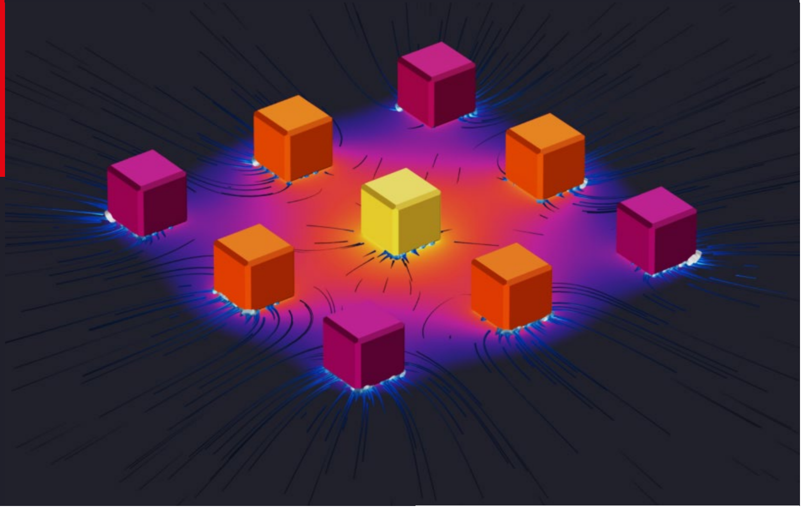
Heat and Mass Transfer
ME-341

Instructor: Giulia Tagliabue



École
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de Lausanne

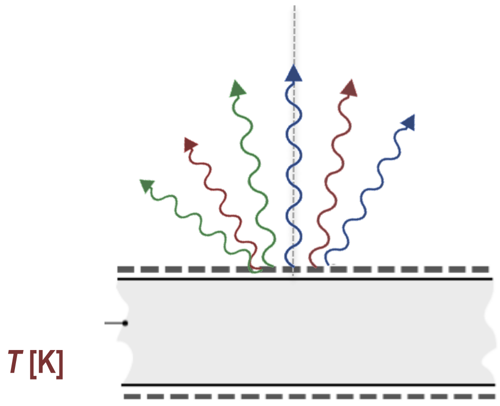
Spring Semester

Previously

- ☑ Introduction to Radiation
- ☑ Emission of Thermal Radiation
  - ☑ Spatial distribution and Diffuse Emitter
  - ☑ Spectral distribution
  - ☑ Stefan-Boltzmann and Wien’s laws
- ☑ Interaction of Thermal Radiation with Matter
  - ☑ Absorptivity, Reflectivity and Transmissivity
  - ☑ Irradiation and Radiosity
- ☑ Black-body
- ☑ Real surfaces: Emissivity, Diffuse & Gray Surfaces, Kirchoff’s Laws

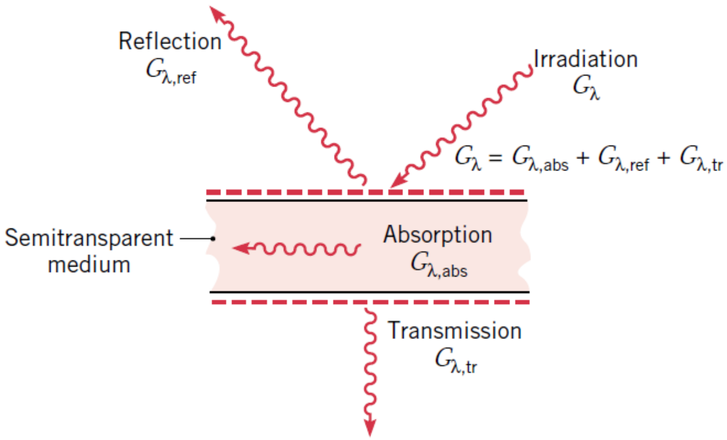
Overview so far

$$I_{\lambda,e}(\lambda, \theta, \Phi, T)$$



Emission of Thermal Radiation

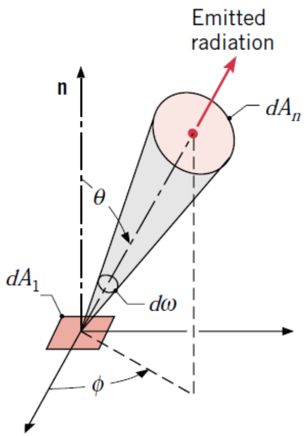
$$1 = \alpha_\lambda + \rho_\lambda + \tau_\lambda$$



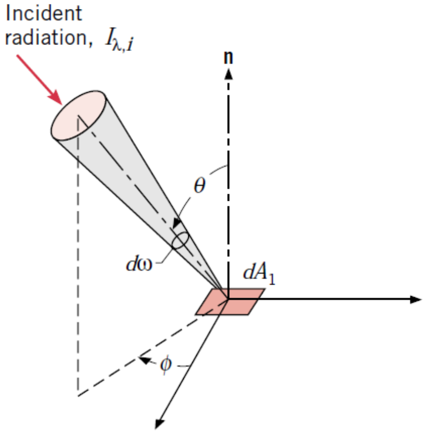
Interaction of Radiation with Matter

Measures of Radiation

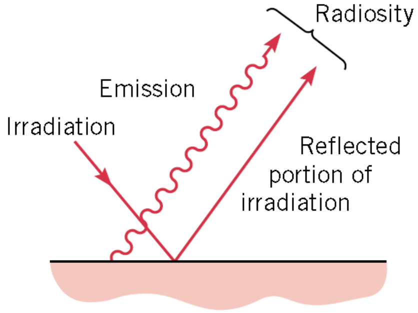
Emission



Irradiation



Radiosity



Radiation spectral intensity can depend on the wavelength (λ), the spatial direction (θ, Φ) and, in the case of emission, the temperature (T) of the surface.

Measures of Radiation

	Spectral Intensity $I_{\lambda,x}$	Spectral X_λ $X_\lambda(\lambda) = \int_0^{2\pi}\int_0^{\pi/2} I_{\lambda,x}(\lambda,\theta,\Phi) cos\theta sin\theta d\Phi d\theta$	Total X $X = \int_0^{\infty} X_\lambda(\lambda) d\lambda$
Emission	$I_{\lambda,e}(\lambda,\theta,\Phi,T)$	$E_\lambda = spectral\ emissive\ power$	$E = emissive\ power$
Irradiation	$I_{\lambda,i}(\lambda,\theta,\Phi)$	$G_\lambda = spectral\ irradiation$	$G = irradiation$
Radiosity	$I_{\lambda,e+r}(\lambda,\theta,\Phi)$	$J_\lambda = spectral\ radiosity$	$J = radiosity$

Diffuse radiation and surfaces = spectral intensity <u>independent</u> of the angular direction

- Diffuse emitter : $I_{\lambda,e}(\lambda,\theta,\Phi,T) = I_{\lambda,e}(\lambda,T)$
- Diffuse irradiation : $I_{\lambda,i}(\lambda,\theta,\Phi) = I_{\lambda,i}(\lambda)$
- Diffuse emitter and diffuse reflector : $I_{\lambda,e+r}(\lambda,\theta,\Phi) = I_{\lambda,e+r}(\lambda)$

⇨ $I_x = \int_0^{\infty} I_{\lambda,x}(\lambda) d\lambda = total\ intensity$

Black-body and Real Surfaces

	Ideal Object (Black-body)	*Real Surfaces*
Emission of Thermal Radiation	$I_{\lambda,b}(\lambda,T) = \frac{2hc_0^2}{\lambda^5[\exp(hc_0/\lambda kT) - 1]}$ $E_{\lambda,b}(\lambda,T) = \pi I_{\lambda,b}(\lambda,T)$ $\boldsymbol{E_b(T) = \sigma T^4}$	$\varepsilon_{\lambda,\theta} = \frac{I_\lambda(\lambda,\theta,\Phi,T)}{I_{\lambda,b}(\lambda,T)}$, $\varepsilon_\lambda(\lambda,\mathrm{T}) = \frac{E_\lambda(\lambda,T)}{E_{\lambda,b}(\lambda,T)}$ $\boldsymbol{\varepsilon(\mathrm{T}) = \frac{E\ (T)}{E_b(T)} = \frac{\int_0^\infty \varepsilon_\lambda(\lambda,T) E_{\lambda,b}(\lambda,T) d\lambda}{E_b(T)}}$ $E(T) = \varepsilon\sigma T^4$
Interaction of Radiation with Matter	Diffuse emitter: $I_{\lambda,e}(\lambda,\theta,\Phi,T) = I_{\lambda,b}(\lambda,T)$ $\alpha_\lambda = 1 \quad \rho_\lambda = \tau_\lambda = 0$	Diffuse surface: $\varepsilon_{\lambda,\theta} = \boldsymbol{\varepsilon_\lambda} = \boldsymbol{\alpha_\lambda} = \alpha_{\lambda,\theta}$ Gray surface: $\varepsilon_\lambda = \boldsymbol{\varepsilon} = \boldsymbol{\alpha} = \alpha_\lambda$ $\alpha \equiv \frac{G_{abs}}{G} = \frac{\int_0^\infty \alpha_\lambda \boldsymbol{E_{\lambda,b}(\lambda,T)} d\lambda}{\int_0^\infty \boldsymbol{E_{\lambda,b}(\lambda,T)} d\lambda}$ $\rho \equiv \frac{G_{ref}}{G} = \frac{\int_0^\infty \rho_\lambda E_{\lambda,b}(\lambda,T) d\lambda}{\int_0^\infty E_{\lambda,b}(\lambda,T) d\lambda}$ $\tau \equiv \frac{G_{tr}}{G} = \frac{\int_0^\infty \tau_\lambda E_{\lambda,b}(\lambda,T) d\lambda}{\int_0^\infty E_{\lambda,b}(\lambda,T) d\lambda}$

Real Surfaces: Kirchoff's Laws

It can be shown that the following relationship is always true:

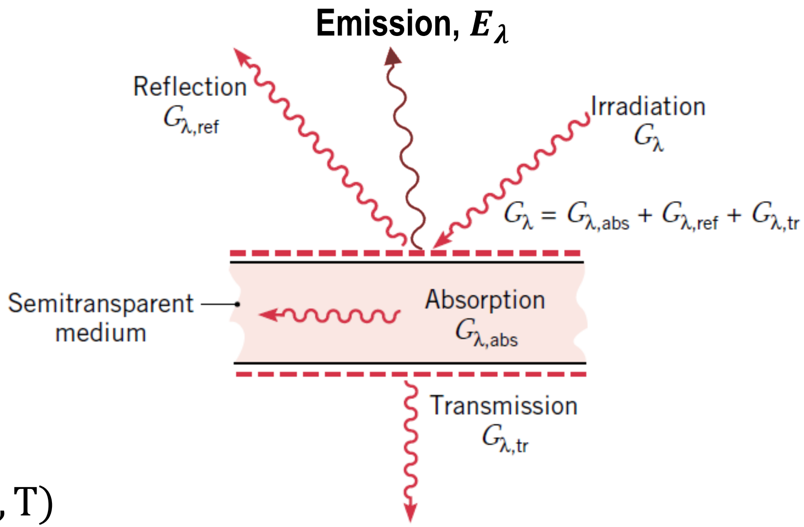
$$\varepsilon_{\lambda,\theta} = \alpha_{\lambda,\theta}$$

In addition, if the irradiation is diffuse ($I_{\lambda,i}(\lambda, \theta, \Phi) = I_{\lambda,i}(\lambda)$)
OR the <u>surface is diffuse</u>:

$$\varepsilon_{\lambda} = \alpha_{\lambda}$$

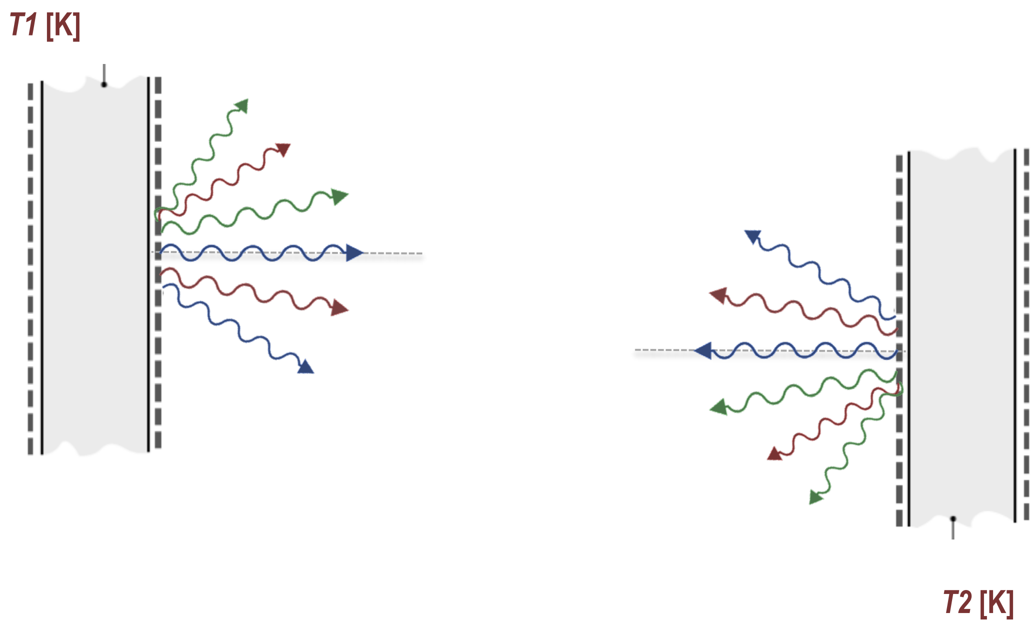
Finally, if the irradiation is a black-body emission ($G_{\lambda}(\lambda) = E_{\lambda,b}(\lambda, \mathrm{T})$
and $\mathrm{G} = E_b(T)$) **OR** the <u>surface is gray</u>:

$$\varepsilon = \alpha$$



These are Kirchoff's Laws and define the conditions under which we can establish simple relationships for emissivity and absorptivity.

This Lecture



Radiation exchange between surfaces

This Lecture

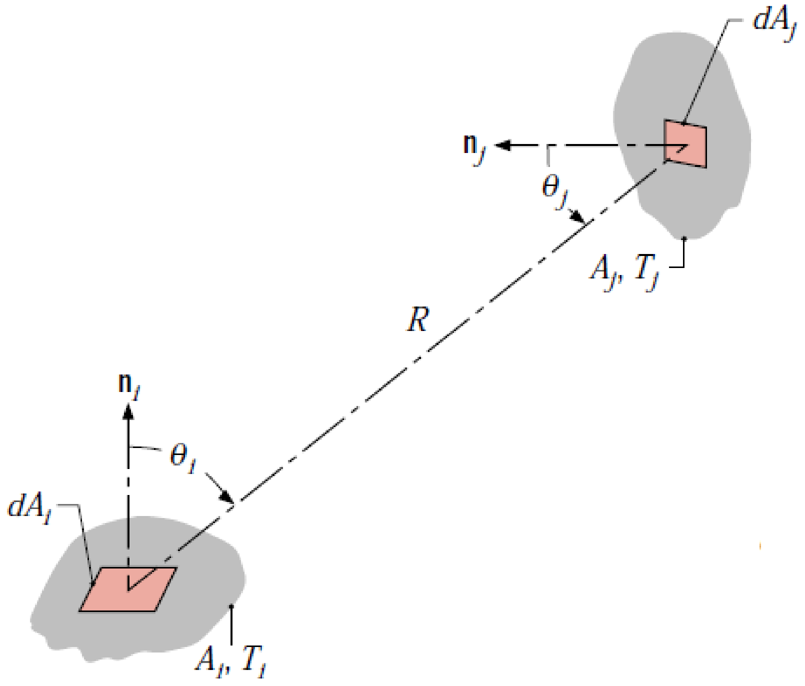
- Radiation exchange between surfaces
  - View factors

Learning Objectives:

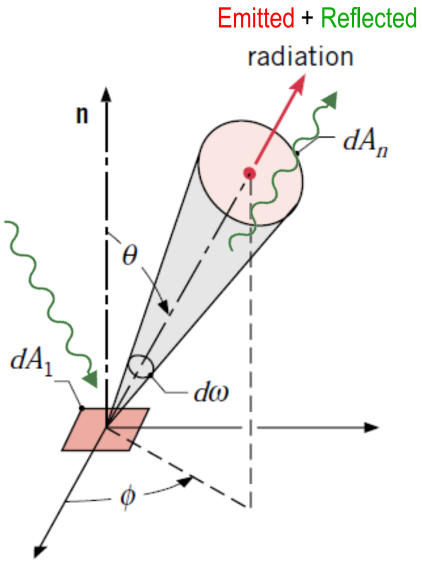
- Calculate the view factor between two surfaces

The View Factor

We consider two surfaces with arbitrary relative orientation (not parallel) and we define the **view factor F_{ij} as the fraction of the radiation leaving surface *i* that is intercepted by surface *j***.



Spectral Intensity of Radiosity



$$I_{\lambda,e+r} = spectral\ intensity\ of\ radiosity$$

rate at which energy leaves the surface at wavelength λ and along the direction (Φ,θ)

- *per unit area of the emitting surface* ***normal to this direction*** $= dA_1 cos\theta$
- per unit solid angle about this direction $= d\omega$
- per unit wavelength interval about λ $= d\lambda$

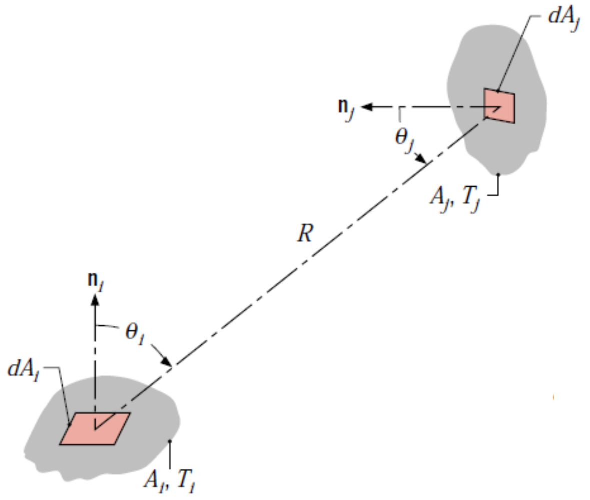
$$I_{\lambda,e+r} = \frac{dQ}{dA_1 cos\theta d\omega d\lambda} = \frac{dq_\lambda}{dA_1 cos\theta d\omega}$$

$$dq_\lambda = I_{\lambda,e+r} dA_1 cos\theta d\omega$$

The View Factor

We consider two surfaces with arbitrary relative orientation (not parallel) and we define the **view factor F_{ij} as the fraction of the radiation leaving surface *i* that is intercepted by surface *j***.

$$dq_{\lambda,i-j} = I_{\lambda,e+r,i} dA_i cos\theta_i d\omega_{i-j} \qquad d\omega_{j-i} = \frac{dA_j cos\theta_j}{R^2}$$



$$\Rightarrow \quad dq_{\lambda,i-j} = I_{\lambda,e+r,i} dA_i cos\theta_i \frac{dA_j cos\theta_j}{R^2}$$

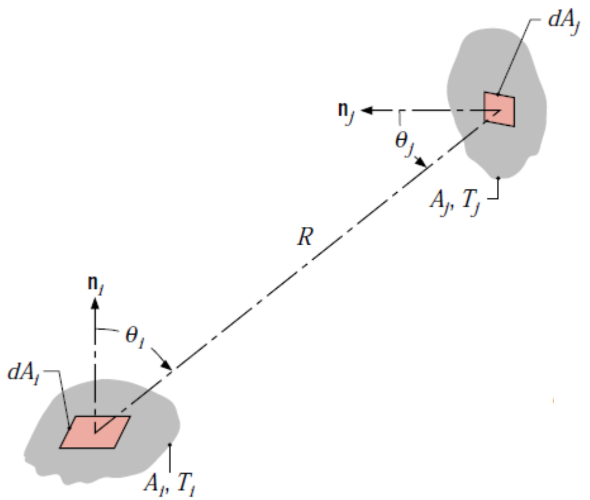
We assume that surface *i* emits and reflects diffusively:

$$J_{\lambda,i} = \pi I_{\lambda,e+r,i}(\lambda) \qquad J = \pi \int_0^\infty I_{\lambda,e+r,i}(\lambda) d\lambda = \pi I_{e+r,i}$$

$$\Rightarrow \quad dq_{\lambda,i-j} = J_{\lambda,i} dA_i cos\theta_i \frac{dA_j cos\theta_j}{\pi R^2}$$

The View Factor

We consider two surfaces with arbitrary relative orientation (not parallel) and we define the **view factor F_{ij} as the fraction of the radiation leaving surface *i* that is intercepted by surface *j***.



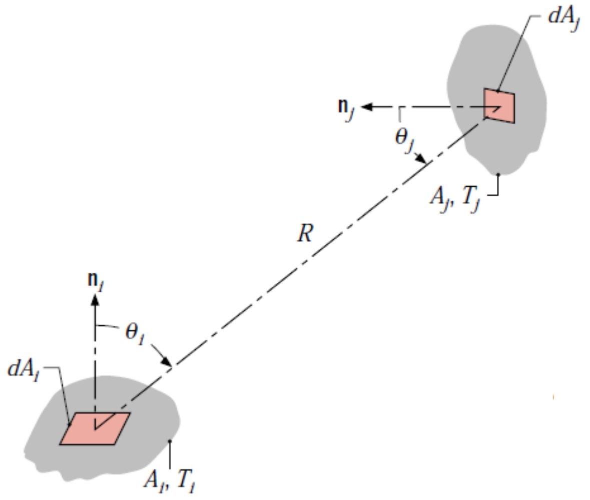
$$dq_{i-j} = \int_0^\infty dq_{\lambda,i-j} d\lambda = J_i \frac{cos\theta_i cos\theta_j}{\pi R^2} dA_i dA_j$$

$$\Rightarrow \quad Q_{i-j} = \int_{A_i}\int_{A_j} J_i \frac{cos\theta_i cos\theta_j}{\pi R^2} dA_j dA_i$$

$$\Rightarrow \quad F_{ij} \equiv \frac{Q_{i-j}}{A_i J_i} = \frac{1}{A_i}\int_{A_i}\int_{A_j} \frac{cos\theta_i cos\theta_j}{\pi R^2} dA_j dA_i$$

The View Factor – Reciprocity of Energy Transfer

We consider two surfaces with arbitrary relative orientation (not parallel) and we define the **view factor F_{ij} as the fraction of the radiation leaving surface *i* that is intercepted by surface *j***.



If both surface *i* and *j* emit and reflect diffusively:

$$F_{ij} \equiv \frac{Q_{i-j}}{A_i J_i} = \frac{1}{A_i} \int_{A_i} \int_{A_j} \frac{\cos\theta_i \cos\theta_j}{\pi R^2} dA_j dA_i$$

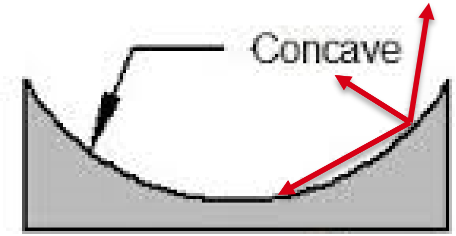
$$F_{ji} \equiv \frac{Q_{j-i}}{A_j J_j} = \frac{1}{A_j} \int_{A_j} \int_{A_i} \frac{\cos\theta_j \cos\theta_i}{\pi R^2} dA_i dA_j$$

➡ $\boldsymbol{A_i F_{ij} = F_{ji} A_j}$

Reciprocity of radiative energy transfer

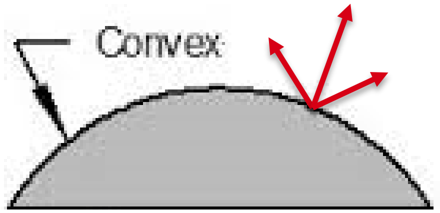
The View Factor – Concave and Convex Surfaces

We consider two surfaces with arbitrary relative orientation (not parallel) and we define the **view factor F_{ij} as the fraction of the radiation leaving surface *i* that is intercepted by surface *j***.



If a surface is **concave** it *sees* itself therefore we have:

$$F_{ii} \neq 0$$

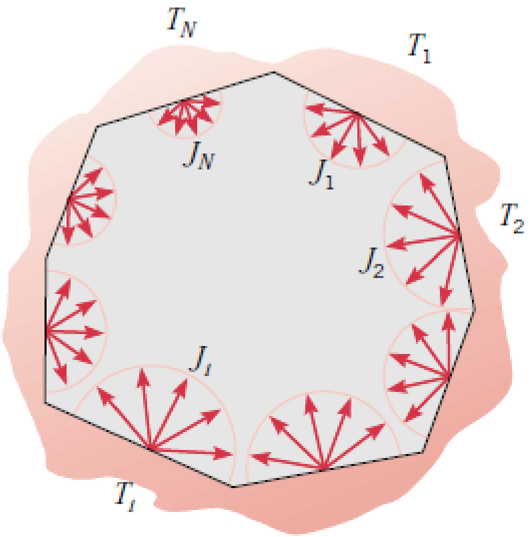


If a surface is **planar or convex** it does not *sees* itself therefore:

$$F_{ii} = 0$$

The View Factor - Enclosures

We consider two surfaces with arbitrary relative orientation (not parallel) and we define the **view factor F_{ij} as the fraction of the radiation leaving surface *i* that is intercepted by surface *j***.



In an **<u>enclosure</u>**, the radiation leaving a surface *i* is entirely intercepted by all of the other surfaces, therefore:

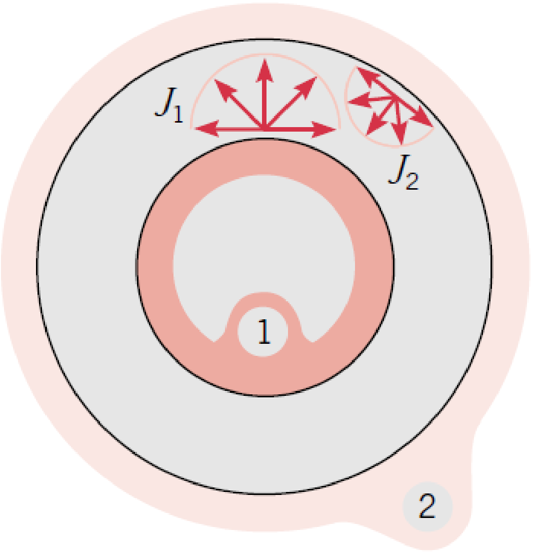
$$\sum_{j=1}^{N} F_{ij} = 1$$

A total of N view factors is needed:

$$\begin{bmatrix} F_{11} & F_{12} & \cdots & F_{1N} \\ F_{21} & F_{22} & \cdots & F_{2N} \\ \vdots & \vdots & & \vdots \\ F_{N1} & F_{N2} & \cdots & F_{NN} \end{bmatrix}$$

The View Factor - Example

We consider two surfaces with arbitrary relative orientation (not parallel) and we define the **view factor F_{ij} as the fraction of the radiation leaving surface *i* that is intercepted by surface *j***.



Example. In an enclosure formed by two concentric spheres we have:

- Surface 1 is convex: $F_{11} = 0$
- Surface 2 intercepts all energy leaving surface 1: $F_{12} = 1$
- For an enclosure we have: $F_{21} + F_{22} = 1$
- From the reciprocity relationship: $A_1 F_{12} = A_2 F_{21}$

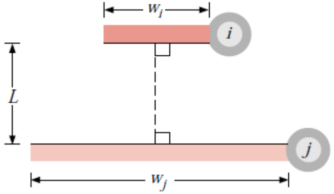
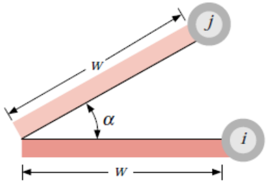
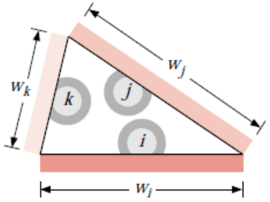
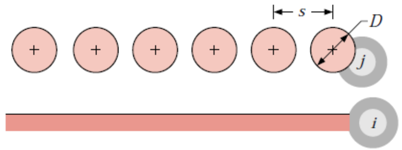
$$F_{21} = \left(\frac{A_1}{A_2}\right) F_{12} = \left(\frac{A_1}{A_2}\right) \qquad F_{22} = 1 - \left(\frac{A_1}{A_2}\right)$$

For an enclosure, we can always write a set of equations to determine all view factors. For other geometries, we must solve the integrals.

The View Factor – Pre-calculated cases

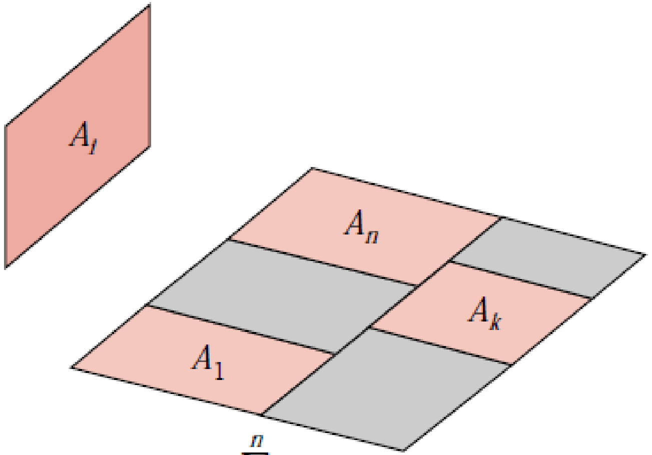
We consider two surfaces with arbitrary relative orientation (not parallel) and we define the **view factor F_{ij} as the fraction of the radiation leaving surface *i* that is intercepted by surface *j***.

TABLE 13.1 View Factors for Two-Dimensional Geometries [4]

Geometry	Relation
Parallel Plates with Midlines Connected by Perpendicular 	$F_{ij} = \dfrac{[(W_i + W_j)^2 + 4]^{1/2} - [(W_j - W_i)^2 + 4]^{1/2}}{2W_i}$ $W_i = w_i/L,\ W_j = w_j/L$
Inclined Parallel Plates of Equal Width and a Common Edge 	$F_{ij} = 1 - \sin\left(\dfrac{\alpha}{2}\right)$
Three-Sided Enclosure 	$F_{ij} = \dfrac{w_i + w_j - w_k}{2w_i}$
Infinite Plane and Row of Cylinders 	$F_{ij} = 1 - \left[1 - \left(\dfrac{D}{s}\right)^2\right]^{1/2} + \left(\dfrac{D}{s}\right)\tan^{-1}\left[\left(\dfrac{s^2 - D^2}{D^2}\right)^{1/2}\right]$

The View Factor – Surface Decomposition

We consider two surfaces with arbitrary relative orientation (not parallel) and we define the **view factor F_{ij} as the fraction of the radiation leaving surface *i* that is intercepted by surface *j***.



$$A_j = \sum_{k=1}^{n} A_k$$

If a surface *j* is decomposed in n sub-surfaces, the overall energy intercepted will remain the same. Therefore:

$$F_{i(j)} = \sum_{k=1}^{n} F_{ik}$$

$$A_j F_{(j)i} = \sum_{k=1}^{n} A_k F_{ki}$$

$$F_{(j)i} = \frac{\sum_{k=1}^{n} A_k F_{ki}}{\sum_{k=1}^{n} A_k}$$

This Lecture

- ❑ Radiation exchange between surfaces
  - ☑ View factors

Learning Objectives:

- ☑ Calculate the view factor between two surfaces

Next Lecture

- Radiation exchange between surfaces
  - Net Radiation Exchange at a Surface
    - Electrical Analogy
  - The two surface enclosure
    - Electrical Analogy

Learning Objectives:

- Use the electrical analogy to calculate the radiation resistance of a surface
- Calculate the radiation exchange from a surface and a 2-surface enclosure