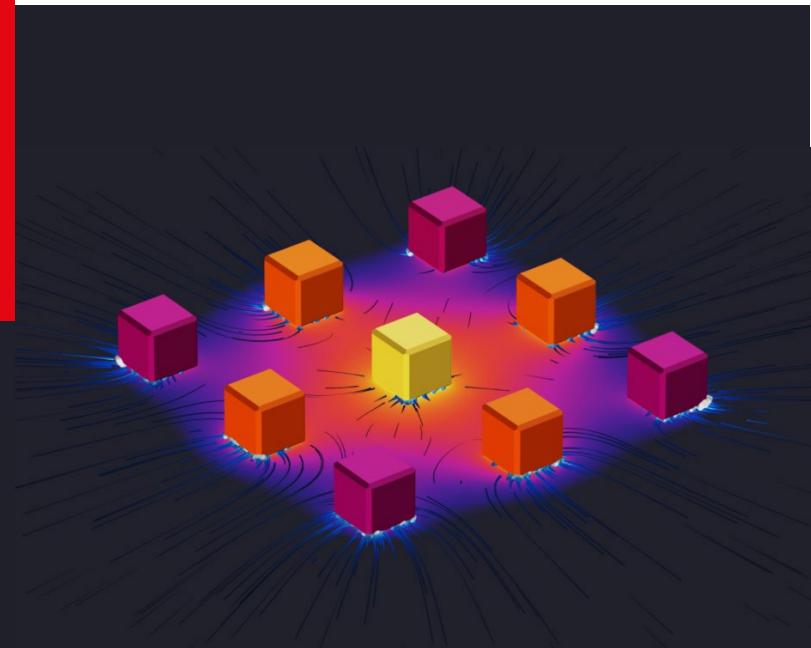


Heat and Mass Transfer

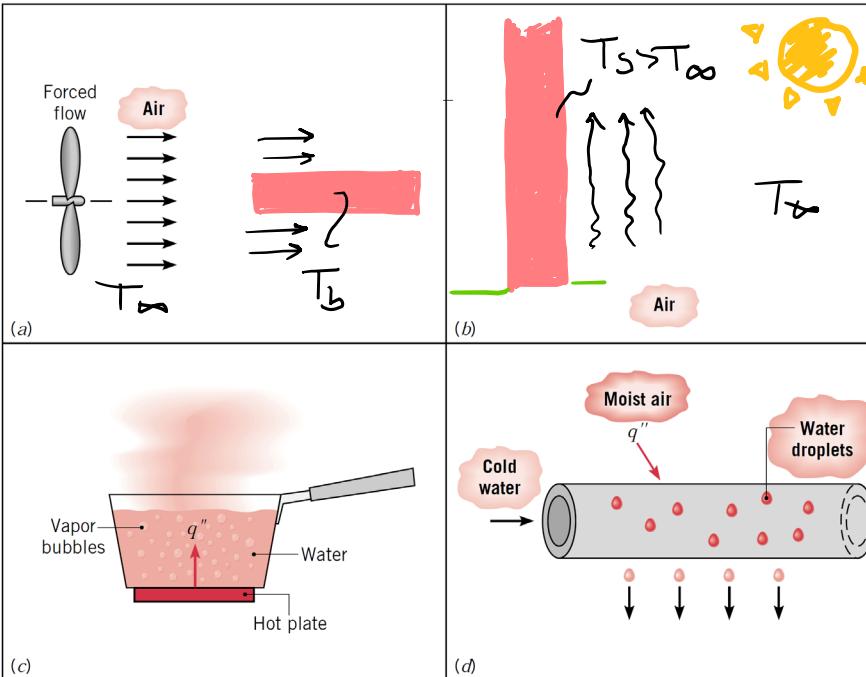
ME-341

Instructor: Giulia Tagliabue



Previously

1. Forced Convection

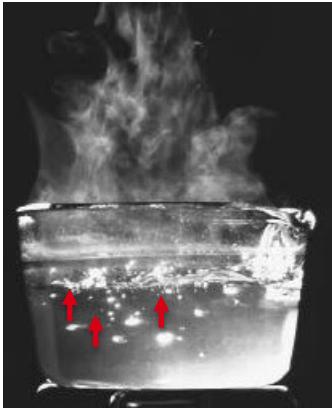


2. Natural (Free) Convection

3. Boiling

4. Condensation

Introduction to Boiling and Condensation



Boiling: heat transfer from the wall to the fluid



Condensation: heat transfer from the fluid to the wall

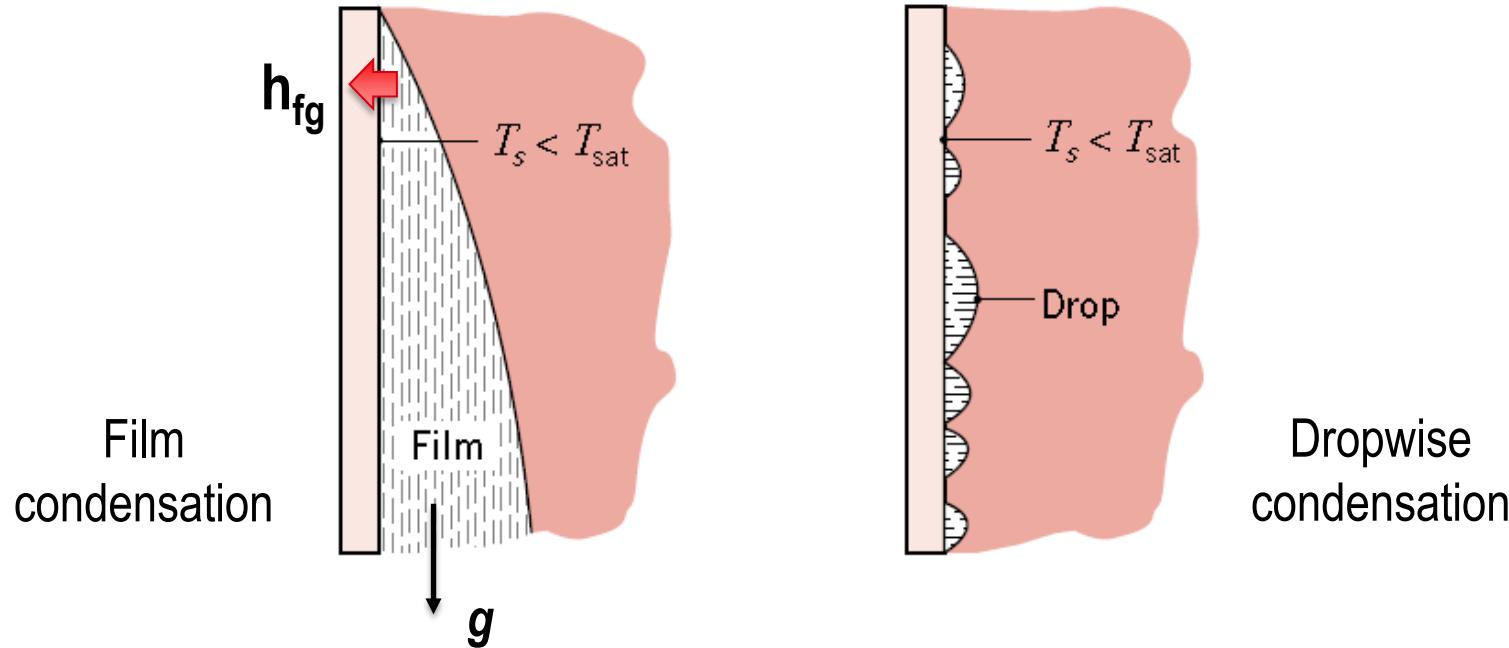
Fluid motion controlled by:

- Surface tension σ at the liquid-vapor interface
- Density difference (buoyancy) between liquid/vapor phases

Phase Transition = Isothermal Process

Latent heat (h_{fg}) is exchanged between solid and fluid
Small solid/fluid ΔT

Condensation



The conditions of the surface (wettability) determine the mode of condensation

Which one is most desirable?

Condensation: Laminar Film on a Vertical Plate

During condensation, gravity forces the liquid to move along the solid.

FLUID DYNAMICS

Mass conservation → Continuity equation

Momentum conservation → Navier-Stokes equations

Flow condition (Laminar/turbulent) → Re



Velocity profile: $\vec{u}(x, y)$



Condensate-film
Thickness δ

- Shear stress τ_w
- Friction coefficient C_f
- Friction factor f

No slip condition $u(x, 0) = 0$

$$Q_{convection} = Q_{cond,film}$$

HEAT TRANSFER

Energy conservation → 1st Law of Thermodynamics

Nusselt assumption = NO ADVECTION TERMS

Boundary Conditions (Temperature)
 Pr

Temperature profile: $T(x, y)$

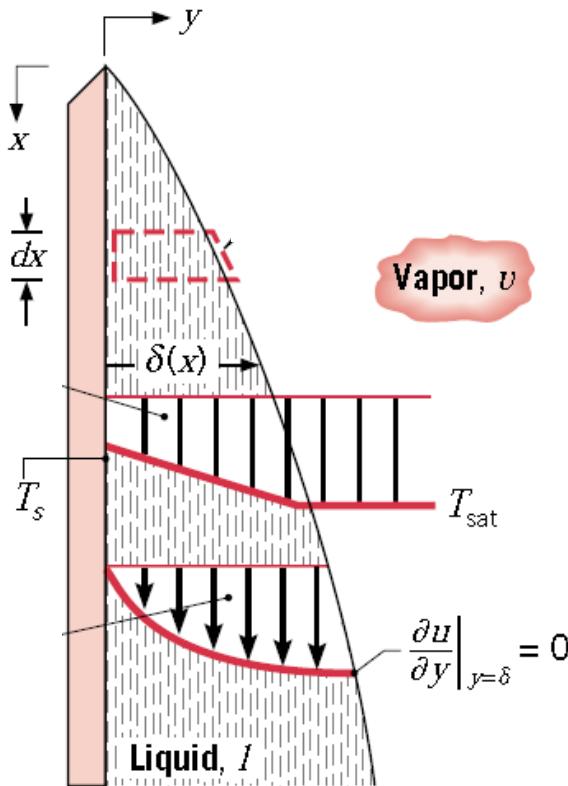


Transport Laws (Newton/Fourier)

$$h(T_{sat} - T_s) = -\frac{k_l(T_{sat} - T_s)}{\delta}$$

Nu

Condensation: Laminar Film on a Vertical Plate



To obtain the local convection coefficient:

$$q_s'' = \frac{k_l(T_{sat} - T_s)}{\delta} = h_x(T_{sat} - T_s) \quad \rightarrow \quad h_x = \frac{k_l}{\delta}$$

$$\rightarrow h(x) = \left[\frac{g \rho_l (\rho_l - \rho_v) k_l^3 h'_{fg}}{4 \mu_l (T_{sat} - T_s) x} \right]^{1/4}$$

$$h'_{fg} = h_{fg} + 0.68 c_{p,l} (T_{sat} - T_s)$$

With liquid properties estimated at $T_f = \frac{(T_s + T_{sat})}{2}$
and ρ_v, h_{fg} estimated at T_{sat}

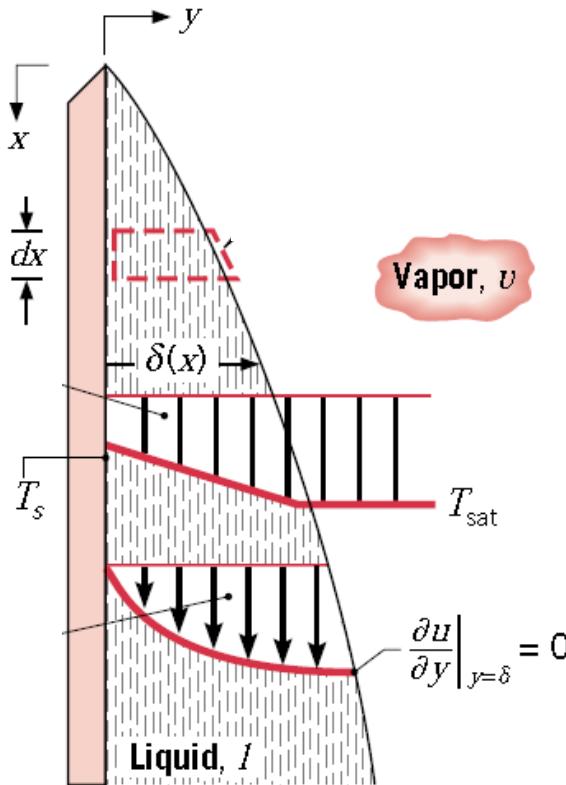
This Lecture

- Film Condensation on a Vertical Plate: Correlations
- Film Condensation on Radial Systems
 - Correlations for Laminar Flow
 - Overall Heat Transfer Coefficient

Learning Objectives:

- Calculate convection coefficient in selected condensation cases

Film Condensation: Average convection coefficient



To describe the overall heat transfer by condensation we need the average convection coefficient over the entire plate. In fact:

$$Q = \overline{h_L} A (T_{sat} - T_s)$$

Also the total condensation rate is:

$$\dot{m} = \frac{Q}{h'_{fg}} = \frac{\overline{h_L} A (T_{sat} - T_s)}{h'_{fg}}$$

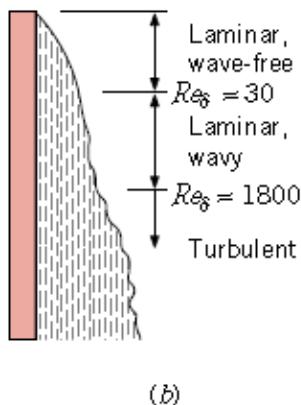
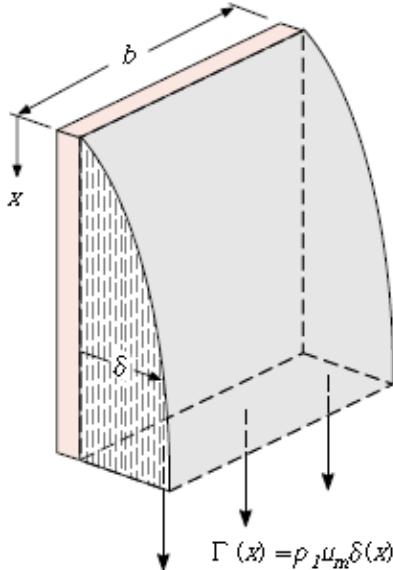
We have to find the appropriate correlations depending on the flow conditions (Laminar vs Turbulent)

Film Condensation on a Vertical Plate: Correlations

We define: $Re_\delta = \frac{4\Gamma}{\mu_l}$  $Re_\delta = \frac{4\dot{m}}{\mu_l b}$

We observe that: $\dot{m} = \rho_l u_{mean} b \delta$

 $Re_\delta = \frac{4\rho_l u_{mean} \delta}{\mu_l}$



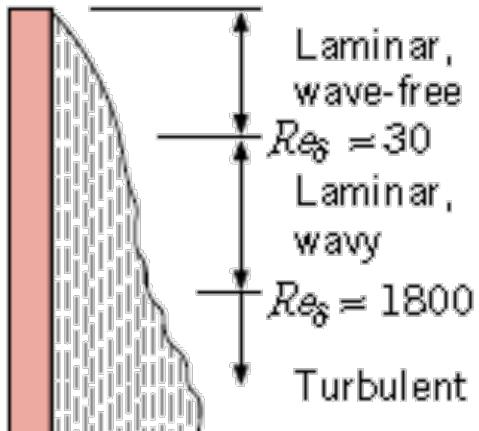
We also have: $\dot{m} = \frac{\overline{h_L} (b \cdot L) (T_{sat} - T_s)}{h'_{fg}} = \frac{Re_\delta \mu_l b}{4}$

 $\overline{h_L} = \frac{Re_\delta \mu_l h'_{fg}}{4L(T_{sat} - T_s)}$

 If we know Re_δ we can obtain $\overline{h_L}$

 We use correlations to find Re_δ

Film Condensation on a Vertical Plate: Correlations



(b)

$$\overline{h_L} = \frac{Re_\delta \mu_l h'_{fg}}{4L(T_{sat} - T_s)}$$

$$Re_\delta = 3.78 \left[\frac{k_l L (T_{sat} - T_s)}{\mu_l h'_{fg} (v_l^2/g)^{1/3}} \right]^{3/4} \quad Re_\delta \leq 30 \text{ (Laminar)}$$

$$Re_\delta = \left[\frac{3.7 k_l L (T_{sat} - T_s)}{\mu_l h'_{fg} (v_l^2/g)^{1/3}} + 4.8 \right]^{0.82} \quad 30 \leq Re_\delta \leq 1800 \text{ (Wavy)}$$

$$Re_\delta = \left[\frac{0.069 k_l L (T_{sat} - T_s)}{\mu_l h'_{fg} (v_l^2/g)^{1/3}} Pr_l^{0.5} - 151 Pr_l^{0.5} + 253 \right]^{4/3} \quad Re_\delta \geq 1800 \text{ (Turbulent)}$$

Where liquid properties are estimated at $T_f = \frac{(T_s + T_{sat})}{2}$

Calculate all three values of Re_δ , decide which one is within the correct range and then substitute in the average convection coefficient.

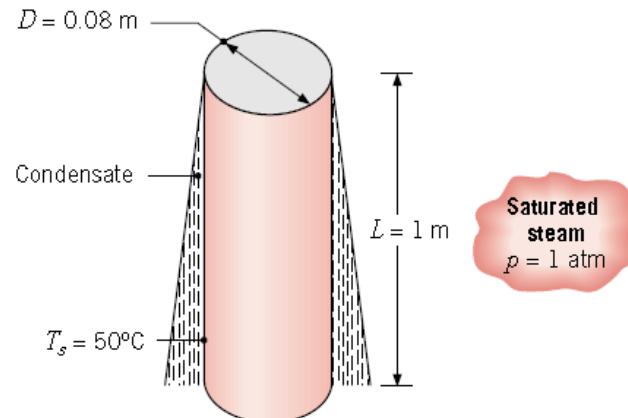
Film Condensation (Example)

The outer surface of a vertical tube, which is 1 m long and has an outer diameter of 80 mm, is exposed to saturated steam at atmospheric pressure and is maintained at 50°C by the flow of cool water through the tube. What is the rate of heat transfer to the coolant, and what is the rate at which steam is condensed at the surface?

$$\text{Liquid properties estimated at } T_f = \frac{(T_s + T_{sat})}{2} = 75\text{C}$$

Properties: Table A.6, saturated vapor ($p = 1.0133$ bars): $T_{sat} = 100\text{C}$, $\rho_v = (1/v_g) = 0.596 \text{ kg/m}^3$, $h_{fg} = 2257 \text{ kJ/kg}$. Table A.6, saturated liquid ($T_f = 75\text{C}$): $\rho_f = (1/v_f) = 975 \text{ kg/m}^3$, $\mu_f = 375 \times 10^{-6} \text{ N} \cdot \text{s/m}^2$, $k_f = 0.668 \text{ W/m} \cdot \text{K}$, $c_{p,f} = 4193 \text{ J/kg} \cdot \text{K}$, $v_f = \mu_f / \rho_f = 385 \times 10^{-9} \text{ m}^2/\text{s}$.

Assumptions: The condensate film thickness is small relative to the cylinder diameter.



Use the vertical plate correlations

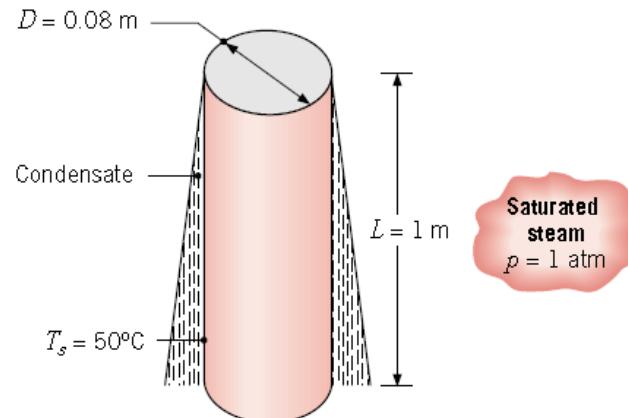
Film Condensation (Example)

The outer surface of a vertical tube, which is 1 m long and has an outer diameter of 80 mm, is exposed to saturated steam at atmospheric pressure and is maintained at 50°C by the flow of cool water through the tube. What is the rate of heat transfer to the coolant, and what is the rate at which steam is condensed at the surface?

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Assumptions: The condensate film thickness is small relative to the cylinder diameter.



Use the vertical plate correlations 

$$h'_{fg} = h_{fg} + 0.68c_{p,l}(T_{sat} - T_s) = h_{fg} \left(1 + 0.68 \frac{c_{p,l}(T_{sat} - T_s)}{h_{fg}} \right) = h_{fg}(1 + 0.68Ja)$$

$$h'_{fg} = h_{fg}(1 + 0.68Ja) = 2257 \text{ kJ/kg} (1.0632) = 2400 \text{ kJ/kg}$$

Film Condensation (Example)

The outer surface of a vertical tube, which is 1 m long and has an outer diameter of 80 mm, is exposed to saturated steam at atmospheric pressure and is maintained at 50°C by the flow of cool water through the tube. What is the rate of heat transfer to the coolant, and what is the rate at which steam is condensed at the surface?

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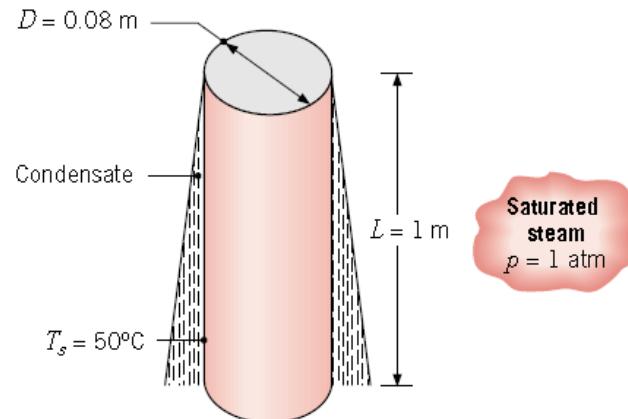
We should now calculate Re_δ based on all three correlations and see which one gives a value correct for its range of applicability. We first try the intermediate one:

$$Re_\delta = \left[\frac{3.7k_l L(T_{\text{sat}} - T_s)}{\mu_l h'_{fg} (v_l^2/g)^{1/3}} + 4.8 \right]^{0.82}$$

$$30 \leq Re_\delta \leq 1800,$$

$$Re_\delta = \left[\frac{3.70 \times 0.668 \text{ W/m} \cdot \text{K} \times 1 \text{ m} \times (100 - 50) \text{ K}}{375 \times 10^{-6} \text{ N} \cdot \text{s/m}^2 \times 2.4 \times 10^6 \text{ J/kg} \left(\frac{(385 \times 10^{-9} \text{ m}^2/\text{s})^2}{9.8 \text{ m/s}^2} \right)^{1/3}} + 4.8 \right]^{0.82}$$

$$= 1177 \checkmark$$

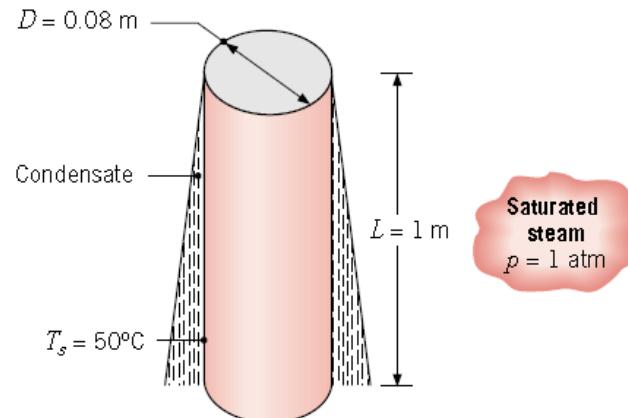


Note: Calculate Re with the other two correlations and verify that it is out of the range of applicability of the correlation ($Re = 910$ and 1019 respectively).

Film Condensation (Example)

The outer surface of a vertical tube, which is 1 m long and has an outer diameter of 80 mm, is exposed to saturated steam at atmospheric pressure and is maintained at 50°C by the flow of cool water through the tube. What is the rate of heat transfer to the coolant, and what is the rate at which steam is condensed at the surface?

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$$\overline{h}_L = \frac{Re \delta \mu_l h'_{fg}}{4L(T_{\text{sat}} - T_s)} \quad \rightarrow \quad \overline{h}_L = \frac{1177 \times 375 \times 10^{-6} \text{ kg/s} \cdot \text{m} \times 2.4 \times 10^6 \text{ J/kg}}{4 \times 1 \text{ m} (100 - 50) \text{ K}} = 5300 \text{ W/m}^2 \cdot \text{K}$$

$$Q = \overline{h}_L A (T_{\text{sat}} - T_s) = \overline{h}_L (\pi D L) (T_{\text{sat}} - T_s) = 5300 \text{ W/m}^2 \cdot \text{K} \times \pi \times 0.08 \text{ m} \times 1 \text{ m} (100 - 50) \text{ K} = 66.6 \text{ kW}$$

$$\dot{m} = \frac{q}{h'_{fg}} = \frac{66.6 \times 10^3 \text{ W}}{2.4 \times 10^6 \text{ J/kg}} = 0.0276 \text{ kg/s}$$

Note: if we use water at 30°C instead of 50°C for cooling the system, how much are the heat flux q and the mass transfer rate? Are they higher or lower?

Film Condensation (Example)

The outer surface of a vertical tube, which is 1 m long and has an outer diameter of 80 mm, is exposed to saturated steam at atmospheric pressure and is maintained at 50°C by the flow of cool water through the tube. What is the rate of heat transfer to the coolant, and what is the rate at which steam is condensed at the surface?

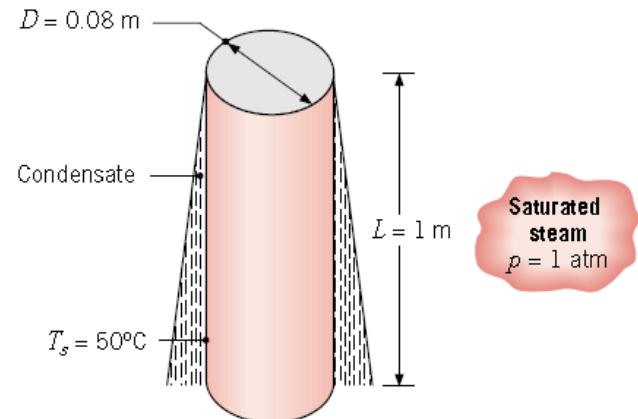
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We can now verify our initial assumption by calculating:

$$\delta(L) = \left[\frac{4k_I \mu_I (T_{\text{sat}} - T_s)L}{g \rho_I (\rho_I - \rho_v) h'_{fg}} \right]^{1/4} \quad (\text{laminar wavy})$$

$$\delta(L) = \left[\frac{4 \times 0.668 \text{ W/m} \cdot \text{K} \times 375 \times 10^{-6} \text{ kg/s} \cdot \text{m} (100 - 50) \text{ K} \times 1 \text{ m}}{9.8 \text{ m/s}^2 \times 975 \text{ kg/m}^3 (975 - 0.596) \text{ kg/m}^3 \times 2.4 \times 10^6 \text{ J/kg}} \right]^{1/4}$$

$$\delta(L) = 2.18 \times 10^{-4} \text{ m} = 0.218 \text{ mm}$$



$$\delta(L) \ll (D/2) \quad \checkmark$$

This Lecture



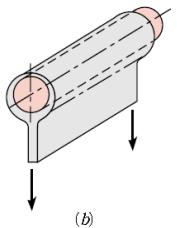
- Film Condensation on a Vertical Plate: Correlations
- Film Condensation on Radial Systems
 - Correlations for Laminar Flow
 - Overall Heat Transfer Coefficient

Learning Objectives:

- Calculate convection coefficient in selected condensation cases

Film Condensation on Radial Systems: Correlations for Laminar Flow

Single Tube

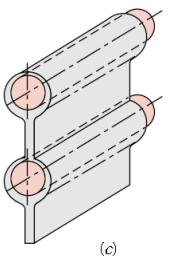


$$\overline{h_D} = C \left[\frac{g \rho_l (\rho_l - \rho_v) k_l^3 h'_{fg}}{\mu_l (T_{sat} - T_s) D} \right]^{1/4}$$

Where $C = 0.862$ for a sphere

$C = 0.729$ for a cylindrical tube

Bank of Tubes



$$\overline{h_D} = 0.729 \left[\frac{g \rho_l (\rho_l - \rho_v) k_l^3 h'_{fg}}{N \mu_l (T_{sat} - T_s) D} \right]^{1/4}$$

Where N =number of tubes in a column

Reduction in h as N increases due to increasing thickness of the film.

The condensed mass-flow rate per unit length of a single tubes is: $\dot{m}'_1 = \frac{Q'}{h'_{fg}} = \frac{\overline{h_D}(\pi D)(T_{sat} - T_s)}{h'_{fg}}$

Condensation: Laminar Film Condensation in Radial Systems

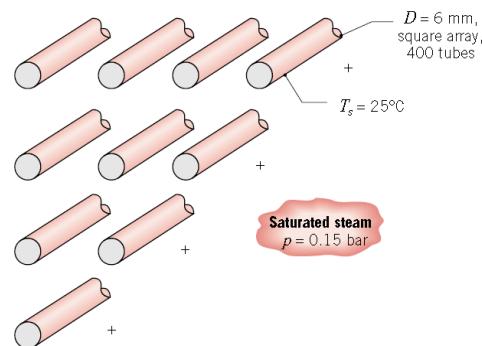
A steam condenser consists of a square array of 400 tubes, each 6 mm in diameter. If the tubes are exposed to saturated steam at a pressure of 0.15 bar and the tube surface temperature is maintained at 25°C, what is the rate at which steam is condensed per unit length of the tubes?

$$\text{Liquid properties estimated at } T_f = \frac{(T_s + T_{sat})}{2}$$

Assumptions:

1. Negligible concentration of noncondensable gases in the steam.
2. Laminar film condensation on the tubes.

Properties: Table A.6, saturated vapor ($p = 0.15$ bar): $T_{sat} = 327$ K = 54°C, $\rho_v = (1/v_g) = 0.098$ kg/m³, $h_{fg} = 2373$ kJ/kg. Table A.6, saturated water ($T_f = 312.5$ K): $\rho_l = (1/v_f) = 992$ kg/m³, $\mu_l = 663 \times 10^{-6}$ N·s/m², $k_l = 0.631$ W/m·K, $c_{p,l} = 4178$ J/kg·K.



Condensation: Laminar Film Condensation in Radial Systems

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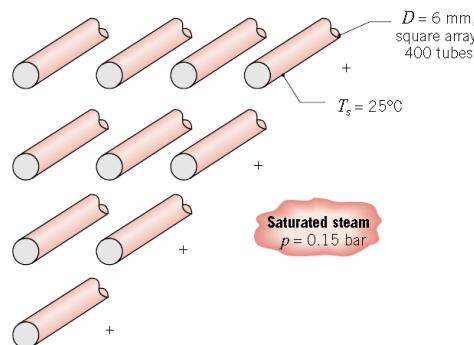
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The condensed mass flow rate per unit length is: $\dot{m}'_1 = \frac{q'_1}{h'_{fg}} = \frac{\bar{h}_{D,N}(\pi D)(T_{sat} - T_s)}{h'_{fg}}$ where

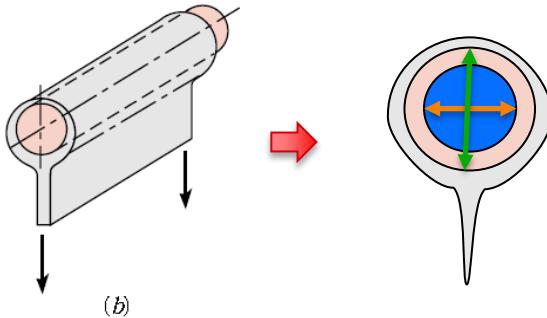


$$\left. \begin{aligned} \bar{h}_{D,N} &= 0.729 \left[\frac{gp_l(\rho_l - \rho_v)k_l^3 h'_{fg}}{N\mu_l(T_{sat} - T_s)D} \right]^{1/4} = 5194 \text{ W/m}^2 \cdot \text{K} \\ N &= 20, \\ h'_{fg} &= h_{fg} + 0.68 c_{p,l} (T_{sat} - T_s) \end{aligned} \right|$$

$$\rightarrow \dot{m}'_1 = \frac{5194 \text{ W/m}^2 \cdot \text{K} (\pi \times 0.006 \text{ m}) (54 - 25) \text{ K}}{2.455 \times 10^6 \text{ J/kg}} = 1.16 \times 10^{-3} \text{ kg/s} \cdot \text{m}$$

$$\rightarrow \dot{m}' = N^2 \dot{m}'_1 = 400 \times 1.16 \times 10^{-3} \text{ kg/s} \cdot \text{m} = 0.464 \text{ kg/s} \cdot \text{m}$$

Film Condensation: Overall Heat Transfer Coefficient



The total heat transfer is equal to:

$$Q = \frac{(T_{sat} - T_m)}{R_{tot}} = U_{out}A_{out}(T_{sat} - T_m) = U_{in}A_{in}(T_{sat} - T_m)$$

Where: T_m = average temperature of the internal fluid

$$R_{tot} = R_{conv,in} + R_{cond} + R_{conv,out}$$

$$U_{out}A_{out} = U_{in}A_{in} = 1/R_{tot} \quad \left. \begin{array}{l} A_{out} = \pi D_{out} L \\ A_{in} = \pi D_{in} L \end{array} \right\} \quad U_{out} = \frac{1}{A_{out}R_{tot}} \neq U_{in} = \frac{1}{A_{in}R_{tot}}$$

The total condensation rate is: $\dot{m} = \frac{Q}{h'_{fg}} = \frac{UA(T_{sat} - T_m)}{h'_{fg}}$

If the total length of the tubes is unknown we can calculate the heat transfer rate and the mass condensation rate per unit length.

This Lecture

- Film Condensation on a Vertical Plate: Correlations
- Film Condensation on Radial Systems
 - Correlations for Laminar Flow
 - Overall Heat Transfer Coefficient

Learning Objectives:

- Calculate convection coefficient in selected condensation cases

RECAP of Boiling and Condensation

Boiling

- ↳ Saturated Pool Boiling
- ↳ Boiling Curve
- ↳ Nucleate and Film Pool Boiling Correlations
- ↳ Forced Convection Boiling
- ↳ External Flow Correlation
- ↳ Two-Phase Flow

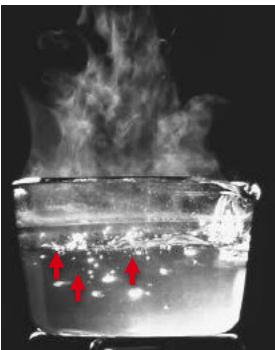
Condensation

- ↳ Vertical Plate
- ↳ Laminar Flow Equations
- ↳ Laminar/Laminar Wavy/Turbulent Correlations
- ↳ Radial Systems (Tubes)
- ↳ Laminar Flow
- ↳ Bank of Tubes

Introduction to Boiling

Boiling occurs when the surface temperature exceeds the saturation temperature at that pressure.

$$q_s'' = h(T_s - T_{sat}) = h\Delta T_e \quad \Delta T_e \text{ excess temperature}$$



Fluid Dynamics

Heat Transfer

Pool boiling

the liquid is initially quiescent and only free convection occurs

Forced boiling

the fluid is moving while it boils (e.g. inside a pipe)

Saturated Boiling

$T_s > T_{sat}$, the bubble must rise

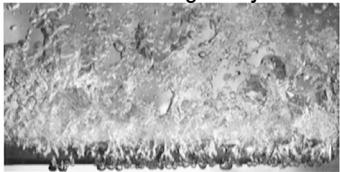
Subcooled Boiling

$T_s < T_{sat}$, the bubbles can re-condense in the liquid

Saturated Pool Boiling

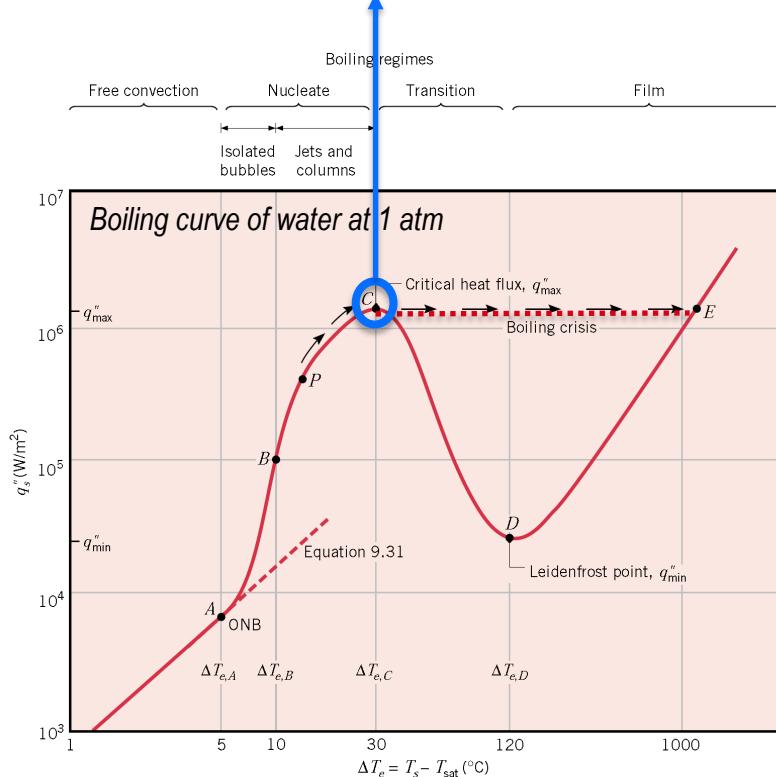
$$q''_{\max} = Ch_{fg}\rho_v \left[\frac{\sigma g(\rho_l - \rho_v)}{\rho_v^2} \right]^{1/4}$$

Nucleate boiling with jets

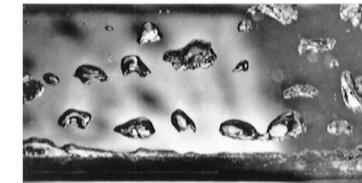


$$q''_s = \mu_I h_{fg} \left[\frac{g(\rho_l - \rho_v)}{\sigma} \right]^{1/2} \left(\frac{c_{p,I} \Delta T_e}{C_{s,I} h_{fg} P r_I^n} \right)^3$$

properties estimated at T_{sat}



Film boiling



$$\overline{Nu_D} = C \left[\frac{g \rho_v (\rho_l - \rho_v) h'_{fg} D^3}{\mu_v k_v (T_s - T_{sat})} \right]^{1/4}$$

Where $h'_{fg} = h_{fg} + 0.8 c_{p,v} (T_s - T_{sat})$

$C = 0.62$ for horizontal cylinders

$C = 0.67$ for spheres

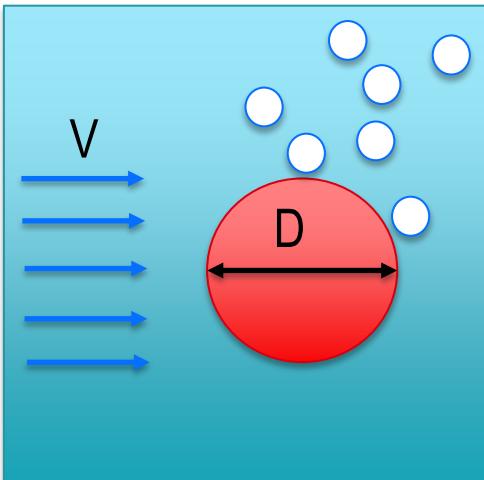
With properties estimated at $T_f = \frac{(T_s + T_{sat})}{2}$

Forced Convection Boiling: External Flow (heat flux)

Nucleate Boiling Regime = Bubbles form on the surface

q''_{max} is the critical heat flux for External Forced Convection Boiling. Beyond q''_{max} a film of vapor will form.

Cylinder



Use both correlations to calculate q''_{max} , then verify the validity conditions and keep the one that is true.

$$\text{Low Velocity, i.e. } \frac{q''_{max}}{\rho_v h_{fg} V} > \left[\frac{0.275}{\pi} \left(\frac{\rho_l}{\rho_v} \right)^{1/2} + 1 \right] :$$

$$\frac{q''_{max}}{\rho_v h_{fg} V} = \frac{1}{\pi} \left[1 + \left(\frac{4}{We_D} \right)^{1/3} \right]$$

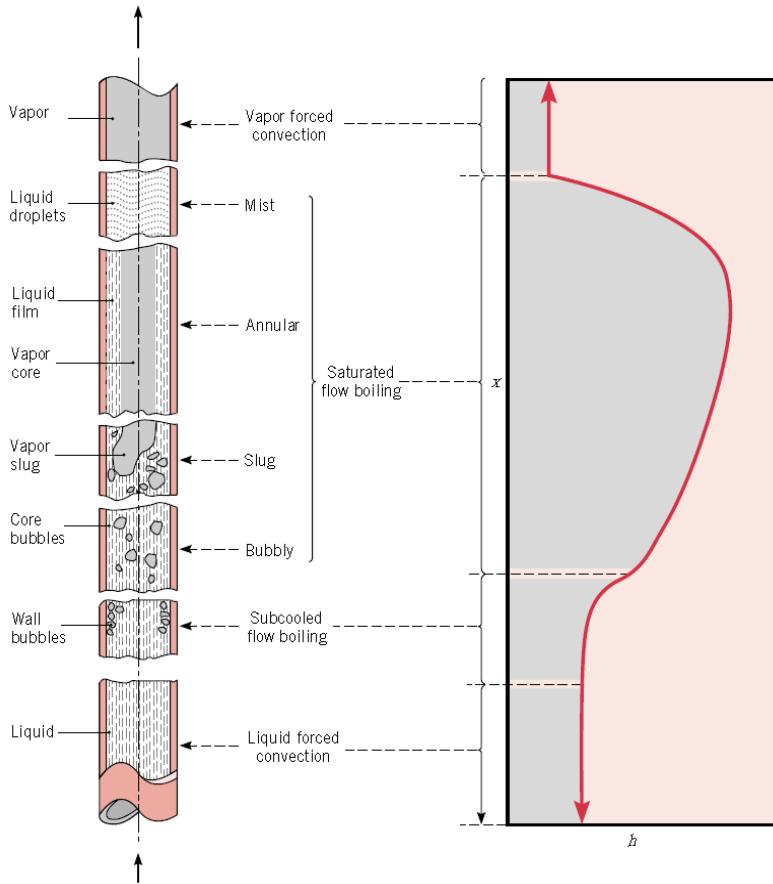
$$\text{High Velocity, i.e. } \frac{q''_{max}}{\rho_v h_{fg} V} < \left[\frac{0.275}{\pi} \left(\frac{\rho_l}{\rho_v} \right)^{1/2} + 1 \right] :$$

$$\frac{q''_{max}}{\rho_v h_{fg} V} = \frac{\left(\frac{\rho_l}{\rho_v} \right)^{3/4}}{169\pi} + \frac{\left(\frac{\rho_l}{\rho_v} \right)^{1/2}}{19.2\pi We_D^{1/3}}$$

$$\text{Where } We_D = \frac{\rho_v V^2 D}{\sigma}$$

V = fluid velocity

Forced Convection Boiling: Internal Flow



$$\frac{h}{h_{sp}} = 0.6683 \left(\frac{\rho_l}{\rho_v} \right)^{0.1} \bar{X}^{0.16} (1 - \bar{X})^{0.64} f(Fr) + 1058 \left(\frac{q''_s}{\dot{m}'' h_{fg}} \right)^{0.7} (1 - \bar{X})^{0.8} G_{s,f}$$

$$\frac{h}{h_{sp}} = 1.136 \left(\frac{\rho_l}{\rho_v} \right)^{0.45} \bar{X}^{0.72} (1 - \bar{X})^{0.08} f(Fr) + 667.2 \left(\frac{q''_s}{\dot{m}'' h_{fg}} \right)^{0.7} (1 - \bar{X})^{0.8} G_{s,f}$$

Where $\bar{X}(x) = \text{vapor quality} = \dot{m}_v / \dot{m}_{tot} = (q''_s \pi D x / h_{fg}) / \dot{m}$

$$\dot{m}'' = \dot{m} / A_c$$

h_{sp} = single-phase convection coefficient. Liquid properties evaluated at T_{sat}

$$Fr = \frac{(\dot{m}'' / \rho_l)^2}{gD}$$

TABLE 10.2 Values of $G_{s,f}$ for various surface–liquid combinations [27, 28].

Fluid in Commercial Copper Tubing	$G_{s,f}$
Kerosene	0.488
Refrigerant R-134a	1.63
Refrigerant R-152a	1.10
Water	1.00
For stainless steel tubing, use $G_{s,f} = 1$.	

$f(Fr)$	Vertical tubes	Horizontal tubes $Fr > 0.04$	Horizontal tubes $Fr < 0.04$
1	1	1	$f(Fr) = 2.63 Fr^{-0.3}$

Film Condensation: Average convection coefficient

To describe the overall heat transfer by condensation we need the average convection coefficient over the solid:

$$Q = \overline{h_{L/D}} A (T_{sat} - T_s)$$

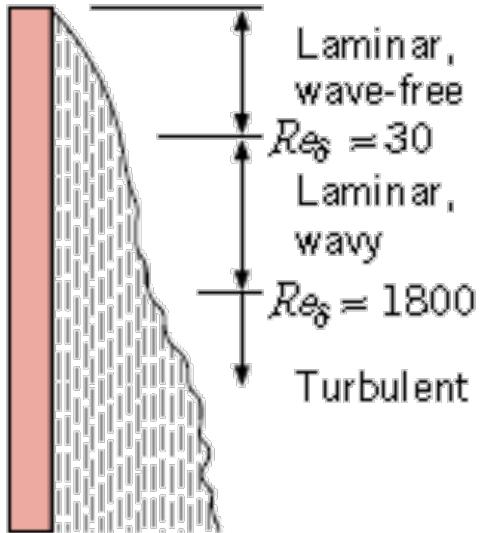
Also the total condensation rate is:

$$\dot{m} = \frac{Q}{h'_{fg}} = \frac{\overline{h_{L/D}} A (T_{sat} - T_s)}{h'_{fg}}$$

$$h'_{fg} = h_{fg} + 0.68c_{p,l}(T_{sat} - T_s)$$

We have to find the appropriate correlations depending on the flow conditions (Laminar vs Turbulent)

Film Condensation on a Vertical Plate: Correlations



(b)

$$\overline{h_L} = \frac{Re_\delta \mu_l h'_{fg}}{4L(T_{sat} - T_s)}$$

$$Re_\delta = 3.78 \left[\frac{k_l L (T_{sat} - T_s)}{\mu_l h'_{fg} (v_l^2/g)^{1/3}} \right]^{3/4} \quad Re_\delta \leq 30 \text{ (Laminar)}$$

$$Re_\delta = \left[\frac{3.7 k_l L (T_{sat} - T_s)}{\mu_l h'_{fg} (v_l^2/g)^{1/3}} + 4.8 \right]^{0.82} \quad 30 \leq Re_\delta \leq 1800 \text{ (Wavy)}$$

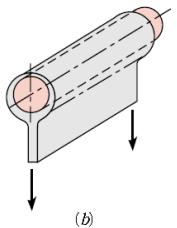
$$Re_\delta = \left[\frac{0.069 k_l L (T_{sat} - T_s)}{\mu_l h'_{fg} (v_l^2/g)^{1/3}} Pr_l^{0.5} - 151 Pr_l^{0.5} + 253 \right]^{4/3} \quad Re_\delta \geq 1800 \text{ (Turbulent)}$$

Calculate all three values of Re_δ , decide which one is within the correct range and then substitute in the average convection coefficient.

Liquid properties are estimated at $T_f = \frac{(T_s + T_{sat})}{2}$
 Vapor properties as well as h'_{fg} are obtained at T_{sat}

Film Condensation on Radial Systems: Correlations for Laminar Flow

Single Tube

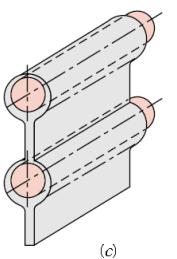


$$\overline{h_D} = C \left[\frac{g \rho_l (\rho_l - \rho_v) k_l^3 h'_{fg}}{\mu_l (T_{sat} - T_s) D} \right]^{1/4}$$

Where $C = 0.862$ for a sphere

$C = 0.729$ for a cylindrical tube

Bank of Tubes



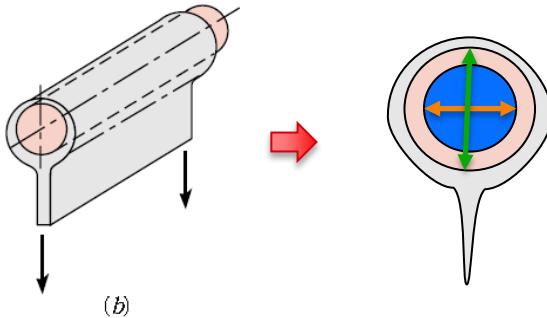
$$\overline{h_D} = 0.729 \left[\frac{g \rho_l (\rho_l - \rho_v) k_l^3 h'_{fg}}{N \mu_l (T_{sat} - T_s) D} \right]^{1/4}$$

Where N =number of tubes in a column

Reduction in h as N increases due to increasing thickness of the film.

The condensed mass-flow rate per unit length of a single tubes is: $\dot{m}'_1 = \frac{Q'}{h'_{fg}} = \frac{\overline{h_D} (\pi D) (T_{sat} - T_s)}{h'_{fg}}$

Film Condensation: Overall Heat Transfer Coefficient



The total heat transfer is equal to:

$$Q = \frac{(T_{sat} - T_m)}{R_{tot}} = U_{out}A_{out}(T_{sat} - T_m) = U_{in}A_{in}(T_{sat} - T_m)$$

Where: T_m = average temperature of the internal fluid

$$R_{tot} = R_{conv,in} + R_{cond} + R_{conv,out}$$

$$U_{out}A_{out} = U_{in}A_{in} = 1/R_{tot} \quad \left. \begin{array}{l} A_{out} = \pi D_{out} L \\ A_{in} = \pi D_{in} L \end{array} \right\} \quad U_{out} = \frac{1}{A_{out}R_{tot}} \neq U_{in} = \frac{1}{A_{in}R_{tot}}$$

The total condensation rate is: $\dot{m} = \frac{Q}{h'_{fg}} = \frac{UA(T_{sat} - T_m)}{h'_{fg}}$

If the total length of the tubes is unknown we can calculate the heat transfer rate and the mass condensation rate per unit length.