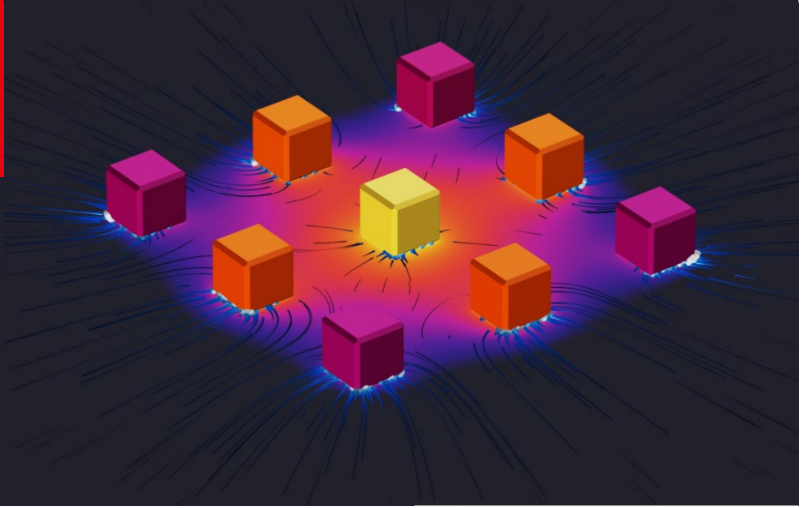
EPFL

Heat and Mass Transfer
ME-341

Instructor: Giulia Tagliabue



École
polytechnique
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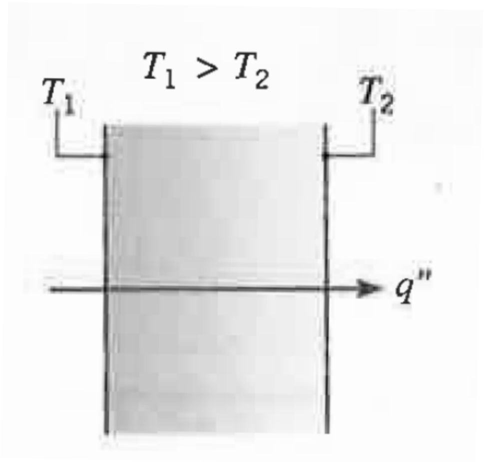
de Lausanne

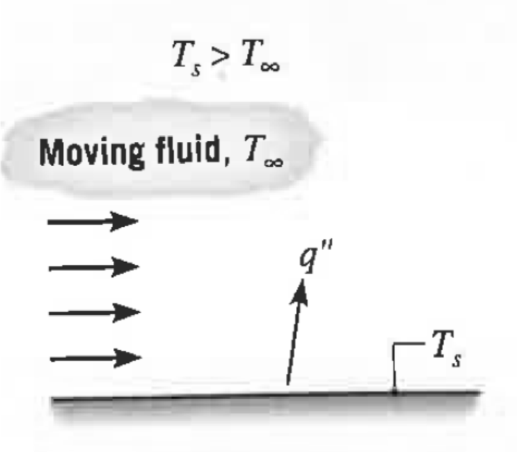
Spring Semester

Heat Transfer Mechanisms

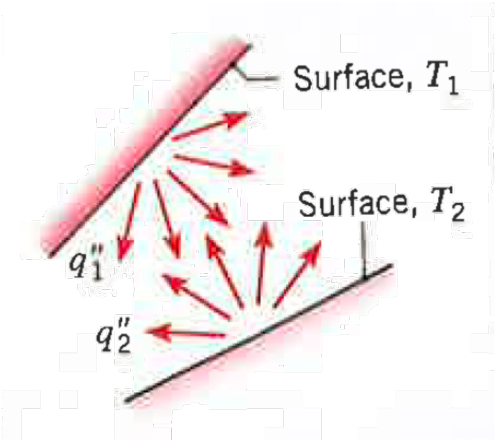
Conduction



Convection



Radiation



Involves mass transport

Involve physical contact

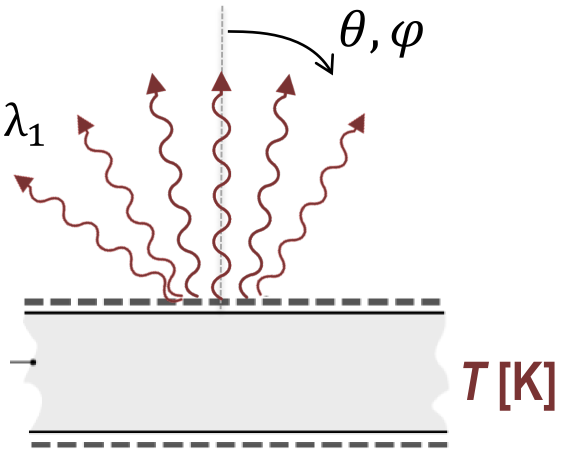
Previously

- ❑ Emission of Thermal Radiation
  - ☑ Spatial distribution and Diffuse Emitter
  - ☑ Spectral distribution
  - ☑ Stefan-Boltzmann and Wien’s laws

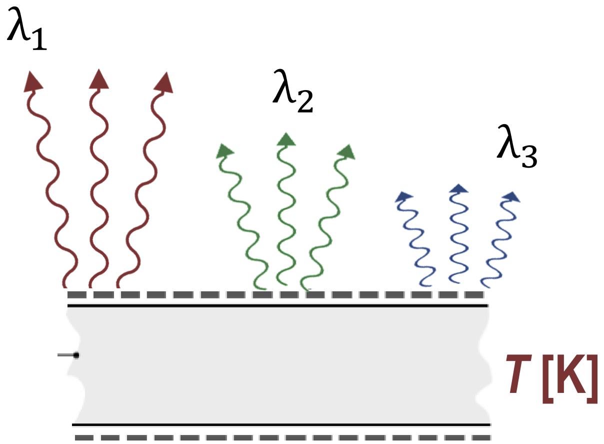
Learning Objective:

- ☑ Understand emission of thermal radiation
- ☑ Quantify the emission of thermal radiation

Emission of Thermal Radiation

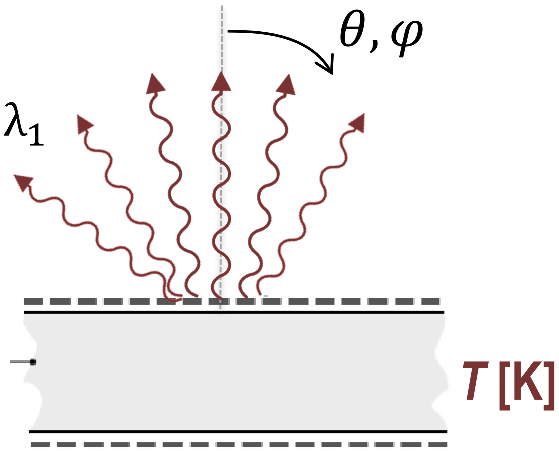


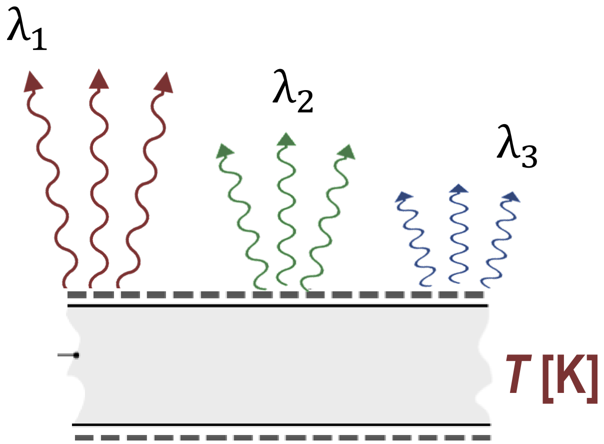
Spatial distribution



Spectral distribution

Emission of Thermal Radiation



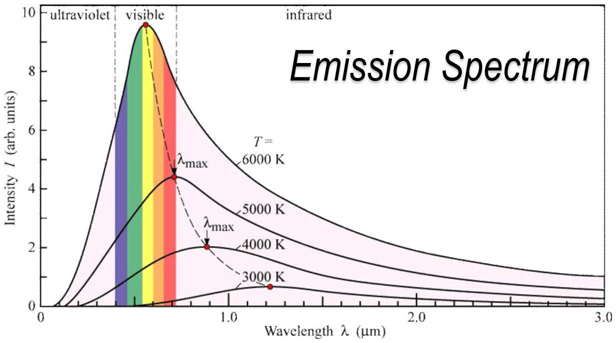


$$E(T) = \int_0^{\infty}\int_0^{2\pi}\int_0^{\pi/2} I_{\lambda,e}(\lambda,\theta,\Phi,T) cos\theta sin\theta d\Phi d\theta d\lambda$$

Spectral distribution

$$E(T) = \pi \int_0^{\infty} \boldsymbol{I_{\lambda,e}(\lambda,T)} d\lambda$$ ***(diffuse emitter)***

Emission of Thermal Radiation: Spectral Distribution



Spectral intensity of a *black-body** $\left[\frac{W}{m^2 sr\ \mu m}\right]$:

$$I_{\lambda,b}(\lambda,T) = \frac{2hc_0^2}{\lambda^5[\exp(hc_0/\lambda kT) - 1]}$$

$$h = 6.626 \times 10^{-34}\ \mathrm{J \cdot s}$$
$$k = 1.381 \times 10^{-23}\ \mathrm{J/K}$$
$$c_o = 2.998 \times 10^{8}\ \mathrm{m/s}$$

Temperatures MUST BE in Kelvin: T[K]

Stefan-Boltzman Law *(total emissive power)*

$$\boldsymbol{E_b(T)} = \pi \int_0^\infty I_{\lambda,b}(\lambda,T)d\lambda = \boldsymbol{\sigma T^4}$$

T = absolute temperature [K]
σ = 5.670367 10^{-8} W/m^2K^4

Total intensity of thermal emission

$$I_b = \frac{E_b}{\pi}$$

Wien's Law (peak emission)

$$(\lambda T)_{e_{\lambda=max}} = 2898\ \mu m \cdot K$$

a black-body is a* ***diffuse emitter* *that absorbs all energy that impinges on it. We discuss this in detail later*

Emission of Thermal Radiation: Spectral Distribution

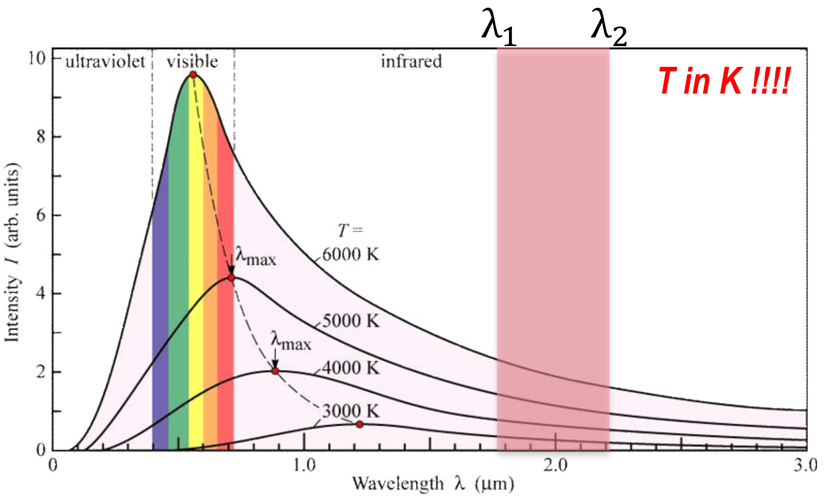


TABLE 12.1 Blackbody Radiation Functions

λT (μm · K)	$F_{(0\to\lambda)}$	$I_{\lambda,b}(\lambda,T)/\sigma T^5$ (μm · K · sr)$^{-1}$	$\frac{I_{\lambda,b}(\lambda,T)}{I_{\lambda,b}(\lambda_{max},T)}$
200	0.000000	0.375034×10^{-27}	0.000000
400	0.000000	0.490335×10^{-13}	0.000000
600	0.000000	0.104046×10^{-8}	0.000014
800	0.000016	0.991126×10^{-7}	0.001372
1,000	0.000321	0.118505×10^{-5}	0.016406
1,200	0.002134	0.523927×10^{-5}	0.072534
1,400	0.007790	0.134411×10^{-4}	0.186082
1,600	0.019718	0.249130	0.344904
1,800	0.039341	0.375568	0.519949

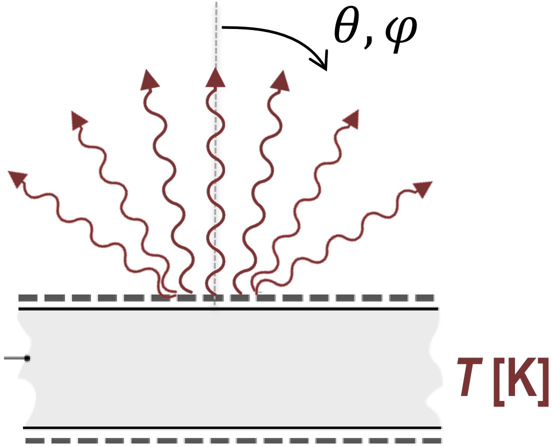
Often we want to know what is the emissive power within a limited wavelength range $[\lambda_1, \lambda_2]$. Therefore we define:

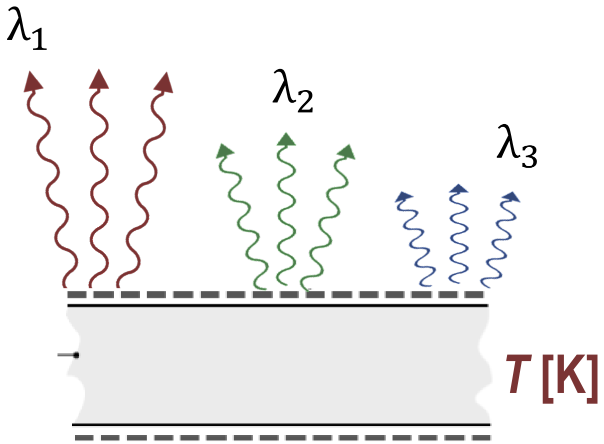
$$F_{(0\to\lambda)} \equiv \frac{\int_0^{\lambda} E_{\lambda,b}\, d\lambda}{\int_0^{\infty} E_{\lambda,b}\, d\lambda} = \frac{\int_0^{\lambda} E_{\lambda,b}\, d\lambda}{\sigma T^4} = \int_0^{\lambda T} \frac{E_{\lambda,b}}{\sigma T^5}\, d(\lambda T) = f(\lambda T)$$

The values of *F* are tabulated. Hence within a given wavelength range we have:

$$F_{(\lambda_1\to\lambda_2)} = \frac{\int_0^{\lambda_2} E_{\lambda,b}\, d\lambda - \int_0^{\lambda_1} E_{\lambda,b}\, d\lambda}{\sigma T^4} = F_{(0\to\lambda_2)} - F_{(0\to\lambda_1)}$$

Emission of Thermal Radiation (Black-body)





$$I_{\lambda,e}(\lambda, \theta, \Phi, T) = I_{\lambda,b}(\lambda, T)$$

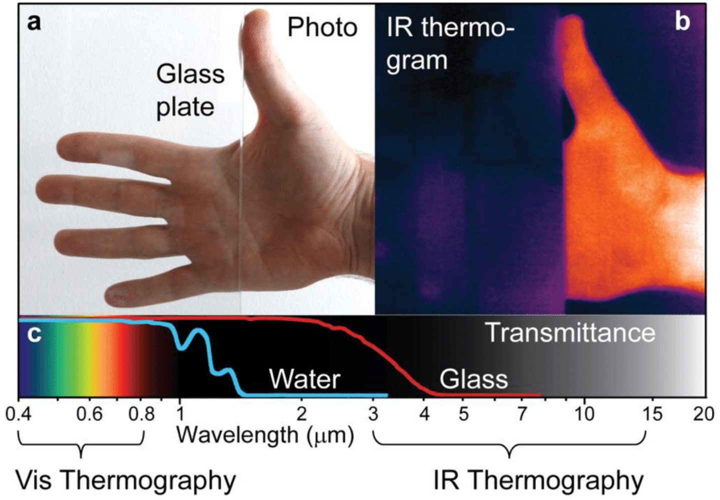
$$I_{\lambda,b}(\lambda, T) = \frac{2hc_0^2}{\lambda^5[\exp(hc_0/\lambda kT) - 1]}$$

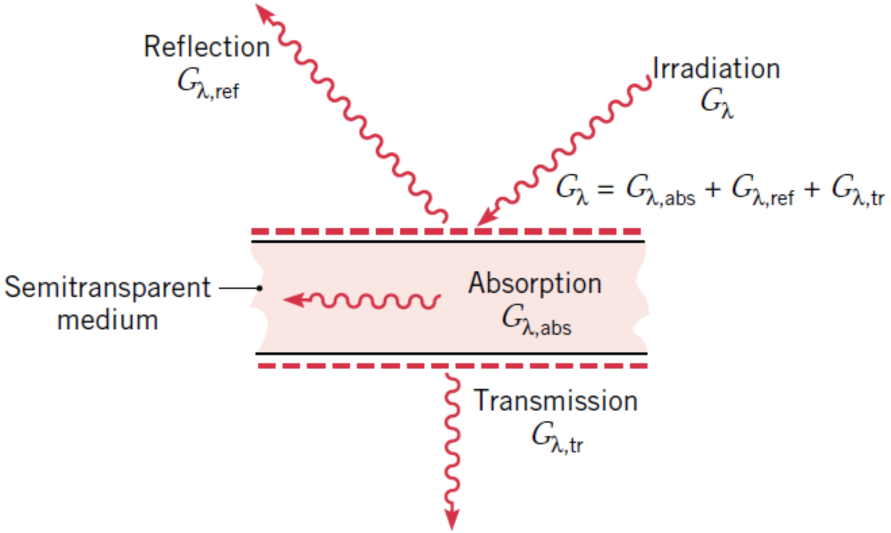
$$E_b(T) = \sigma T^4$$

This Lecture

- Interaction of Thermal Radiation with Matter
  - Absorptivity, Reflectivity and Transmissivity
  - Irradiation and Radiosity
- Black-body
- Real surfaces: Emissivity, Diffuse & Gray Surfaces, Kirchoff’s Laws

Interaction of Thermal Radiation with Matter





How does thermal radiation interacts with matter?

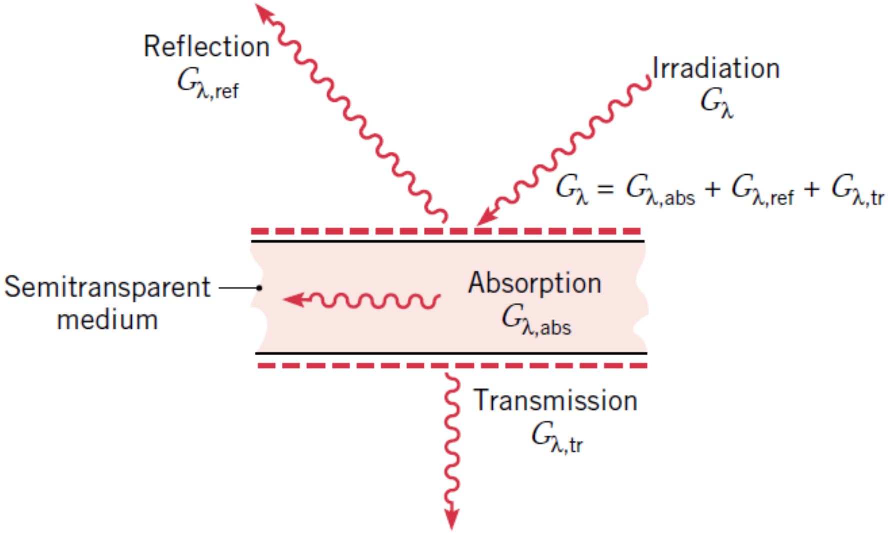
*High-resolution remote thermography using luminescent low-dimensional tin-halide perovskites

Absorptivity, Reflectivity, Transmissivity

$G_{\lambda,ref} = \rho_\lambda G_\lambda$ $\quad$ $\rho_\lambda = reflectivity$

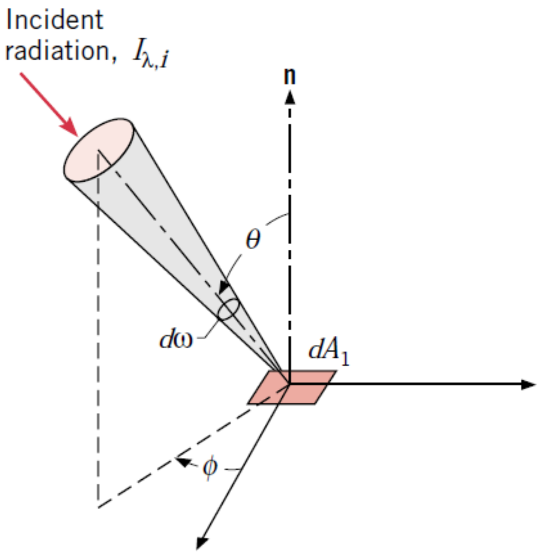
$G_{\lambda,tr} = \tau_\lambda G_\lambda$ $\quad$ $\tau_\lambda = transmissivity$

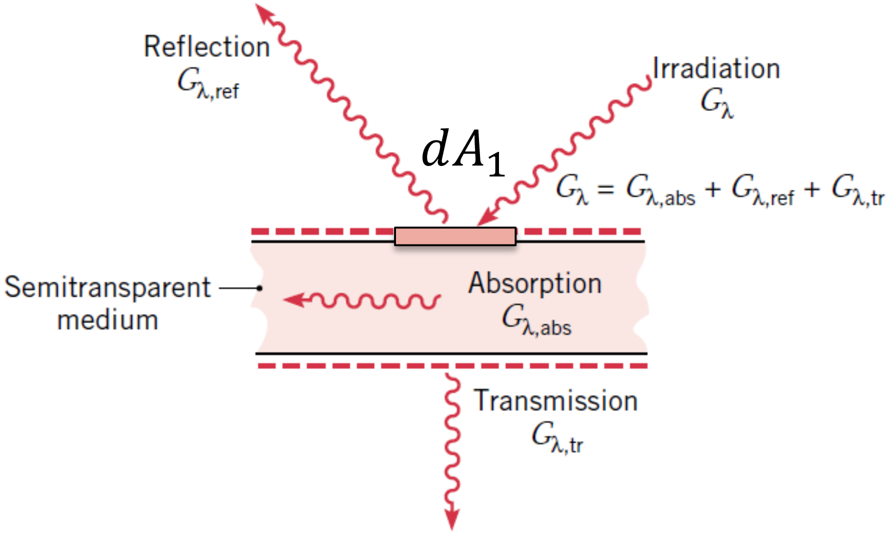
$G_{\lambda,abs} = \alpha_\lambda G_\lambda$ $\quad$ $\alpha_\lambda = absorptivity$



$$G_\lambda = G_{\lambda,abs} + G_{\lambda,ref} + G_{\lambda,tr} \Rightarrow 1 = \alpha_\lambda + \rho_\lambda + \tau_\lambda$$

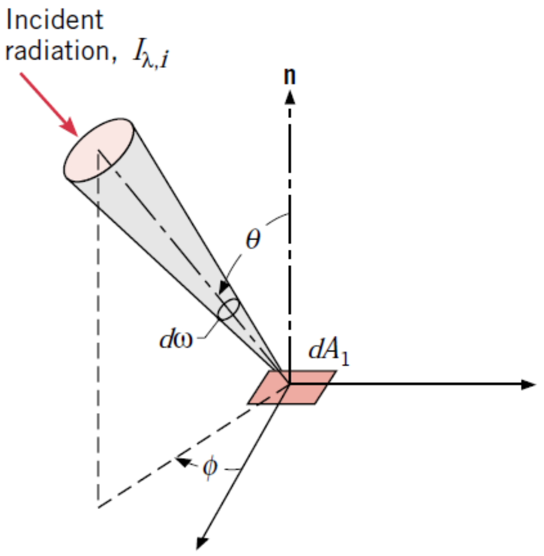
Irradiation G





Similarly to emission, irradiation can also have spatial and spectral distributions.

Irradiation G



Spatial distribution of the incident radiation $\rightarrow$ $I_{\lambda,i}(\lambda, \theta, \Phi)$

- *Spectral irradiation:*

$$G_\lambda(\lambda) = \int_0^{2\pi} \int_0^{\pi/2} I_{\lambda,i}(\lambda, \theta, \Phi) cos\theta sin\theta d\Phi d\theta$$

- *Total irradiation:*

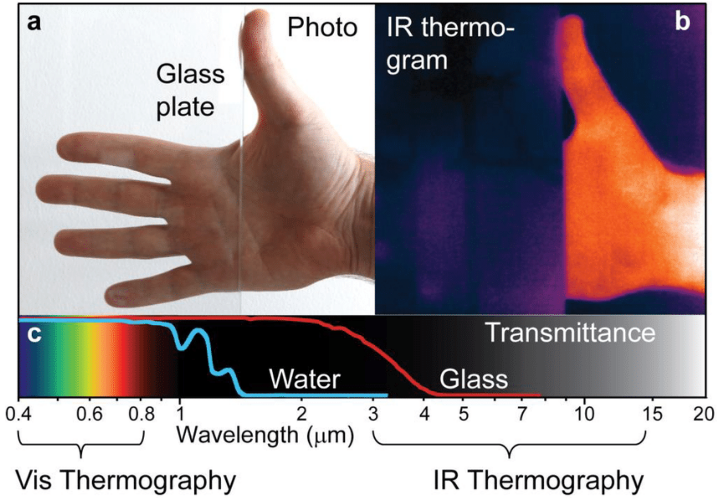
$$G = \int_0^{\infty} G_\lambda(\lambda) d\lambda = \int_0^{\infty} \int_0^{2\pi} \int_0^{\pi/2} I_{\lambda,i}(\lambda, \theta, \Phi) cos\theta sin\theta d\Phi d\theta d\lambda$$

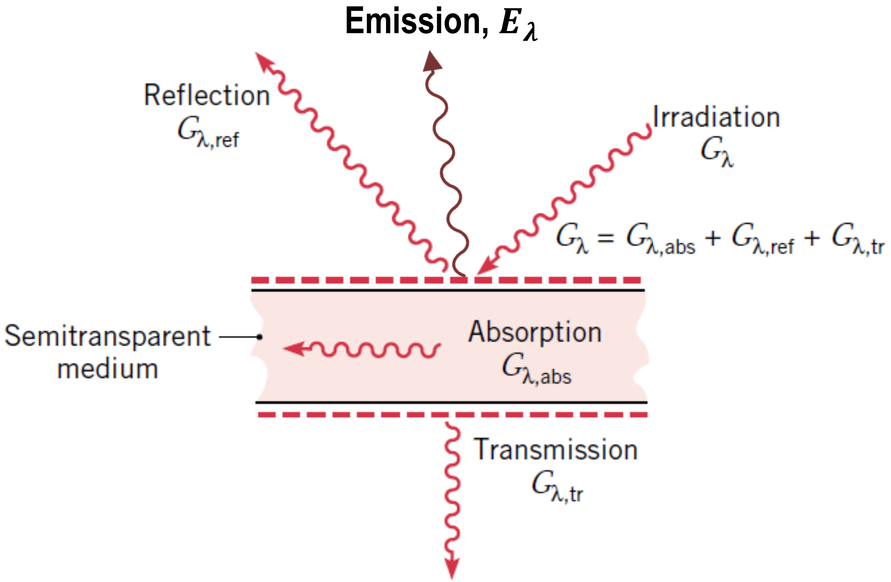
Diffuse irradiation: $I_{\lambda,i}(\lambda, \theta, \Phi) = I_{\lambda,i}(\lambda)$

$$G = \pi \int_0^{\infty} I_{\lambda,i}(\lambda) d\lambda = \pi I_i$$

$I_i = \int_0^{\infty} I_{\lambda,i}(\lambda) d\lambda = total\ intensity$ of irradiation

Radiosity J



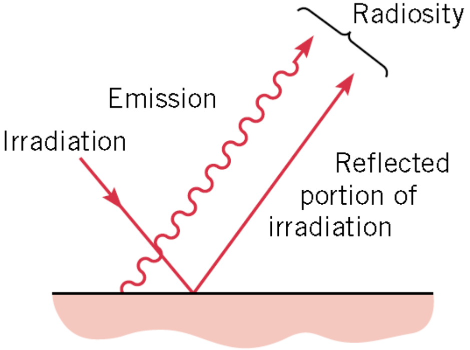


When we look at a surface we detect BOTH the emitted and reflected radiation !

*High-resolution remote thermography using luminescent low-dimensional tin-halide perovskites

Radiosity J

Radiosity (*J*) accounts for ALL the energy leaving the surface along all spatial directions. It thus **combines emission and reflection of irradiation**.



- *Spectral radiosity:*

$$J_\lambda(\lambda) = \int_0^{2\pi} \int_0^{\pi/2} I_{\lambda,e+r}(\lambda, \theta, \Phi) cos\theta sin\theta d\Phi d\theta$$

- *Total radiosity:*

$$J = \int_0^{\infty} J_\lambda(\lambda) d\lambda = \int_0^{\infty} \int_0^{2\pi} \int_0^{\pi/2} I_{\lambda,e+r}(\lambda, \theta, \Phi) cos\theta sin\theta d\Phi d\theta d\lambda$$

Diffuse emitter AND diffuse reflector: $I_{\lambda,e+r}(\lambda, \theta, \Phi) = I_{\lambda,e+r}(\lambda)$

$$J = \pi \int_0^{\infty} I_{\lambda,e+r}(\lambda) d\lambda = \pi I_{e+r}$$

$I_{e+r} = \int_0^{\infty} I_{\lambda,e+r}(\lambda) d\lambda = total\ intensity$ of radiosity

Measures of Radiation

	Intensity $I_{\lambda,x}$	Spectral X_λ $X_\lambda(\lambda) = \int_0^{2\pi} \int_0^{\pi/2} I_{\lambda,x}(\lambda,\theta,\Phi) cos\theta sin\theta d\Phi d\theta$	Total X $X = \int_0^{\infty} X_\lambda(\lambda) d\lambda$
Emission	$I_{\lambda,e}(\lambda,\theta,\Phi)$	$E_\lambda = spectral\ emissive\ power$	$E = emissive\ power$
Irradiation	$I_{\lambda,i}(\lambda,\theta,\Phi)$	$G_\lambda = spectral\ irradiation$	$G = irradiation$
Radiosity	$I_{\lambda,e+r}(\lambda,\theta,\Phi)$	$J_\lambda = spectral\ radiosity$	$J = radiosity$

Diffuse radiation and surfaces = spectral intensity independent of the angular direction

- Diffuse emitter : $I_{\lambda,e}(\lambda,\theta,\Phi) = I_{\lambda,e}(\lambda)$
- Diffuse irradiation : $I_{\lambda,i}(\lambda,\theta,\Phi) = I_{\lambda,i}(\lambda)$
- Diffuse emitter and diffuse reflector : $I_{\lambda,e+r}(\lambda,\theta,\Phi) = I_{\lambda,e+r}(\lambda)$

$$\Rightarrow I_x = \int_0^{\infty} I_{\lambda,x}(\lambda) d\lambda = total\ intensity$$

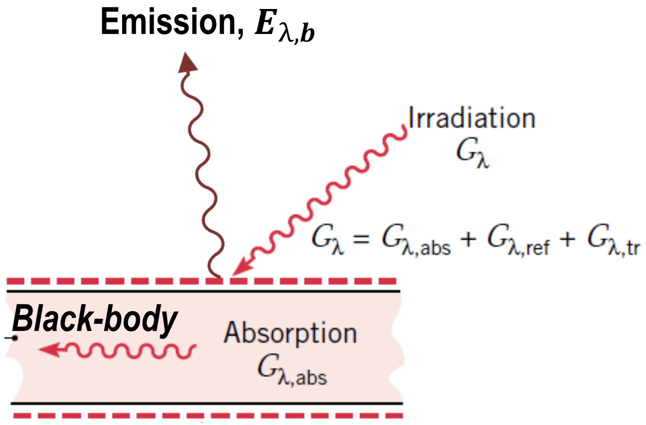
This Lecture

- ☑ Interaction of Thermal Radiation with Matter
  - ☑ Absorptivity, Reflectivity and Transmissivity
  - ☑ Irradiation and Radiosity
- ❑ Black-body
- ❑ Real surfaces: Emissivity, Diffuse & Gray Surfaces, Kirchoff’s Laws

Black-body

A black-body is an object that, for all wavelengths, absorbs all the radiation that is impinging on it:

$$\alpha_\lambda = 1 \quad \rho_\lambda = \tau_\lambda = 0$$

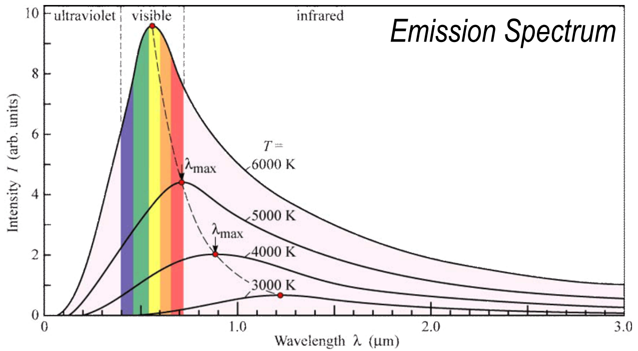


$$G_\lambda = G_{\lambda,abs} = E_{\lambda,b}$$

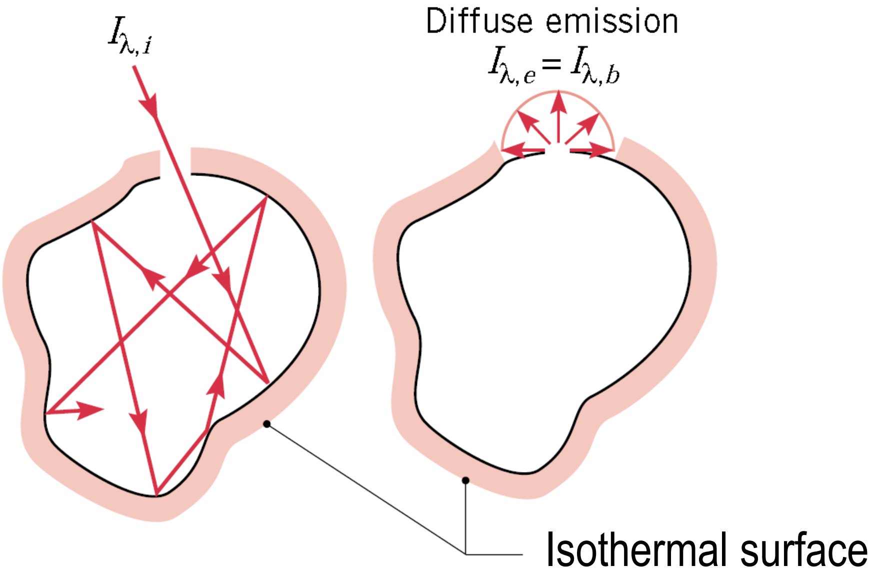
A black-body is by definition a diffuse emitter:

$$I_{\lambda,b}(\lambda,\theta,\Phi) = I_{\lambda,b}(\lambda) = \frac{2hc_0^2}{\lambda^5[\exp(hc_0/\lambda kT)-1]}$$

$$\boldsymbol{E_b(T)} = \pi \int_0^\infty I_{\lambda,b}(\lambda,T)d\lambda = \boldsymbol{\sigma T^4}$$



Black-body



$I_{\lambda,i}$
Diffuse emission
$I_{\lambda,e} = I_{\lambda,b}$
Isothermal surface

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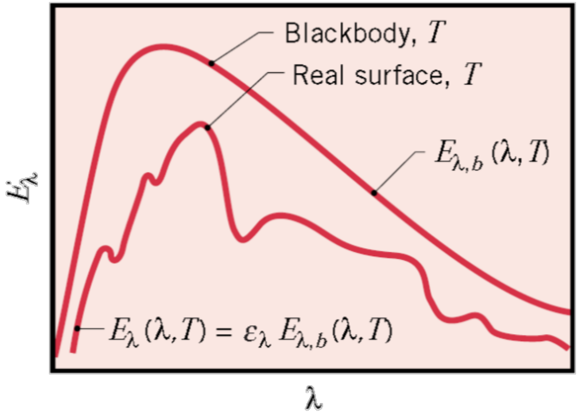
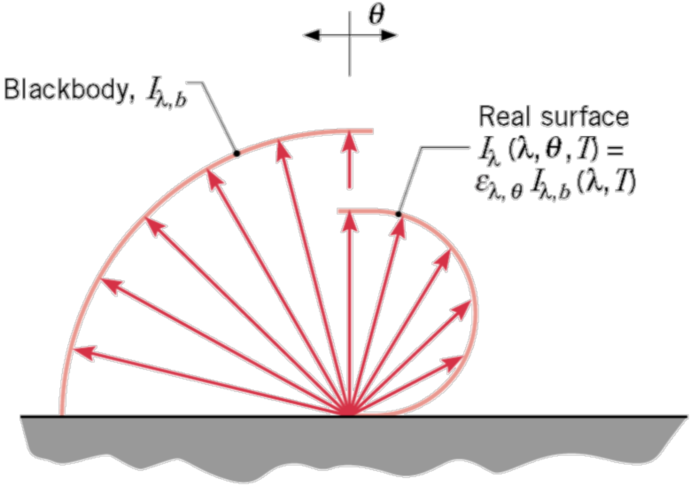
Real Surfaces: Emissivity

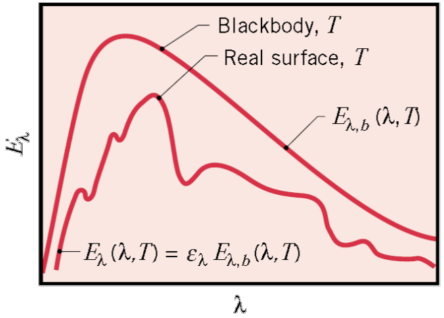


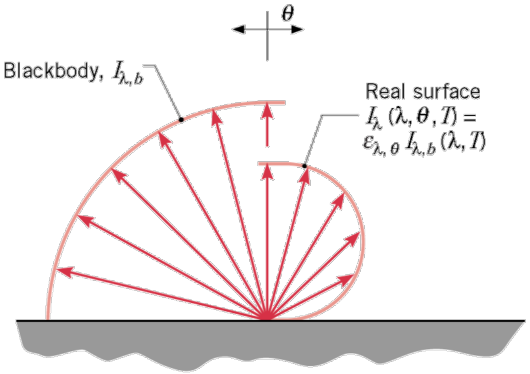


We describe the emission of a real surface with respect to the ideal blackbody introducing a "correction" factor called the ***surface emissivity ε***, such that:

$$E_\lambda(\lambda, T) = \varepsilon_\lambda E_{\lambda,b}(\lambda, T) \qquad I_\lambda(\lambda, \theta, \Phi, T) = \varepsilon_{\lambda,\theta} I_{\lambda,b}(\lambda, T)$$

Real Surfaces: Emissivity





$$\varepsilon_{\lambda,\theta} = \frac{I_\lambda(\lambda, \theta, \Phi, T)}{I_{\lambda,b}(\lambda, T)}$$

$$\varepsilon_\lambda(\lambda, \mathrm{T}) = \frac{E_\lambda(\lambda, T)}{E_{\lambda,b}(\lambda, T)}$$

Spectral hemispherical emissivity (directional average at a given wavelength)

$$\varepsilon(\mathrm{T}) = \frac{E\ (T)}{E_b(T)} = \frac{\int_0^\infty \varepsilon_\lambda(\lambda, \mathrm{T}) E_{\lambda,b}(\lambda, T) d\lambda}{E_b(T)}$$

Total hemispherical emissivity

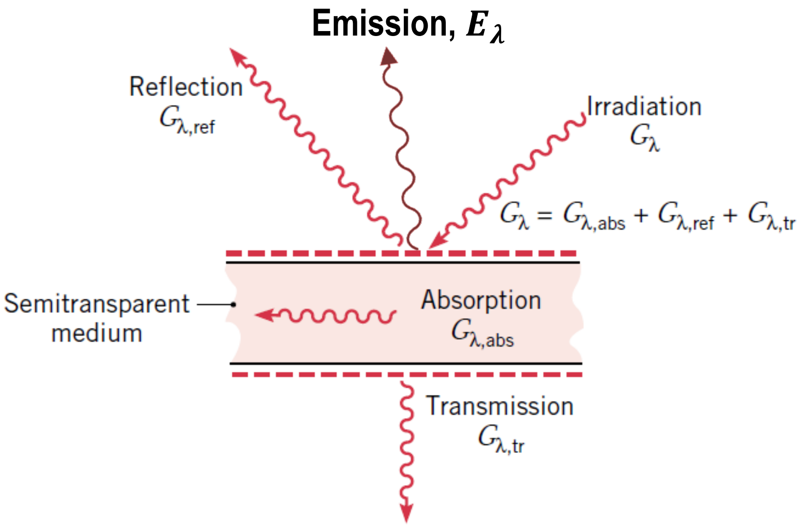
$$\boldsymbol{E(T) = \varepsilon E_b(T) = \varepsilon\sigma T^4} \qquad 0 < \varepsilon < 1$$

Real Surfaces: Diffuse & Gray Surfaces

Black-body : $\alpha_\lambda = 1, \;\; \rho_\lambda = \tau_\lambda = 0$

Real surface : $\alpha_\lambda < 1, \;\; \rho_\lambda > 0, \;\; \tau_\lambda > 0$

$$1 = \alpha_\lambda + \rho_\lambda + \tau_\lambda$$



For a real surface, we can then calculate:

$$\alpha \equiv \frac{G_{abs}}{G} = \frac{\int_0^\infty \alpha_\lambda G_\lambda d\lambda}{\int_0^\infty G_\lambda d\lambda} \qquad \rho \equiv \frac{G_{ref}}{G} = \frac{\int_0^\infty \rho_\lambda G_\lambda d\lambda}{\int_0^\infty G_\lambda d\lambda} \qquad \tau \equiv \frac{G_{tr}}{G} = \frac{\int_0^\infty \tau_\lambda G_\lambda d\lambda}{\int_0^\infty G_\lambda d\lambda}$$

Note: We typically assume that $G_\lambda \propto E_{\lambda,b}(\lambda, T_{surr})$ so that G_λ can be replaced by $E_{\lambda,b}$ in the above expressions.

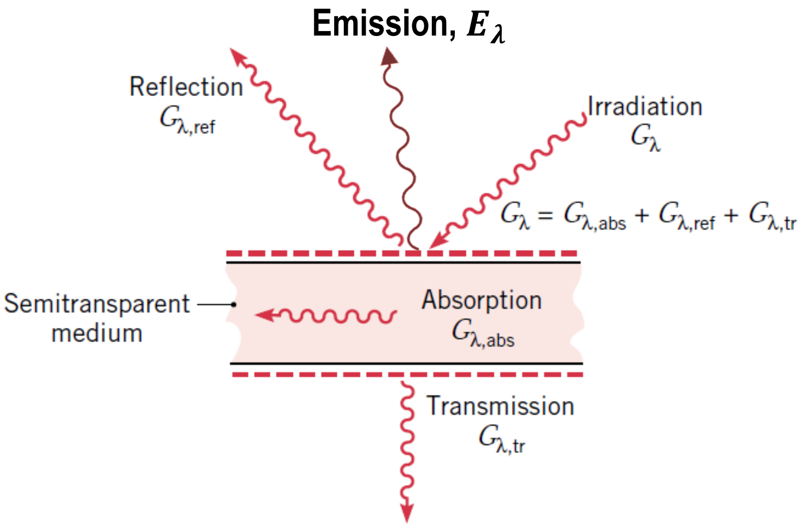
Real Surfaces: Diffuse & Gray Surfaces

Black-body : $\alpha_\lambda = 1, \ \ \rho_\lambda = \tau_\lambda = 0$

Real surface : $\alpha_\lambda < 1, \ \ \rho_\lambda > 0, \ \ \tau_\lambda > 0$

$$1 = \alpha_\lambda + \rho_\lambda + \tau_\lambda$$

$$G_{\lambda,abs} = \alpha_\lambda G_\lambda = E_\lambda = \varepsilon_\lambda E_{\lambda,b}$$



Both absorption and emission occur at the surface of a body.
Is there any relationship between spectral absorptivity, α_λ , and spectral emissivity, ε_λ ?

Real Surfaces: Diffuse and Gray Surfaces

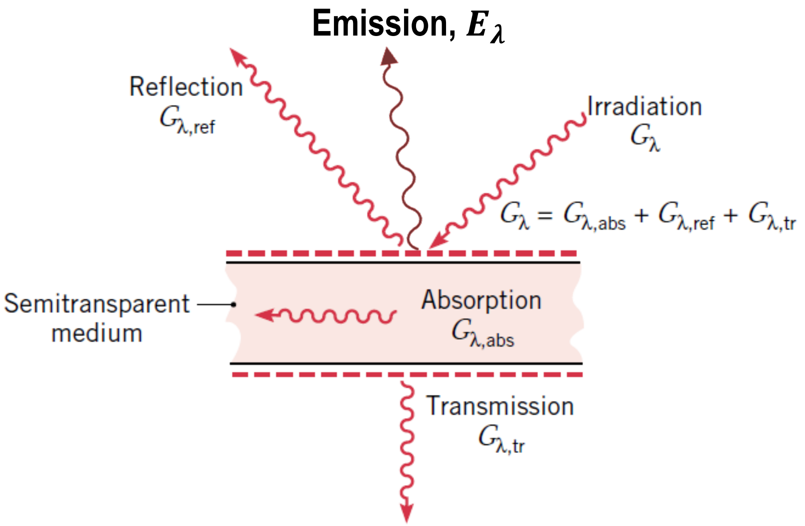
We define the following special cases for real surfaces:

- **Diffuse surface**: $\alpha_{\lambda,\theta}$ and $\varepsilon_{\lambda,\theta}$ are independent of θ, φ

  ➡ $\varepsilon_{\lambda,\theta} = \varepsilon_\lambda \qquad \alpha_{\lambda,\theta} = \alpha_\lambda$

- **Gray surface**: α_λ and ε_λ are independent of λ

  ➡ $\varepsilon_\lambda = \varepsilon \qquad \alpha_\lambda = \alpha$



Real Surfaces: Kirchoff's Laws

It can be shown that the following relationship is always true:

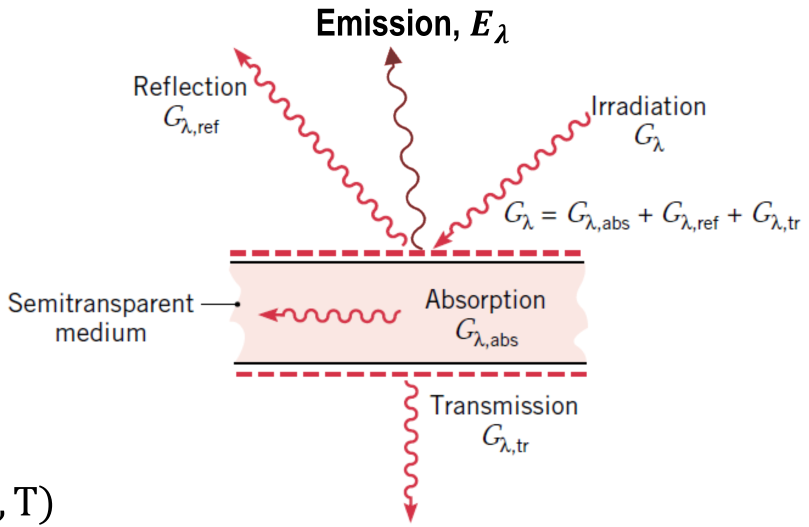
$$\varepsilon_{\lambda,\theta} = \alpha_{\lambda,\theta}$$

In addition, if the irradiation is diffuse ($I_{\lambda,i}(\lambda, \theta, \Phi) = I_{\lambda,i}(\lambda)$)
OR the <u>surface is diffuse</u>:

$$\varepsilon_{\lambda} = \alpha_{\lambda}$$

Finally, if the irradiation is a black-body emission ($G_{\lambda}(\lambda) = E_{\lambda,b}(\lambda, \mathrm{T})$
and $\mathrm{G} = E_b(T)$) **OR** the <u>surface is gray</u>:

$$\varepsilon = \alpha$$



These are Kirchoff's Laws and define the conditions under which we can establish simple relationships for emissivity and absorptivity.

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    - ☑ Irradiation and Radiosity
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Next Lecture

- Examples