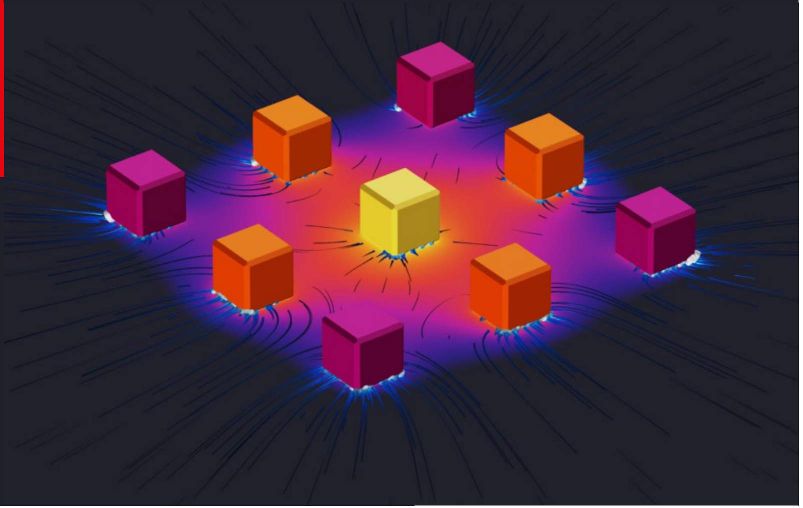
Heat and Mass Transfer
ME-341

Instructor: Giulia Tagliabue



École
polytechnique
fédérale
de Lausanne

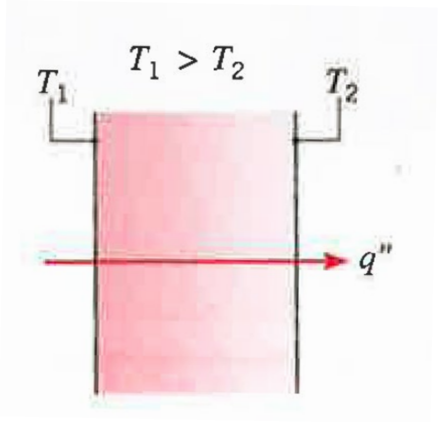
Spring Semester

Previously

- ☑ Heat Conduction
- ☑ Introduction to Convection
    - ☑ Forced Convection
        - ☑ External Forced Convection
        - ☑ Internal Forced Convection

Transport Laws

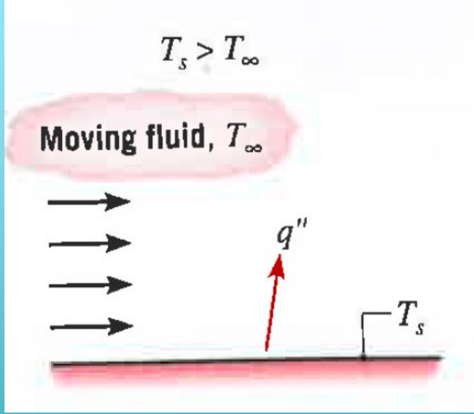
Conduction



Fourier's Law

$$q'' = -k\frac{dT}{dx}$$

Convection



Newton's Law

$$q'' = \bar{h}\,(T_s - T_\infty)$$

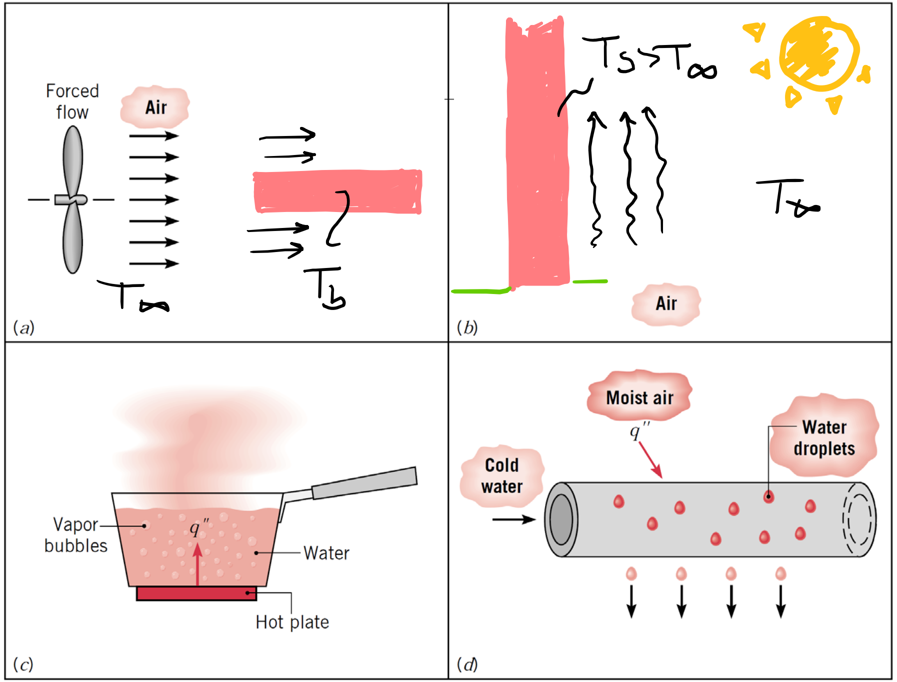
Initially it was only a boundary condition and h was given, now we want to calculate it.

Introduction to Convection

Convection refers to the heat transfer between a **solid** and a **fluid in motion** when they are at **different temperatures**.

1. Forced Convection

We impose the velocity/ temperature of the fluid



2. Natural (Free) Convection

3. Boiling

4. Condensation

Forced Convection

Convection refers to the heat transfer between a **solid** and a **fluid in motion** when they are at **different temperatures**.

Forced convection occurs when we can impose **the initial velocity** (or mass flow rate) and **temperature** of the fluid.

FLUID DYNAMICS

Mass conservation → Continuity equation
Momentum conservation → Navier-Stokes equations

Flow condition (Laminar/turbulent) → $\boldsymbol{Re}$

Velocity profile: $\vec{\boldsymbol{u}}(\boldsymbol{x},\boldsymbol{y})$

- Shear stress τ_w
- Friction coefficient C_f
- Friction factor f

HEAT TRANSFER

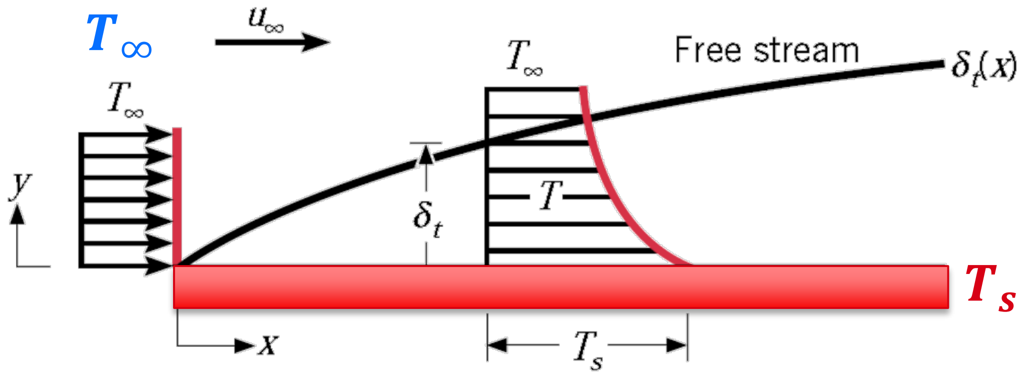
Energy conservation → 1st Law of Thermodynamics

Boundary Conditions (Heat flux/Temperature)
$\boldsymbol{Pr}$

Heat transfer includes advection!

Temperature profile: $\boldsymbol{T}(\boldsymbol{x},\boldsymbol{y})$

Forced Convection – Boundary Layer Equations



$$T_s > T_\infty$$

$$h = \frac{-k_f \left.\frac{\partial T}{\partial y}\right|_{y=0}}{(T_s - T_\infty)}$$

- ❑ Conservation of mass $\quad \dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} = 0$
- ❑ Conservation of momentum $\quad u\dfrac{\partial u}{\partial x} + v\dfrac{\partial u}{\partial y} = -\dfrac{1}{\rho}\dfrac{\partial p}{\partial x} + \nu\dfrac{\partial^2 u}{\partial y^2}$
- ❑ Conservation of energy $\quad u\dfrac{\partial T}{\partial x} + v\dfrac{\partial T}{\partial y} = \alpha\dfrac{\partial^2 T}{\partial y^2}$

Velocity boundary layer

The geometry and flow characteristics are critical for convective heat transfer

Thermal boundary layer

Forced Convection

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No slip condition $u(x, 0) = 0$

$$Q_{conv} = Q_{cond,wall}$$

Transport Laws (Newton/Fourier)

$$h(T_s - T_\infty) = -k_f \left.\frac{\partial \boldsymbol{T}}{\partial \boldsymbol{y}}\right|_{y=0}$$

$\boldsymbol{Nu}$

Forced Convection -

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1. Forced Convection

1. External Forced Convection:

- Horizontal plate → **Flow Condition (Re)**: Laminar vs Turbulent; **Thermal BC**: Heat Flux vs Temperature
- Cross-flow around a cylinder
- Bank of tubes

2. Internal Forced Convection

- Circular pipe
- Non-circular pipe

→ **Flow Condition (Re)**: Laminar vs Turbulent; **Thermal BC**: Heat Flux vs Temperature

Forced Convection – Laminar Flow

Fully developed flow ($x>x_{fd,t}$)

Internal Forced Convection

$Re_D<2300$

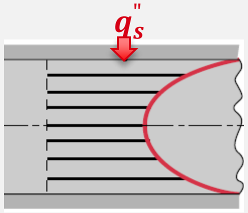
$$T_f = T_m = \frac{(T_{m,i} + T_{m,o})}{2}$$

External Forced Convection

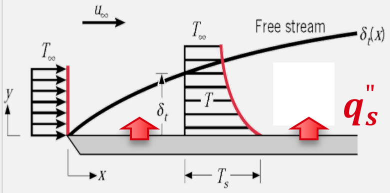
$Re_x<5 \cdot 10^5$

$$T_f = \frac{(T_s + T_\infty)}{2}$$

A. Constant heat flux

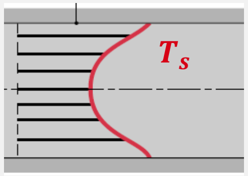


$$Nu_D = \frac{hD}{k_f} = 4.36$$

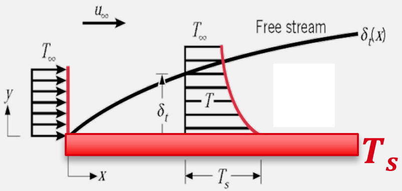


$$Nu_x = 0.435 Re_x^{1/2} Pr^{1/3}$$

B. Constant T_s



$$Nu_D = \frac{hD}{k_f} = 3.66$$



$$Nu_x = 0.332 Re_x^{1/2} Pr^{1/3}$$

$$\overline{Nu_x} = 0.664 Re_x^{1/2} Pr^{1/3}$$

This Lecture

- ❑ Introduction to Free Convection
- ❑ Governing Equations of Free Convection
- ❑ Grashof number and Rayleigh number

Learning Objectives:

- ❑ Understand free convection
- ❑ Derive the equation of free convection

Introduction to Convection

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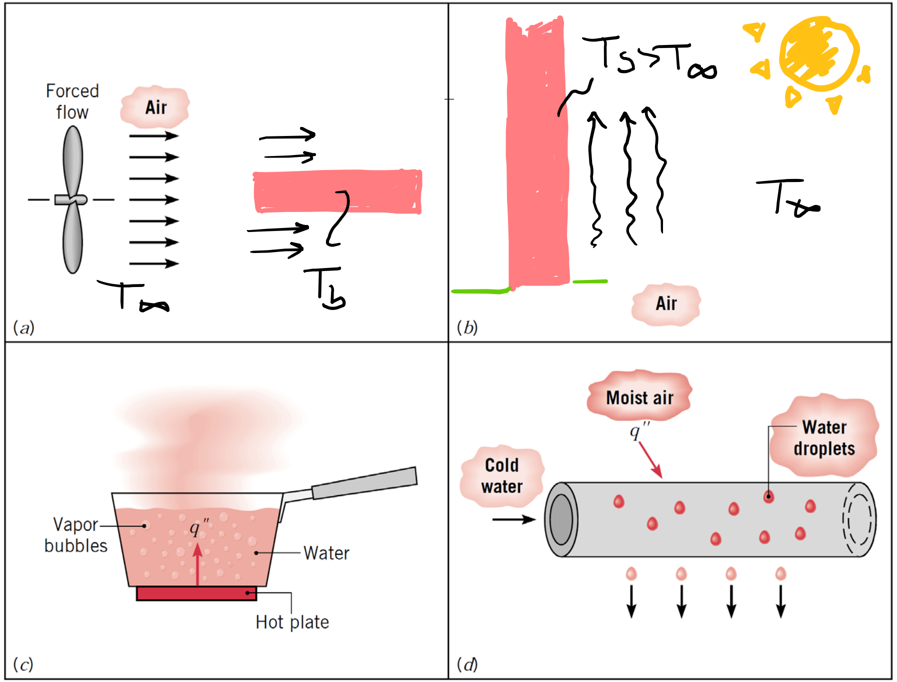
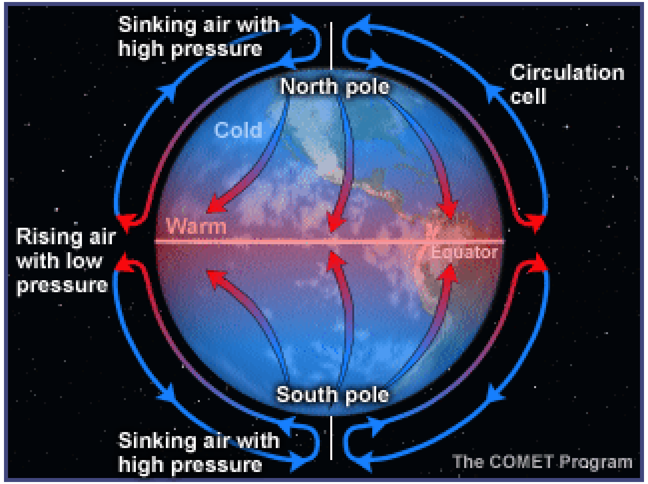


Introduction to Free Convection



Introduction to Free Convection

Convection refers to the heat transfer between a **solid** and a **fluid in motion** when they are at **different temperatures**.

Forced convection occurs when we can impose **the initial velocity** (or mass flow rate) and **temperature** of the fluid.

A solid is in contact with a fluid at a different temperature. Free convection occurs when temperature-driven density changes result in a net-buoyancy force that sets the fluid in motion.

- The velocity and the temperature of the fluid are controlled by the heat exchange with the solid.
- Free (Natural) Convection

Free Convection

Free convection occurs when temperature-driven density changes result in a net-buoyancy force that sets the fluid in motion.

FLUID DYNAMICS

Mass conservation → Continuity equation
Momentum conservation → Navier-Stokes equations

Flow condition (Laminar/turbulent) → $\boldsymbol{Re}$

HEAT TRANSFER

Energy conservation → 1st Law of Thermodynamics

Boundary Conditions (Temperature)
$\boldsymbol{Pr}$

Heat transfer includes advection!

Velocity profile: $\vec{\boldsymbol{u}}(\boldsymbol{x}, \boldsymbol{y})$

Temperature profile: $\boldsymbol{T}(\boldsymbol{x}, \boldsymbol{y})$

Temperature-driven fluid-motion!

- Shear stress τ_w
- Friction coefficient C_f
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No slip condition $u(x, 0) = 0$

$$Q_{conv} = Q_{cond,wall}$$

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$$h(T_s - T_\infty) = -k_f \left.\frac{\partial \boldsymbol{T}}{\partial \boldsymbol{y}}\right|_{y=0}$$

$\boldsymbol{Nu}$

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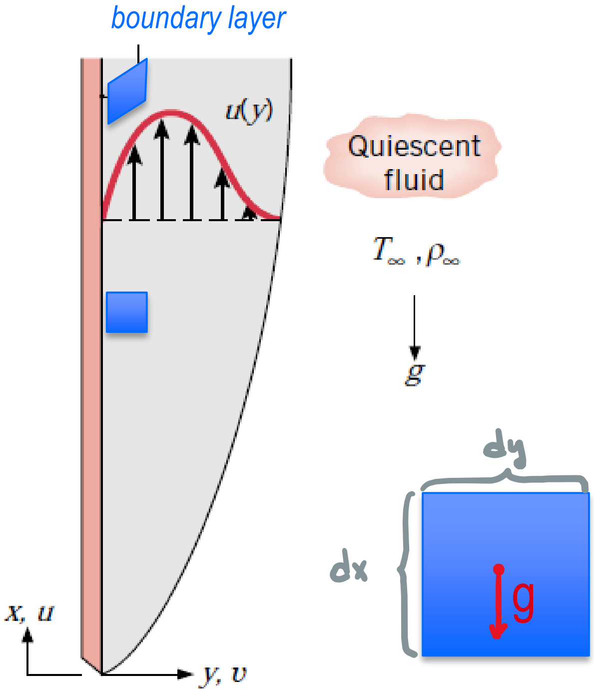
Governing Equations

FLUID DYNAMICS
(momentum conservation)



$$\sum F = \Delta momentum_{x,in-out}$$

Assumptions:

1. Steady-state
2. Gravity is along negative y-direction
3. Apart from the effect on buoyancy, the fluid is incompressible
4. Boundary layer approximation is valid

Forces F:

- Shear stresses
- Pressure
- Gravity (volume force)

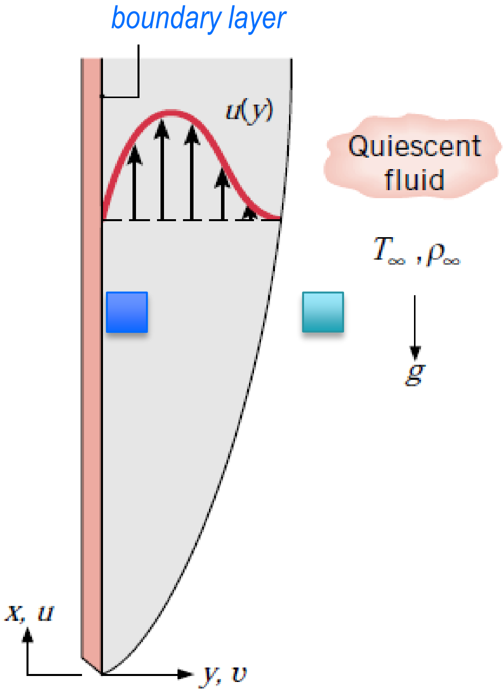
Momentum$_x$:

- $\dot{m}_x u = (\rho u dy)u$
- $\dot{m}_y u = (\rho v dx)u$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{dp_\infty}{dx} - \boldsymbol{g} + \nu\frac{\partial^2 u}{\partial y^2} \qquad (*)$$

**See appendix D2 for complete derivation* **Compare to W5L1-1h slide 16*

Governing Equations



FLUID DYNAMICS
(momentum conservation)

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{dp_\infty}{dx} - \boldsymbol{g} + \nu\frac{\partial^2 u}{\partial y^2}$$

In the **quiescent fluid**, away from the wall, $u = 0$ and $v = 0$:

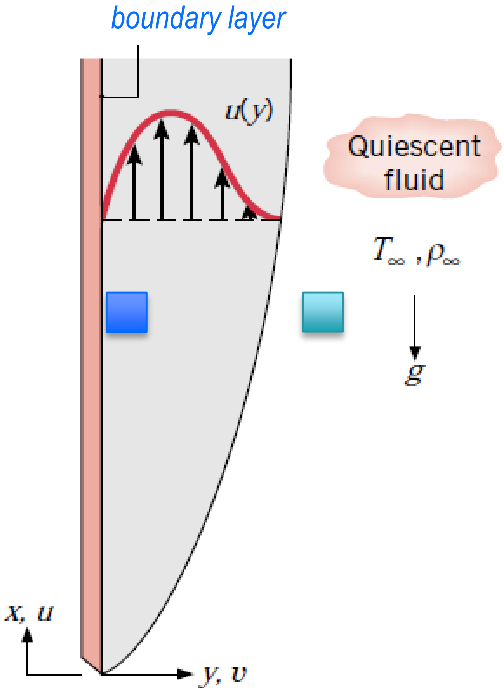
$$0 = -\frac{1}{\rho_\infty}\frac{dp_\infty}{dx} - g \quad \Rightarrow \quad \frac{dp_\infty}{dx} = -\rho_\infty g$$

Using the same pressure-gradient in the **boundary layer**:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \boldsymbol{g}\frac{\boldsymbol{\Delta\rho}}{\boldsymbol{\rho}} + \nu\frac{\partial^2 u}{\partial y^2} \qquad \Delta\rho = \rho_\infty - \rho$$

HEAT TRANSFER
(Temperature-driven density changes)

Governing Equations – Boussinesq Approximation



$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = g\frac{\Delta\rho}{\rho} + \nu\frac{\partial^2 u}{\partial y^2}$$

$$\Delta\rho = \rho_\infty - \rho$$

Volumetric thermal expansion coefficient: $\beta = -\frac{1}{\rho}\left(\frac{\partial\rho}{\partial T}\right)_p$

Boussinesq Approximation: $\beta \approx -\frac{1}{\rho}\frac{\Delta\rho}{\Delta T} = -\frac{1}{\rho}\frac{\rho_\infty - \rho}{T_\infty - T}$

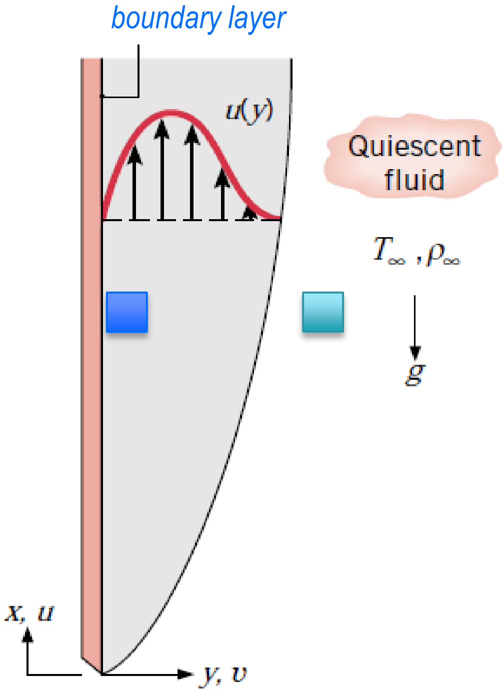
$$\Rightarrow \quad \frac{\rho_\infty - \rho}{\rho} \approx \beta(T - T_\infty)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \boldsymbol{g\beta(T - T_\infty)} + \nu\frac{\partial^2 u}{\partial y^2}$$

Ideal gas: $\beta = \frac{1}{T}$

Other fluids (water-vapor included!): *see Tables for* β

Governing Equations



$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

(Mass conservation)

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = g\beta(T - T_\infty) + \nu\frac{\partial^2 u}{\partial y^2}$$

(Momentum conservation)

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}$$

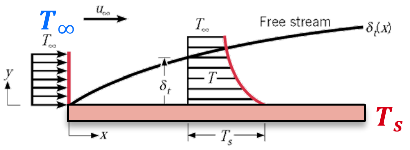
(Energy conservation)

Temperature-driven fluid-motion!

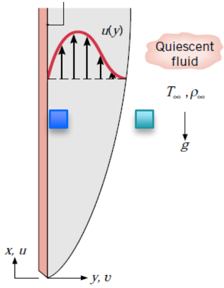
Heat transfer includes advection!

Governing Equations

Forced Convection



Free Convection



- ❑ Conservation of mass

Forced convection: $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Free convection: $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

- ❑ Conservation of momentum

Forced convection: $$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2}$$

Free convection: $$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \boldsymbol{g\beta(T - T_\infty)} + \nu\frac{\partial^2 u}{\partial y^2}$$

- ❑ Conservation of energy

Forced convection: $$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}$$

Heat transfer includes advection!

Free convection: $$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}$$

Temperature-driven fluid-motion!

Heat transfer includes advection!

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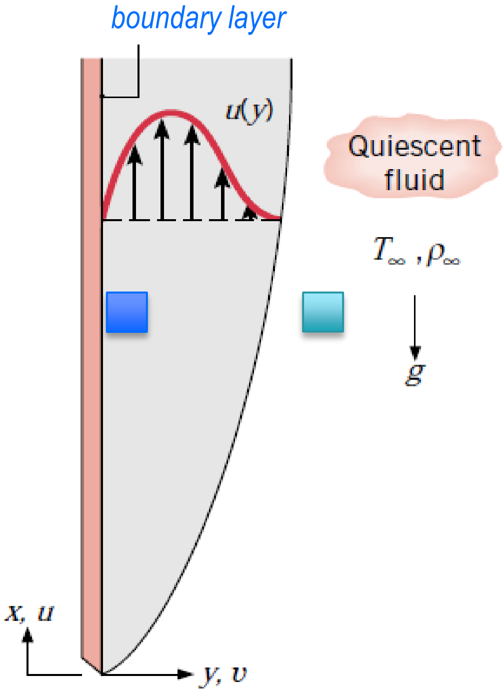
Governing Equations – Grashof Number



Non-dimensional variables: $x^* = \frac{x}{L}$ $y^* = \frac{y}{L}$ $u^* = \frac{u}{u_0}$ $v^* = \frac{v}{u_0}$ $T^* = \frac{T - T_s}{T_\infty - T_s}$

$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = \frac{g\beta(T_s - T_\infty)L}{u_0^2} T^* + \frac{1}{Re_L} \frac{\partial^2 u^*}{\partial y^{*2}}$$

(Momentum conservation)

$$u^* \frac{\partial T^*}{\partial x^*} + v^* \frac{\partial T^*}{\partial y^*} = \frac{1}{Re_L Pr} \frac{\partial^2 T^*}{\partial y^{*2}}$$

(Energy conservation)

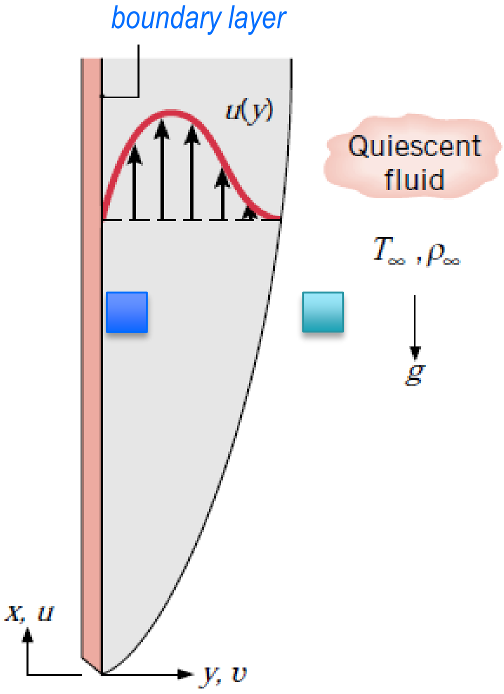
The choice of u_0 is arbitrary (no clear reference velocity): $u_0^2 \equiv g\beta(T_s - T_\infty)L$

$$Re_L = \frac{\rho u_0 L}{\mu} = \sqrt{\frac{g\beta(T_s - T_\infty)L^3}{\nu^2}} = \sqrt{Gr_L}$$

$$Gr_L \equiv \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}$$

Grashof number – replaces Re in free convection problems (it accounts for thermal effects on the flow)

Governing Equations – Rayleigh Number & Flow Conditions



$$Gr_x \equiv \frac{g\beta(T_s - T_\infty)x^3}{\nu^2}$$

$$Ra_x \equiv Gr_x Pr = \frac{g\beta(T_s - T_\infty)x^3}{\nu\alpha}$$

Rayleigh number – determines the flow conditions

$Ra_x < 10^9$ Laminar Flow

$Ra_x > 10^9$ Turbulent Flow

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