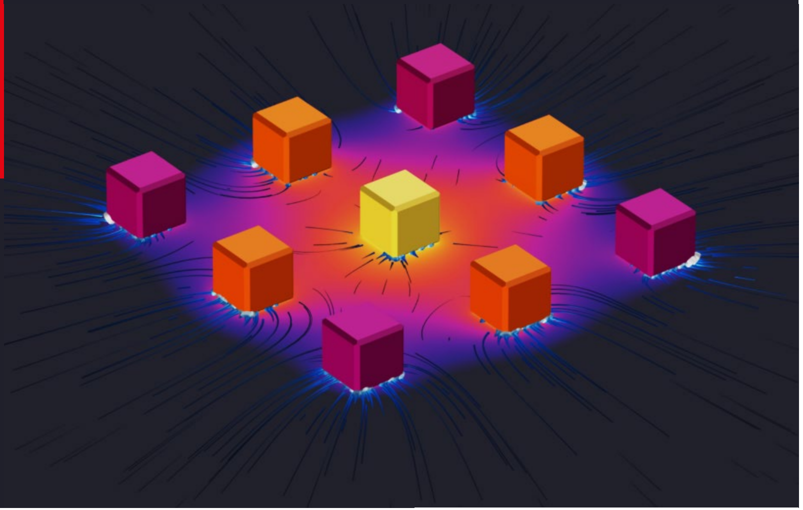
Heat and Mass Transfer
ME-341

Instructor: Giulia Tagliabue



École
polytechnique
fédérale
de Lausanne

Spring Semester

Previously

- ☑ Introduction to Free Convection
- ☑ Governing Equations of Free Convection
- ☑ Grashof number and Rayleigh number

Learning Objectives:

- ☑ Understand free convection
- ☑ Derive the equation of free convection

Introduction to Convection

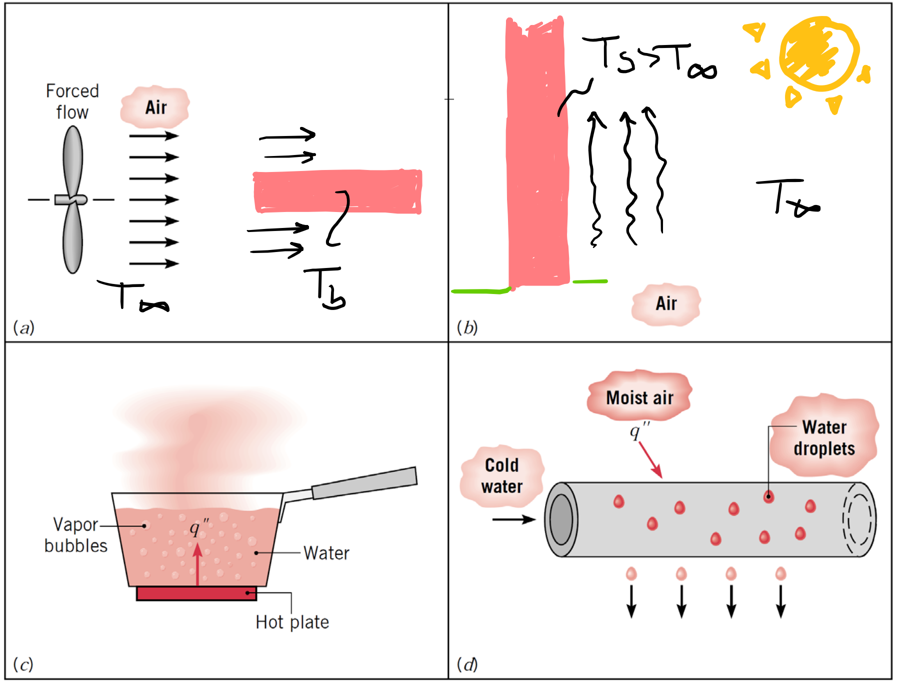
Convection refers to the heat transfer between a **solid** and a **fluid in motion** when they are at **different temperatures**.

1. Forced Convection

2. Natural (Free) Convection

3. Boiling

4. Condensation



Introduction to Free Convection

Convection refers to the heat transfer between a **solid** and a **fluid in motion** when they are at **different temperatures**.

Forced convection occurs when we can impose **the initial velocity** (or mass flow rate) and **temperature** of the fluid.

A solid is in contact with a fluid at a different temperature. Free convection occurs when temperature-driven density changes result in a net-buoyancy force that sets the fluid in motion.

- The velocity and the temperature of the fluid are controlled by the heat exchange with the solid.

- Free (Natural) Convection

Free Convection

Free convection occurs when temperature-driven density changes result in a net-buoyancy force that sets the fluid in motion.

FLUID DYNAMICS

Mass conservation → Continuity equation
Momentum conservation → Navier-Stokes equations

Flow condition (Laminar/turbulent) → $\boldsymbol{Re}$

HEAT TRANSFER

Energy conservation → 1st Law of Thermodynamics

Boundary Conditions (Temperature)
$\boldsymbol{Pr}$

Velocity profile: $\vec{\boldsymbol{u}}(\boldsymbol{x}, \boldsymbol{y})$

Temperature profile: $\boldsymbol{T}(\boldsymbol{x}, \boldsymbol{y})$

Heat transfer includes advection!

Temperature-driven fluid-motion!

- Shear stress τ_w
- Friction coefficient C_f
- Friction factor f

No slip condition $u(x, 0) = 0$

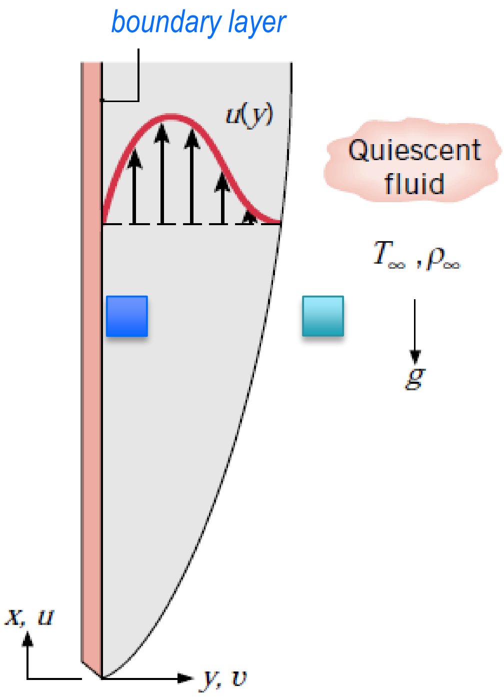
$Q_{conv} = Q_{cond,wall}$

Transport Laws (Newton/Fourier)

$$h(T_s - T_\infty) = -k_f \left.\frac{\partial \boldsymbol{T}}{\partial \boldsymbol{y}}\right|_{y=0}$$

$\boldsymbol{Nu}$

Governing Equations



$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

(Mass conservation)

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = g\beta(T - T_\infty) + \nu\frac{\partial^2 u}{\partial y^2}$$

(Momentum conservation)

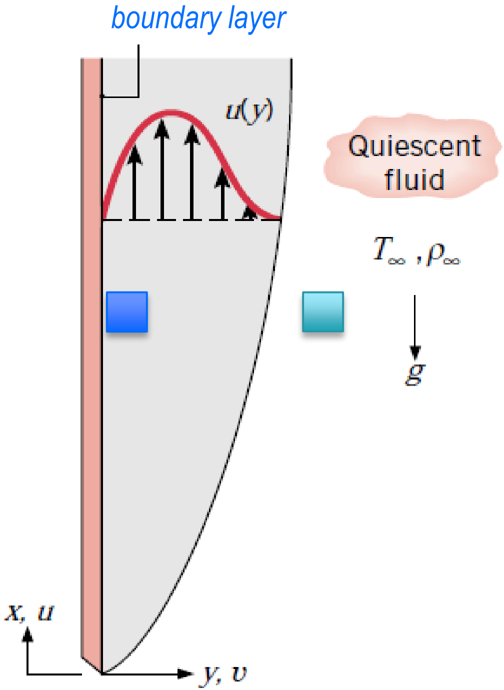
$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}$$

(Energy conservation)

Temperature-driven fluid-motion!

Heat transfer includes advection!

Governing Equations – Grashof Number



Non-dimensional variables: $x^* = \frac{x}{L}$ $y^* = \frac{y}{L}$ $u^* = \frac{u}{u_0}$ $v^* = \frac{v}{u_0}$ $T^* = \frac{T - T_s}{T_\infty - T_s}$

$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = T^* + \frac{1}{\sqrt{Gr_L}} \frac{\partial^2 u^*}{\partial {y^*}^2}$$

(Momentum conservation)

$$u^* \frac{\partial T^*}{\partial x^*} + v^* \frac{\partial T^*}{\partial y^*} = \frac{1}{\sqrt{Gr_L} Pr} \frac{\partial^2 T^*}{\partial {y^*}^2}$$

(Energy conservation)

$$Gr_x \equiv \frac{g\beta(T_s - T_\infty)x^3}{\nu^2}$$

Grashof number – replaces Re in free convection problems (it accounts for thermal effects on the flow)

$$Ra_x \equiv Gr_x Pr = \frac{g\beta(T_s - T_\infty)x^3}{\nu\alpha}$$

Rayleigh number – determines the flow conditions

$Ra_x < 10^9$ → Laminar Flow

$Ra_x > 10^9$ → Turbulent Flow

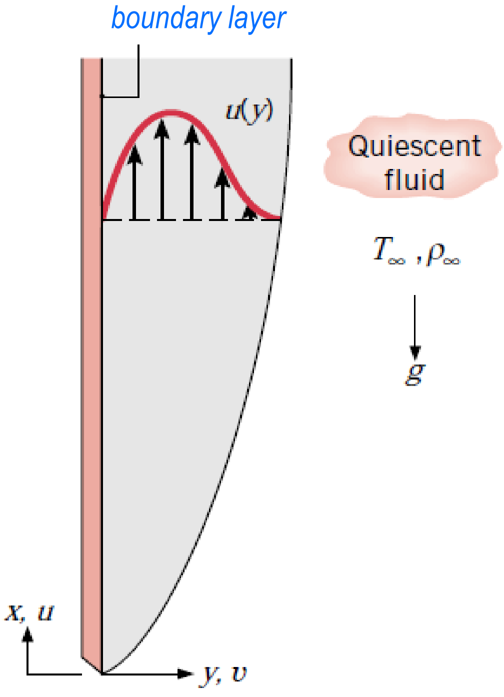
This Lecture

- ❑ Free Convection on a Vertical Plate
- ❑ Correlations for Free Convection

Learning Objectives:

- ❑ Calculate the convection coefficient for free convection

Free Convection on a Vertical Plate – Laminar Flow



(Mass conservation) $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

(Momentum conservation) $$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = g\beta(T - T_\infty) + \nu\frac{\partial^2 u}{\partial y^2}$$

(Energy conservation) $$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}$$

Boundary conditions:

$y = 0$: $u = v = 0$ $T = T_s$ **Temperature BC**

$y \to \infty$: $u \to 0$ $T \to T_\infty$

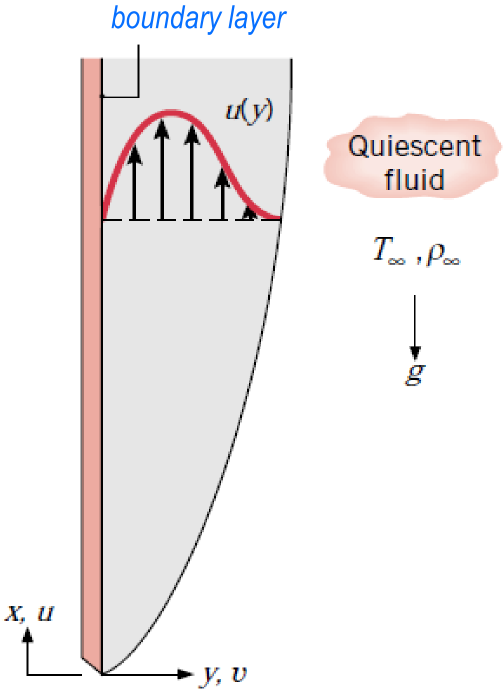
A solution to the equations can be obtained identifying a similarity parameter η and a single-variable function $f(\eta)$ that are related via the streamline function $\psi(x, y)$:

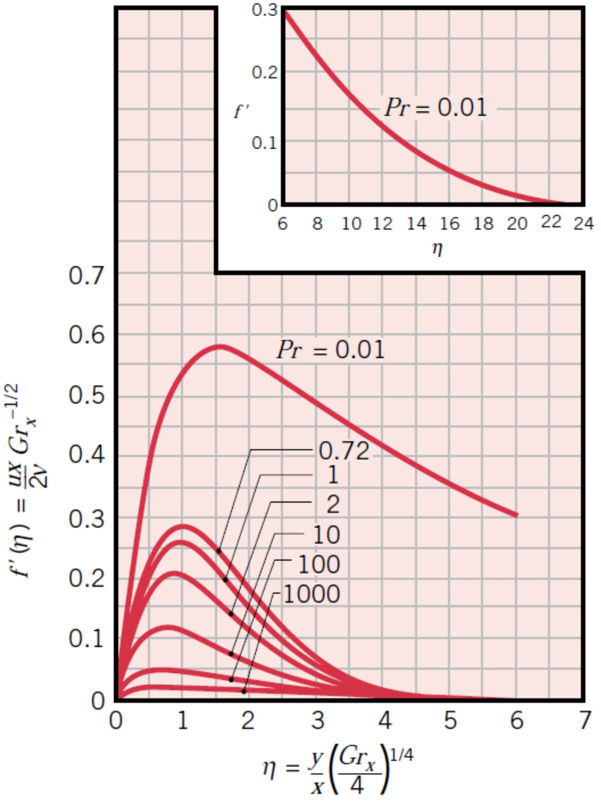
$$\eta \equiv \frac{y}{x}\left(\frac{Gr_x}{4}\right)^{1/4} \qquad \psi(x,y) \equiv f(\eta)\left[4\nu\left(\frac{Gr_x}{4}\right)^{1/4}\right]$$

$$f''' + 3ff'' - 2(f')^2 + T^* = 0$$ *(Momentum conservation)*

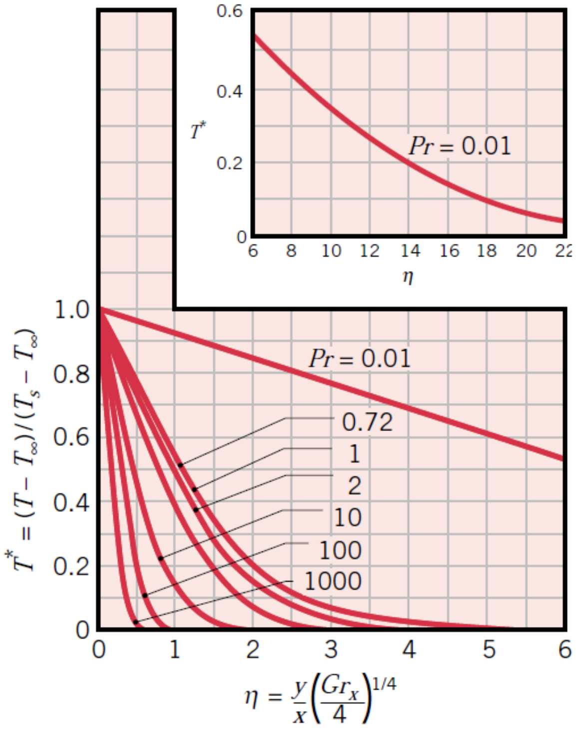
$$T^{*''} + 3Pr\, f\, T^{*'} = 0$$ *(Energy conservation)*

Free Convection on a Vertical Plate – Laminar Flow





FLUID DYNAMICS



HEAT TRANSFER

Free Convection

Free convection occurs when temperature-driven density changes result in a net-buoyancy force that sets the fluid in motion.

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Transport Laws (Newton/Fourier)

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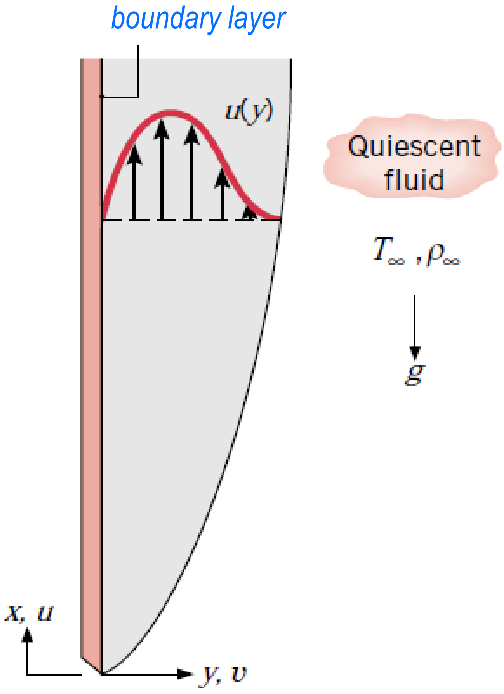
$\boldsymbol{Nu}$

Free Convection on a Vertical Plate – Laminar Flow



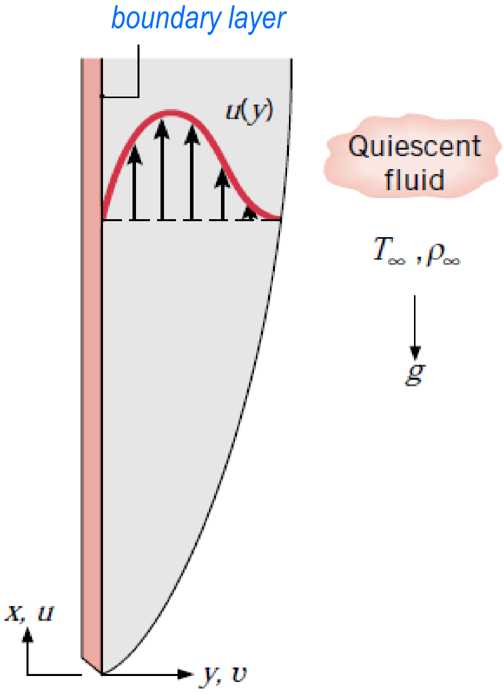
$$q_s'' = -k\frac{\partial T}{\partial y}\bigg|_{y=0} = -\frac{k}{x}(T_s - T_\infty)\left(\frac{Gr_x}{4}\right)^{1/4}\frac{dT^*}{d\eta}\bigg|_{\eta=0} = h(T_s - T_\infty)$$

$$Nu_x = \frac{hx}{k_f} = -\left(\frac{Gr_x}{4}\right)^{1/4}\frac{dT^*}{d\eta}\bigg|_{\eta=0}$$

Local coefficient:

$$Nu_x = \frac{hx}{k_f} = \left(\frac{Gr_x}{4}\right)^{1/4} g(Pr) \qquad g(Pr) = \frac{0.75Pr^{1/2}}{(0.609 + 1.221Pr^{1/2} + 1.238Pr)^{1/4}}$$

Free Convection on a Vertical Plate – Average h



Average coefficient (laminar and turbulent):

$$\overline{Nu_L} = \left\{0.825 + \frac{0.387\,Ra_L^{1/6}}{[1 + (0.492/Pr)^{9/16}]^{8/27}}\right\}^2$$

Where we use $$T_f = \frac{(T_s + T_\infty)}{2}$$

This Lecture

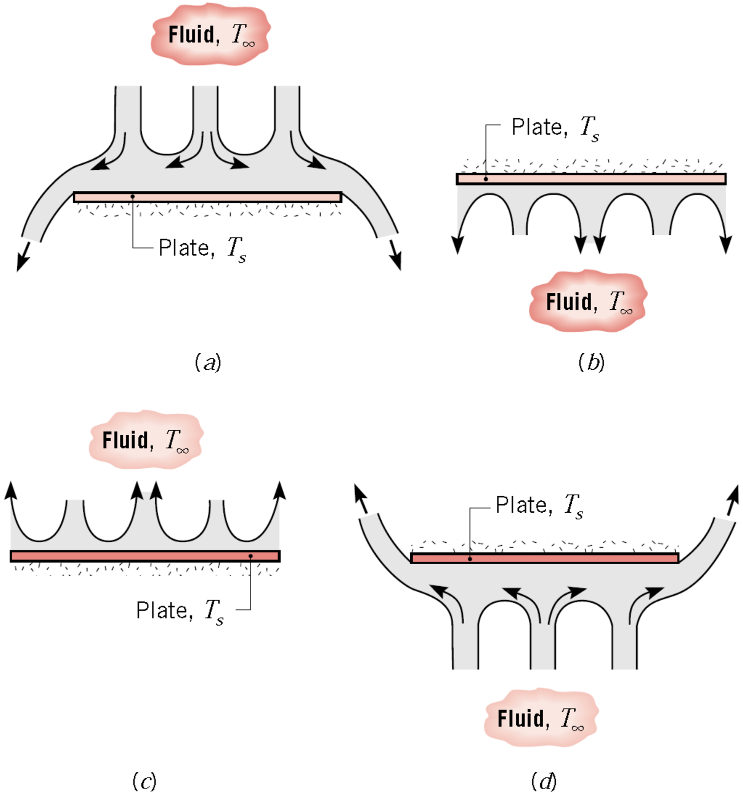
- ☑ Free Convection on a Vertical Plate
- ❑ Correlations for Free Convection

Learning Objectives:

- ❑ Calculate the convection coefficient for free convection

Correlations for Natural Convection – External Flows

A. Horizontal Plates



$$L \equiv \frac{A_s}{P}$$

Upper Surface of Hot Plate or Lower Surface of Cold Plate:

$$\overline{Nu}_L = 0.54 Ra_L^{1/4} \quad (10^4 \lesssim Ra_L \lesssim 10^7) \tag{9.30}$$

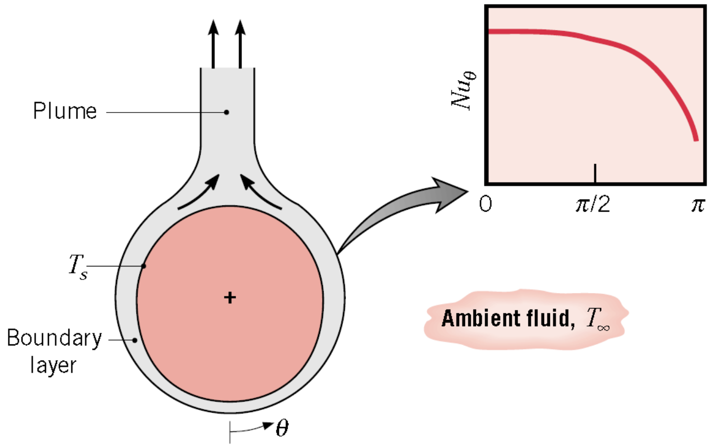
$$\overline{Nu}_L = 0.15 Ra_L^{1/3} \quad (10^7 \lesssim Ra_L \lesssim 10^{11}) \tag{9.31}$$

Lower Surface of Hot Plate or Upper Surface of Cold Plate:

$$\overline{Nu}_L = 0.27 Ra_L^{1/4} \quad (10^5 \lesssim Ra_L \lesssim 10^{10}) \tag{9.32}$$

Correlations for Natural Convection – External Flows

B. Long horizontal Cylinder



$$\overline{Nu}_D = \left\{ 0.60 + \frac{0.387\, Ra_D^{1/6}}{\left[1 + (0.559/Pr)^{9/16}\right]^{8/27}} \right\}^2 \qquad Ra_D \lesssim 10^{12} \qquad (9.34)$$

This Lecture

- ☑ Free Convection on a Vertical Plate
- ☑ Correlations for Free Convection

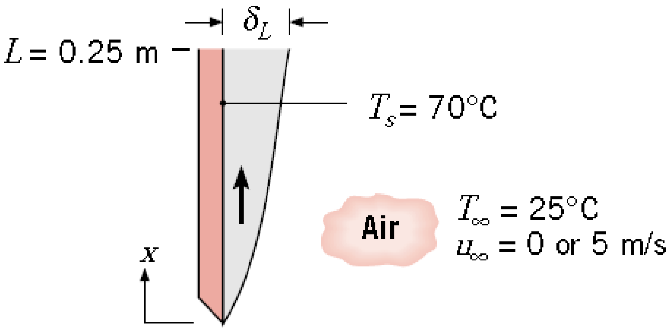
Learning Objectives:

- ❑ Calculate the convection coefficient for free convection

General methodology for calculating the convection coefficient

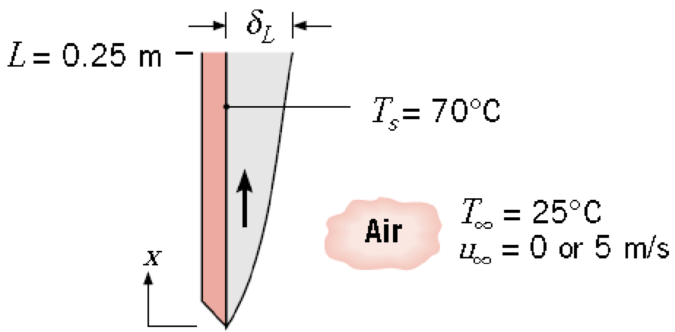
0. Identify the type of convection (Forced/External, Forced/Internal, Natural, Boiling/Condensation) **[Conv]**
1. Recognize the flow geometry (plate, cylinder, inner/outer flow etc.) **[GEOM]**
2. Specify the appropriate reference temperature and evaluate the pertinent fluid properties at that temperature $\boldsymbol{T_f}$
3. Calculate the Reynolds/Rayleigh number (be careful to use the right characteristic dimension - x, L, D – and velocity - u_m, u_∞) and determine the flow conditions (laminar/turbulent) **[FLOW]**
4. Decide whether a local or surface average coefficient is required **[Loc/Ave]**
5. Calculate Pr or get it from the table **[Pr]**
6. Select the appropriate correlation, determine Nu and the convection coefficient **[Nu, h]**

Free Convection - Example



Consider a 0.25-m-long vertical plate that is at 70°C. The plate is suspended in air that is at 25°C. Estimate the boundary layer thickness at the trailing edge of the plate if the air is quiescent. How does this thickness compare with that which would exist if the air were flowing over the plate at a free stream velocity of 5 m/s?

Free Convection - Example



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[Conv]

a) Free Convection (air is quiescent)

b) Forced Convection (there is an externally imposed flow velocity)

[GEOM]: Vertical plate

$$T_f = \frac{(T_s + T_\infty)}{2}$$

→ ***Properties:*** Table A.4, air ($T_f = 320.5$ K): $\nu = 17.95 \times 10^{-6}$ m²/s, $Pr = 0.7$, $\beta = T_f^{-1} = 3.12 \times 10^{-3}\ \mathrm{K^{-1}}$.

[FLOW]

a) Rayleigh

b) Reynolds

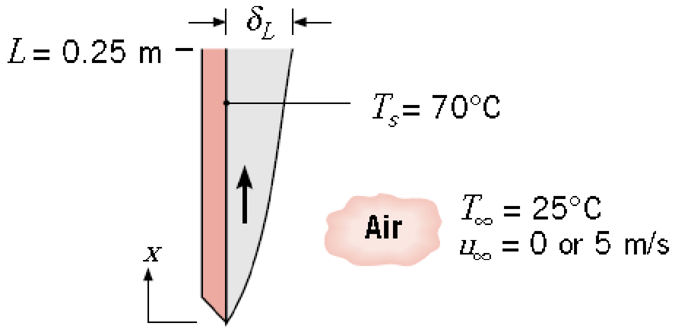
$$Gr_L = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}$$

$$= \frac{9.8\ \mathrm{m/s^2} \times (3.12 \times 10^{-3}\ \mathrm{K^{-1}})(70 - 25)°\mathrm{C}(0.25\ \mathrm{m})^3}{(17.95 \times 10^{-6}\ \mathrm{m^2/s})^2} = 6.69 \times 10^7$$

→ $Ra_L = Gr_L\, Pr = 4.68 \times 10^7 < 10^9$

Laminar Flow

Free Convection - Example



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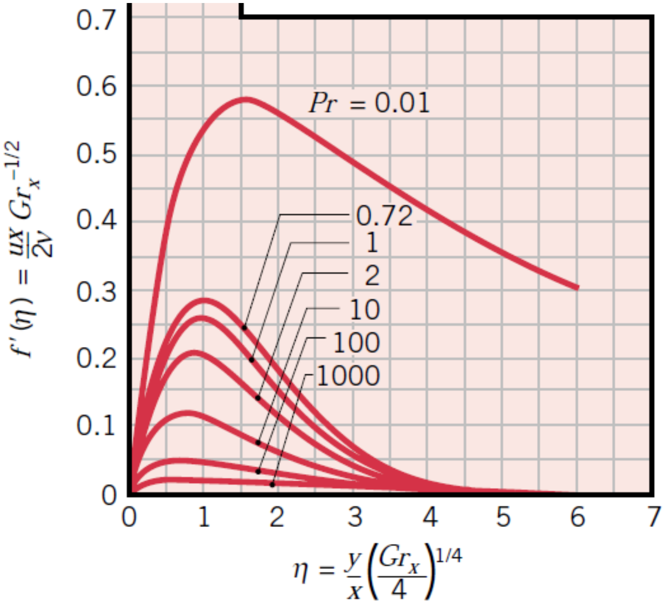
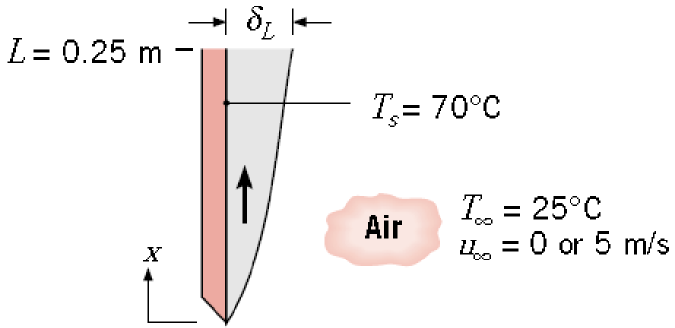


The boundary layer ends where $f'(\eta) \sim 0$

$Pr = 0.7$, ➡ $\eta \equiv \frac{y}{x}\left(\frac{Gr_x}{4}\right)^{\frac{1}{4}} \sim 6$ ➡ $\delta_L = y(\eta_{x=L} \sim 6)$

$$\delta_L \approx \frac{6L}{(Gr_L/4)^{1/4}} = \frac{6(0.25\text{ m})}{(1.67 \times 10^7)^{1/4}} = 0.024\text{ m}$$

Free Convection - Example



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[Conv]

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b) Forced Convection (there is an externally imposed flow velocity)

[GEOM]: Vertical plate

$$T_f = \frac{(T_s + T_\infty)}{2}$$

Properties: Table A.4, air (T_f = 320.5 K): $\nu = 17.95 \times 10^{-6}$ m^2/s, $Pr = 0.7$, $\beta = T_f^{-1} = 3.12 \times 10^{-3}$ K^{-1}.

[FLOW]

a) Rayleigh

b) Reynolds

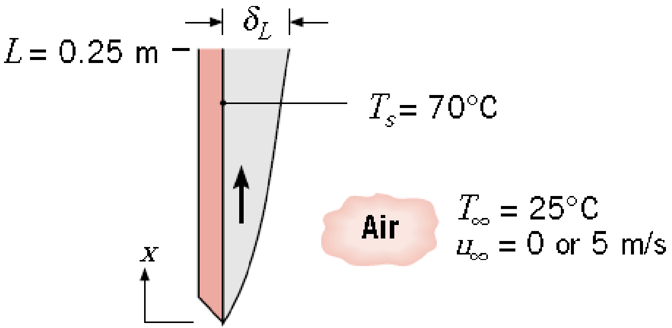
$$Re_L = \frac{u_\infty L}{\nu} = \frac{(5 \text{ m/s}) \times 0.25 \text{ m}}{17.95 \times 10^{-6} \text{ m}^2/\text{s}} = 6.97 \times 10^4$$

$$Re_L < 5 \cdot 10^5$$

Laminar Flow

Free Convection - Example

Consider a 0.25-m-long vertical plate that is at 70°C. The plate is suspended in air that is at 25°C. Estimate the boundary layer thickness at the trailing edge of the plate if the air is quiescent. How does this thickness compare with that which would exist if the air were flowing over the plate at a free stream velocity of 5 m/s?



For the laminar boundary layer over a plate we recall (W5L1 – slide 20)

$$\frac{\delta}{x} = \frac{4.92}{\sqrt{Re_x}} \Rightarrow \delta_L \approx \frac{5L}{Re_L^{1/2}} = \frac{5(0.25\text{ m})}{(6.97 \times 10^4)^{1/2}} = 0.0047\text{ m}$$

Observe that the boundary layer thickness for free convection is much larger than for forced convection. This is often the case.

This Lecture

- ☑ Free Convection on a Vertical Plate
- ☑ Correlations for Free Convection

Learning Objectives:

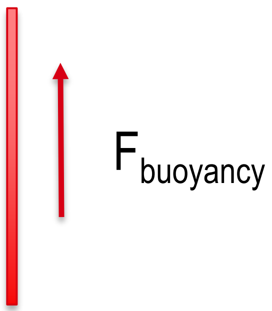
- ☑ Calculate the convection coefficient for free convection

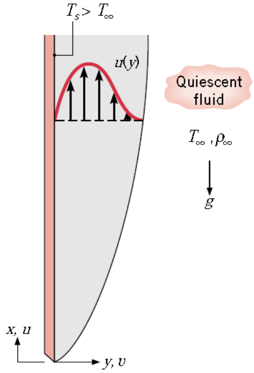
Supplementary Slides

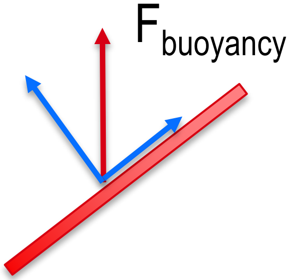
Correlations for Natural Convection – External Flows

C. Inclined Plates

For a vertical plate, heated (or cooled) relative to an ambient fluid, the plate is aligned with the gravitational vector, and the buoyancy force acts exclusively to induce fluid motion in the upward (or downward) direction. However, if the plate is inclined with respect to gravity, the buoyancy force has a component normal, as well as parallel, to the plate surface. With a reduction in the buoyancy force parallel to the surface, there is a reduction in fluid velocities along the plate, and one might expect there to be an attendant reduction in convection heat transfer. Whether, in fact, there is such a reduction depends on whether one is interested in heat transfer from the top or bottom surface of the plate.







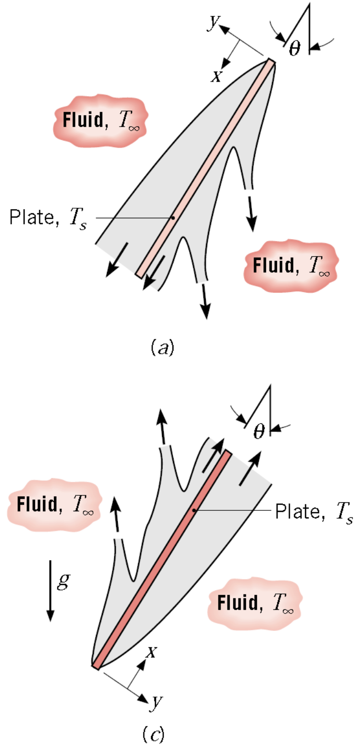
$u_{//} < u(y)_{max}$

?

Correlations for Natural Convection – External Flows

C. Inclined Plates

As shown in Figure 9.6*a*, if the plate is cooled, the *y* component of the buoyancy force, which is normal to the plate, acts to maintain the descending boundary layer flow in contact with the top surface of the plate. Since the *x* component of the gravitational acceleration is reduced to $g \cos \theta$, fluid velocities along the plate are reduced and there is an attendant reduction in convection heat transfer to the top surface. However, at the bottom surface, the *y* component of the buoyancy force acts to move fluid from the surface, and boundary layer development is interrupted by the discharge of parcels of cool fluid from the surface (Figure 9.6*a*). The resulting flow is three-dimensional, and, as shown by the spanwise (*z*-direction) variations of Figure 9.6*b*, the cool fluid discharged from the bottom surface is continuously replaced by the warmer ambient fluid. The displacement of cool boundary layer fluid by the warmer ambient and the attendant reduction in the thermal boundary layer thickness act to increase convection heat transfer to the bottom surface. In fact, heat transfer enhancement due to the three-dimensional flow typically exceeds the reduction associated with the reduced *x* component of *g*, and the combined effect is to increase heat transfer to the bottom surface. Similar trends characterize a heated plate (Figure 9.6*c,d*), and the three-dimensional flow is now associated with the upper surface, from which parcels of warm fluid are discharged.



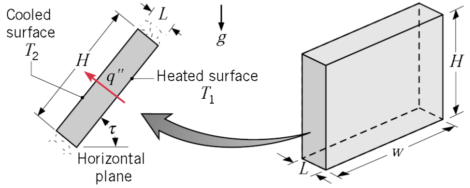
Break-up of the boundary layer enhances heat transfer!!

For θ<60° for the top and bottom surfaces of a cooling or heating plate, respectively, use equation of vertical plate (slide 23) but with:

$$g' = g cos\theta$$

Correlations for Natural Convection – Enclosures

D. Rectangular Cavities



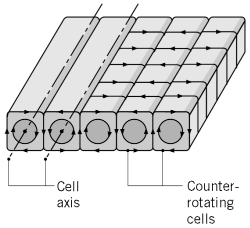
average temperature, $\overline{T} \equiv (T_1 + T_2)/2.$

a. Horizontal cavities (τ = 0 , T1 > T2)

$$Ra_L \equiv \frac{g\beta(T_1 - T_2)L^3}{\alpha\nu} < 1708$$

No onset of flow (only conduction)

$$1708 < Ra_L \lesssim 5 \times 10^4,$$



$Ra_L > 5 \times 10^4$

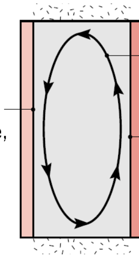
Turbulent flow

$$\overline{Nu}_L = \frac{\overline{h}L}{k} = 0.069 Ra_L^{1/3}\, Pr^{0.074} \qquad 3 \times 10^5 \lesssim Ra_L \lesssim 7 \times 10^9 \qquad (9.49)$$

Note: for τ = 180 , no flow is created and only conduction occurs

Correlations for Natural Convection – Enclosures

b. Vertical cavities (τ = 90)



In the vertical rectangular cavity ($\tau = 90°$), the vertical surfaces are heated and cooled, while the horizontal surfaces are adiabatic. As shown in Figure 9.12, fluid motion is characterized by a recirculating or cellular flow for which fluid ascends along the hot wall and descends along the cold wall. For small Rayleigh numbers, $Ra_L \lesssim 10^3$, the buoyancy-driven flow is weak and heat transfer is primarily by conduction across the fluid. Hence, from Fourier's law, the Nusselt number is again $Nu_L = 1$. With increasing Rayleigh number, the cellular flow intensifies and becomes concentrated in thin boundary layers adjoining the sidewalls. The core becomes nearly stagnant, although additional cells can develop in the corners and the sidewall boundary layers eventually undergo transition to turbulence. For aspect ratios in the range $1 \lesssim (H/L) \lesssim 10$, the following correlations have been suggested [26]:

$$\overline{Nu}_L = 0.22\left(\frac{Pr}{0.2 + Pr} Ra_L\right)^{0.28}\left(\frac{H}{L}\right)^{-1/4} \quad (9.50)$$

$$\left[\begin{array}{l} 2 \lesssim \frac{H}{L} \lesssim 10 \\ Pr \lesssim 10^5 \\ 10^3 \lesssim Ra_L \lesssim 10^{10} \end{array}\right]$$

$$\overline{Nu}_L = 0.18\left(\frac{Pr}{0.2 + Pr} Ra_L\right)^{0.29} \quad (9.51)$$

$$\left[\begin{array}{l} 1 \lesssim \frac{H}{L} \lesssim 2 \\ 10^{-3} \lesssim Pr \lesssim 10^5 \\ 10^3 \lesssim \frac{Ra_L Pr}{0.2 + Pr} \end{array}\right]$$

mean temperature, $(T_1 + T_2)/2$.

while for larger aspect ratios, the following correlations have been proposed [30]:

$$\overline{Nu}_L = 0.42 Ra_L^{1/4} Pr^{0.012}\left(\frac{H}{L}\right)^{-0.3} \quad \left[\begin{array}{l} 10 \lesssim \frac{H}{L} \lesssim 40 \\ 1 \lesssim Pr \lesssim 2 \times 10^4 \\ 10^4 \lesssim Ra_L \lesssim 10^7 \end{array}\right] \quad (9.52)$$

$$\overline{Nu}_L = 0.046 Ra_L^{1/3} \quad \left[\begin{array}{l} 1 \lesssim \frac{H}{L} \lesssim 40 \\ 1 \lesssim Pr \lesssim 20 \\ 10^6 \lesssim Ra_L \lesssim 10^9 \end{array}\right] \quad (9.53)$$

Correlations for Natural Convection – Enclosures

E. Concentric Cylinders

Free convection heat transfer in the annular space between *long,* horizontal concentric cylinders (Figure 9.13) has been considered by Raithby and Hollands [37]. Flow in the annular region is characterized by two cells that are symmetric about the vertical midplane. If the inner cylinder is heated and the outer cylinder is cooled ($T_i > T_o$), fluid ascends and descends along the inner and outer cylinders, respectively. If $T_i < T_o$, the cellular flows are reversed. The heat transfer rate (W) between the two cylinders, each of length L, is expressed by Equation 3.27 (with an *effective thermal conductivity,* k_{eff}, replacing the molecular thermal conductivity, k) as

$$q = \frac{2\pi L k_{\text{eff}} (T_i - T_o)}{\ln(r_o/r_i)} \qquad (9.58)$$

We see that the effective conductivity of a fictitious *stationary* fluid will transfer the same amount of heat as the actual *moving* fluid. The suggested correlation for k_{eff} is

$$\frac{k_{\text{eff}}}{k} = 0.386\left(\frac{Pr}{0.861 + Pr}\right)^{1/4} Ra_c^{1/4} \qquad (9.59)$$

where the length scale in Ra_c is given by

$$L_c = \frac{2[\ln(r_o/r_i)]^{4/3}}{(r_i^{-3/5} + r_o^{-3/5})^{5/3}} \qquad (9.60)$$

Equation 9.59 may be used for the range $0.7 \lesssim Pr \lesssim 6000$ and $Ra_c \lesssim 10^7$. Properties are evaluated at the mean temperature, $T_m = (T_i + T_o)/2$. Of course, the minimum heat transfer rate between the cylinders cannot fall below the conduction limit; therefore, $k_{\text{eff}} = k$ if the value of k_{eff}/k predicted by Equation 9.59 is less than unity. A more detailed correlation, which accounts for cylinder eccentricity effects, has been developed by Kuehn and Goldstein [38].