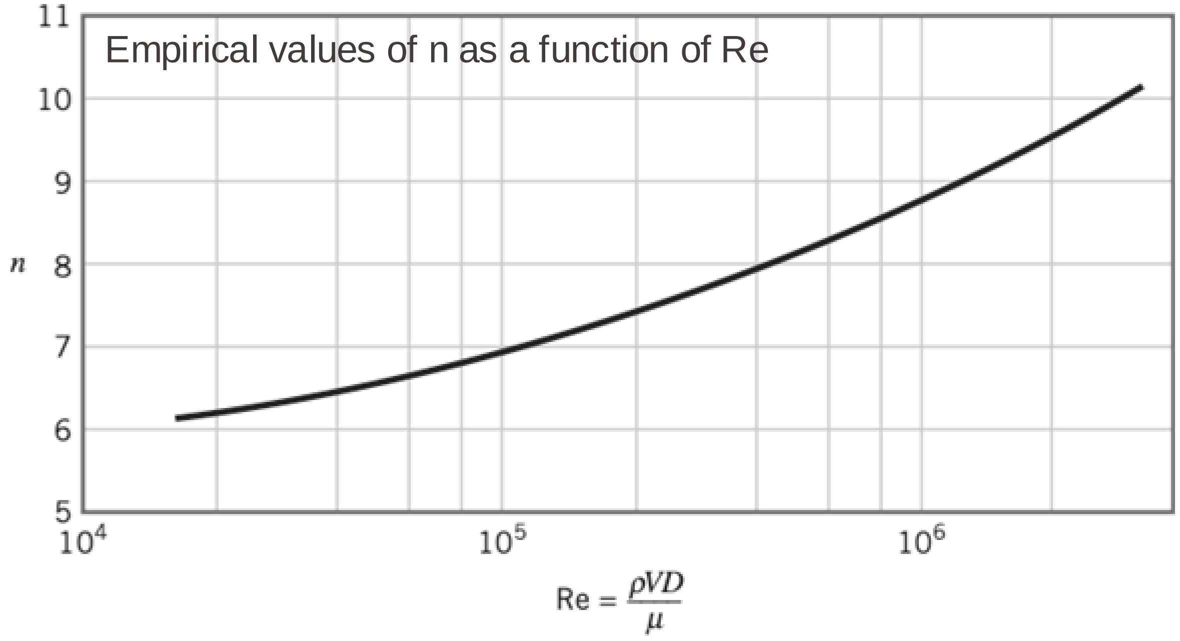
$$\boxed{\frac{(V_c - \bar{u})}{u^*} = 2.5 \ln\left(\frac{R}{y}\right)}$$ or $$\boxed{\frac{\bar{u}}{V_c} = \left(1 - \frac{r}{R}\right)^{1/n}}$$

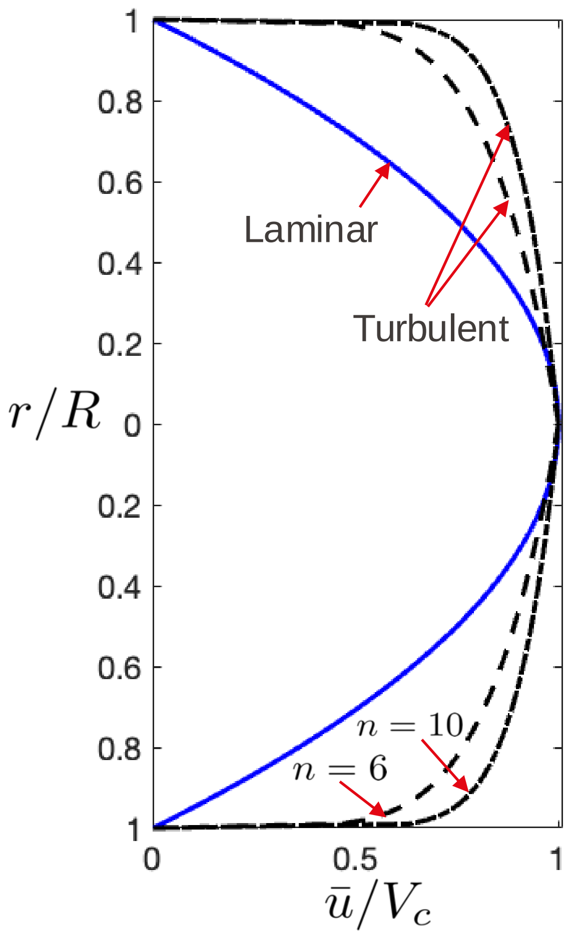


Velocity profile in turbulent pipe

- **Power-law velocity profile**

$$\boxed{\frac{\bar{u}}{V_c} = \left(1 - \frac{r}{R}\right)^{1/n}}$$





Examine the power-law velocity profile and discuss if it makes sense at :

- Validity r=R
- Validity r=0

Next lecture

- Head loss estimations for various type of pipes.
- Other fluids phenomena.