



Chapter 10: Hydraulic Turbines 2



ME-342 Introduction to
turbomachinery

Prof. Eunok Yim, HEAD-lab.

Impulse or reaction : No matter the working fluid, turbines can be broadly classified into two types based on the mechanism of the fluid interaction

Impulse turbine: the force on the blades is produced solely by turning the fluid, without appreciable pressure drop in the blade passage, with all of the pressure drop occurring in a fixed nozzle.

Reaction turbine: some of the fluid-vane force is from fluid turning and some of the force is a reaction to acceleration of the fluid relative to the vane. In reaction blading, a pressure drop occurs in both a fixed nozzle and the moving vane.

Turbine blading is characterized by the **degree of reaction (R) (or simply reaction)**, which is the ratio of the drop in static pressure (or enthalpy) across the moving blade to the overall drop in static pressure (or enthalpy) across the fixed nozzle plus the moving blade. Impulse turbines have $R = 0$ while reaction turbines typically have $0.1 < R < 0.7$.

Change in **static enthalpy** across the **rotor** divided by the static enthalpy change across the entire **stage**

(static) Enthalpy: internal energy and flow work

$$\check{h} = \check{u} + \frac{p}{\rho} \quad [\text{J/kg}]$$

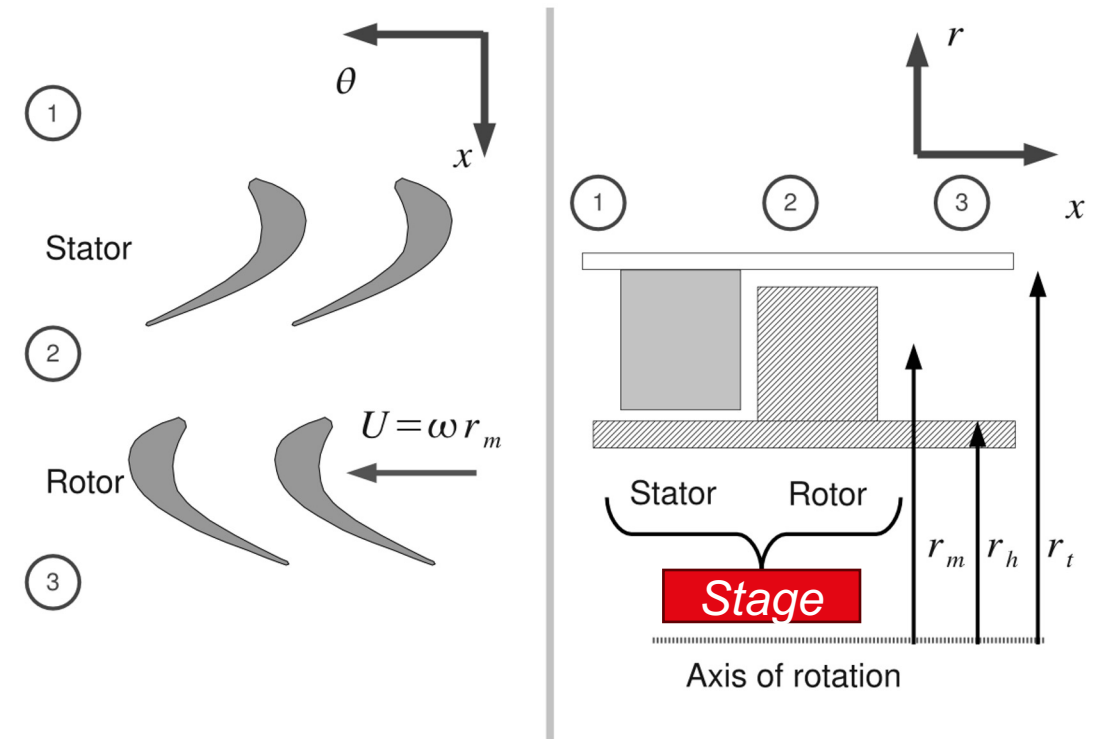
Internal energy

$\text{Pa} \cdot \text{m}^3/\text{kg} \equiv \text{J/kg}$

Stagnation enthalpy: sum of enthalpy, kinetic energy and potential energy

$$\check{h}_0 = \check{h} + \frac{V^2}{2} + gz$$

Achtung! $\check{h}$ is enthalpy, not head h or h_a

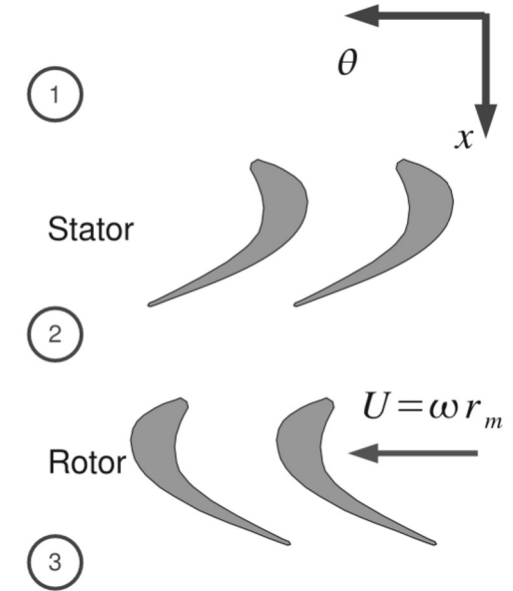


Recall- Degree of reaction or Reaction (R)

This concept is much used in axial flow machines as a measure of the relative proportions of energy transfer obtained by static and dynamic pressure change

(static) Enthalpy: internal energy and flow work

$$\check{h} = \check{u} + \frac{p}{\rho} \quad [\text{J/kg}]$$



$$R = \frac{\text{energy change due to, or resulting from, static pressure change in the **rotor**}}{\text{total energy change for a **stage**}}$$

$$R = \frac{\text{static enthalpy change in **rotor**}}{\text{static enthalpy change in **stage**}}$$

$$= \frac{\check{h}_2 - \check{h}_3}{\check{h}_1 - \check{h}_3}$$

If no internal energy is changed, incompressible,

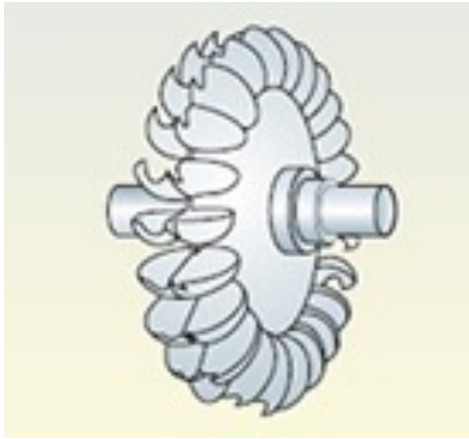
$$R \simeq \frac{p_2 - p_3}{p_1 - p_3}$$

$$R = \frac{U_2^2 - U_3^2 + W_3^2 - W_2^2}{V_2^2 - V_1^2 + U_2^2 - U_3^2 + W_3^2 - W_2^2}$$

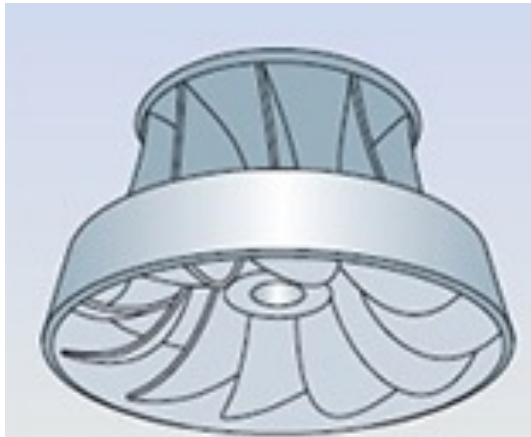
- If $U_2=U_3$ (axial machine), $W_3=W_2 \rightarrow R=0$
- If $V_1=V_2 \rightarrow R=1$

Recall-Types of hydraulic turbines

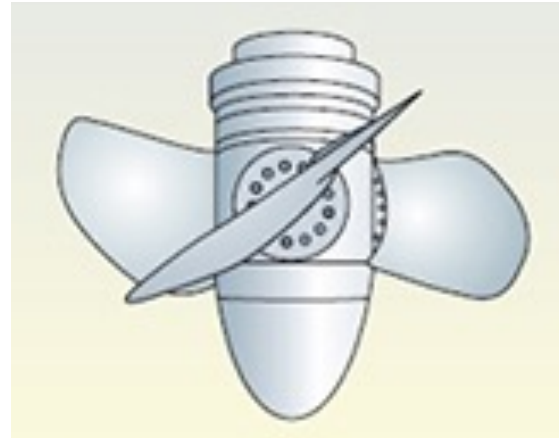
- Impulse (Action) turbines: Pelton turbines, $R \sim 0$
- Reaction turbines: $R \sim [0.1, 0.7]$
 - Francis turbines (radial et axial), Kaplan turbines (axial)
 - Propeller turbines (similar to Kaplan turbines with fixed pitch)



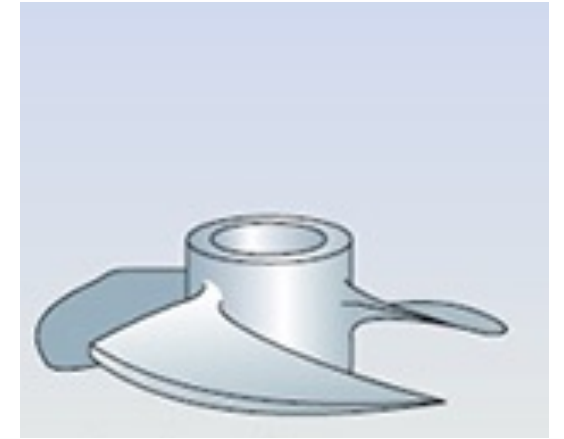
Pelton



Francis

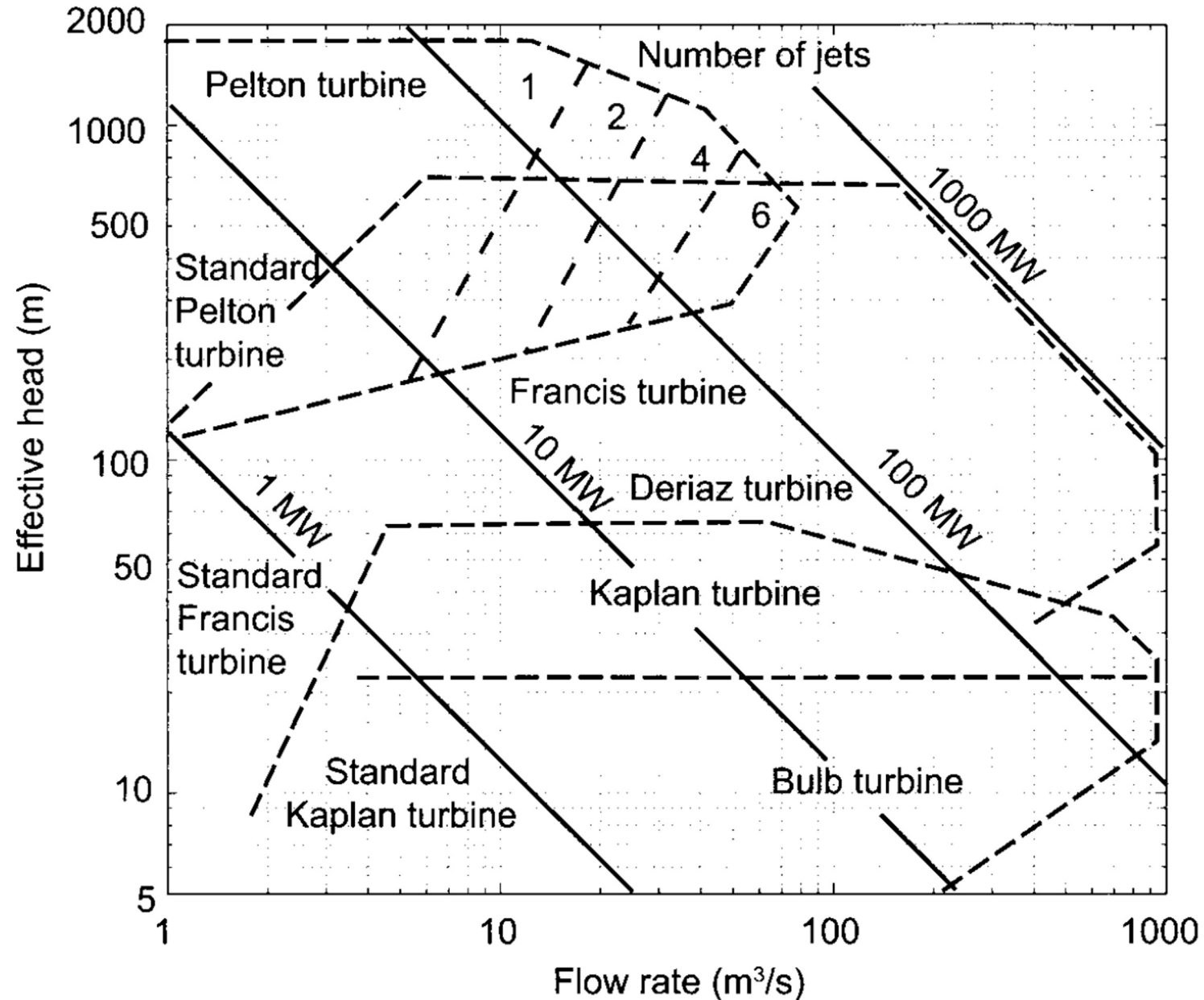


Kaplan & Bulb



Fixed pitch propeller

Recall-Types of hydraulic turbines



Impulse turbines

Reaction turbines

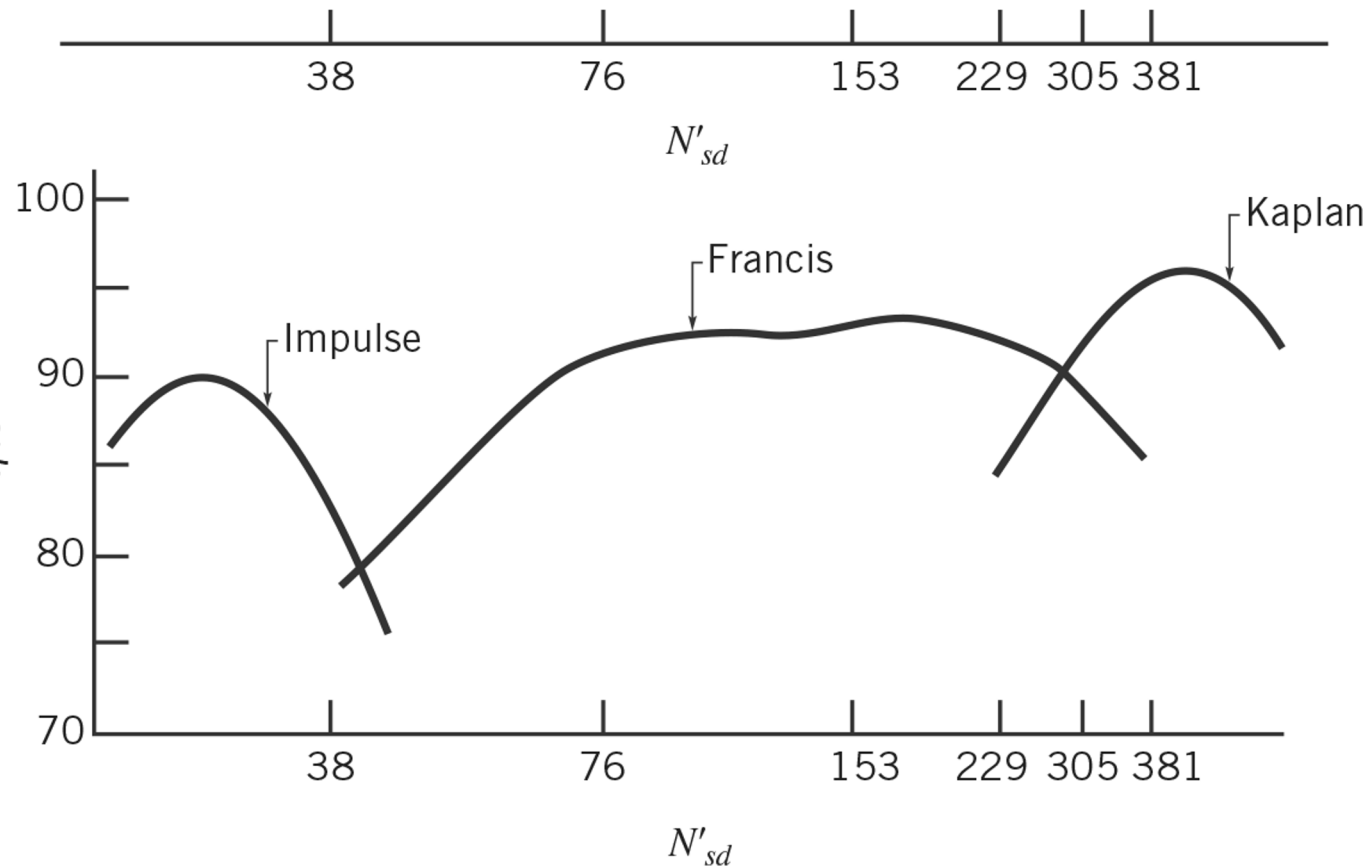
Radial-flow

Mixed-flow

Axial-flow

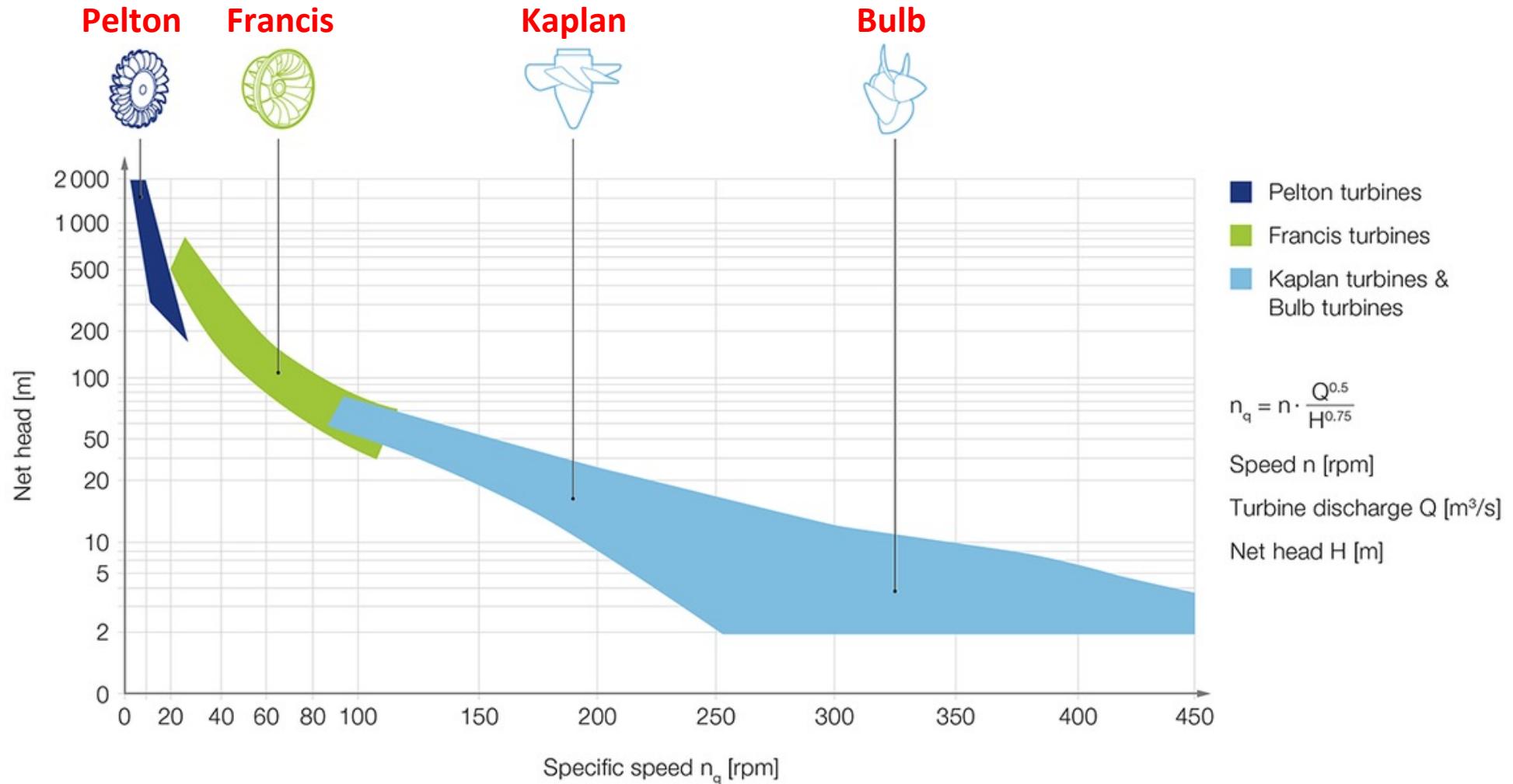
Power specific speed:

$$N'_s = \frac{\omega \sqrt{\dot{W}_{\text{shaft}} / \rho}}{(gh_a)^{5/4}}$$



Type	N'_s	$\eta\%$
Pelton wheel		
Single jet	0.02 – 0.18	88 – 90
Twin jet	0.09 – 0.26	89 – 92
Three jet	0.10 – 0.30	89 – 92
Four jet	0.12 – 0.36	86
Francis		
Low-speed	0.39 – 0.65	90 – 92
Medium-speed	0.65 – 1.2	93
High-speed	1.2 – 1.9	93 – 96
Extreme-speed	1.9 – 2.3	89 – 91
Kaplan turbine	1.55 – 5.17	87 – 94
Bulb turbine	3 – 8	

- Classification of turbine types as a function of the head and unit specific speed



Hydraulic Turbines – Reaction turbines

The primary features :

- (i) Only **part of the overall pressure drop** has occurred up to turbine entry; the **remaining pressure drop** takes place **in the turbine** itself
- (ii) The flow **completely fills all of the passages in the runner**, unlike the Pelton turbine where, for each jet, only one or two of the buckets at a time are in contact with the water
- (iii) Pivotal **guide vanes** are used to control and direct the flow
- (iv) A **draft tube** is normally added to the turbine exit; this is considered an integral part of the turbine

As water flows through the runner, its pressure gradually decreases. This pressure drop is what gives the turbine its name: a reaction turbine.

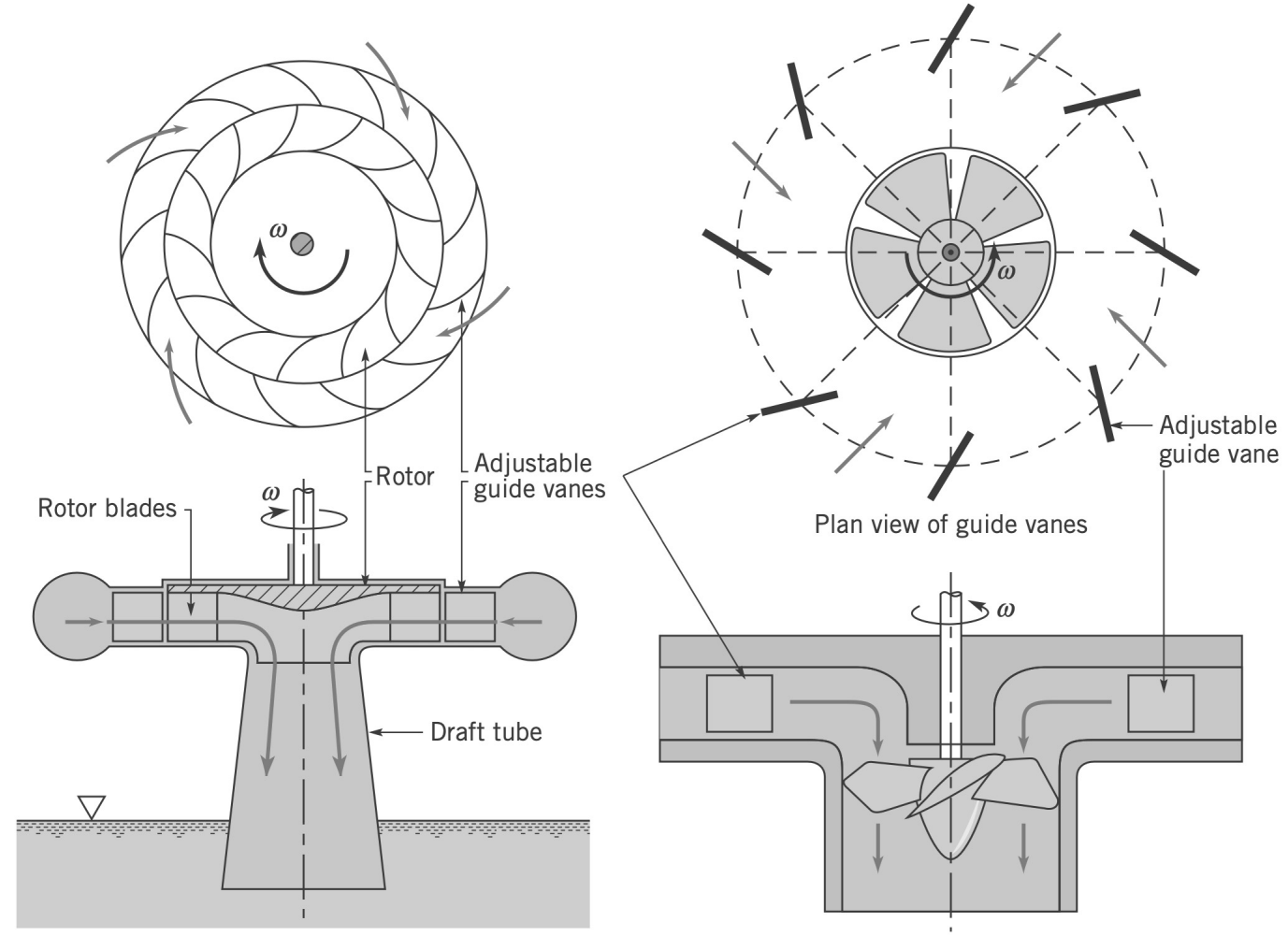
The **angular momentum, pressure, and velocity of the fluid decrease** as it flows through the turbine rotor—the turbine rotor extracts energy from the fluid.

Reaction turbines are best suited for **higher flow rates** and **lower head** situations.

As with pumps, turbines are manufactured in a variety of configurations—radial-flow, mixed-flow, and axial-flow types.

Typical radial- and mixed-flow hydraulic turbines are called **Francis** turbines, named after James B. Francis, an American engineer.

The Kaplan turbine, named after Victor **Kaplan**, a German professor, is an efficient axial-flow hydraulic turbine with adjustable blades.

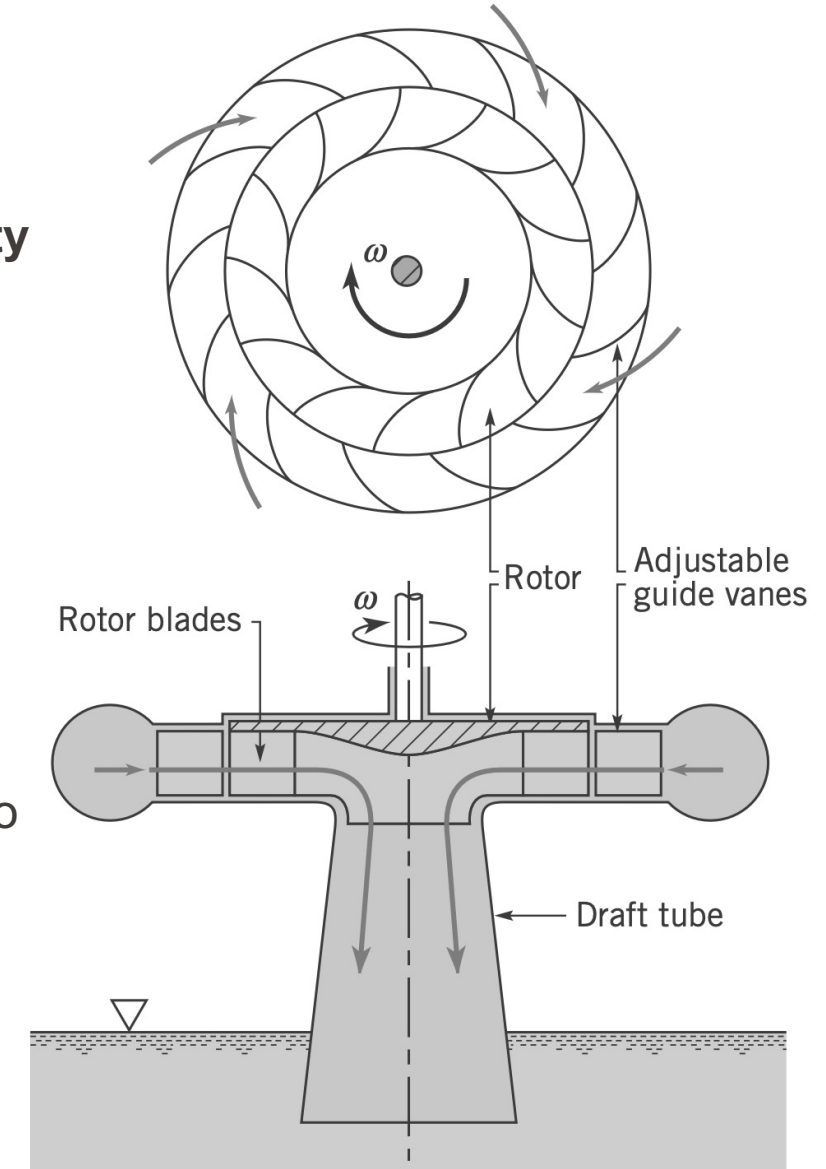


- Flow across rotor blades has a major **radial** component
- **Adjustable inlet guide vanes** direct water with a **tangential velocity** into the rotor
- Water exits rotor **with no tangential velocity component**
- Rotor:
 - **Decreases angular momentum** of the fluid
 - Fluid exerts **torque on rotor** in direction of rotation
 - Rotor **extracts energy from fluid**
- **Euler turbomachine equation** and **power equation** apply, similar to a centrifugal pump

$$T_{\text{shaft}} = -\dot{m}_1 (r_1 V_{\theta 1}) + \dot{m}_2 (r_2 V_{\theta 2})$$

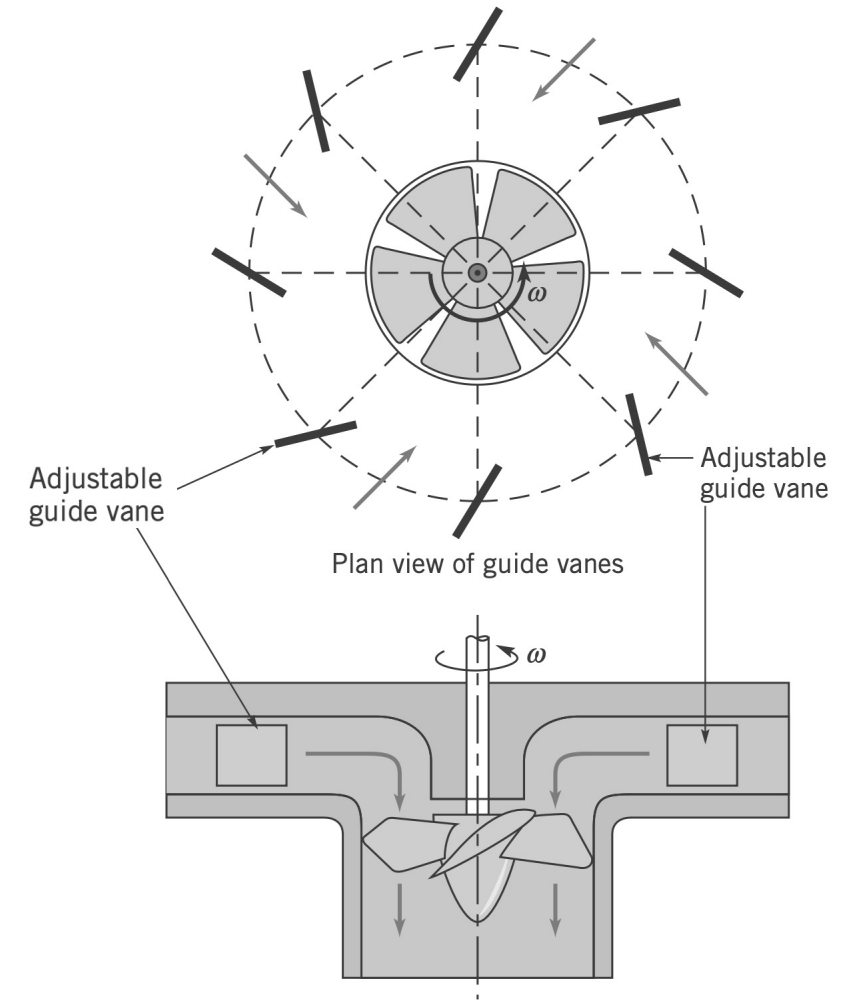
$$\dot{W}_{\text{shaft}} = -\dot{m}_1 (U_1 V_{\theta 1}) + \dot{m}_2 (U_2 V_{\theta 2})$$

$$w_{\text{shaft}} = - (U_1 V_{\theta 1}) + (U_2 V_{\theta 2})$$



Axial-Flow Kaplan Turbine

- Fluid flows through **inlet guide vanes (wicket gates)**, gaining **tangential velocity (vortex/swirl)**
- Flow across rotor has a major **axial** component
- Both **inlet guide vanes** and **turbine blades** are **adjustable** for optimal performance
- Adjustment accommodates:
 - Changes in **operating head** (seasonal variations)
 - Variations in **flow rate** through the rotor

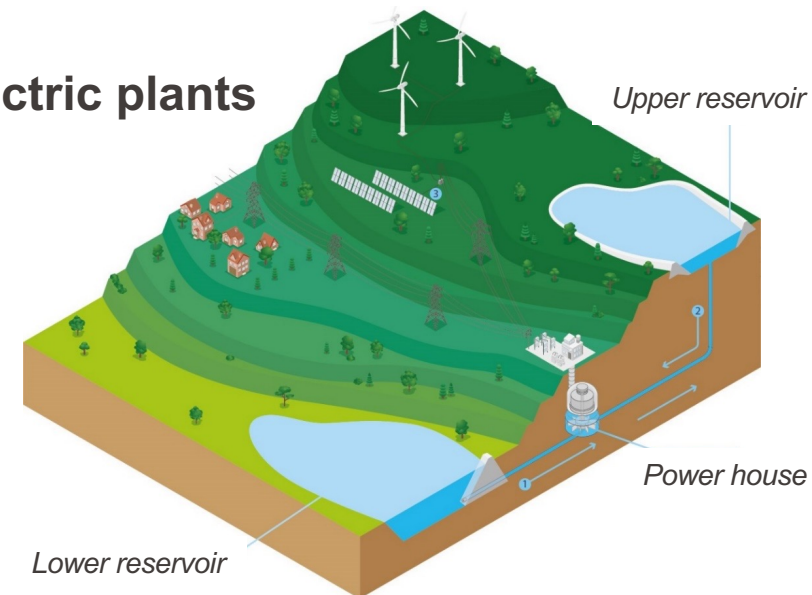


Pumps vs. Turbines

- Pumps and turbines are “inverse” machines
- Pumps add energy to the fluid
- Turbines remove energy from the fluid
- Example:
 - Propeller on an **outboard motor (pump)** vs. propeller on a **Kaplan turbine**
 - **Geometrically similar**, but perform **opposite tasks**
 - Similar comparison: **centrifugal pumps vs. Francis turbines**

Pump storage

- Some large **pump/turbines** are used in **pumped storage hydroelectric plants**
- Run as **turbines** during **high-power demand** (day)
- Run as **pumps** during **low-power demand** (night)
- Used to **resupply upstream reservoir**
- Often, **each pump type has a corresponding turbine type**



As with pumps, incompressible flow turbine performance is often specified in terms of appropriate dimensionless parameters. The flow coefficient, the head coefficient, and the power coefficient are defined in the same way for pumps and turbines

Flow coefficient : $C_Q = \frac{Q}{\omega D^3}$

Head coefficient : $C_H = \frac{gh_a}{\omega^2 D^2}$

Power coefficient : $C_{\mathcal{P}} = \frac{\dot{W}_{\text{shaft}}}{\rho \omega^3 D^5}$

On the other hand, turbine efficiency, η , is the inverse of pump efficiency. That is, the efficiency is the ratio of the **shaft power output** to the **power available in the flowing fluid**:

$$\eta_{\text{turbine}} = \frac{\dot{W}_{\text{shaft}}}{P_f} \quad \eta_{\text{pump}} = \frac{P_f}{\dot{W}_{\text{shaft}}}$$

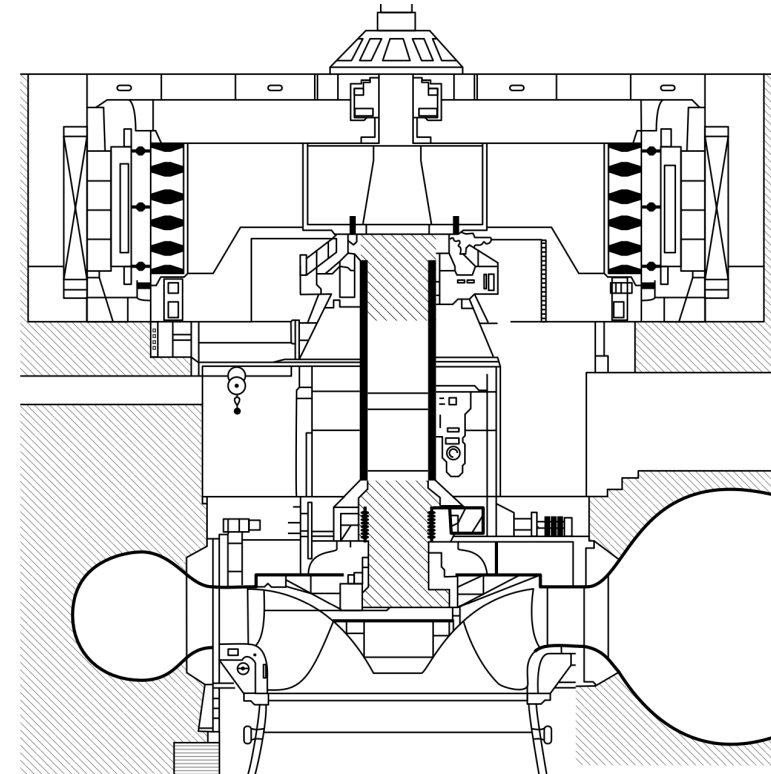
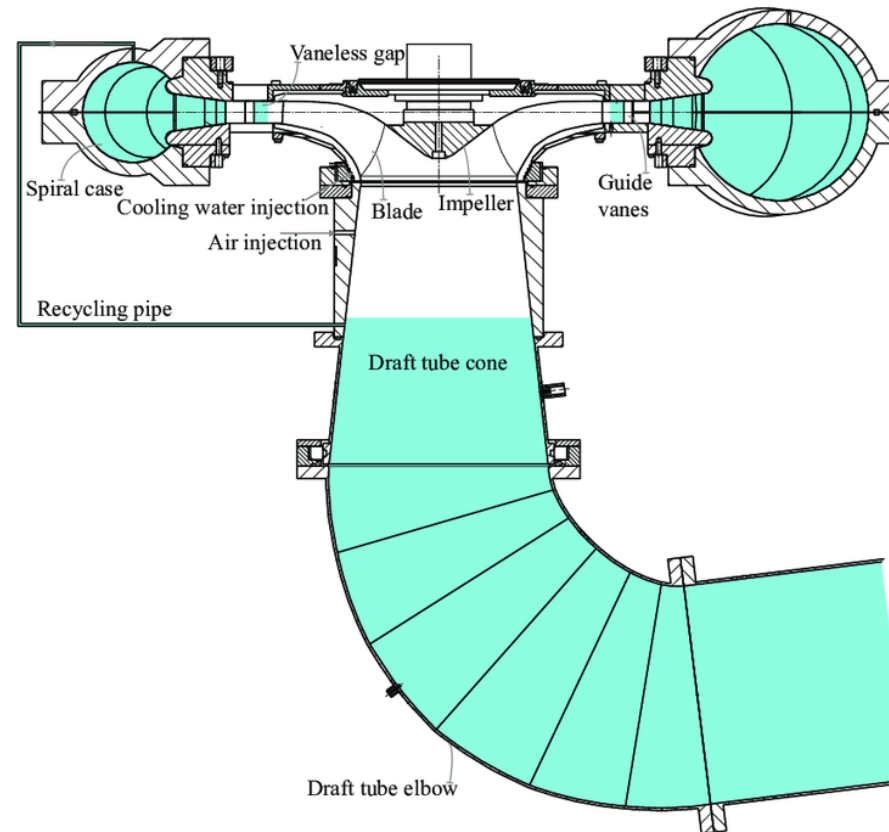
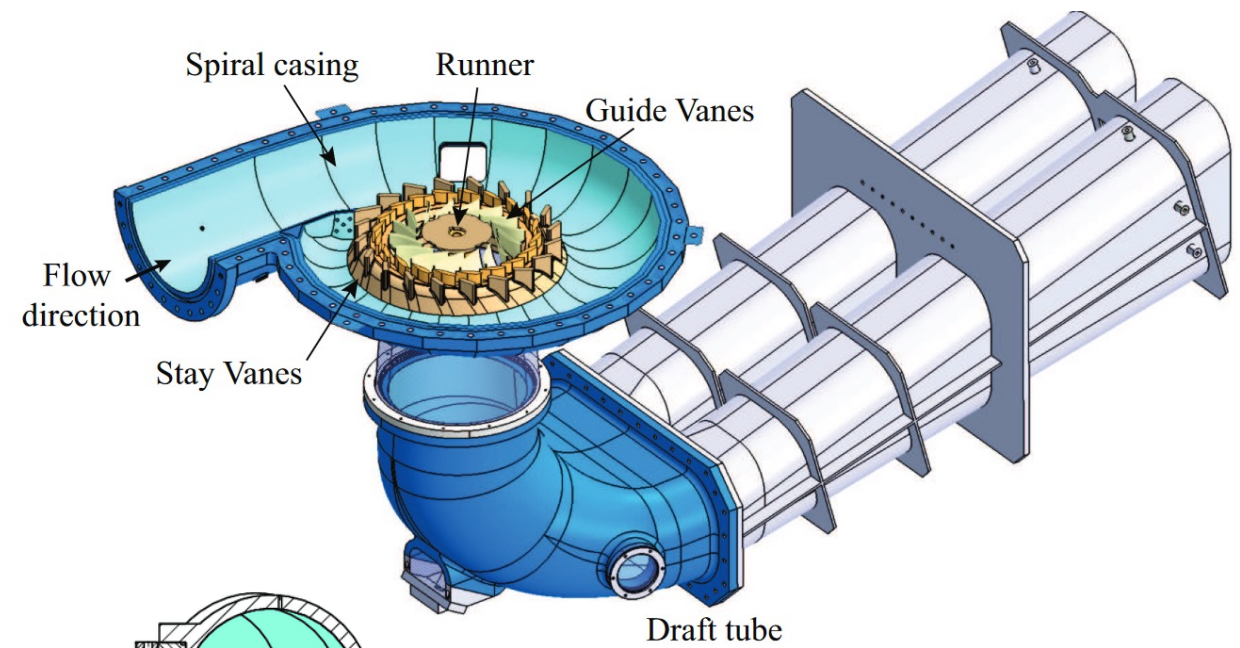
$$P_f = \gamma Q h_a$$

Hydraulic Turbines – Reaction turbines

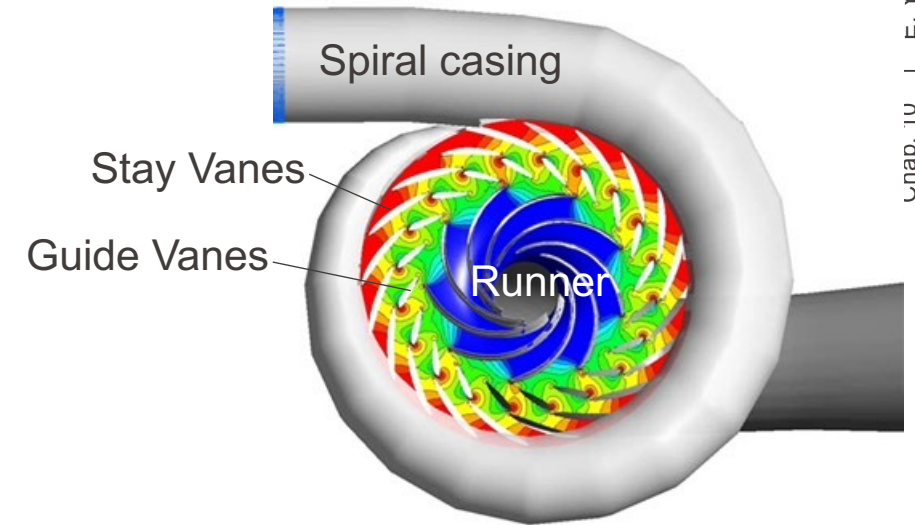
Francis turbine

Francis Turbine

- **Main components:**
 - Spiral casing
 - Stay vanes
 - Wicket gates (Distributor)
 - Runner
 - Draft tube



- **Spiral casing:**
 - Connects the turbine to the incoming flow (from the penstock)
 - Creates a uniform and pre-rotating flow
- **Stay vanes:**
 - Set of fixed vanes: Direct the flow towards the runner
- **Wicket gates (Guide vanes):**
 - Set of adjustable vanes: Direct the flow further and adjust the flowrate (from ~0% to 100%)
 - Actioned by a servomotor with the help of a gates ring



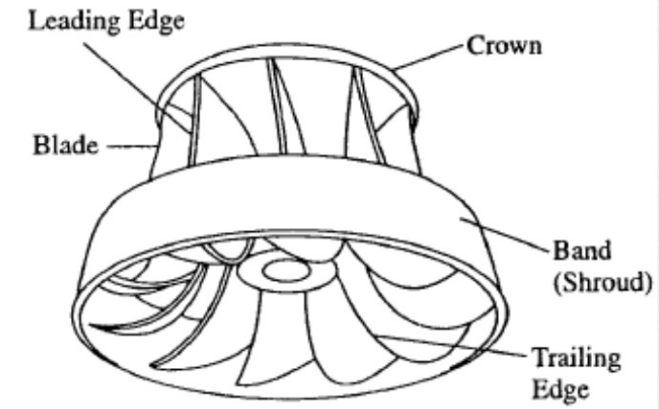
Francis Turbine

- **Runner:**

- The most sensitive component, made of a set of sophisticated 3D blades, a crown and a band (shroud)
- Specific design for every (large) project (Nominal head and flowrate)
- Highly efficient: more than 95%
- At optimum conditions, the flow leaves the runner axially. Otherwise, a swirl develops at the runner outlet

- **Draft Tube:**

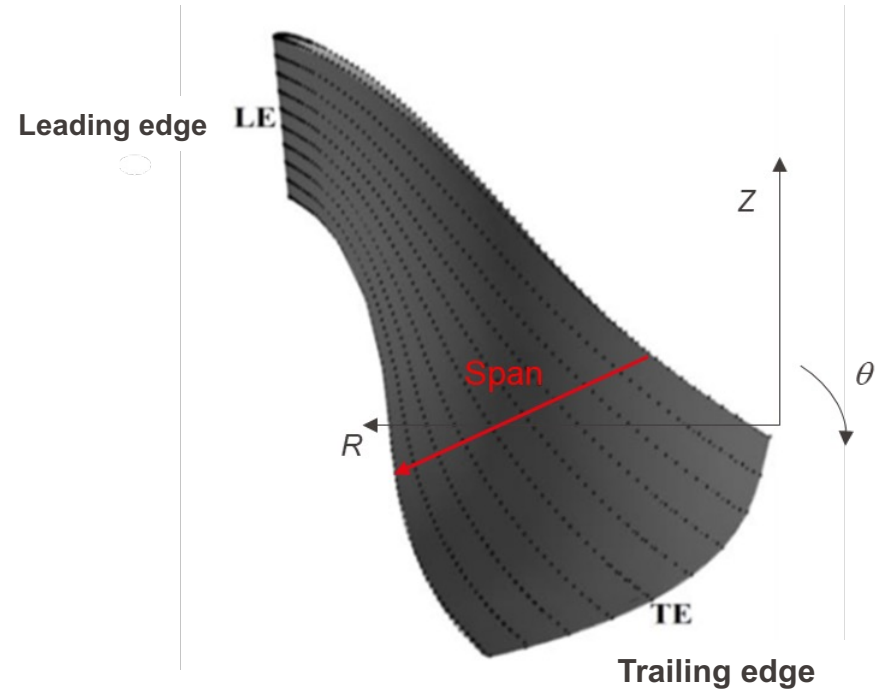
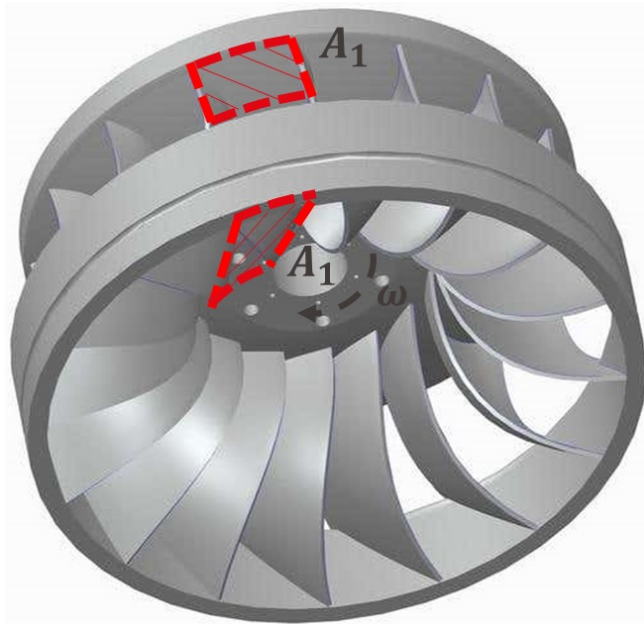
- Connects the turbine to the downstream pipe
- Raises the pressure and slow down the flow

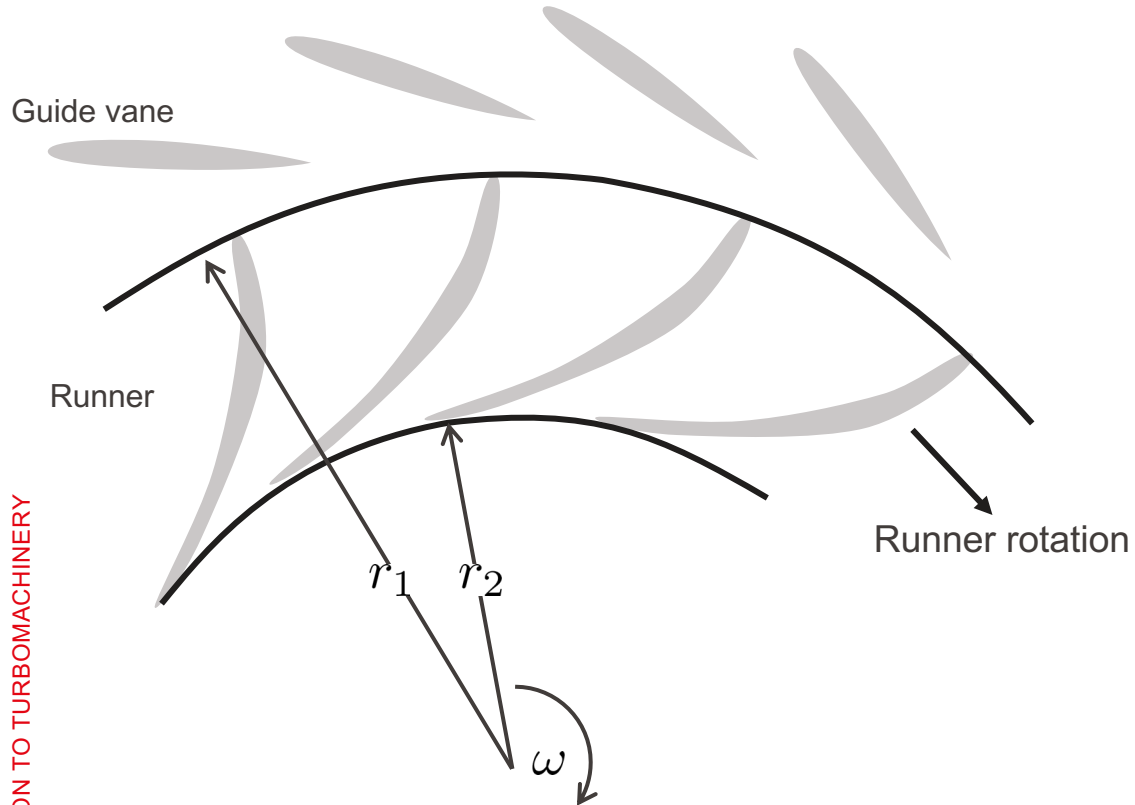


Francis-turbine runner (schematic sketch)

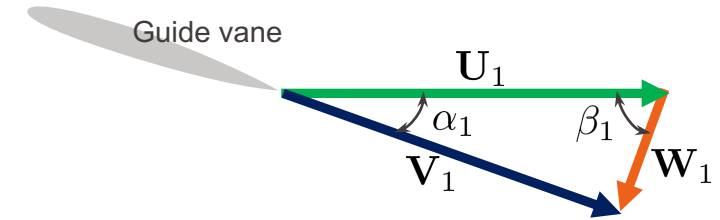


Francis Turbine – Velocity triangle

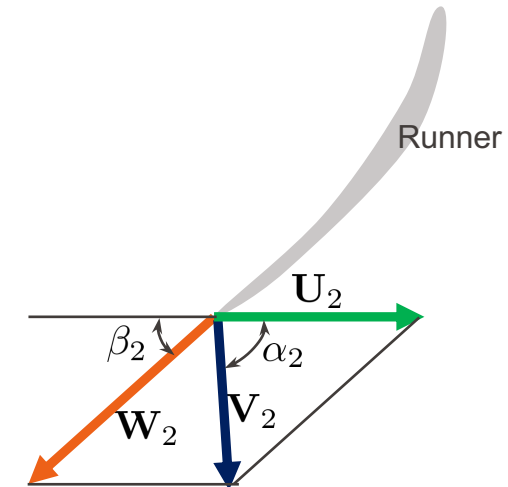




- Velocity triangle at the runner inlet



- Velocity triangle at the runner outlet



$\alpha_1 = \widehat{(\vec{V}_1, \vec{U}_1)}$: imposed by the guide vanes (trailing edge)

$\beta_2 = \widehat{(\vec{W}_2, \vec{U}_2)}$ is imposed by the runner blades (trailing edge)

β_1 depends on the blade inlet geometry and operating conditions

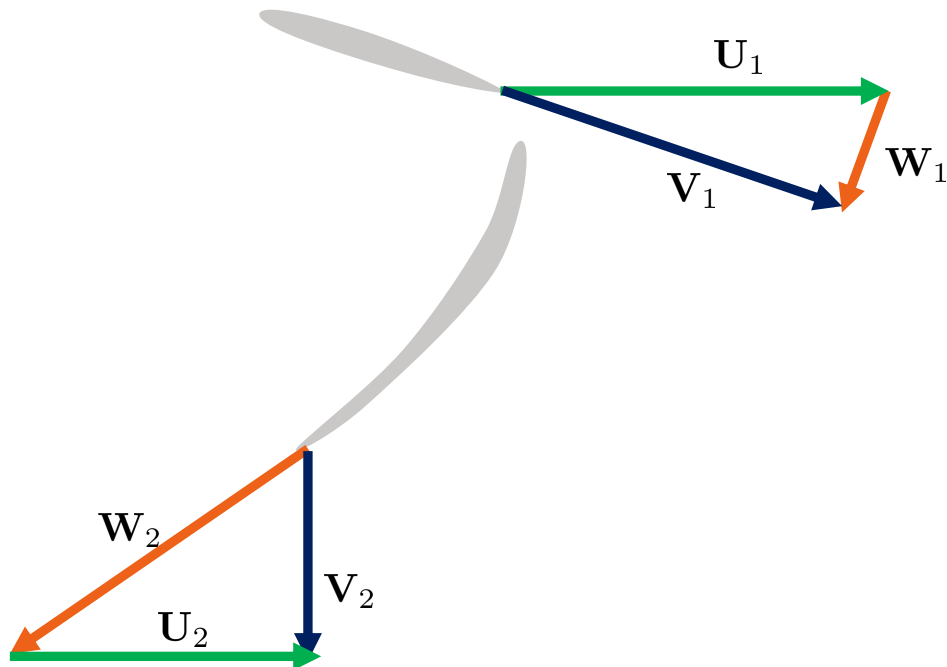
α_2 depends on the blade inlet geometry and operating conditions

In most simple analyses of the Francis turbine it is assumed that there is no exit swirl.

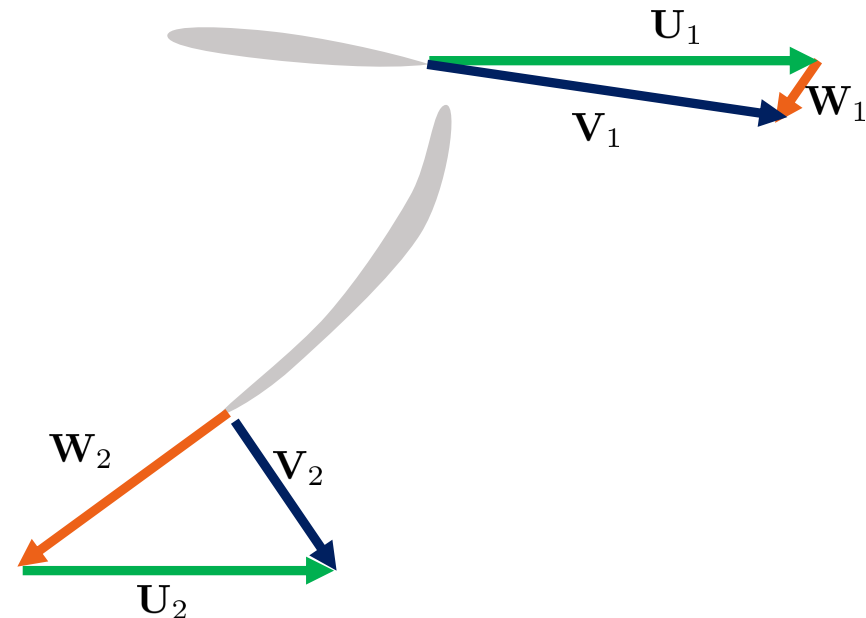
When a Francis turbine is required to operate at part load, the power output is reduced by swiveling the guide vanes to restrict the flow, i.e., Q is reduced, while the blade speed is maintained constant.

The relative flow at runner entry is at a high incidence and at runner exit the absolute flow has a large component of swirl. Both of these flow conditions give rise to high head losses.

Design point operation



Part load operation



Francis Turbine

The rule also applies to Kaplan turbine

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 \pm h_s - \sum h_L = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

If the flow at runner exit is without swirl ($V_{\theta 2} \simeq 0$) then the shaft work per unit mass (specific work) reduces to

$$w_{\text{shaft}} = -(U_1 V_{\theta 1}) + (U_2 V_{\theta 2}) = -(U_1 V_{\theta 1})$$

Then, the shaft head, h_s : $h_s = \frac{U_1 V_{\theta 1}}{g}$

Energy equation between 0 and 3

$$\frac{p_0}{\gamma} + h_g - h_s - \sum h_L = \frac{p_3}{\gamma} + \frac{V_3^2}{2g}$$

$$p_0 \approx p_3 \approx p_{\text{atm}}$$

$$h_s = h_g - \sum h_L - \frac{V_3^2}{2g}$$

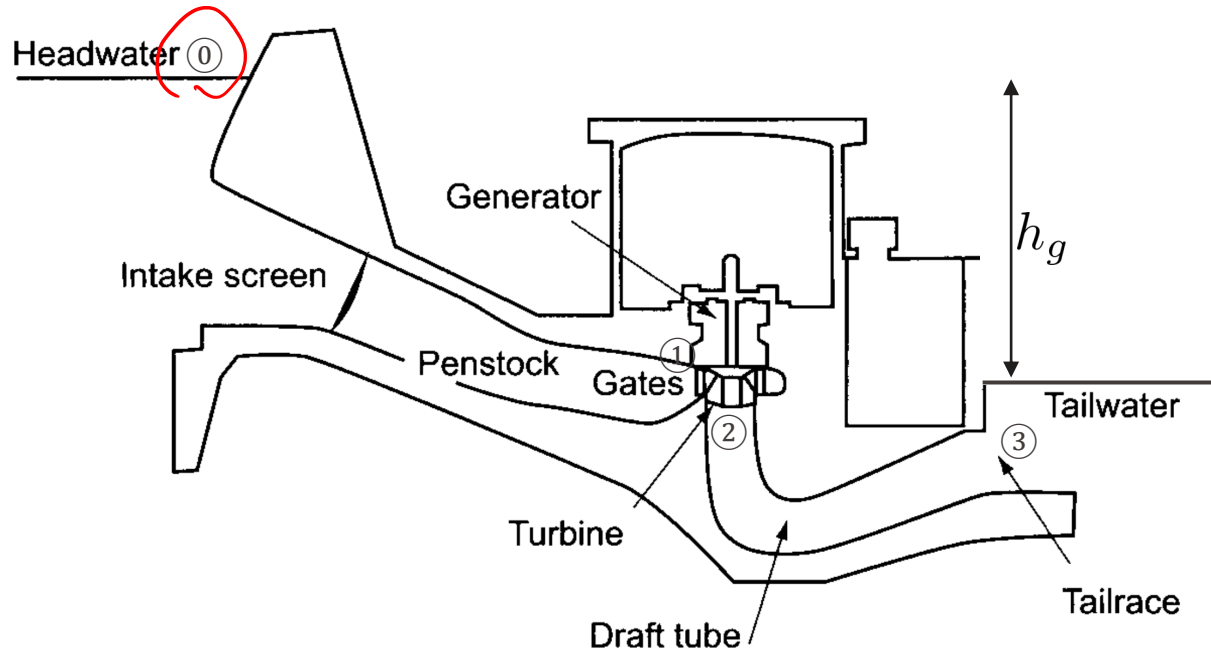
Losses in the flow passage

h_{LP} : the loss of head due to friction in penstock

h_{LV} : the loss of head due to friction in the volute and guide vanes

h_{LR} : the loss of head due to friction in runner

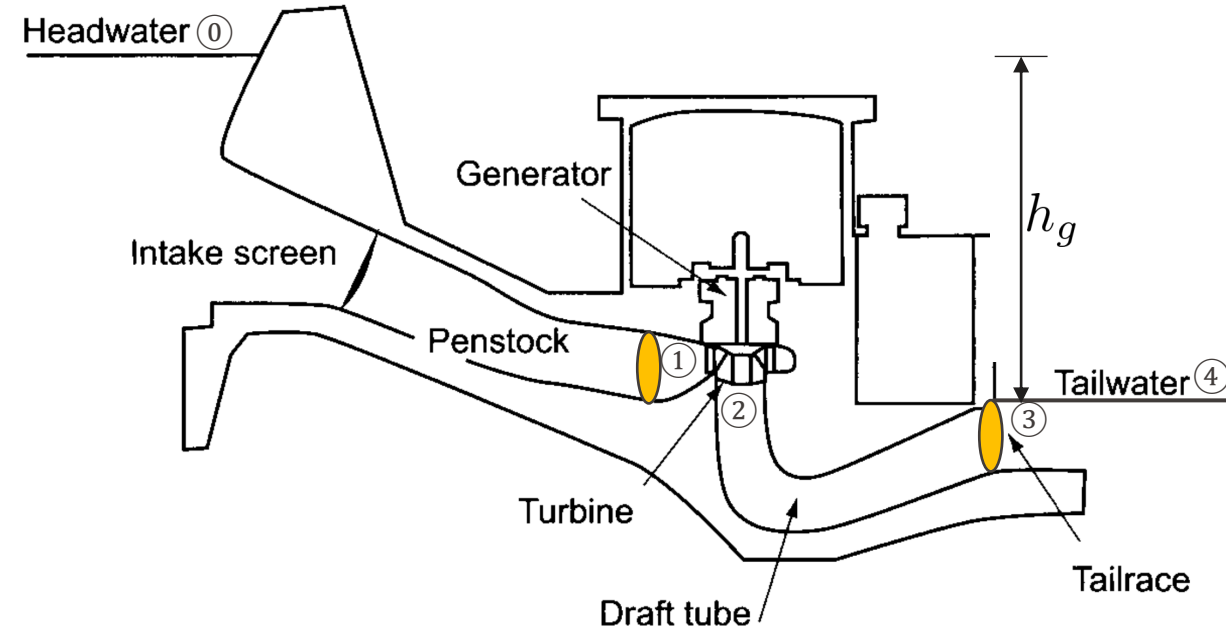
h_{LD} : the loss of head due to friction in draft tube



Francis Turbine

The rule also applies to Kaplan turbine

Efficiency of turbine : $\eta = \eta_m \eta_h \eta_v$



Pump performance characteristics

- Overall efficiency $\eta = \frac{\text{power gained by the fluid}}{\text{shaft power driving the pump}} = \frac{P_f}{\dot{W}_{\text{shaft}}}$
= ratio of power actually gained by the fluid to the shaft power supplied

- Hydraulic losses: Skin friction, flow separation, 3D and unsteady effects
→ Hydraulic efficiency η_h
- Mechanical losses: bearing and sealing losses
→ Mechanical efficiency η_m
- Volumetric losses: flow leakage components
→ Volumetric efficiency η_v

$$\eta = \eta_h \eta_m \eta_v$$

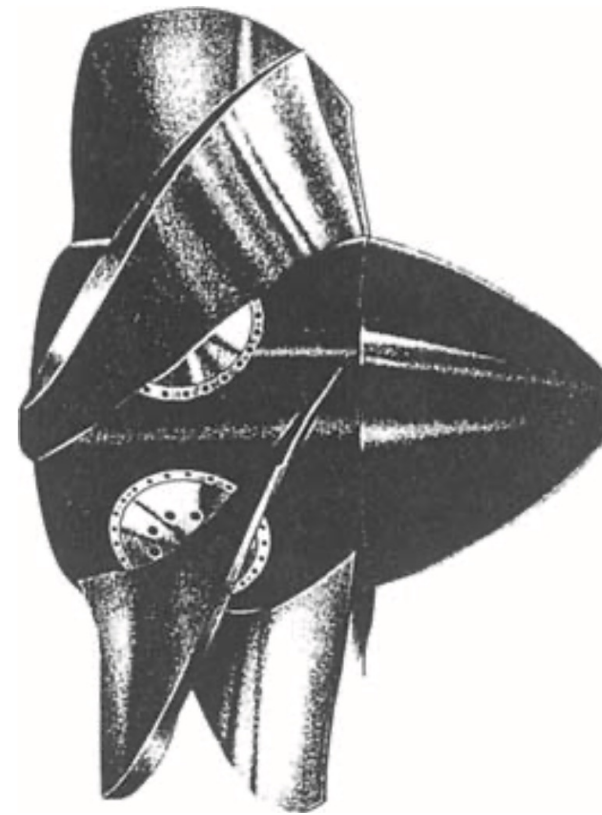
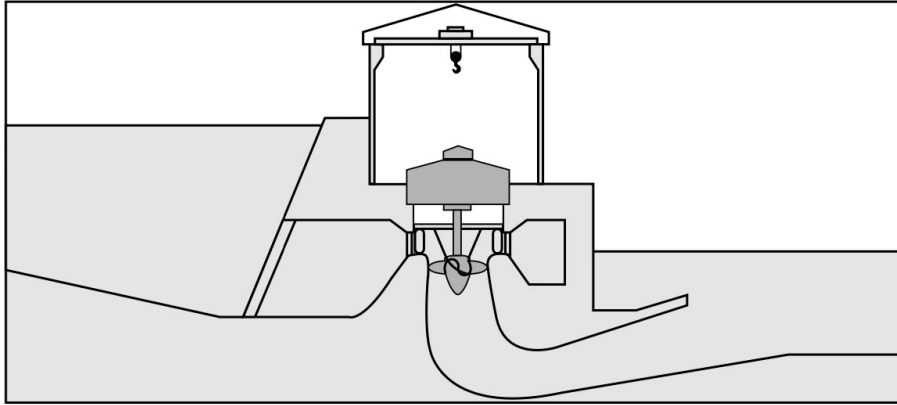
$$\eta_h = \frac{|w_{\text{shaft}}|}{gH_E} = \frac{|-U_1 V_{\theta 1} + U_2 V_{\theta 2}|}{gH_E} = \frac{U_1 V_{\theta 1}}{gH_E}$$

H_E is the total head available at the turbine **inlet** relative to the surface of the tailrace → $H_E = h_g - h_{LP}$

$$\eta_h = \frac{\text{shaft power driving the turbine}}{\text{power gained by the fluid}} = \frac{\dot{W}_{\text{shaft}}}{P_f}$$

$$P_f = \gamma Q h_a$$

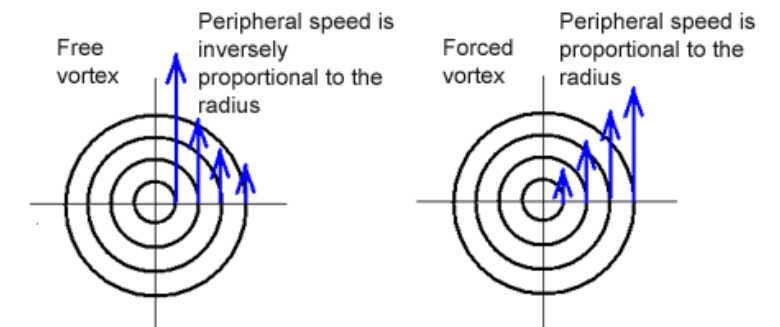
Hydraulic Turbines – Reaction turbines Kaplan and Bulb turbines

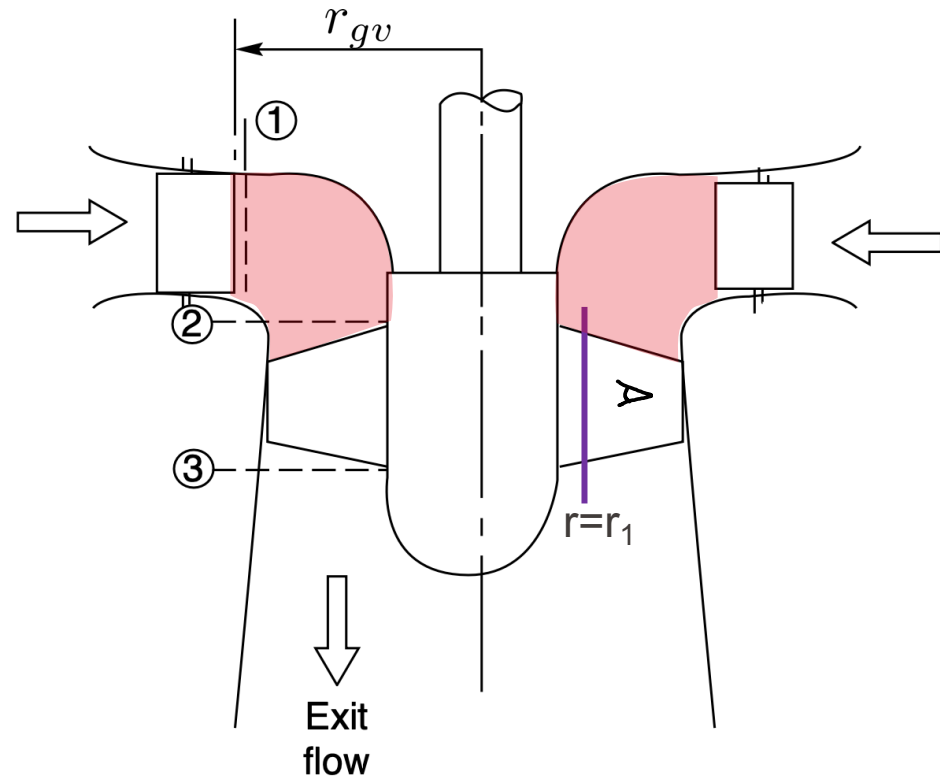


The flow leaving the guide vanes is forced by the shape of the passage into an axial direction and the swirl becomes essentially a **free vortex**:

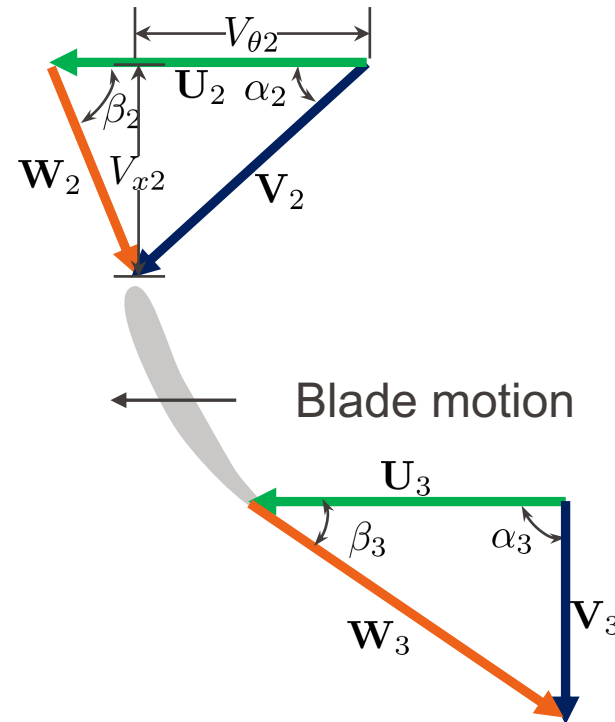
$$rV_{\theta} = \text{const.}$$

- Angular momentum is constant over the radius
- Small r , large V_{θ}
- Large r , small V_{θ}





Free vortex $rV_\theta = \text{const.} \rightarrow V_\theta = \frac{K}{r}$



Blade angles w.r.t. horizontal line

$$\tan \beta_2 = \frac{V_{x2}}{U_2 - V_{\theta 2}}$$

$$\tan \alpha_2 = \frac{V_{t2}}{V_{oc}}$$

Well-designed blade $\alpha_3 = \pi/2$, $V_{\theta 3} = 0$

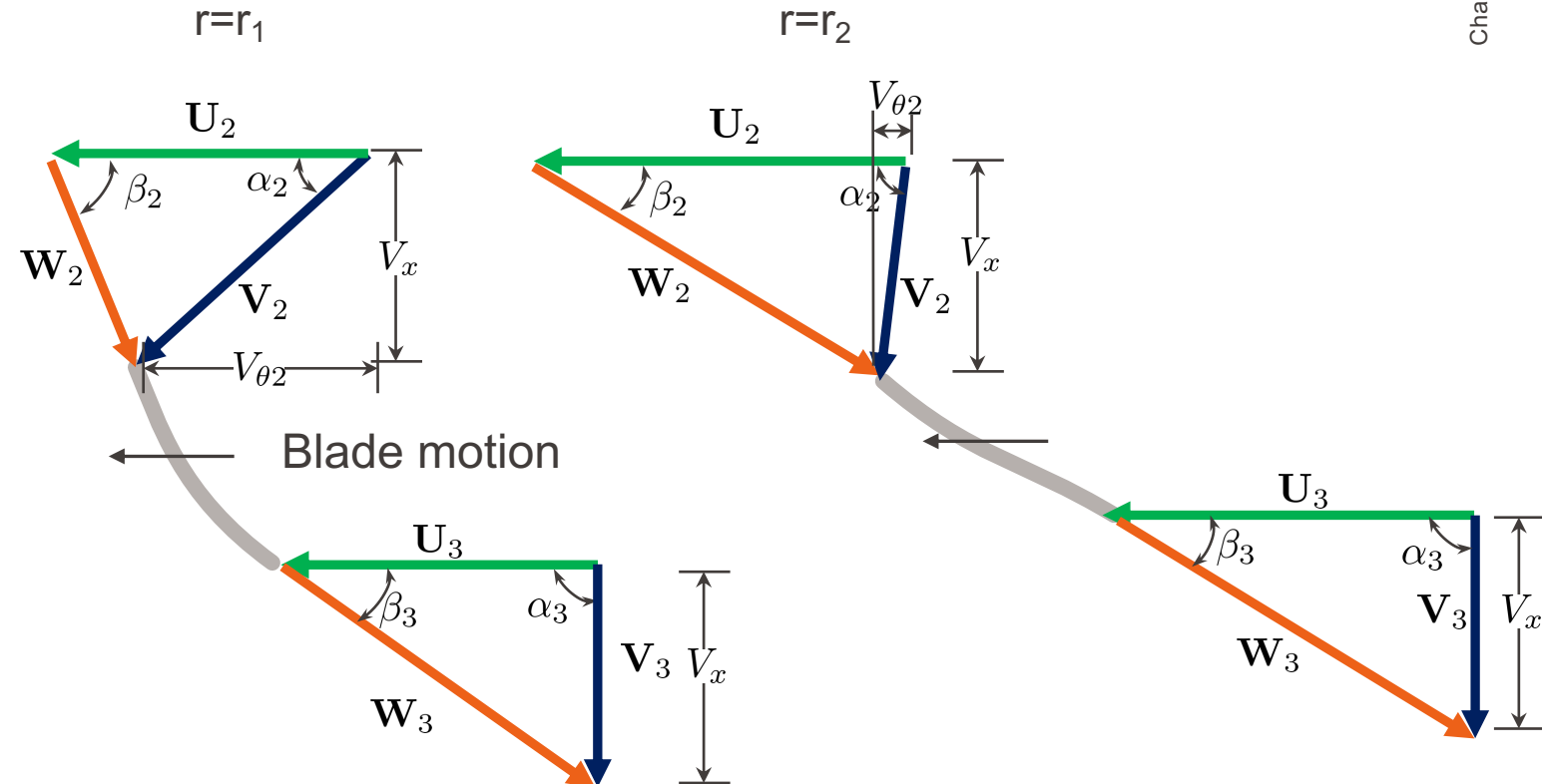
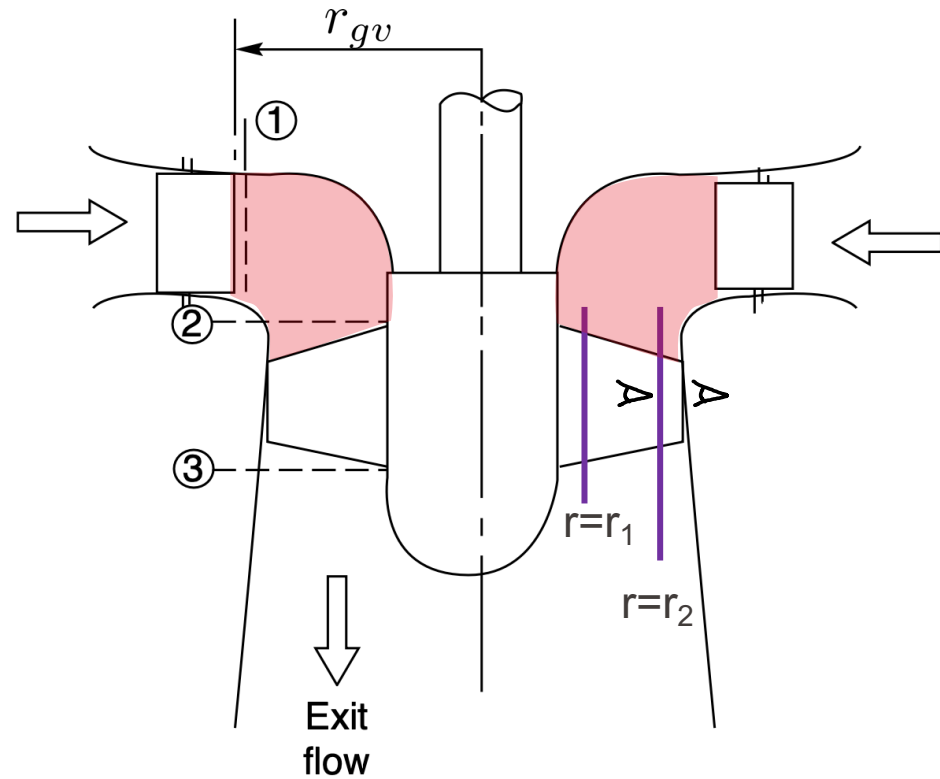
$$\rightarrow V_3 = V_x$$

$$\alpha_3 = \frac{\pi}{2}$$

$$\tan \beta_3 = \frac{V_3 = V_x = V_{x2}}{U_3}$$

$$V_\theta = \frac{K}{r}$$

Free vortex, axial velocity constant



Kaplan turbine



Exercise

$$\rho = 1000 \text{ kg/m}^3, \eta_h = \frac{\dot{W}_{\text{shaft}}}{\gamma Q h_a}, \eta_h = \frac{U_1 V_{\theta 1}}{g H_E}$$

A small-scale Kaplan turbine has a power output of 8 MW, an available head (H_E) at turbine entry of 13.4 m, and a rotational speed of 200 rev/min. The inlet guide vanes have a length of 1.6 m and the diameter at the trailing edge surface is 3.1 m. The runner diameter is 2.9 m and the hub-tip ratio ($=D_{\text{hub}}/D_{\text{tip}}$) is 0.4. Assuming the hydraulic efficiency is 92% (ignore mech. & volumetric losses) and the runner design is “free-vortex”, determine

- (i) The radial component of velocity at exit from the guide vanes

$$0.92 = \frac{8 \times 10^6}{9.81 \times 1000 \times Q \times 13.4} \Rightarrow Q = 66.15 \text{ m}^3/\text{s}$$

$$V_r = \frac{Q}{A} = \frac{Q}{2\pi r_{gv} \cdot L} \approx 4.25 \text{ m/s}$$



- (ii) The component of axial velocity at the runner and the azimuthal velocity of the runner edge

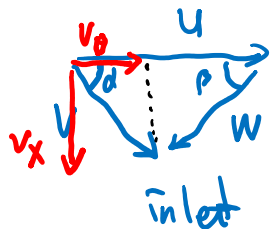


$$V_x = \frac{Q}{A} = 11.09 \text{ m/s}$$

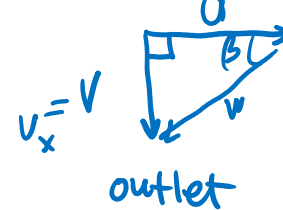
$$\pi r_o^2 - \pi r_i^2 = \frac{\pi}{4} (D_o^2 - D_i^2) = \frac{\pi}{4} D_o^2 (1 - 0.4^2)$$

$$0.92 = \frac{r_o \cdot \omega \cdot V_{\theta 1}}{g \cdot 13.4} \rightarrow V_{\theta 1} \approx 3.98 \text{ m/s}$$

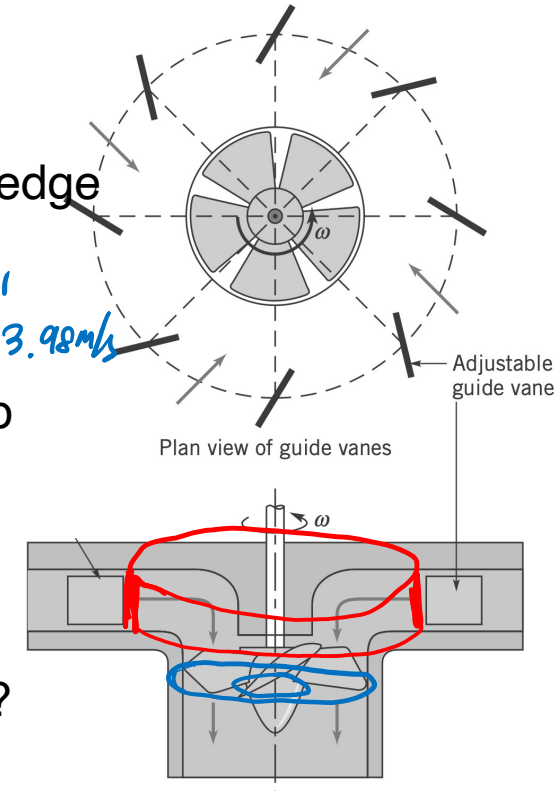
- (iii) the absolute and relative flow angles upstream and downstream of the runner at the tip



$$\tan \alpha = \frac{V_x}{V_{\theta}}, \tan \beta = \frac{V_x}{U - V_{\theta}}$$



$$\tan \beta = \frac{V_x}{U}$$



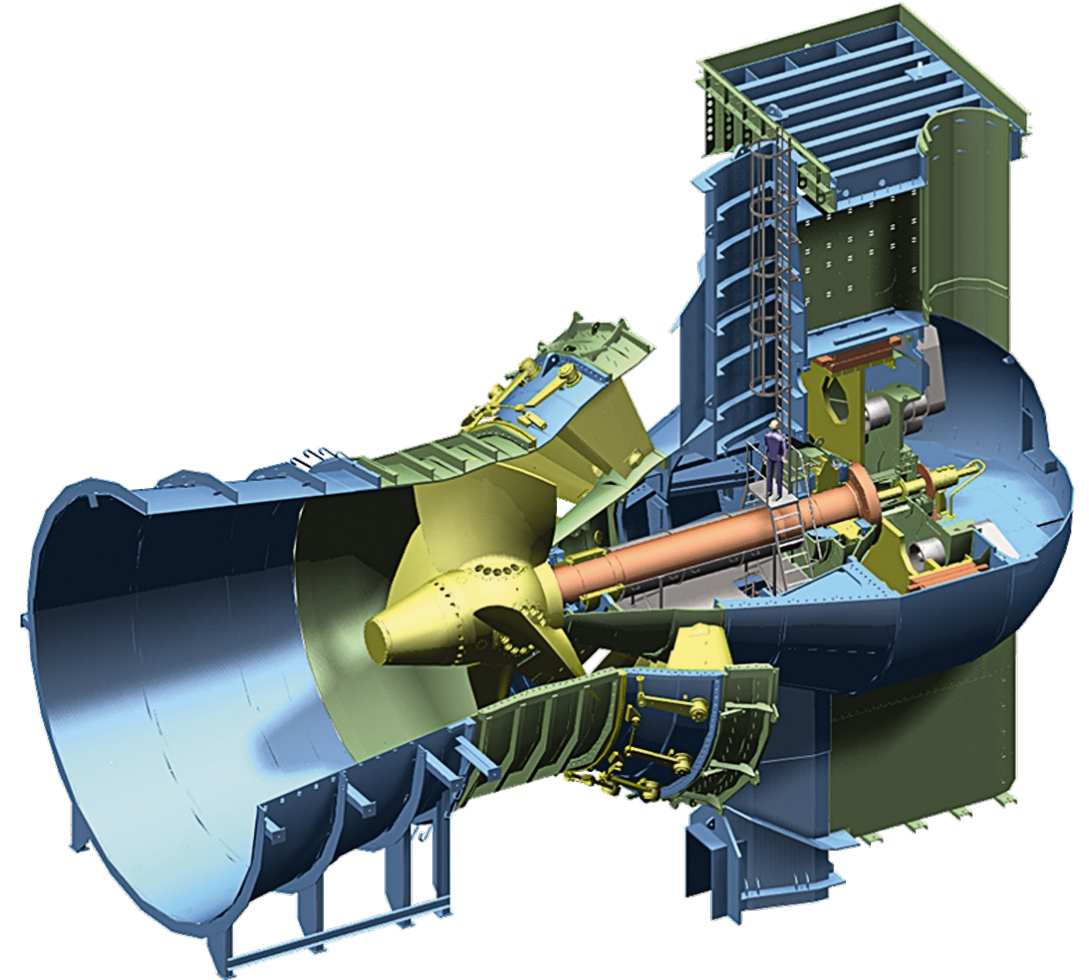
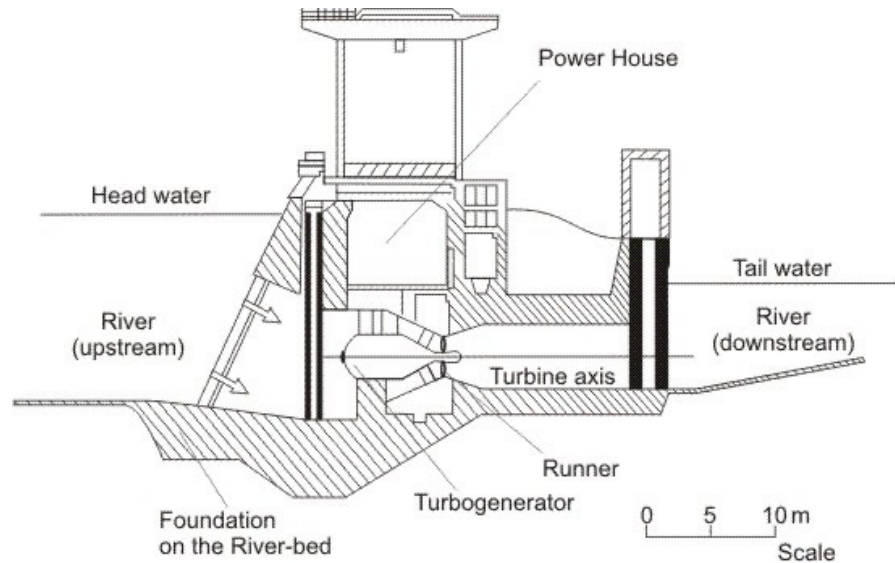
From the free vortex relation, what is the azimuthal velocity of the outlet of the guide vane?

$$r V_{\theta} = \text{const}$$

$$r_1 V_{\theta 1} \rightarrow r_2 \rightarrow V_{\theta 2}$$

Bulb turbine

- Kaplan turbine in horizontal configuration. Suited for very low heads and high flowrates
- The turbine and the generator are fitted inside a bulb with fully axial flow
→ low construction cost
- No spiral casing, no stay vanes
- Straight draft tube → higher efficiency

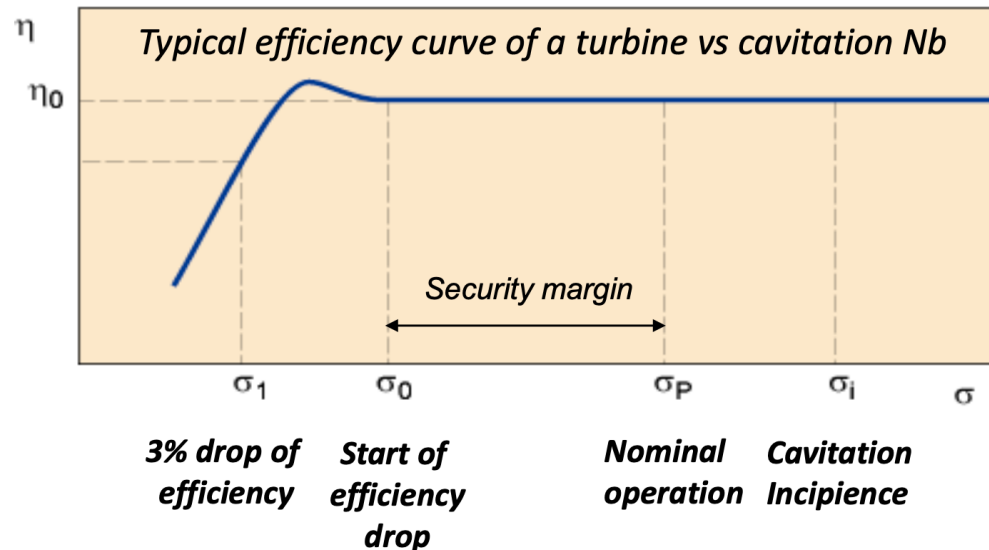


Cavitation Effects

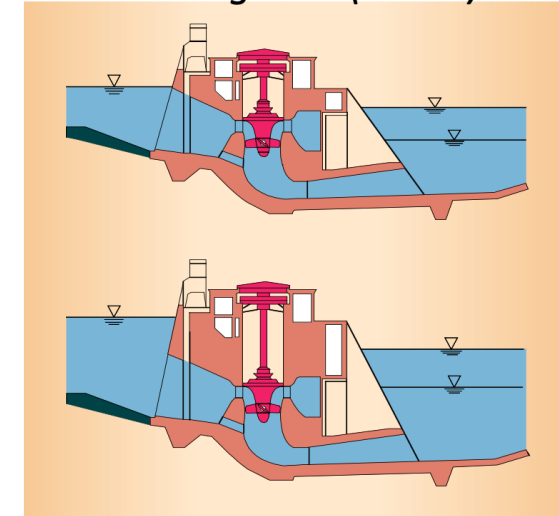
- Effects of cavitation:

1. Alteration of hydrodynamic performances:

The presence of the gas phase within a flowing liquid causes an increase of drag and a decrease of lift

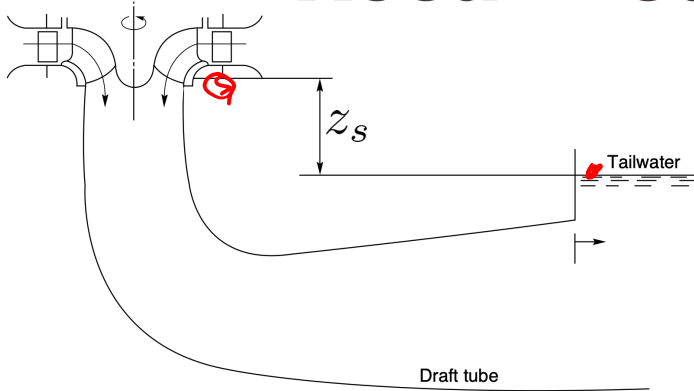


Solution: Increase of the setting level (Cost ?)



As for submarines, a hydraulic turbine may be placed deeper in earth to increase the pressure and avoid cavitation. Nevertheless, this comes with an increase of the construction cost.

Recall - Cavitation



Thoma coefficient : $\sigma = \frac{\text{NPSH}_A}{H_E}$

Note! It is not the same σ from Ch9 with the dynamic pressure, it shows more local behaviour near the blades. Thoma number is for a global hydropower plant location.

NPSH_A : the amount of head needed to avoid cavitation

The net positive suction pressure head (NPSH_A) is defined as the sum of the **static pressure head** and the **dynamic head** minus the **vapor pressure head** of water at the ambient temperature

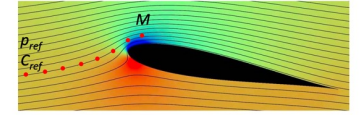
$$\text{NPSH}_A = \frac{p_s - p_v}{\gamma} + \frac{V_s^2}{2g}$$

Handwritten notes: The first term is circled in green. The second term has a red 'X' and '2g' written next to it. A red arrow points from the first term to the second, with the word 'negligible' written above it. Below the arrow, the equation is rewritten as: $\frac{p_s}{\gamma} + \frac{V_s^2}{2g} + z_s = \frac{p_{\text{atm}}}{\gamma} + \frac{V_s^2}{2g} + \sum h_L$

$$\sigma = \frac{p_{\text{atm}} - p_v}{\gamma H_e} - \frac{z_s}{H_e} + \frac{\sum h_L}{H_e}$$

- Condition for cavitation occurrence:

$$p_M < p_v(T) \Leftrightarrow c_p(M) < -\sigma$$



where $c_p(M)$ is the pressure coefficient and σ is the cavitation number

$$c_p(M) = \frac{p_M - p_{\text{ref}}}{\frac{1}{2} \rho C_{\text{ref}}^2} \quad \sigma = \frac{p_{\text{ref}} - p_v(T)}{\frac{1}{2} \rho C_{\text{ref}}^2}$$

- σ represents a non-dimensional margin between the liquid pressure and vapor pressure
→ The risk of cavitation increases for a decreasing cavitation number

NPSH

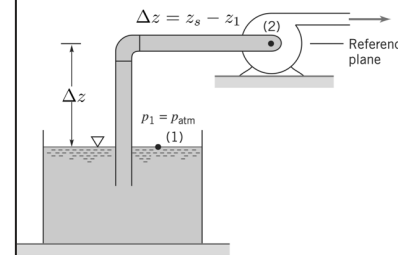
suction side, near the pump impeller inlet

$$\frac{p_{\text{atm}}}{\gamma} + z_1 = \frac{p_s}{\gamma} + \frac{V_s^2}{2g} + z_s + \sum h_L$$

$$\frac{p_s}{\gamma} + \frac{V_s^2}{2g} = \frac{p_{\text{atm}}}{\gamma} - \Delta z - \sum h_L$$

Available NPSH,

$$\text{NPSH}_A = \frac{p_{\text{atm}}}{\gamma} - \Delta z - \sum h_L - \frac{p_v}{\gamma}$$



For proper pump operation, it is necessary:

$$\text{NPSH}_A \geq \text{NPSH}_R$$

If Δz increases, NPSH_A decreases

→ matching $\text{NPSH}_A = \text{NPSH}_R$

The pump operates with cavitation

Cavitation number

Power specific speed and cavitation number

$$N'_s = \frac{\omega \sqrt{\dot{W}_{\text{shaft}} / \rho}}{(gh_a)^{5/4}}$$

A Francis turbine with the effective head of $H_E = 80$ m and operates $N'_s = 1$, ignore other losses.

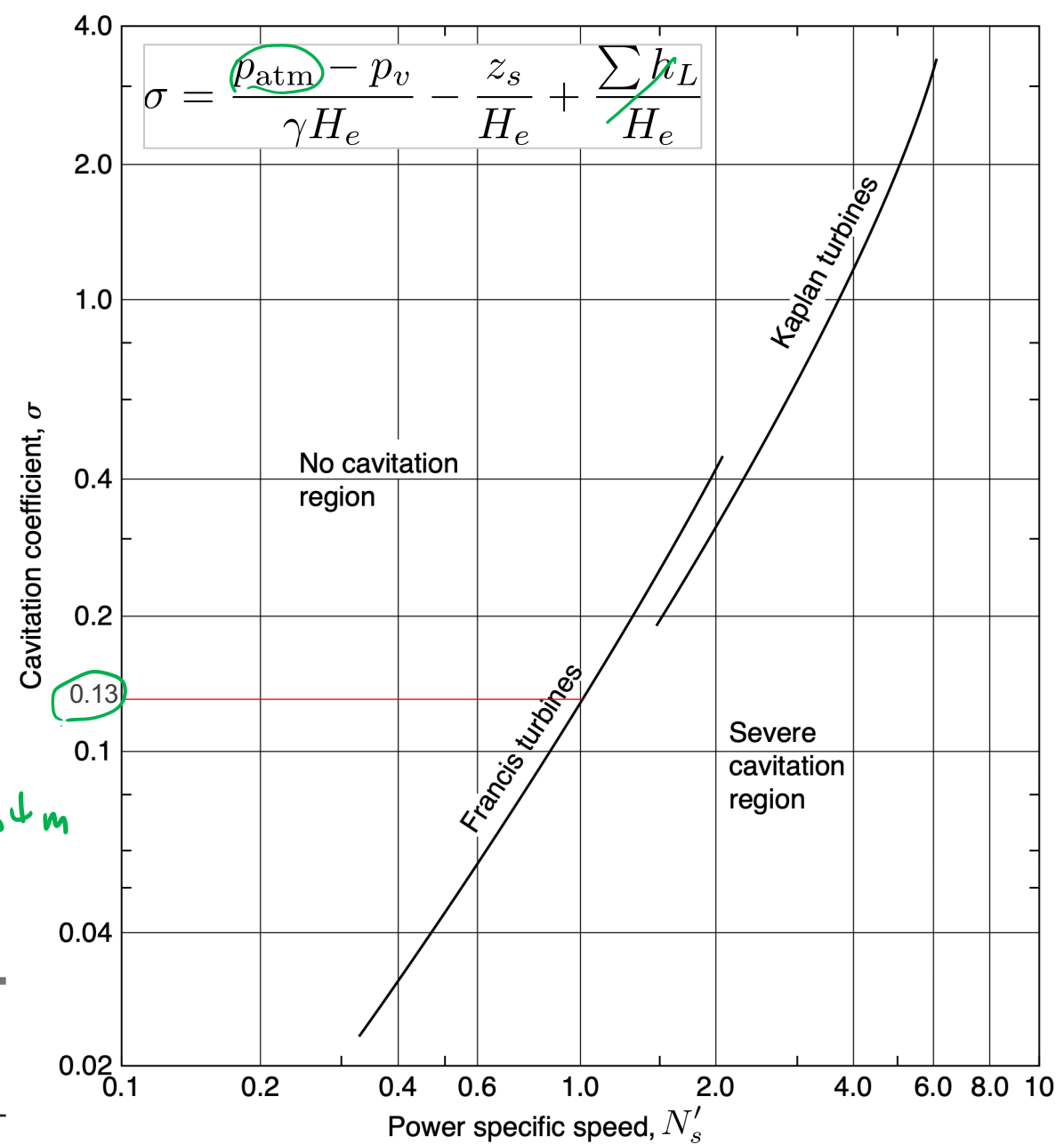
If the level difference of turbine outlet is +5 m, will the cavitation occur? yes

If so, what will be the critical level difference of the turbine to avoid it? $0.13 = 0.1242 - \frac{x}{80} \Rightarrow x = -0.464 \text{ m}$

The water temperature is 30 °C, 101.3 kPa

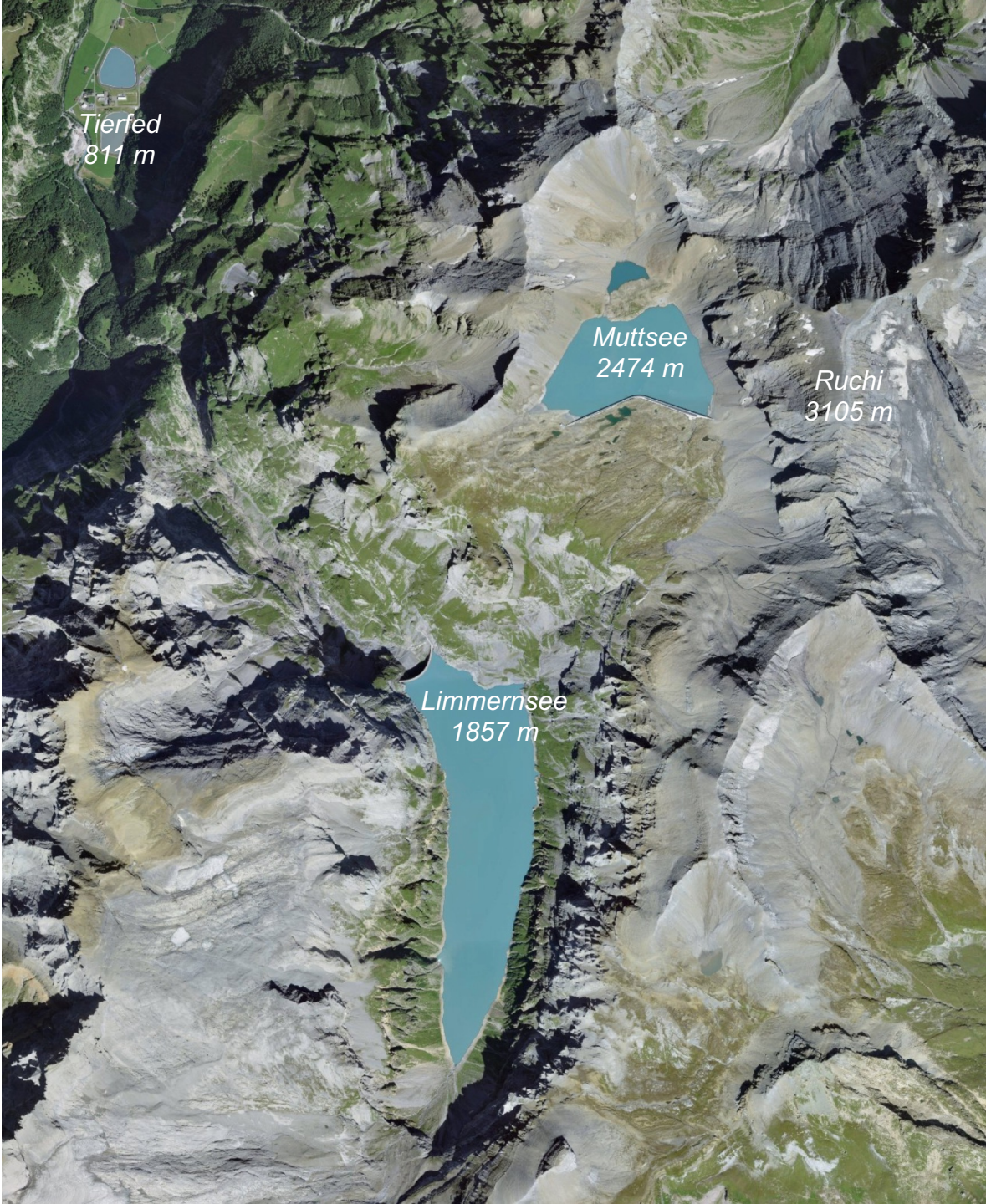
Physical Properties of Water (SI Units)^a

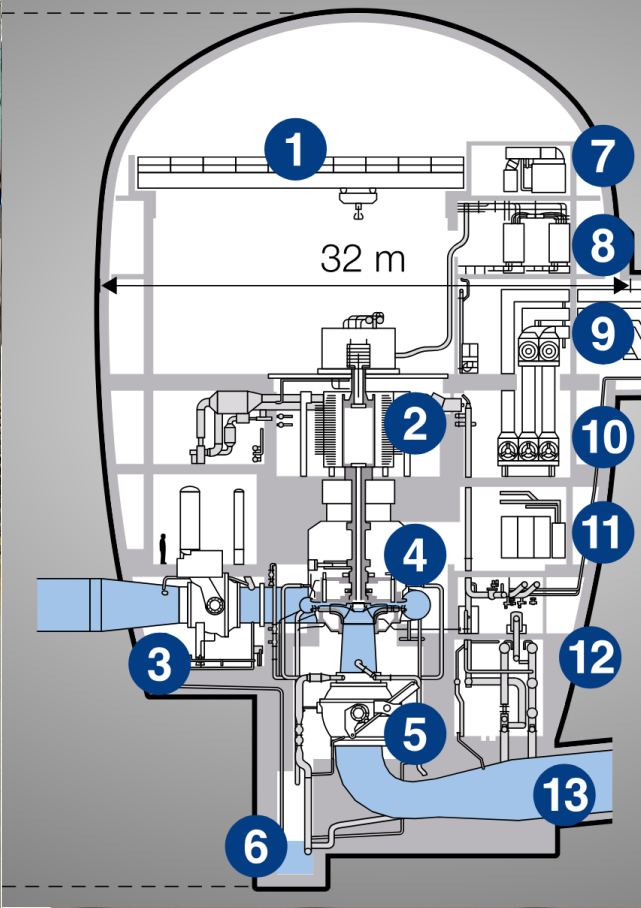
Temperature (°C)	Density, ρ (kg/m ³)	Specific Weight ^b , γ (kN/m ³)	Dynamic Viscosity, μ (N·s/m ²)	Kinematic Viscosity, ν (m ² /s)	Surface Tension ^c , σ (N/m)	Vapor Pressure, p_v [N/m ² (abs)]
0	999.9	9.806	1.787 E - 3	1.787 E - 6	7.56 E - 2	6.105 E + 2
5	1000.0	9.807	1.519 E - 3	1.519 E - 6	7.49 E - 2	8.722 E + 2
10	999.7	9.804	1.307 E - 3	1.307 E - 6	7.42 E - 2	1.228 E + 3
20	998.2	9.789	1.002 E - 3	1.004 E - 6	7.28 E - 2	2.338 E + 3
30	995.7	9.765	7.975 E - 4	8.009 E - 7	7.12 E - 2	<u>4.243 E + 3</u>



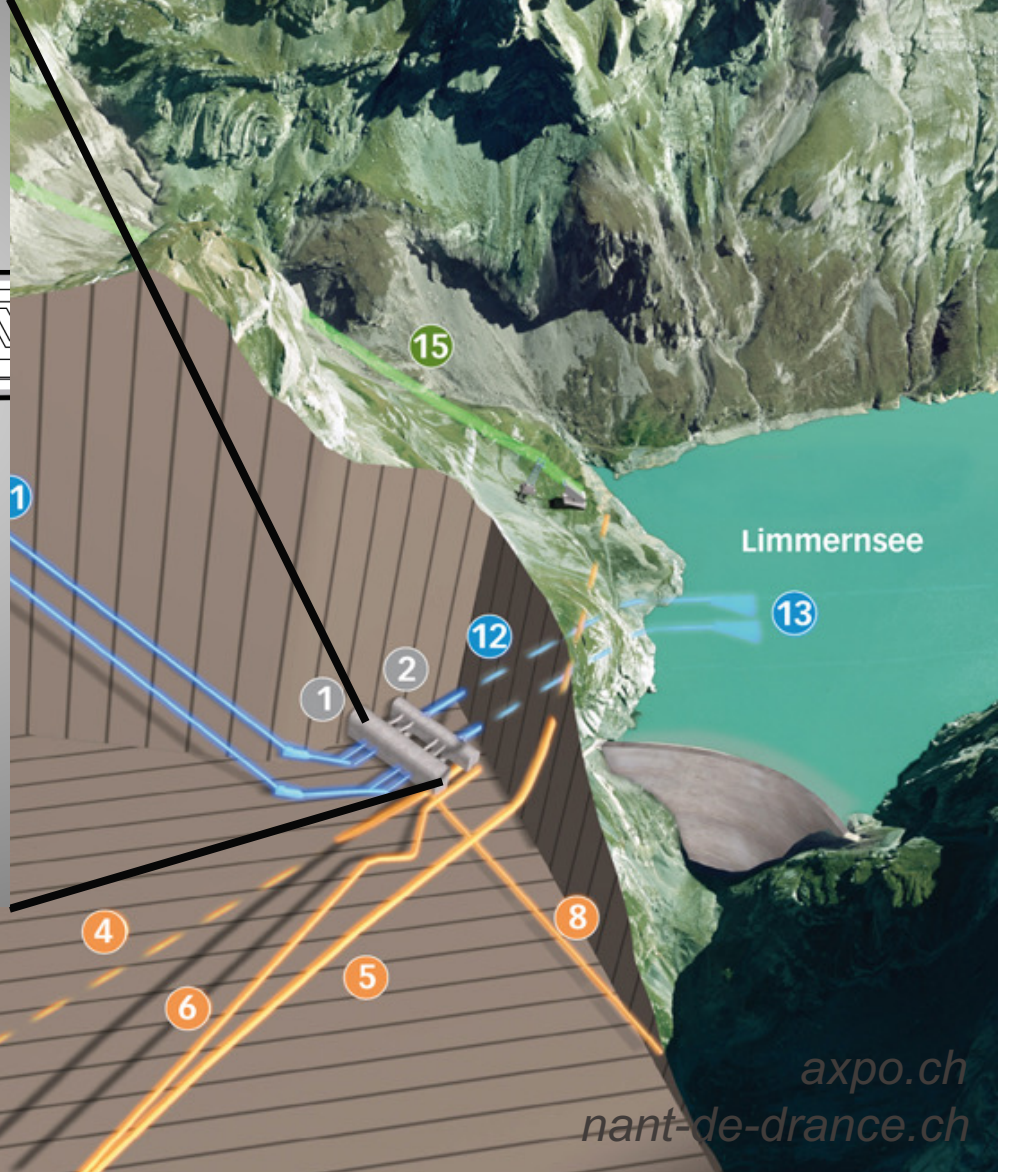
$$\sigma = \frac{101.3 \times 10^3 - 4.243 \times 10^3}{9.765 \times 10^3 \cdot 80} - \frac{5}{80} = 0.1242 - \frac{5}{80} = 0.0117$$

Pump storage





- 12 Unterwasserdruckstollen
- 13 Ein- und Auslaufbauwerk
- 14 Hochwasserentlastung und Grundablass
- 15 Bauseilbahn 2 Ochsenstäfeli-Muttsee





<https://www.nant-de-drance.ch/en/>



Appendix

Recall-Pump performance characteristics

- Overall efficiency $\eta = \frac{\text{power gained by the fluid}}{\text{shaft power driving the pump}} = \frac{P_f}{\dot{W}_{\text{shaft}}}$

= ratio of power actually gained by the fluid to the shaft power supplied

- Hydraulic losses: Skin friction, flow separation, 3D and unsteady effects
→ Hydraulic efficiency η_h
- Mechanical losses: bearing and sealing losses
→ Mechanical efficiency η_m
- Volumetric losses: flow leakage components
→ Volumetric efficiency η_v

$$\eta = \eta_h \eta_m \eta_v$$