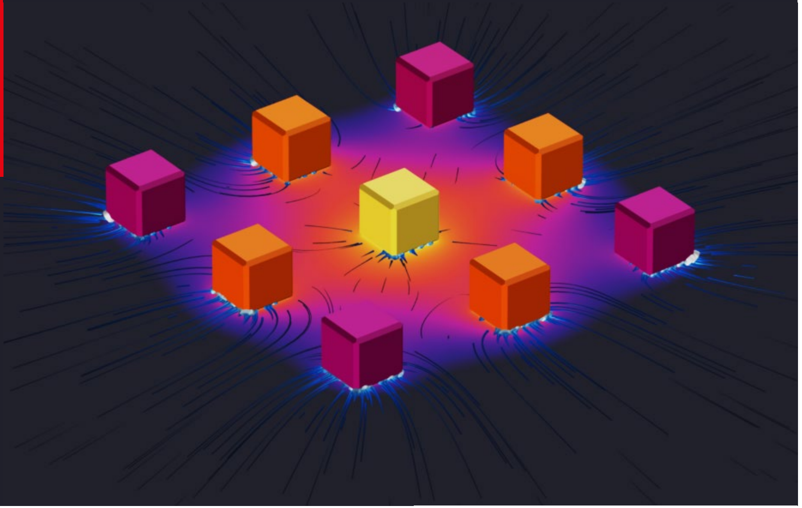
Heat and Mass Transfer ME-341

Instructor: Giulia Tagliabue

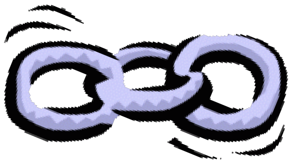


École polytechnique fédérale de Lausanne

Spring Semester

From Thermodynamics to Heat Transfer

Thermodynamics

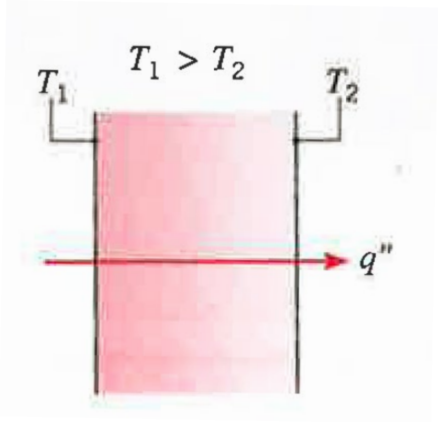


Heat Transfer

Although thermodynamics may be used to determine the amount of heat needed for a system to pass from one equilibrium state to another it does not acknowledge that heat transfer is inherently a non-equilibrium process. In fact, for heat transfer to occur there MUST be a TEMPERATURE GRADIENT. (Incropera, Ch. 1.3)

Transport Laws

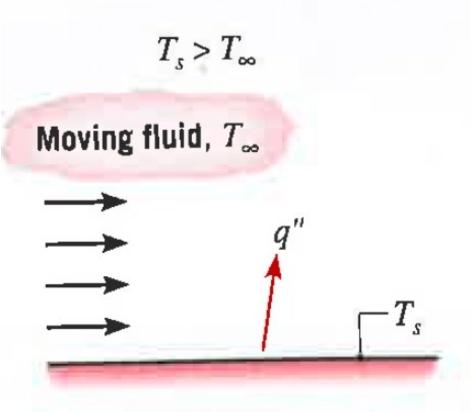
Conduction



Fourier's Law

$$q'' = -k\frac{dT}{dx}$$

Convection



Newton's Law

$$q'' = \bar{h}\,(T_s - T_\infty)$$

Heat Transfer Mechanisms and Transport Laws

Conduction

- Planar and radial systems
- With and without heat sources
- Steady-state and Transient

Convection

- Forced convection (External and Internal)
- Free convection
- Boiling and Condensation

- Thermal resistances/Thermal circuits/Heat transfer coefficient U
- Dimensionless numbers (Re, Nu, Pr, Bi, Ra, Gr)
- Characteristic dimensions & reference temperature
- Mass/Momentum/Energy conservation equations

$$\frac{1}{\alpha}\frac{\partial T}{\partial t} = \nabla^2 T + \frac{\dot{q}}{k}$$

Conduction →Only Diffusion
(Closed system)

$$\rho c_p \left(\frac{\partial T}{\partial t} + \vec{u} \cdot \nabla T \right) = k \nabla^2 T + \dot{q}$$

Convection → Diffusion + Advection
(Open system)

Design a Heat Exchanger

This lecture

- ❑ Introduction to Heat Exchangers
- ❑ The problem of the overall heat transfer coefficient
- ❑ RECAP of critical concepts

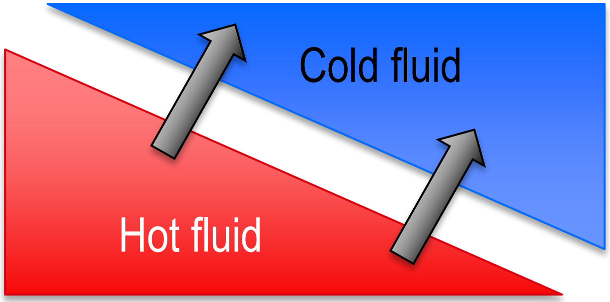
Learning Objectives:

- ❑ Understand the concept and possible design of heat exchangers

Introduction to Heat Exchangers

Air-conditioning &
Waste-heat recovery

Power production
(boilers/condensers)



Natural Systems

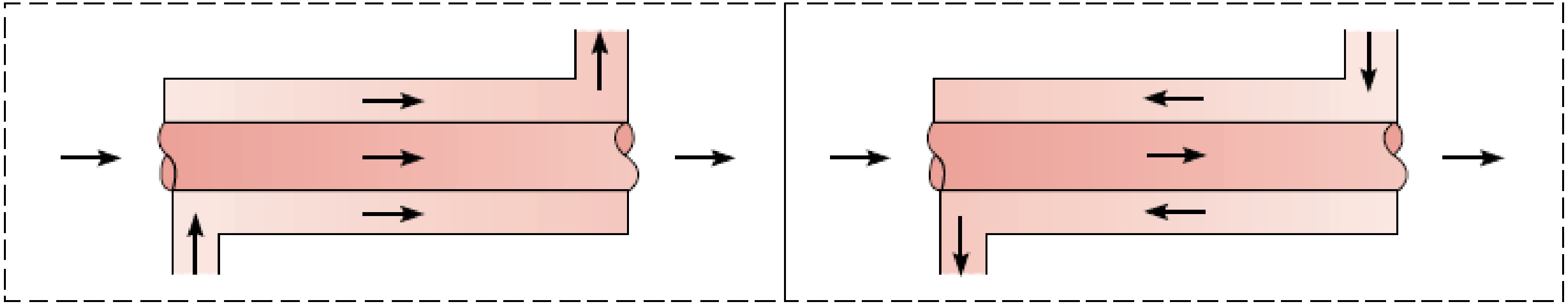
Chemical
Processing

Introduction to Heat Exchangers

Flow arrangement ?

Introduction to Heat Exchangers

A. Concentric Flow

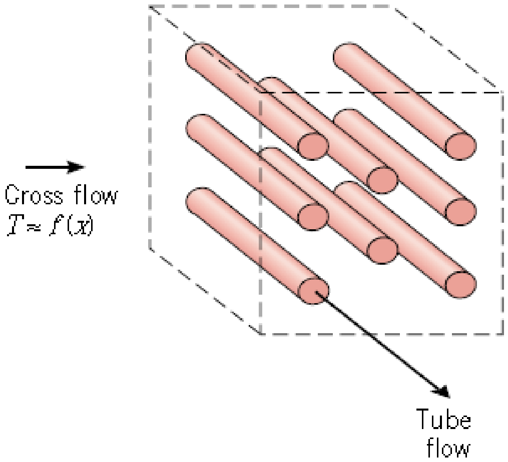


Parallel Flow

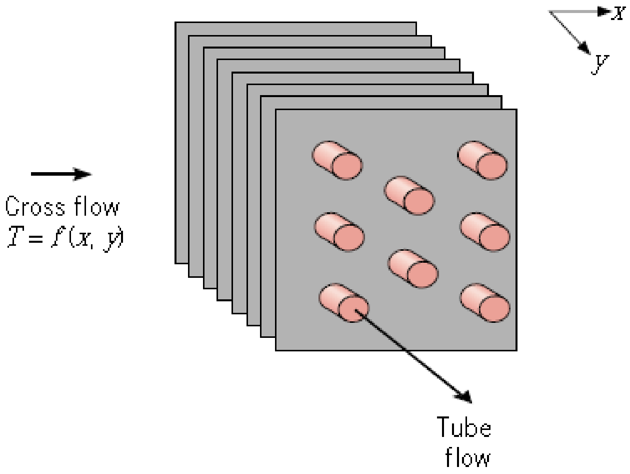
Counter Flow

Introduction to Heat Exchangers

B. Cross-Flow



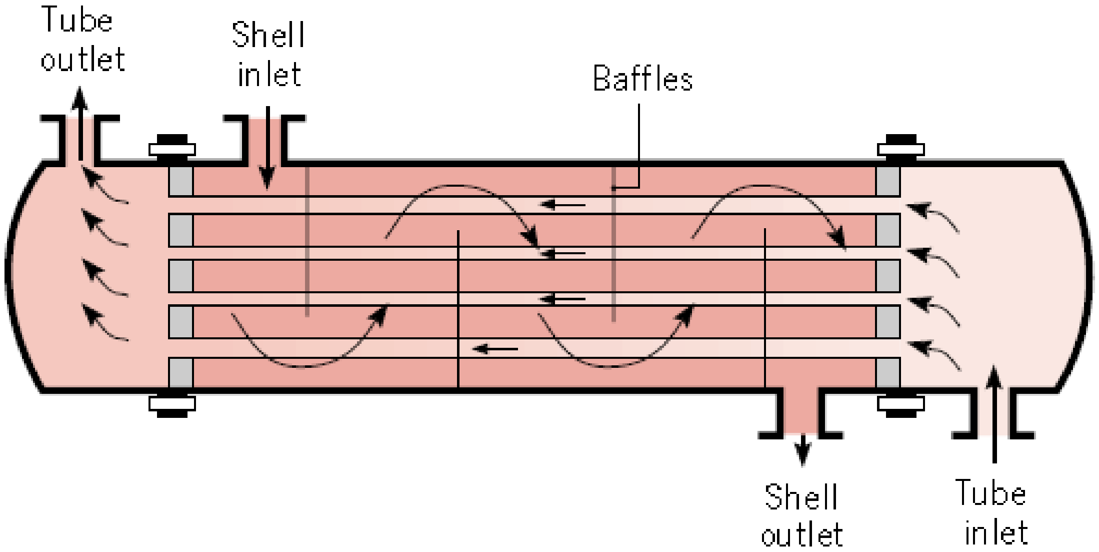
Mixed Flow (Unfinned)



Unmixed Flow (Finned)

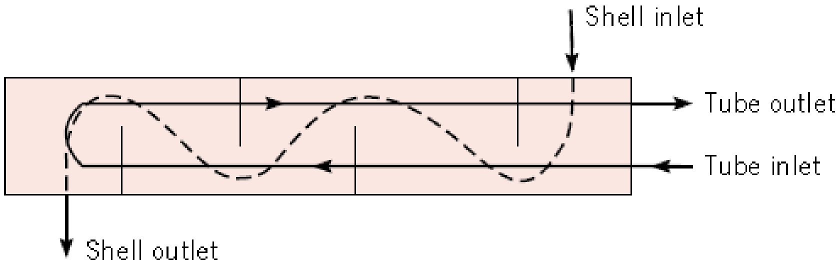
Introduction to Heat Exchangers

C. Shell-and-Tube

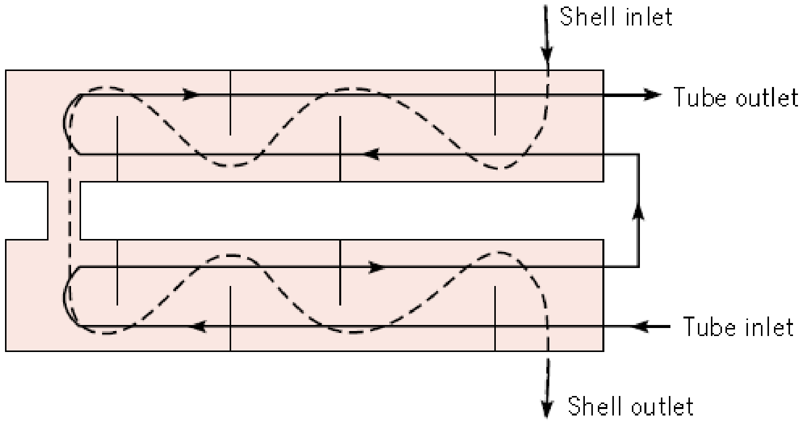


Introduction to Heat Exchangers

c. Shell-and-Tube



1 Shell pass
2 Tube passes



2 Shell passes
4 Tube passes

This lecture

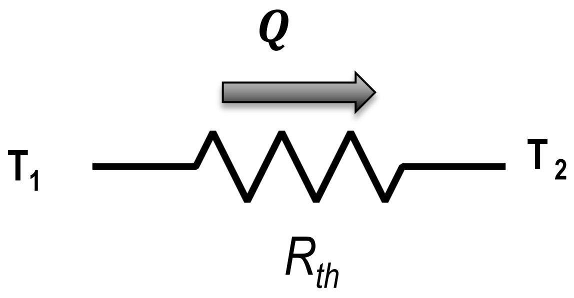
- [x] Introduction to Heat Exchangers
- [ ] The problem of the overall heat transfer coefficient
- [ ] RECAP of critical concepts

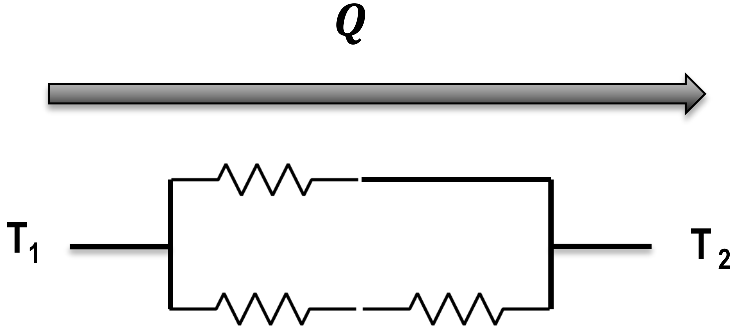
Learning Objectives:

- [x] Understand the concept and possible design of heat exchangers

Overall Heat Transfer Coefficient

Thermal Resistance & Thermal Circuits



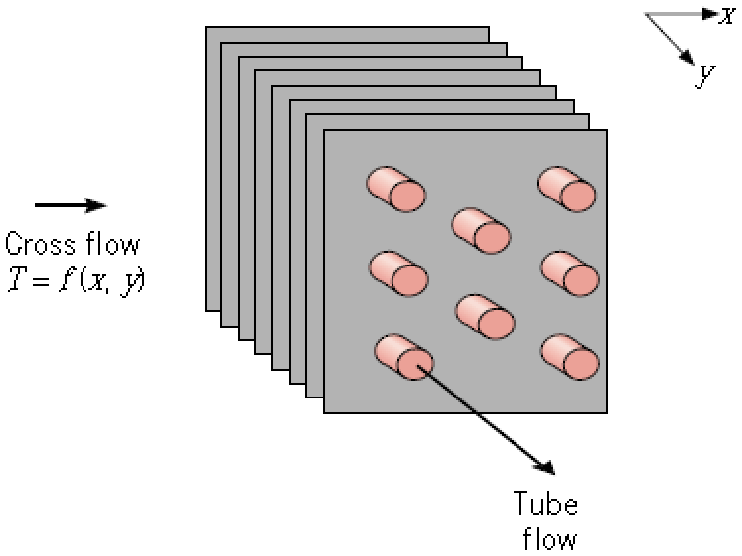


$$Q = \frac{(T_1 - T_2)}{R_{th}}$$

$$Q = \frac{(T_1 - T_2)}{R_{tot}}$$

Overall Heat Transfer Coefficient

$$Q = \frac{\Delta T}{R_{tot}} = \boldsymbol{U} A \Delta T \Rightarrow U \equiv \frac{1}{R_{tot} A} \Rightarrow \frac{1}{UA} \equiv R_{tot}$$



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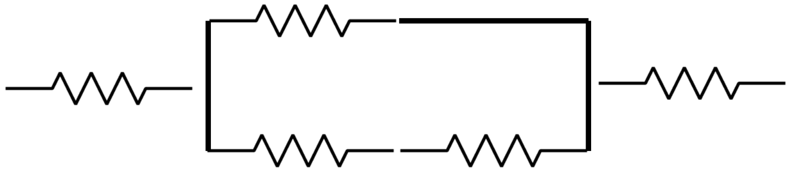


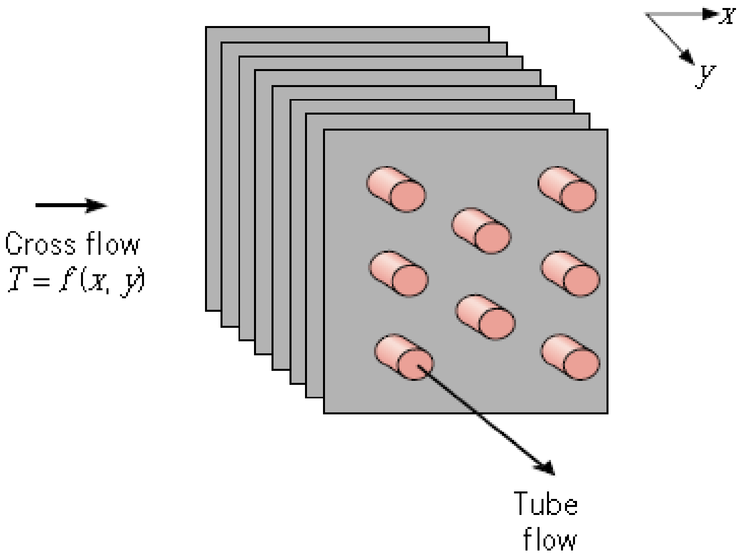
Note: we will discuss the expression for ΔT later

Overall Heat Transfer Coefficient

$$Q = \frac{\Delta T}{R_{tot}} = \boldsymbol{U} A \Delta T$$

$$U \equiv \frac{1}{R_{tot} A}$$

$$\frac{1}{UA} \equiv R_{tot}$$



- Conduction thermal resistances
  - Planar/radial conduction
- Convection thermal resistances
  - Internal and external
- Array of Fins
- Fouling

This lecture

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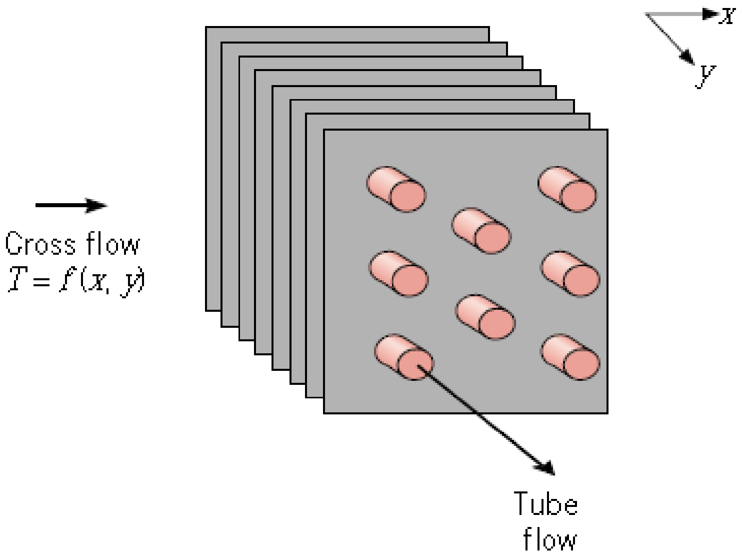
Overall Heat Transfer Coefficient

$$Q = \frac{\Delta T}{R_{tot}} = \boldsymbol{U} A \Delta T \quad \Rightarrow \quad U \equiv \frac{1}{R_{tot} A} \quad \Rightarrow \quad \frac{1}{UA} \equiv R_{tot}$$



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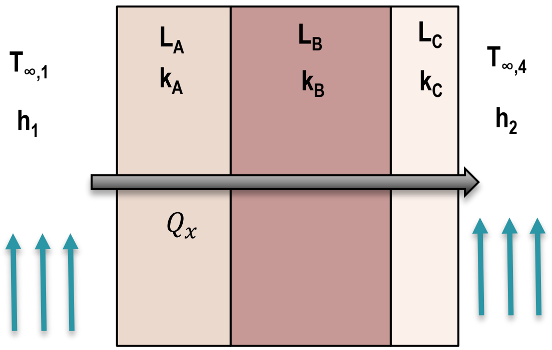
Conduction Thermal Resistance

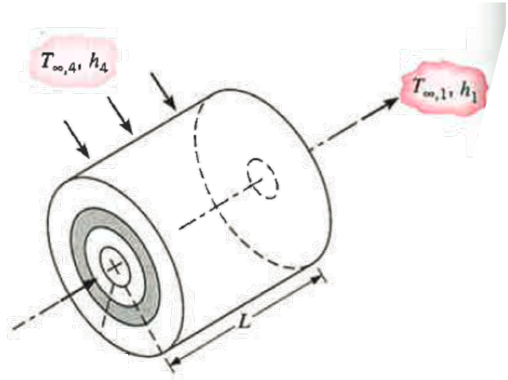
Planar wall

$$R_{th,cond} = \frac{L}{kA} \qquad [\,K/W\,]$$



Radial System

$$R_{th,cond-cyl} = \frac{ln(r_2/r_1)}{2\pi Lk} \qquad [\,K/W\,]$$



What is the critical assumption underlying these expressions ?

Convection Thermal Resistances

Planar wall

$$R_{th,conv} = \frac{1}{hA} \quad [\text{ K/W }]$$

Radial System

$$R_{th,conv} = \frac{1}{h2\pi rL} \quad [\text{ K/W }]$$

Where *h* must be determined with the appropriate correlations!!

Determine the right correlation

General methodology for calculating the convection coefficient

0. Identify the type of convection (Forced/External, Forced/Internal, Free, Boiling/Condensation)

1. Recognize the flow geometry (plate, cylinder, inner/outer flow etc.) [GEOM]

2. Specify the appropriate reference temperature and evaluate the pertinent fluid properties at that temperature $\boldsymbol{T_f}$
   - External convection over a plate/cylinder: $\boldsymbol{T_f = (T_s + T_\infty)/2}$
   - External convection over a bank of tubes: $\boldsymbol{T_f = (T_{in} + T_{out})/2}$
   - Internal convection: $\boldsymbol{T_f = (T_{m,i} + T_{m,o})/2}$
   - Free convection: $\boldsymbol{T_f = (T_s + T_\infty)/2}$
   - Boiling: a) film pool boiling $\boldsymbol{T_l = T_{sat}}$, $\boldsymbol{T_v = T_f = (T_s + T_{sat})/2}$; b) all other types of boiling $\boldsymbol{T_l = T_v = T_{sat}}$
   - Condensation (film): $\boldsymbol{T_v = T_{sat}}$, $\boldsymbol{T_l = T_f = (T_s + T_{sat})/2}$

Note: If the necessary temperatures are unknown, we can use T_∞, T_{in} or T_{sat} to estimate all of the fluid properties. Once we obtain T_s, $T_{out}, T_{m,o}$ we need to check whether it was reasonable.

3. Calculate the Reynolds number (be careful to use the right characteristic dimension - x, L, D – and velocity - u_m, u_∞) or the Gr&Ra numbers. Determine the flow conditions (laminar/turbulent) [FLOW]

4. Decide whether a local or surface average coefficient is required [Loc/Ave]

5. Calculate Pr or get it from the table [Pr]

6. Select the appropriate correlation, determine Nu and the convection coefficient or directly h [Nu, h]

(Radiation Thermal Resistances)

(Radiation Thermal Resistance)

Planar wall

$$R_{th,rad} = \frac{1}{h_{rad}A} \quad [\text{ K/W }]$$

Radial System

$$R_{th,rad} = \frac{1}{h_{rad}2\pi rL} \quad [\text{ K/W }]$$

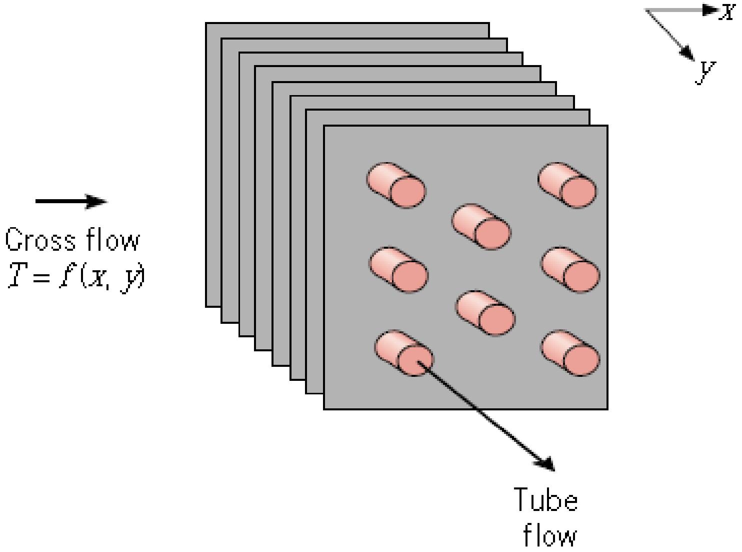
where

$$Q_x = A\varepsilon\sigma(T_s^4 - T_{sur}^4) = Ah_{rad}(T_s - T_{sur})$$

$$\Rightarrow h_{rad} = \varepsilon\sigma(T_s^2 + T_{sur}^2)(T_s + T_{sur})$$

Overall Heat Transfer Coefficient

$$Q = \frac{\Delta T}{R_{tot}} = \boldsymbol{U} A \Delta T \quad \Rightarrow \quad U \equiv \frac{1}{R_{tot} A} \quad \Rightarrow \quad \frac{1}{UA} \equiv R_{tot}$$



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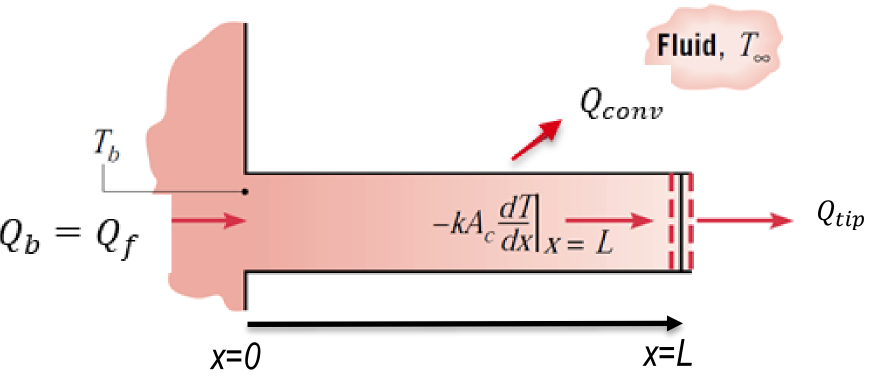
Heat Transfer from Fin of Uniform Cross-Section



$$\frac{d^2\theta}{dx^2} - m^2\theta = 0 \Rightarrow \boldsymbol{\theta = C_1 e^{mx} + C_2 e^{-mx}}$$

Boundary conditions:

- Fin base

$$T(x=0) = T_b$$

$$\Rightarrow \theta_{x=0} = \theta_b$$

- Fin tip
  1. Adiabatic $\quad \frac{d\theta}{dx}\Big|_{x=L} = 0$
  2. Convection $\quad -kA_c \frac{d\theta}{dx}\Big|_{x=L} = hA_c(T(L) - T_\infty)$
  3. Temperature $\quad \theta_{x=L} = \theta_L$
  4. Infinite fin $\quad \theta_L \to 0$

Heat Transfer from Fin of Uniform Cross-Section - Recap

TABLE 3.4 Temperature distribution and heat loss for fins of uniform cross section

Case	Tip Condition ($x = L$)	Temperature Distribution θ/θ_b	Fin Heat Transfer Rate Q_f
A	Convection heat transfer: $h\theta(L) = -k\,d\theta/dx\vert_{x=L}$	$\dfrac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + (h/mk)\sinh mL}$ (3.70)	$M\dfrac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL}$ (3.72)
B	Adiabatic $d\theta/dx\vert_{x=L} = 0$	$\dfrac{\cosh m(L-x)}{\cosh mL}$ (3.75)	$M\tanh mL$ (3.76)
C	Prescribed temperature: $\theta(L) = \theta_L$	$\dfrac{(\theta_L/\theta_b)\sinh mx + \sinh m(L-x)}{\sinh mL}$ (3.77)	$M\dfrac{(\cosh mL - \theta_L/\theta_b)}{\sinh mL}$ (3.78)
D	Infinite fin ($L \to \infty$): $\theta(L) = 0$	e^{-mx} (3.79)	M (3.80)

$\theta \equiv T - T_\infty$ $\qquad m^2 \equiv hP/kA_c$

$\theta_b = \theta(0) = T_b - T_\infty$ $\qquad M \equiv \sqrt{hPkA_c}\,\theta_b$

Efficiency and Thermal Resistance (1 Fin)

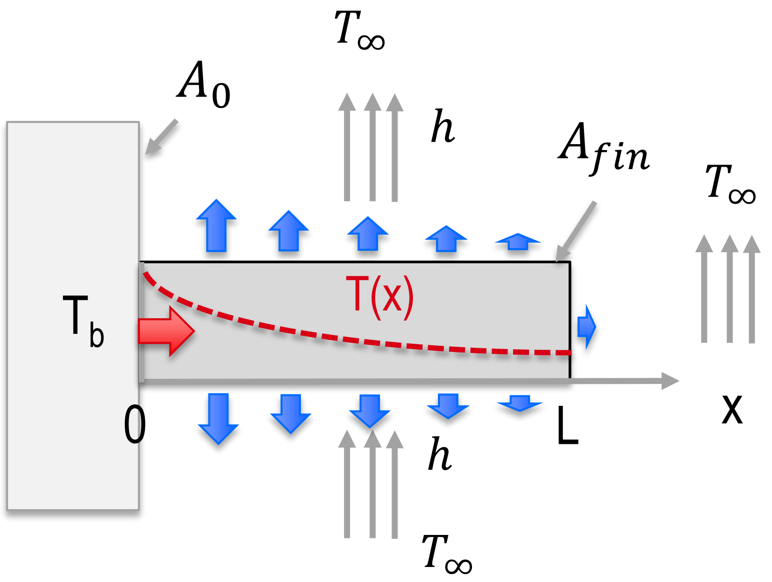
$$Q_f = Q_{b,cond} = -kA\left.\frac{d\theta}{dx}\right|_{x=0}$$

If T_b is the temperature of the fin base:

➡ $$\eta_f \equiv \frac{Q_f}{Q_{f,max}} = \frac{Q_f}{hA_f(T_b - T_\infty)} = \frac{Q_f}{hA_f\theta_b}$$

➡ $$Q_f = \eta_f hA_f(T_b - T_\infty)$$

➡ $$\boldsymbol{R_f \equiv \frac{(T_b - T_\infty)}{Q_f} = \frac{1}{hA_f\eta_f}}$$



Efficiency and Thermal Resistance (1 Fin)

TABLE 3.4 Temperature distrib

Case	Tip Condition ($x = L$)	Fin Heat Transfer Rate Q_f
A	Convection heat transfer: $h\theta(L) = -k\,d\theta/dx\rvert_{x=L}$	$M\dfrac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL}$ (3.72)
B	Adiabatic $d\theta/dx\rvert_{x=L} = 0$	$M\tanh mL$ (3.76)
C	Prescribed temperature: $\theta(L) = \theta_L$	$M\dfrac{(\cosh mL - \theta_L/\theta_b)}{\sinh mL}$ (3.78)
D	Infinite fin ($L \rightarrow \infty$): $\theta(L) = 0$	M (3.80)

$\theta \equiv T - T_\infty$ $\quad m^2 \equiv hP/kA_c$

$\theta_b = \theta(0) = T_b - T_\infty$ $\quad M \equiv \sqrt{hPkA_c}\,\theta_b$

$$M = \sqrt{hPkA_c}\,\theta_b$$

$$\eta_f \equiv \frac{Q_f}{hA_f\theta_b} = \frac{M\tanh mL}{hPL\theta_b} = \frac{\tanh mL}{mL}$$

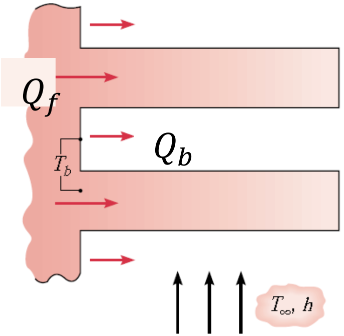
$$R_f \equiv \frac{1}{hA_f\eta_f}$$

$$\eta_f \equiv \frac{Q_f}{hA_f\theta_b} = \frac{M}{hA_f\theta_b} = \frac{1}{mL}$$

$$R_f \equiv \frac{\theta_b}{Q_f} = \frac{\theta_b}{M} = \frac{1}{\sqrt{hPkA_c}}$$

Efficiency and Thermal Resistance (Fin Array)

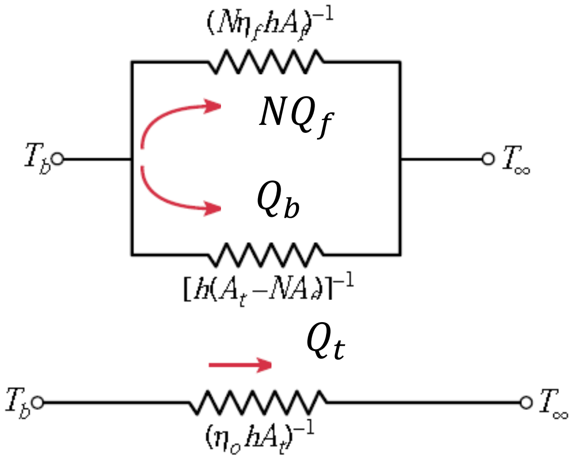
$$\eta_o \equiv \frac{Q_t}{Q_{t,max}} = \frac{Q_t}{hA_t(T_b - T_\infty)} = \frac{NQ_f + Q_b}{h(NA_f + A_b)(T_b - T_\infty)}$$



$$\eta_o \equiv 1 - \frac{NA_f}{A_t}(1 - \boldsymbol{\eta_f})$$ *Overall efficiency*

Single fin efficiency

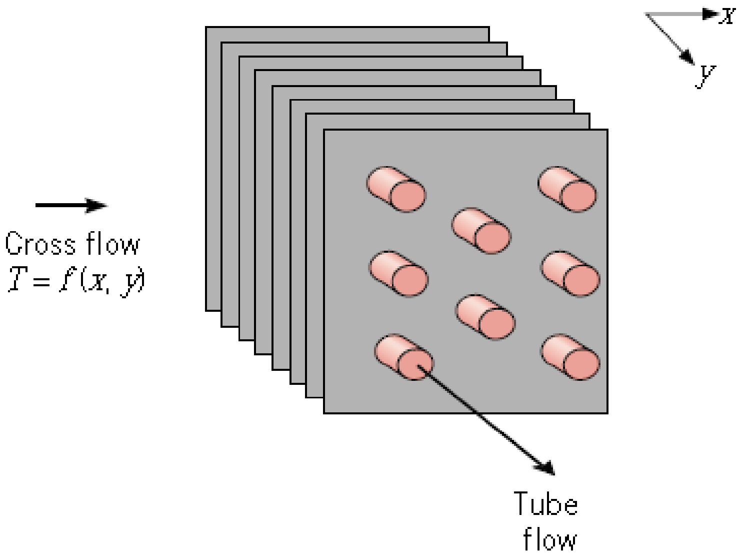
$$R_o \equiv \frac{(T_0 - T_\infty)}{Q_t} = \frac{1}{\eta_o h A_t}$$



Note: for the overall efficiency another common notation is η_t

Overall Heat Transfer Coefficient

$$Q = \frac{\Delta T}{R_{tot}} = \boldsymbol{U} A \Delta T \quad \Rightarrow \quad U \equiv \frac{1}{R_{tot} A} \quad \Rightarrow \quad \frac{1}{UA} \equiv R_{tot}$$



- Conduction thermal resistances
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Overall Heat Transfer Coefficient

Fouling

Dramatic increase in thermal resistance due to poor conduction through the scaling layer

- Introduce a *fouling* resistance per unit area (fouling factor) $R_f^{''}$

- $R_f^{''} \equiv AR_{foul}$ ⇨ $R_{foul} = \dfrac{R_f^{''}}{A}$

This lecture

- ❑ Fouling
- ❑ The overall heat transfer coefficient

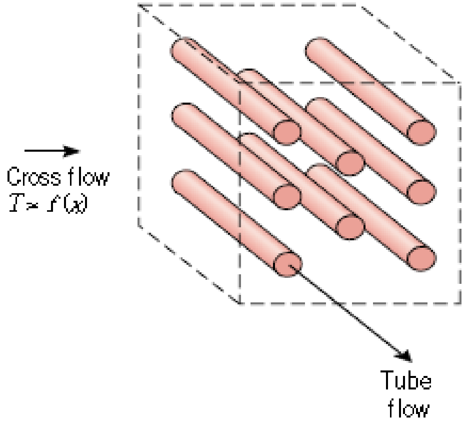
Learning Objectives:

- ❑ Calculate the overall heat transfer coefficient

Overall Heat Transfer Coefficient

$$Q = \frac{\Delta T}{R_{tot}} = \boldsymbol{U} A \Delta T \Rightarrow U \equiv \frac{1}{R_{tot} A} \Rightarrow \frac{1}{UA} \equiv R_{tot}$$

Example 1a: Without fouling



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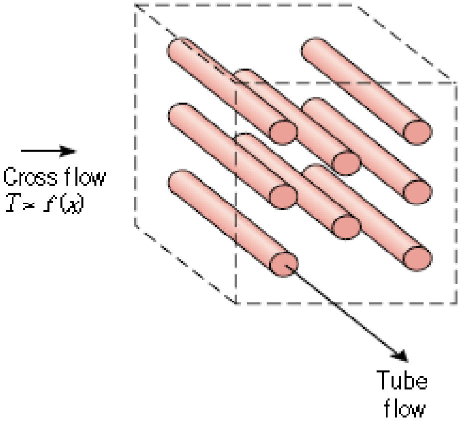


$$\frac{1}{UA} = R_{conv,out} + R_{cond} + R_{conv,in}$$

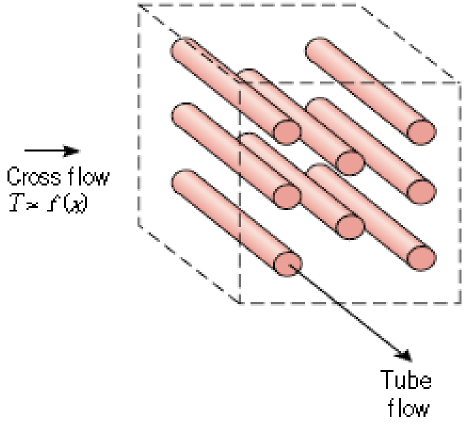
$$\frac{1}{UA} = \frac{1}{h_{out} A_{out}} + \frac{\ln(r_{out}/r_{in})}{2\pi k L} + \frac{1}{h_{in} A_{in}}$$

h_{out} forced convection external flow
h_{in} forced convection internal flow

Overall Heat Transfer Coefficient

$$Q = \frac{\Delta T}{R_{tot}} = \boldsymbol{U} A \Delta T \quad \Rightarrow \quad U \equiv \frac{1}{R_{tot} A} \quad \Rightarrow \quad \frac{1}{UA} \equiv R_{tot}$$

Example 1a: With fouling



Overall Heat Transfer Coefficient

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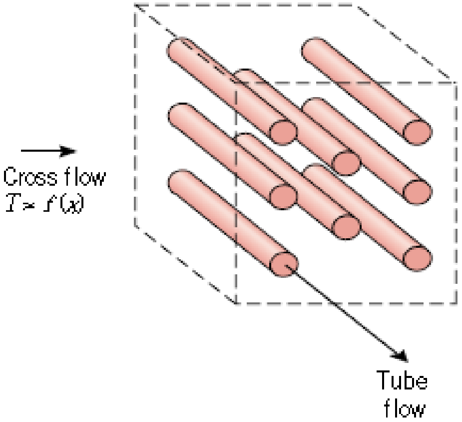


$$\frac{1}{UA} = R_{conv,out} + R_{foul,out} + R_{cond} + R_{foul,in} + R_{conv,in}$$

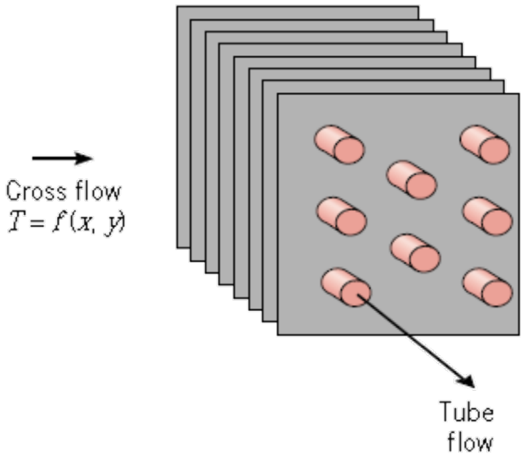
$$R_f'' \equiv A R_{foul} \quad \Rightarrow \quad R_{foul} = \frac{R_f''}{A}$$

$$\frac{1}{UA} = \frac{1}{h_{out} A_{out}} + \frac{R_{f,o}''}{A_{out}} + \frac{\ln(r_{out}/r_{in})}{2\pi k L} + \frac{R_{f,i}''}{A_{in}} + \frac{1}{h_{in} A_{in}}$$

Overall Heat Transfer Coefficient

$$Q = \frac{\Delta T}{R_{tot}} = \boldsymbol{U} A \Delta T \quad \Rightarrow \quad U \equiv \frac{1}{R_{tot} A} \quad \Rightarrow \quad \frac{1}{UA} \equiv R_{tot}$$

Example 2a: Finned, without fouling



Overall Heat Transfer Coefficient

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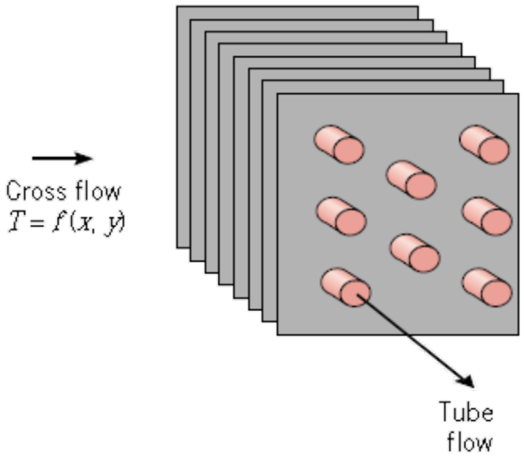

$$\frac{1}{UA} = R_{conv,out} + R_{cond} + R_{conv,in}$$

$$R_{conv,out} = R_{fin\ array} \equiv \frac{(T_0 - T_\infty)}{Q_t} = \frac{1}{\eta_o h A_{total}}$$

$$\frac{1}{UA} = \frac{1}{\eta_o h_{out} A_{out}} + \frac{\ln(r_{out}/r_{in})}{2\pi k L} + \frac{1}{h_{in} A_{in}}$$

Overall Heat Transfer Coefficient

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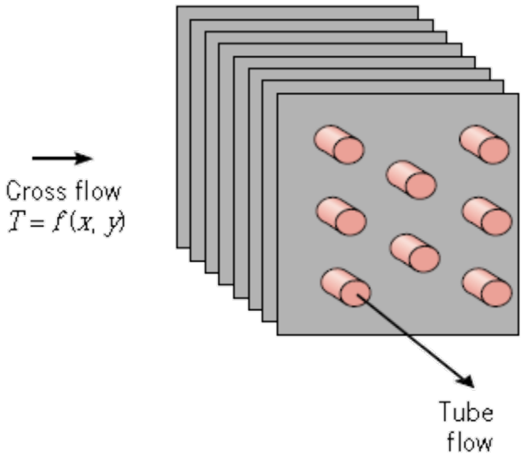
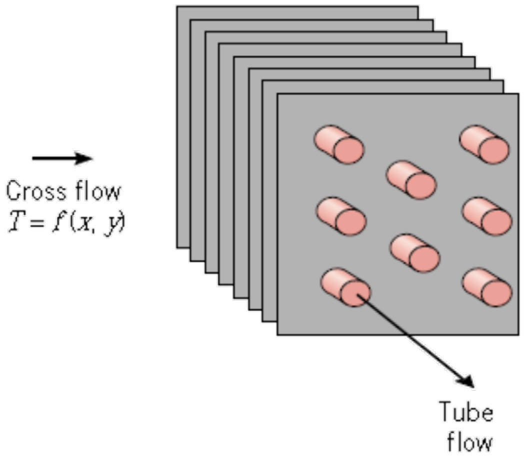


Overall Heat Transfer Coefficient

$$Q = \frac{\Delta T}{R_{tot}} = \boldsymbol{U} A \Delta T \quad \Rightarrow \quad U \equiv \frac{1}{R_{tot} A} \quad \Rightarrow \quad \frac{1}{UA} \equiv R_{tot}$$

Example 2a: Finned, with fouling



$$\frac{1}{UA} = R_{conv,out} + R_{foul,out} + R_{cond} + R_{foul,in} + R_{conv,in}$$

$$R_{overall\ fin\ array} \equiv \frac{(T_0 - T_\infty)}{Q_t} = \frac{1}{\eta_o h A_t} = \frac{1}{h(\eta_o A_t)} = \frac{1}{h A_t'}$$

$$R_{foul} = \frac{R_f''}{A_t'} = \frac{R_f''}{\eta_o A_t}$$

$$\frac{1}{UA} = \frac{1}{\eta_o h_{out} A_{out}} + \frac{R_{f,o}''}{\eta_o A_{out}} + \frac{\ln(r_{out}/r_{in})}{2\pi k L} + \frac{R_{f,i}''}{A_{in}} + \frac{1}{h_{in} A_{in}}$$

Overall Heat Transfer Coefficient

$$Q = \frac{\Delta T}{R_{tot}} = \boldsymbol{U} A \Delta T \quad \Rightarrow \quad U \equiv \frac{1}{R_{tot} A} \quad \Rightarrow \quad \frac{1}{UA} \equiv R_{tot}$$

In the most general case we could have fins present also on the inner side:

$$\frac{1}{UA} = \frac{1}{\eta_{o,out} h_{out} A_{out}} + \frac{R''_{f,o}}{\eta_{o,out} A_{out}} + \underbrace{R_{cond}}_{\text{Account for all the layers of conduction!!}} + \frac{R''_{f,i}}{\eta_{o,in} A_{in}} + \frac{1}{\eta_{o,in} h_{in} A_{in}}$$

Overall Heat Transfer Coefficient

$$Q = \frac{\Delta T}{R_{tot}} = \boldsymbol{U} A \Delta T \quad \Rightarrow \quad U \equiv \frac{1}{R_{tot} A} \quad \Rightarrow \quad \frac{1}{UA} \equiv R_{tot}$$

Note:

$$\frac{\mathbf{1}}{\boldsymbol{UA}} \equiv \boldsymbol{R_{tot}} = \frac{1}{U_{in} A_{in}} = \frac{1}{U_{out} A_{out}} = \frac{1}{U_{cold} A_{cold}} = \frac{1}{U_{hot} A_{hot}}$$

$$U_{in} \equiv \frac{1}{R_{tot} A_{in}} \neq U_{out} \equiv \frac{1}{R_{tot} A_{out}} \qquad U_{cold} \equiv \frac{1}{R_{tot} A_{cold}} \neq U_{hot} \equiv \frac{1}{R_{tot} A_{hot}}$$

Calculate first the total heat transfer resistance R_{tot} then multiply by a specific area to determine the overall heat transfer coefficient with respect to a certain surface.

Note: Do not calculate U starting from R''_{tot} because it is likely you will mess up the areas (you can do it but you have to be extra careful as each term of the specific resistance will be normalized by a different area…)

This lecture

- ☑ Introduction to Heat Exchangers
- ☑ The problem of the overall heat transfer coefficient
- ☑ RECAP of critical concepts

Learning Objectives:

- ☑ Understand the concept and possible design of heat exchangers

Next lecture

- ❑ The overall heat transfer coefficient

Learning Objectives:

- ❑ Calculate the overall heat transfer coefficient