Heat and Mass Transfer ME-341

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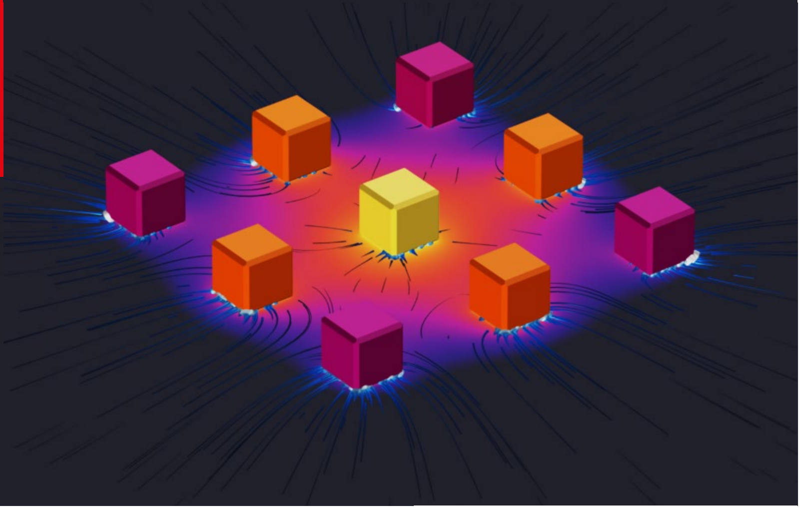


École polytechnique fédérale de Lausanne

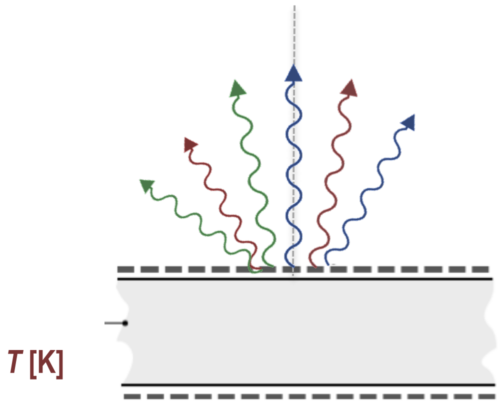
Spring Semester

Previously

- ☑ Emission of Thermal Radiation
- ☑ Interaction of Radiation with Matter
- ☑ Radiation exchange between surfaces
  - ☑ View factors

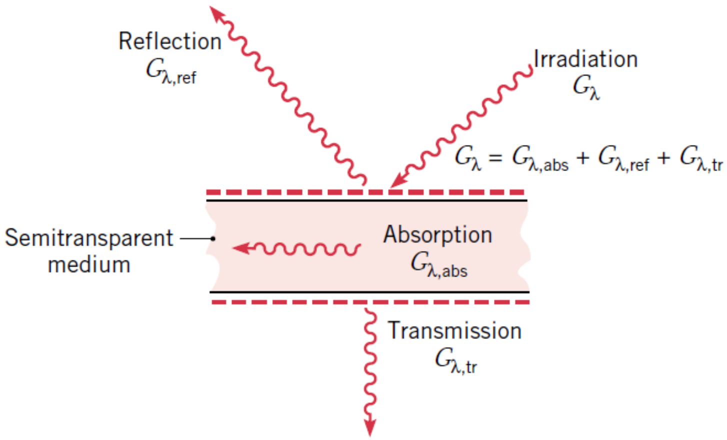
Thermal Radiation

$$I_{\lambda,e}(\lambda, \theta, \Phi, T)$$



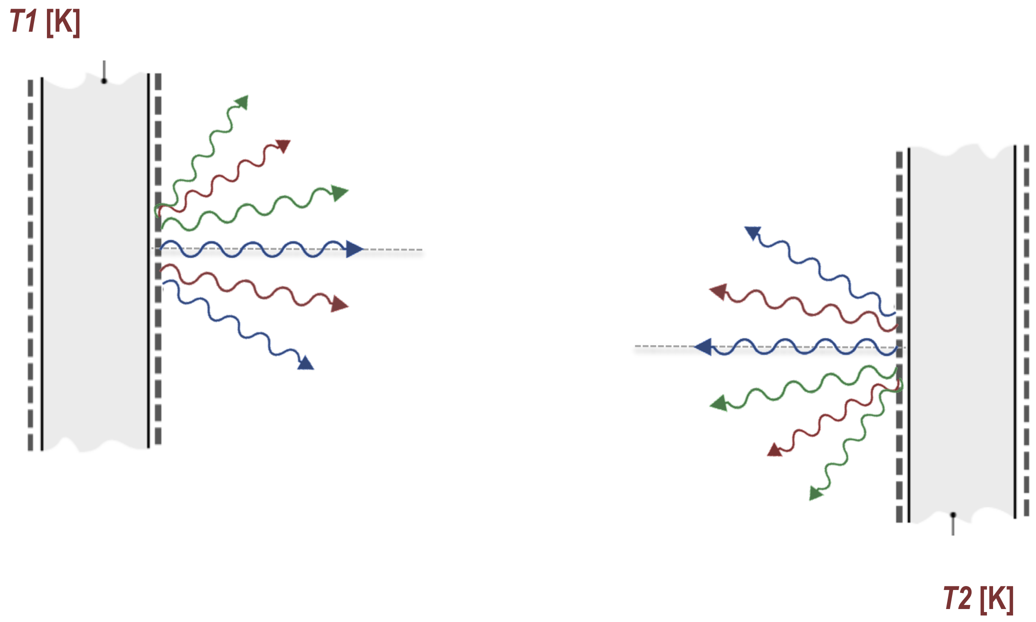
Emission of Thermal Radiation

$$1 = \alpha_\lambda + \rho_\lambda + \tau_\lambda$$



Interaction of Radiation with Matter

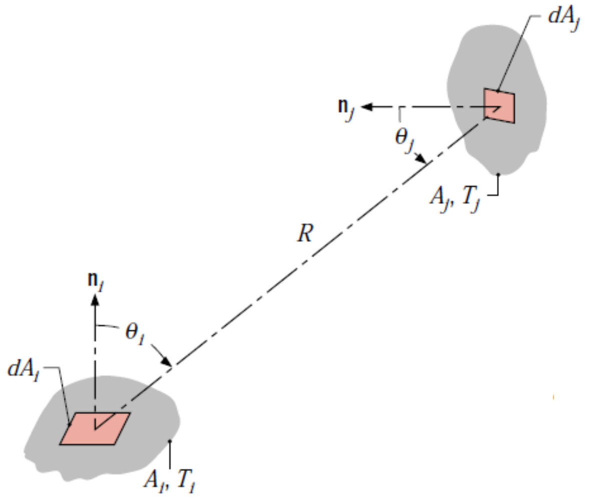
Radiation Exchange



Radiation exchange between surfaces

The View Factor – Reciprocity of Energy Transfer

We consider two surfaces with arbitrary relative orientation (not parallel) and we define the **view factor F_{ij} as the fraction of the radiation leaving surface *i* that is intercepted by surface *j*.**



If both surface *i* and *j* emit and reflect diffusively:

$$F_{ij} \equiv \frac{Q_{i-j}}{A_i J_i} = \frac{1}{A_i} \int_{A_i} \int_{A_j} \frac{cos\theta_i cos\theta_j}{\pi R^2} dA_j dA_i$$

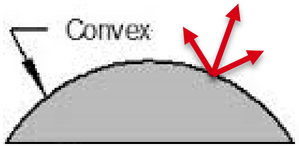
$$F_{ji} \equiv \frac{Q_{j-i}}{A_j J_j} = \frac{1}{A_j} \int_{A_j} \int_{A_i} \frac{cos\theta_j cos\theta_i}{\pi R^2} dA_i dA_j$$

➡ $\boldsymbol{A_i F_{ij} = F_{ji} A_j}$

Reciprocity of radiative energy transfer

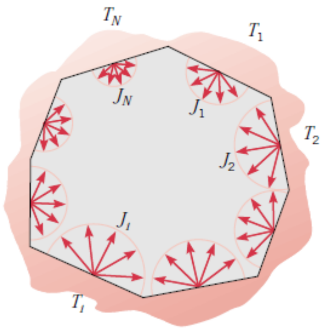
The View Factor

We consider two surfaces with arbitrary relative orientation (not parallel) and we define
the **view factor F_{ij} as the fraction of the radiation leaving surface *i* that is intercepted by surface *j***.



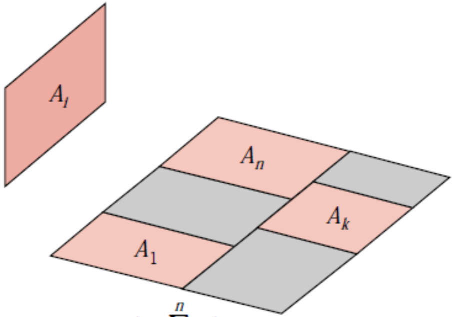
If a surface is **planar or convex** it does not *sees* itself therefore:

$$F_{ii} = 0$$



In an **enclosure**, the radiation leaving a surface *i* is entirely intercepted by all of the other surfaces, therefore:

$$\sum_{j=1}^{N} F_{ij} = 1$$



$$A_j = \sum_{k=1}^{n} A_k$$

A surface can be **decomposed** into sub-surfaces and the view factors are:

$$F_{(j)i} = \frac{\sum_{k=1}^{n} A_k F_{ki}}{\sum_{k=1}^{n} A_k}$$

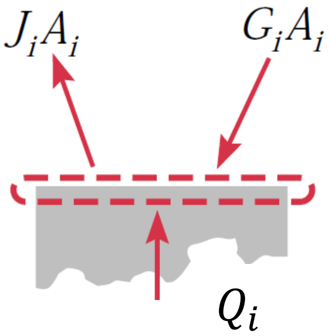
This Lecture

- ❑ Radiation exchange between surfaces
  - ❑ Net Radiation Exchange at a Surface
    - ❑ Electrical Analogy
  - ❑ The two surface enclosure
    - ❑ Electrical Analogy

Learning Objectives:

- ❑ Use the electrical analogy to calculate the radiation resistance of a surface
- ❑ Calculate the radiation exchange from a surface and a 2-surface enclosure

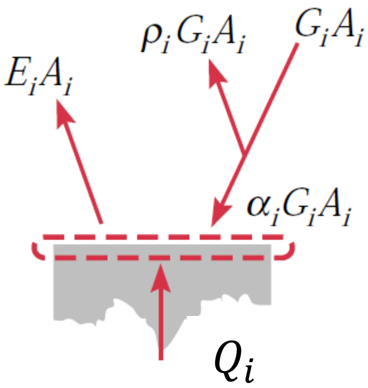
Net Radiation Exchange at a Surface



The net energy that leaves an isothermal, diffuse surface A_i solely via radiation:

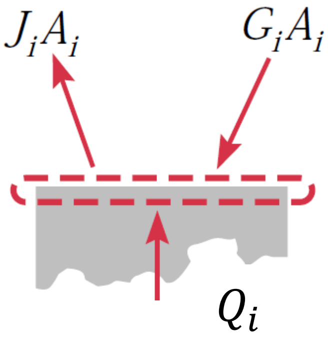
$$Q_i = A_i(J_i - G_i)$$

$$J_i = E_i + \rho_i G_i = \varepsilon_i E_b(T_i) + \rho_i G_i$$



$$\Rightarrow \quad Q_i = A_i\left(J_i - \frac{J_i - \varepsilon_i E_b(T_i)}{\rho_i}\right)$$

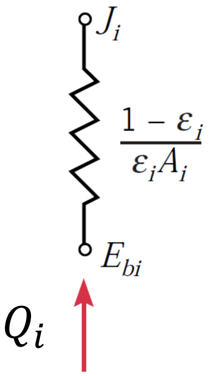
Net Radiation Exchange at a Surface – Electrical Analogy



If we consider an **isothermal**, **opaque**, **diffuse** and **gray** surface:

$$\rho_i = 1 - \alpha_i = 1 - \varepsilon_i \qquad [\text{opaque} \rightarrow \tau_i = 0; \text{gray } \alpha_i = \varepsilon_i]$$

$$\Rightarrow Q_i = A_i \left(J_i - \frac{J_i - \varepsilon_i E_b(T_i)}{1 - \varepsilon_i} \right)$$



$$\Rightarrow Q_i = \frac{E_b(T_i) - J_i}{(1 - \varepsilon_i)/\varepsilon_i A_i} = \frac{E_b(T_i) - J_i}{R_{surf,rad}}$$

$$R_{surf,rad} = \frac{(1 - \varepsilon_i)}{\varepsilon_i A_i}$$

$E_b(T_i) > J_i$ Radiative heat transfer from the surface (loose energy)

$E_b(T_i) < J_i$ Radiative heat transfer to the surface (gain energy)

If $A_i \gg$ then $R_{surf,rad} \sim 0$
$\rightarrow$ Blackbody behavior of surface i

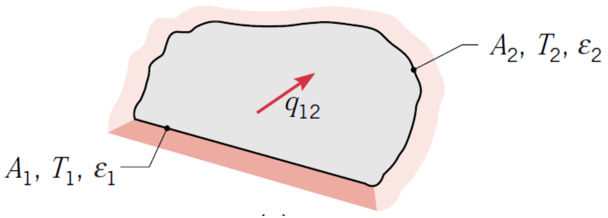
This Lecture

- ❑ Radiation exchange between surfaces
  - ✓ Net Radiation Exchange at a Surface
    - ✓ Electrical Analogy
  - ❑ The two surface enclosure
    - ❑ Electrical Analogy

Learning Objectives:

- ✓ Use the electrical analogy to calculate the radiation resistance of a surface
- ❑ Calculate the radiation exchange from a surface and a 2-surface enclosure

The Two Surface Enclosure

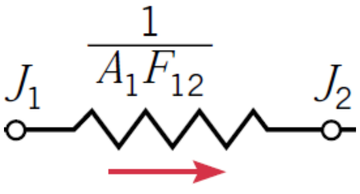


$$Q_1 = A_1(J_1 - G_1) = Q_{12} = -Q_2$$

The irradiation of surface 1 must be related to the radiosity of surface 2 scaled by the view factor between the two surfaces: ⇒ $A_1 G_1 = \sum_{j=1}^{2} F_{j1} A_j J_j$

From the reciprocity relation of the view factors: $F_{j1} A_j = F_{1j} A_1$ ⇒ $A_1 G_1 = \sum_{j=1}^{2} A_1 F_{1j} J_j$

From the summation rule of view factors in an enclosure: $\sum_{j=1}^{2} F_{1j} = 1$

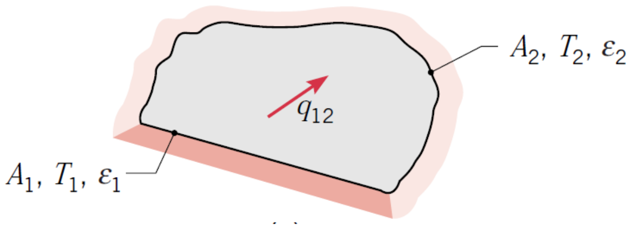


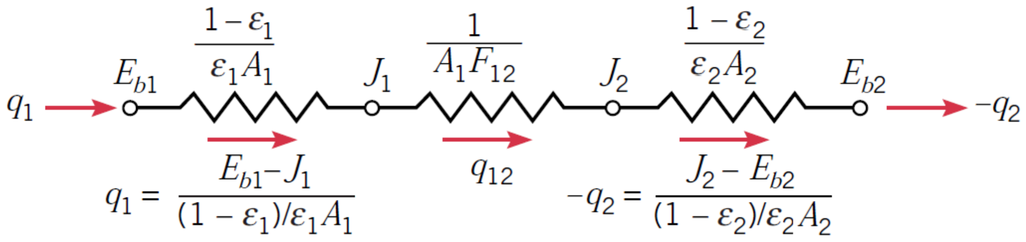
⇒ $$Q_{12} = A_1\left(\sum_{j=1}^{2} F_{1j} J_1 - \sum_{j=1}^{2} F_{1j} J_j\right) = A_1 \sum_{j=1}^{2} F_{1j}\,(J_1 - J_j)$$

⇒ $$Q_{12} = \frac{J_1 - J_j}{1/A_1 F_{12}} = \frac{J_1 - J_j}{R_{geom}}$$

$$R_{geom,12} = \frac{1}{A_1 F_{12}}$$

The Two Surface Enclosure – Electrical Analogy





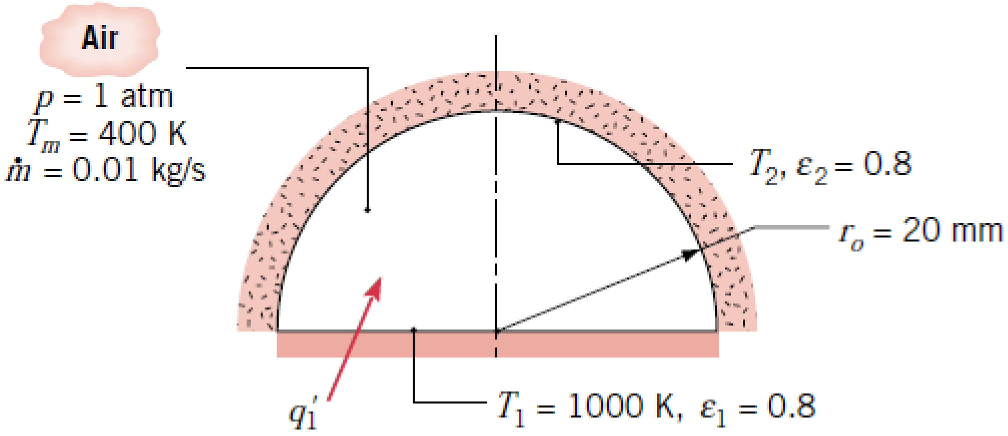
$$Q_1 = Q_{12} = \frac{\sigma(T_1^4 - T_2^4)}{\frac{(1-\varepsilon_1)}{\varepsilon_1 A_1} + \frac{1}{A_1 F_{12}} + \frac{(1-\varepsilon_2)}{\varepsilon_2 A_2}}$$

Example

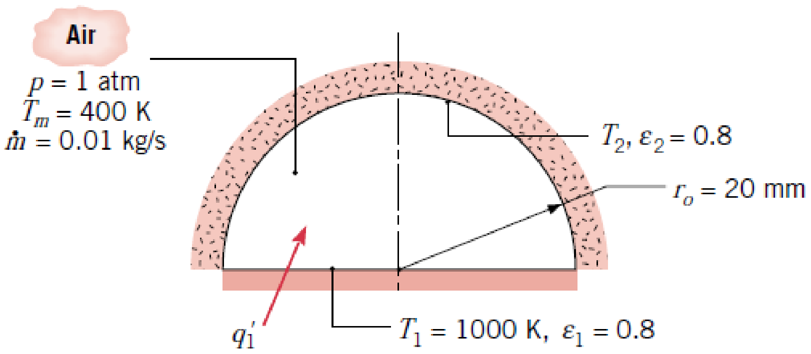
Consider an air heater consisting of a semicircular tube for which the plane surface is maintained at 1000 K and the other surface is well insulated. The tube radius is 20 mm, and both surfaces have an emissivity of 0.8. If atmospheric air flows through the tube at 0.01 kg/s and $T_m = 400$ K, what is the rate at which heat must be supplied per unit length to maintain the plane surface at 1000 K? What is the temperature of the insulated surface?

Assumptions:

1. Steady-state conditions.
2. Diffuse, gray surfaces.
3. Negligible tube end effects and axial variations in gas temperature.
4. Fully developed flow.



Example



Energy balance on surface 1:

- External energy input because of the heater
- Removal of energy via radiation towards surface 2
- Removal of energy via convection

$$Q_{1,ext} - Q_{1,rad} - Q_{1,conv} = 0$$

From the radiative energy balance for a 2-surface enclosure:

$$Q_{1,rad} = -Q_{2,rad} = Q_{12,rad}$$

Energy balance on surface 2:

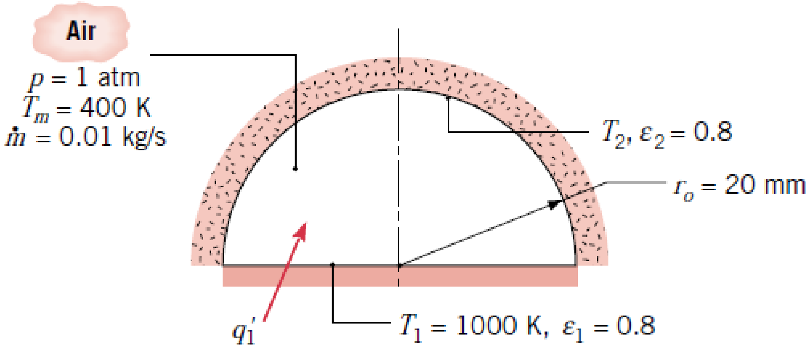
- Input energy via radiation from heater surface
- Removal of energy via convection

$$+(-Q_{2,rad}) - Q_{2,conv} = 0 \quad \Rightarrow \quad -Q_{2,rad} = Q_{2,conv}$$

⇒ *Energy balance on surface 1*: $Q_{1,ext} = Q_{2,conv} + Q_{1,conv}$

⇒ $Q_{1,ext} = hA_2\,(\boldsymbol{T_2} - T_m) + hA_1(T_1 - T_m)$

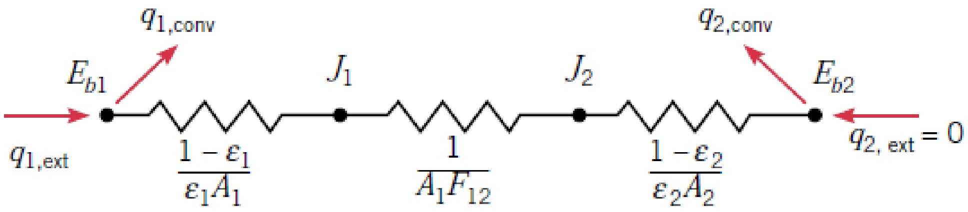
Example



Energy balance on surface 2:

- Input energy via radiation from heater surface
- Removal of energy via convection

$$Q_{2,rad} + Q_{2,conv} = 0 \Rightarrow -Q_{2,rad} = Q_{2,conv}$$

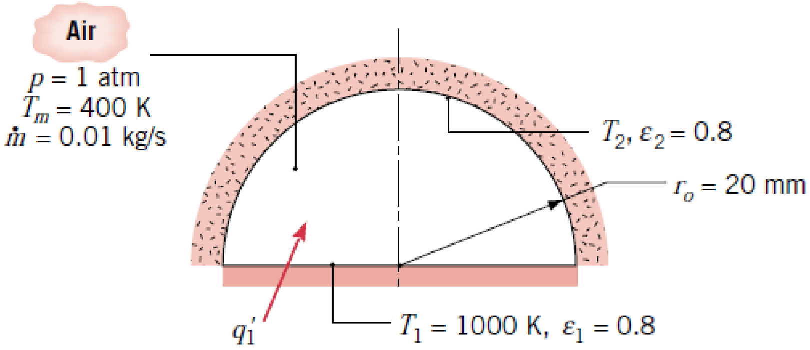


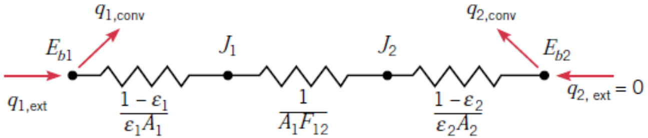
From the radiative energy balance for a 2-surface enclosure:

$$-Q_{2,rad} = Q_{1,rad} = Q_{12,rad}$$

$$\Rightarrow \frac{\sigma(T_1^4 - T_2^4)}{\frac{1-\varepsilon_1}{\varepsilon_1 A_1} + \frac{1}{A_1 F_{12}} + \frac{1-\varepsilon_2}{\varepsilon_2 A_2}} = hA_2(T_2 - T_m)$$

Example





$$\frac{\sigma(T_1^4 - T_2^4)}{\frac{1-\varepsilon_1}{\varepsilon_1 A_1} + \frac{1}{A_1 F_{12}} + \frac{1-\varepsilon_2}{\varepsilon_2 A_2}} = hA_2(T_2 - T_m)$$

Forced internal convection:

Properties: Table A.4, air (1 atm, 400 K): $k = 0.0338$ W/m · K, $\mu = 230 \times 10^{-7}$ kg/s · m, $c_p = 1014$ J/kg · K, $Pr = 0.69$.

$$A_1 = 2r_o \qquad A_2 = \pi r_o.$$

$$Re_D = \frac{\rho u_m D_h}{\mu} = \frac{\dot{m} D_h}{A_c \mu} = \frac{\dot{m} D_h}{(\pi r_o^2/2)\mu}$$

the hydraulic diameter is

$$D_h = \frac{4A_c}{P} = \frac{2\pi r_o}{\pi + 2} = \frac{0.04\pi \text{ m}}{\pi + 2} = 0.0244 \text{ m}$$

Hence

$$Re_D = \frac{0.01 \text{ kg/s} \times 0.0244 \text{ m}}{(\pi/2)\ (0.02 \text{ m})^2 \times 230 \times 10^{-7} \text{ kg/s} \cdot \text{m}} = 16{,}900$$

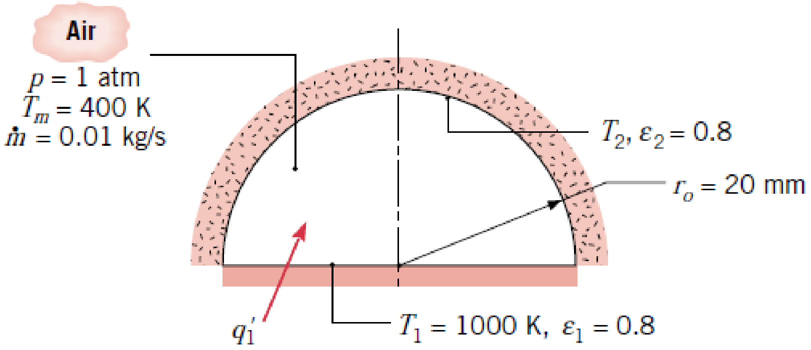
From the Dittus–Boelter equation,

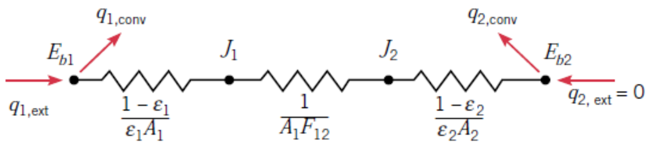
$$Nu_D = 0.023 Re_D^{4/5}\, Pr^{0.4}$$

$$Nu_D = 0.023(16{,}900)^{4/5}(0.69)^{0.4} = 47.8$$

$$h = \frac{k}{D_h} Nu_D = \frac{0.0338 \text{ W/m} \cdot \text{K}}{0.0244 \text{ m}} 47.8 = 66.2 \text{ W/m}^2 \cdot \text{K}$$

Example





$$\frac{\sigma(T_1^4 - T_2^4)}{\frac{1-\varepsilon_1}{\varepsilon_1 A_1} + \frac{1}{A_1 F_{12}} + \frac{1-\varepsilon_2}{\varepsilon_2 A_2}} = hA_2(T_2 - T_m)$$

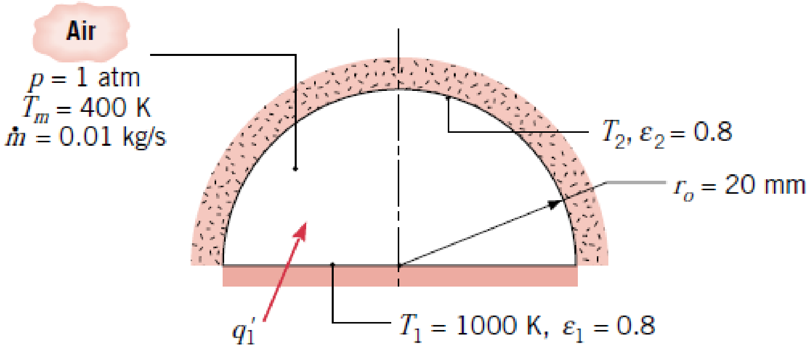
$$F_{11} = 0 \quad \Rightarrow \quad F_{12} = 1$$

$$\frac{5.67 \times 10^{-8}\,\mathrm{W/m^2 \cdot K^4}\,[(1000)^4 - T_2^4]\,\mathrm{K}^4}{\frac{1-0.8}{0.8} + 1 + \frac{1-0.8}{0.8}\frac{2}{\pi}} = 66.2\,\frac{\pi}{2}\,(T_2 - 400)\ \mathrm{W/m^2}$$

A trial-and-error solution yields

$$T_2 = 696\ \mathrm{K}$$

Example



Energy balance on surface 1: $Q_{1,ext} = Q_{2,conv} + Q_{1,conv}$

$$Q_{1,ext} = hA_2\,(T_2 - T_m) + hA_1(T_1 - T_m)$$

on a unit length basis,

$$q'_{1,\text{ext}} = h\pi r_o(T_2 - T_m) + h2r_o(T_1 - T_m)$$

$$q'_{1,\text{ext}} = 66.2 \times 0.02[\pi\,(696 - 400) + 2(1000 - 400)]\ \text{W/m}$$

$$q'_{1,\text{ext}} = (1231 + 1589)\ \text{W/m} = 2820\ \text{W/m}$$

This Lecture

- ❑ Radiation exchange between surfaces
  - ✓ Net Radiation Exchange at a Surface
    - ✓ Electrical Analogy
  - ✓ The two surface enclosure
    - ✓ Electrical Analogy

Learning Objectives:

- ✓ Use the electrical analogy to calculate the radiation resistance of a surface
- ✓ Calculate the radiation exchange from a surface and a 2-surface enclosure

Next Lecture

- ❑ Radiation exchange between surfaces
  - ❑ A Multi-surface Enclosure

Learning Objectives:

- ❑ Calculate the radiation exchange from a multi-surface enclosure