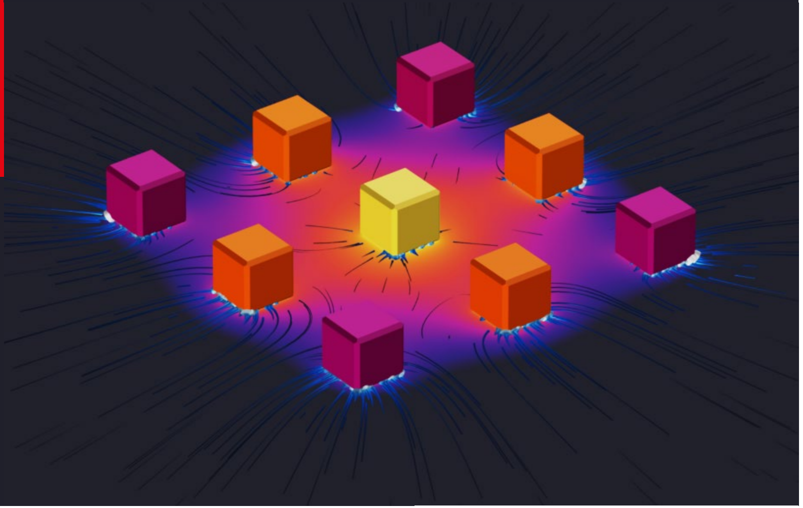
Heat and Mass Transfer
ME-341

Instructor: Giulia Tagliabue



École
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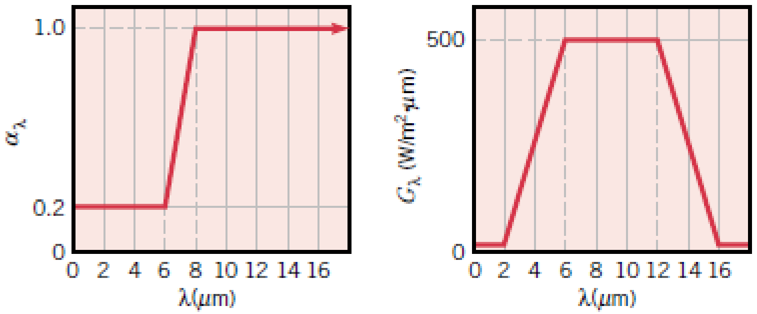
Spring Semester

Examples

Example 1

The spectral, hemispherical absorptivity of an opaque surface and the spectral irradiation at the surface are as shown.



How does the spectral, hemispherical reflectivity vary with wavelength? What is the total, hemispherical absorptivity of the surface? If the surface is initially at 500 K and has a total, hemispherical emissivity of 0.8, how will its temperature change upon exposure to the irradiation?

Assumptions:

1. Surface is opaque.
2. Surface convection effects are negligible.
3. Back surface is insulated.

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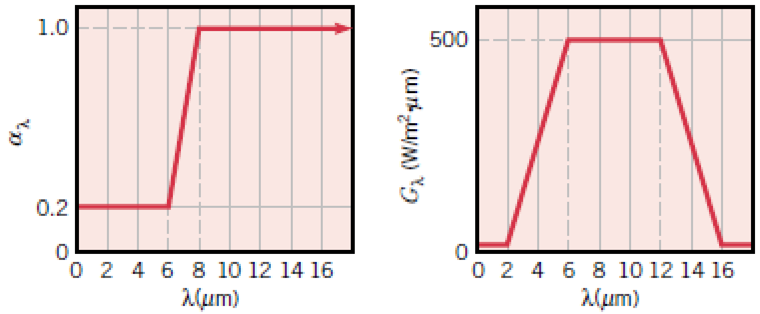

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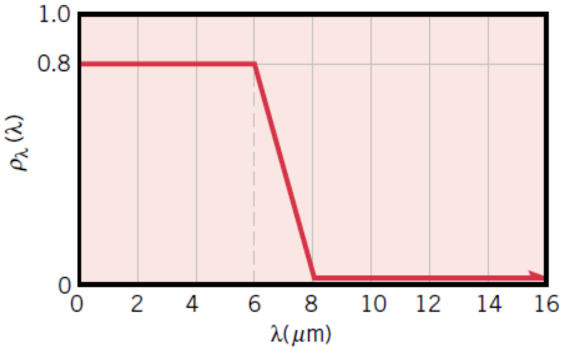
$$\rho_\lambda = 1 - \alpha_\lambda$$



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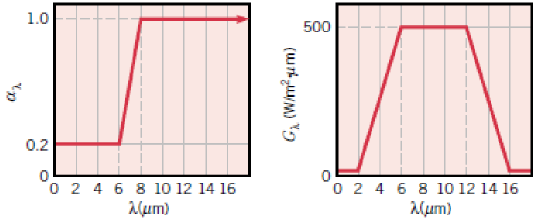

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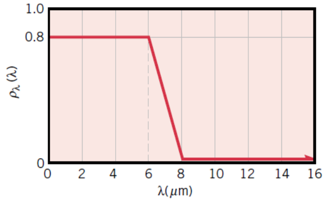


$$\alpha = \frac{G_{abs}}{G} = \frac{\int_0^\infty \alpha_\lambda G_\lambda \, d\lambda}{\int_0^\infty G_\lambda \, d\lambda}$$

or, subdividing the integral into parts,

$$\alpha = \frac{0.2\int_2^6 G_\lambda d\lambda + 500\int_6^8 \alpha_\lambda \, d\lambda + 1.0\int_8^{16} G_\lambda \, d\lambda}{\int_2^6 G_\lambda \, d\lambda + \int_6^{12} G_\lambda \, d\lambda + \int_{12}^{16} G_\lambda \, d\lambda}$$

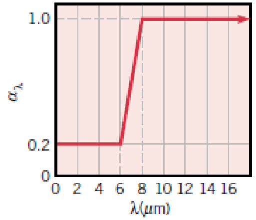
$$\begin{aligned}
\alpha = \{&0.2(\tfrac{1}{2})500 \text{ W/m}^2 \cdot \mu\text{m}\,(6-2)\,\mu\text{m} \\
&+ 500 \text{ W/m}^2 \cdot \mu\text{m}\,[0.2(8-6)\,\mu\text{m} + (1-0.2)(\tfrac{1}{2})(8-6)\,\mu\text{m}] \\
&+ [1 \times 500 \text{ W/m}^2 \cdot \mu\text{m}\,(12-8)\,\mu\text{m} \\
&+ 1(\tfrac{1}{2})500 \text{ W/m}^2 \cdot \mu\text{m}\,(16-12)\,\mu\text{m}]\} \\
&\div [(\tfrac{1}{2})500 \text{ W/m}^2 \cdot \mu\text{m}\,(6-2)\,\mu\text{m} + 500 \text{ W/m}^2 \cdot \mu\text{m}\,(12-6)\,\mu\text{m} \\
&+ (\tfrac{1}{2})500 \text{ W/m}^2 \cdot \mu\text{m}\,(16-12)\,\mu\text{m}]
\end{aligned}$$

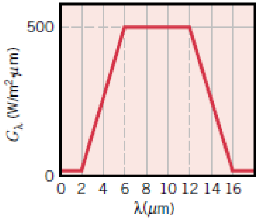
Hence

$$\alpha = \frac{G_{abs}}{G} = \frac{(200 + 600 + 3000) \text{ W/m}^2}{(1000 + 3000 + 1000) \text{ W/m}^2} = \frac{3800 \text{ W/m}^2}{5000 \text{ W/m}^2} = 0.76$$

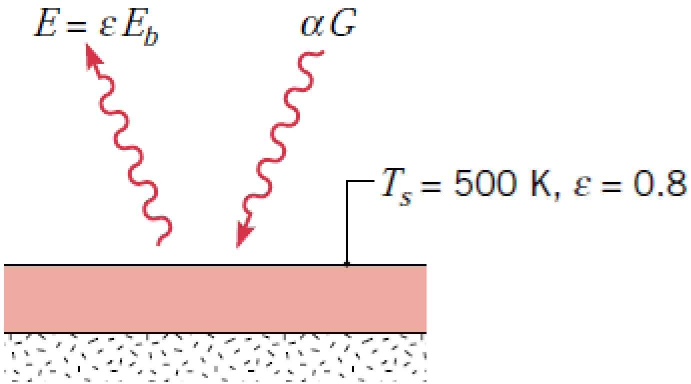
Example 1

The spectral, hemispherical absorptivity of an opaque surface and the spectral irradiation at the surface are as shown.





How does the spectral, hemispherical reflectivity vary with wavelength? What is the total, hemispherical absorptivity of the surface? If the surface is initially at 500 K and has a total, hemispherical emissivity of 0.8, how will its temperature change upon exposure to the irradiation?



Assumptions:

1. Surface is opaque.
2. Surface convection effects are negligible.
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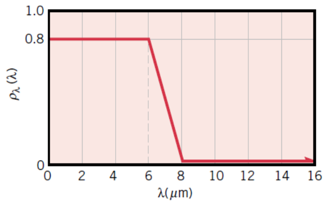


Neglecting convection effects, the net heat flux *to* the surface is

$$q''_{net} = \alpha G - E = \alpha G - \varepsilon \sigma T^4$$

Hence

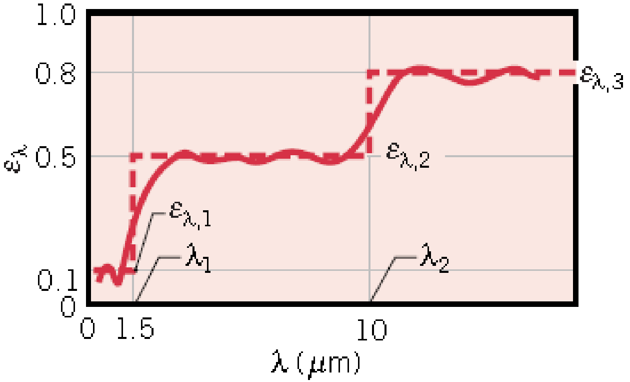
$$q''_{net} = 0.76(5000 \text{ W/m}^2) - 0.8 \times 5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4 (500 \text{ K})^4$$

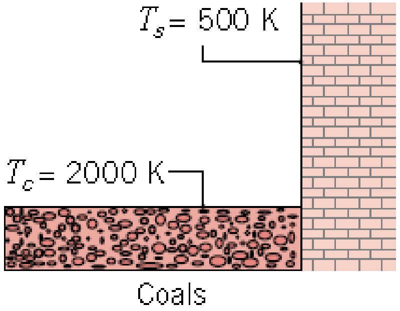
$$q''_{net} = 3800 - 2835 = 965 \text{ W/m}^2$$

Since $q''_{net} > 0$, the surface temperature will *increase* with time.

Example 2

A diffuse, fire brick wall of temperature $T_s = 500$ K has the spectral emissivity shown and is exposed to a bed of coals at 2000 K.





Determine the total, hemispherical emissivity and emissive power of the fire brick wall. What is the total absorptivity of the wall to irradiation resulting from emission by the coals?

Example 2

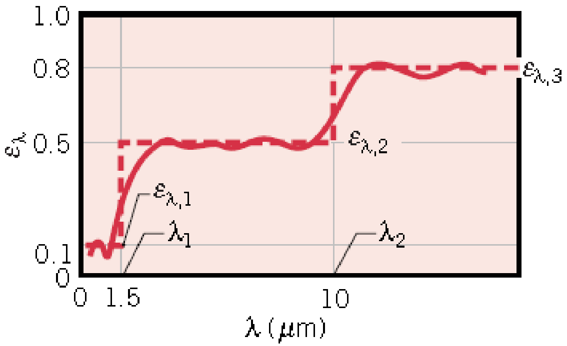
$$\lambda_1 T_s = 1.5\ \mu\text{m} \times 500\ \text{K} = 750\ \mu\text{m}\cdot\text{K}: \qquad F_{(0\to\lambda_1)} = 0.000$$

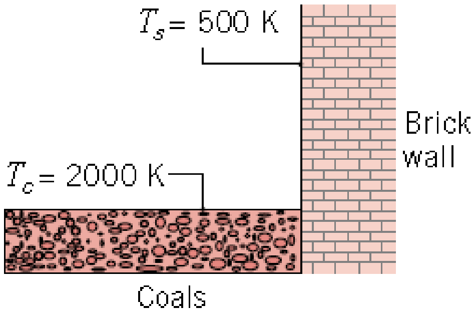
$$\lambda_2 T_s = 10\ \mu\text{m} \times 500\ \text{K} = 5000\ \mu\text{m}\cdot\text{K}: \qquad F_{(0\to\lambda_2)} = 0.634$$

TABLE 12.1 Blackbody Radiation Functions

λT (μm · K)	$F_{(0\to\lambda)}$	$I_{\lambda,b}(\lambda, T)/\sigma T^5$ (μm · K · sr)$^{-1}$	$\dfrac{I_{\lambda,b}(\lambda, T)}{I_{\lambda,b}(\lambda_{max}, T)}$
200	0.000000	0.375034×10^{-27}	0.000000
400	0.000000	0.490335×10^{-13}	0.000000
600	0.000000	0.104046×10^{-8}	0.000014
800	0.000016	0.991126×10^{-7}	0.001372
1,000	0.000321	0.118505×10^{-5}	0.016406
1,200	0.002134	0.523927×10^{-5}	0.072534
1,400	0.007790	0.134411×10^{-4}	0.186082
1,600	0.019718	0.249130	0.344904
1,800	0.039341	0.375568	0.519949
2,000	0.066728	0.493432	0.683123
2,200	0.100888	0.589649×10^{-4}	0.816329
2,400	0.140256	0.658866	0.912155
2,600	0.183120	0.701292	0.970891
2,800	0.227897	0.720239	0.997123
2,898	0.250108	0.722318×10^{-4}	1.000000
3,000	0.273232	0.720254×10^{-4}	0.997143
3,200	0.318102	0.705974	0.977373
3,400	0.361735	0.681544	0.943551
3,600	0.403607	0.650396	0.900429
3,800	0.443382	0.615225×10^{-4}	0.851737
4,000	0.480877	0.578064	0.800291
4,200	0.516014	0.540394	0.748139
4,400	0.548796	0.503253	0.696720
4,600	0.579280	0.467343	0.647004
4,800	0.607559	0.433109	0.599610
5,000	0.633747	0.400813	0.554898

Example 2





Assumptions:

1. Brick wall is opaque and diffuse.
2. Spectral distribution of irradiation at the brick wall approximates that due to emission from a blackbody at 2000 K.

a) Total hemispherical emissivity

$$\varepsilon(T_s) = \frac{\int_0^\infty \varepsilon_\lambda(\lambda) E_{\lambda,b}(\lambda, T_s)\, d\lambda}{E_b(T_s)}$$

$$\varepsilon(T_s) = \varepsilon_{\lambda,1} \frac{\int_0^{\lambda_1} E_{\lambda,b}\, d\lambda}{E_b} + \varepsilon_{\lambda,2} \frac{\int_{\lambda_1}^{\lambda_2} E_{\lambda,b}\, d\lambda}{E_b} + \varepsilon_{\lambda,3} \frac{\int_{\lambda_2}^{\infty} E_{\lambda,b}\, d\lambda}{E_b}$$

$$\varepsilon(T_s) = \varepsilon_{\lambda,1} F_{(0\to\lambda_1)} + \varepsilon_{\lambda,2}[F_{(0\to\lambda_2)} - F_{(0\to\lambda_1)}] + \varepsilon_{\lambda,3}[1 - F_{(0\to\lambda_2)}]$$

Example 2

$$\lambda_1 T_s = 1.5\ \mu m \times 500\ K = 750\ \mu m \cdot K: \qquad F_{(0\rightarrow\lambda_1)} = 0.000$$

$$\lambda_2 T_s = 10\ \mu m \times 500\ K = 5000\ \mu m \cdot K: \qquad F_{(0\rightarrow\lambda_2)} = 0.634$$

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λT ($\mu m \cdot K$)	$F_{(0\rightarrow\lambda)}$	$I_{\lambda,b}(\lambda,T)/\sigma T^5$ ($\mu m \cdot K \cdot sr)^{-1}$	$\dfrac{I_{\lambda,b}(\lambda,T)}{I_{\lambda,b}(\lambda_{max},T)}$
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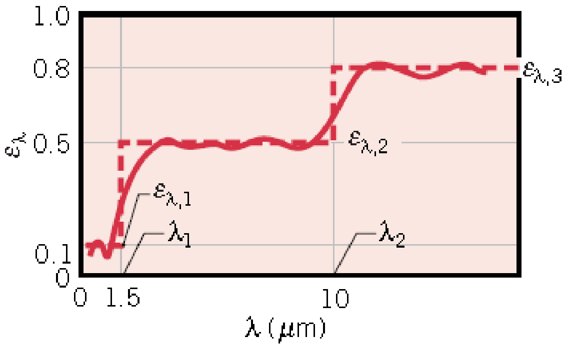
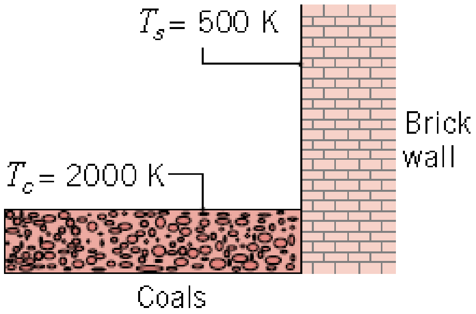




Assumptions:

1. Brick wall is opaque and diffuse.
2. Spectral distribution of irradiation at the brick wall approximates that due to emission from a blackbody at 2000 K.

$$\varepsilon(T_s) = \varepsilon_{\lambda,1} F_{(0\rightarrow\lambda_1)} + \varepsilon_{\lambda,2}[F_{(0\rightarrow\lambda_2)} - F_{(0\rightarrow\lambda_1)}] + \varepsilon_{\lambda,3}[1 - F_{(0\rightarrow\lambda_2)}]$$

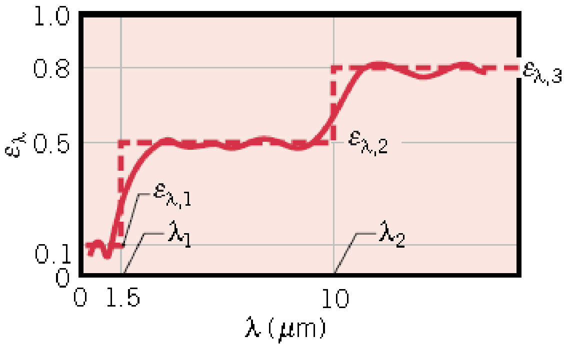
$$\varepsilon(T_s) = 0.1 \times 0 + 0.5 \times 0.634 + 0.8\,(1 - 0.634) = 0.610$$

b) Total emissive power

$$E(T_s) = \varepsilon(T_s)E_b(T_s) = \varepsilon(T_s)\sigma T_s^4$$

$$E(T_s) = 0.61 \times 5.67 \times 10^{-8}\ \mathrm{W/m^2 \cdot K^4}(500\ \mathrm{K})^4 = 2162\ \mathrm{W/m^2}$$

Example 2



Assumptions:

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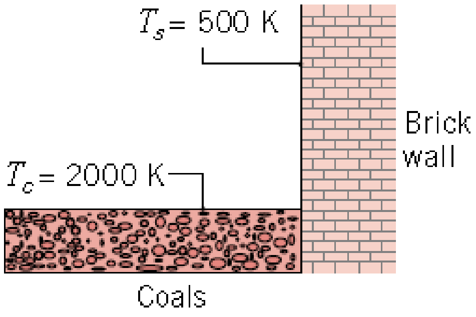
c) Total absorptivity due to coal irradiation

$$\alpha = \frac{\int_0^\infty \alpha_\lambda(\lambda) G_\lambda(\lambda)\, d\lambda}{\int_0^\infty G_\lambda(\lambda)\, d\lambda}$$

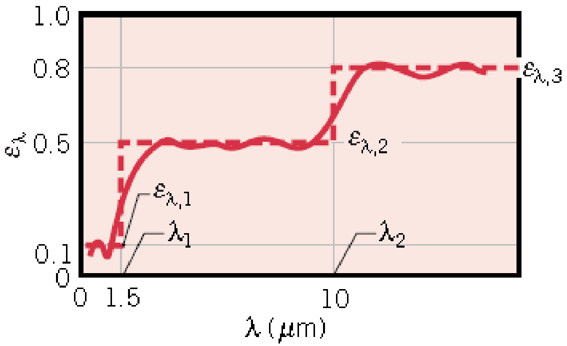


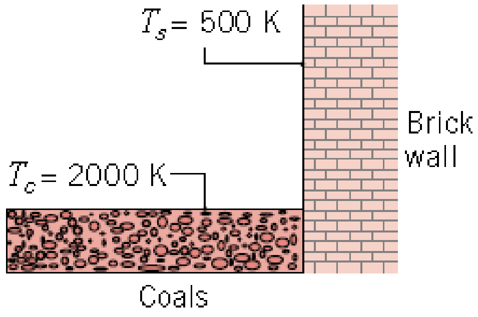
Surface is diffuse: $\alpha_\lambda(\lambda) = \varepsilon_\lambda(\lambda)$

Coal emission approximate blackbody: $G_\lambda(\lambda) \propto E_{\lambda,b}(\lambda, T_c)$

$$\alpha = \frac{\int_0^\infty \varepsilon_\lambda(\lambda) E_{\lambda,b}(\lambda, T_c)\, d\lambda}{\int_0^\infty E_{\lambda,b}(\lambda, T_c)\, d\lambda}$$

Example 2





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$$\alpha = \frac{\int_0^\infty \varepsilon_\lambda(\lambda) E_{\lambda,b}(\lambda, T_c)\, d\lambda}{\int_0^\infty E_{\lambda,b}(\lambda, T_c)\, d\lambda}$$

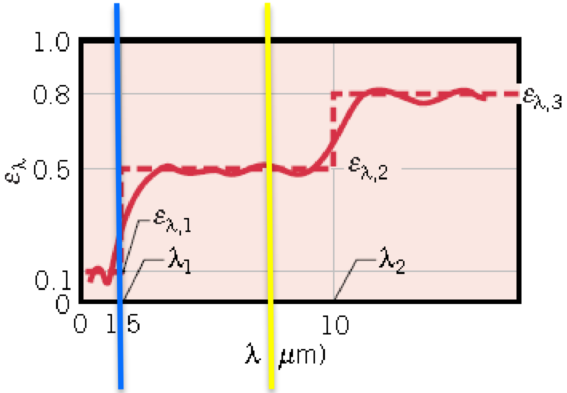
$$\alpha = \varepsilon_{\lambda,1} F_{(0\to\lambda_1)} + \varepsilon_{\lambda,2}[F_{(0\to\lambda_2)} - F_{(0\to\lambda_1)}] + \varepsilon_{\lambda,3}[1 - F_{(0\to\lambda_2)}]$$

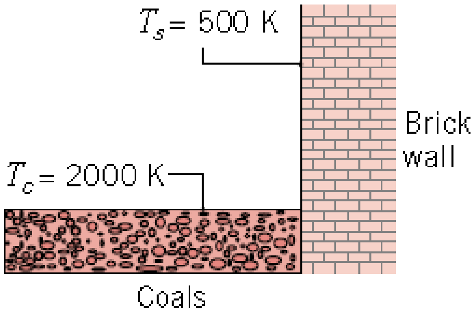
$$\lambda_1 T_c = 1.5\ \mu\text{m} \times 2000\ \text{K} = 3000\ \mu\text{m}\cdot\text{K}: \qquad F_{(0\to\lambda_1)} = 0.273$$

$$\lambda_2 T_c = 10\ \mu\text{m} \times 2000\ \text{K} = 20{,}000\ \mu\text{m}\cdot\text{K}: \qquad F_{(0\to\lambda_2)} = 0.986$$

$$\alpha = 0.1 \times 0.273 + 0.5(0.986 - 0.273) + 0.8(1 - 0.986) = 0.395$$

Example 2





The surface is not gray, $\alpha \neq \varepsilon$

We note that the coal BB emission will peak at:

$$(\lambda_{max} 2000K) \;\; = 2898\; \mu m \cdot K \qquad \boxed{\lambda_{\max} \approx 1.5\; \mu\mathrm{m}}$$

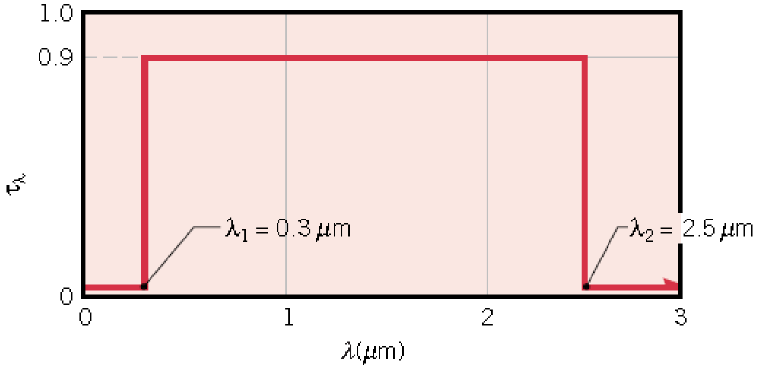
The brick wall emission will peak at:

$$(\lambda_{max} 500K) \;\; = 2898\; \mu m \cdot K \qquad \boxed{\lambda_{\max} \approx \; 6\; \mu\mathrm{m}}$$

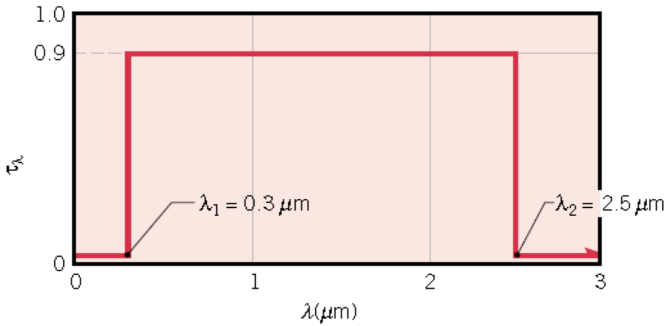
Because the coal BB emission peak $\lambda = 1.5 \mu m$ there will be a significant fraction of coal BB power for shorter λ . In an extended range it is not true that ε_λ is constant and therefore it cannot be a gray surface

Real Surfaces : Example

The cover glass on a flat-plate solar collector has a low iron content, and its spectral transmissivity may be approximated by the following distribution.



What is the total transmissivity of the cover glass to solar radiation?



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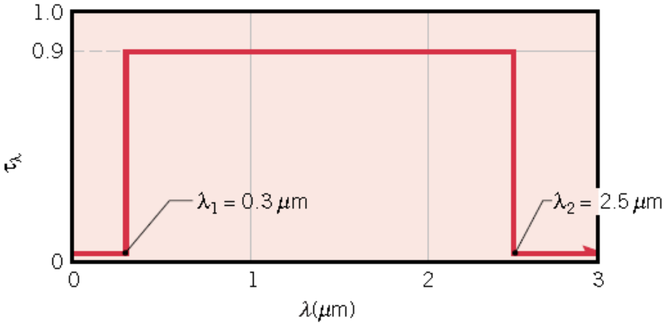

Assumptions: Spectral distribution of solar irradiation is proportional to that of blackbody emission at 5800 K.

$$\tau = \frac{\int_0^\infty \tau_\lambda G_\lambda\, d\lambda}{\int_0^\infty G_\lambda\, d\lambda}$$

$$G_\lambda(\lambda) \propto E_{\lambda,b}(5800\text{ K})$$

$$\tau = 0.90 \frac{\int_{0.3}^{2.5} E_{\lambda,b}(5800\text{ K})\, d\lambda}{E_b(5800\text{ K})}$$

$$\tau = 0.90[F_{(0\to\lambda_2)} - F_{(0\to\lambda_1)}] = 0.84$$

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λT ($\mu m \cdot K$)	$F_{(0\to\lambda)}$	$I_{\lambda,b}(\lambda,T)/\sigma T^5$ $(\mu m \cdot K \cdot sr)^{-1}$	$\frac{I_{\lambda,b}(\lambda,T)}{I_{\lambda,b}(\lambda_{max},T)}$
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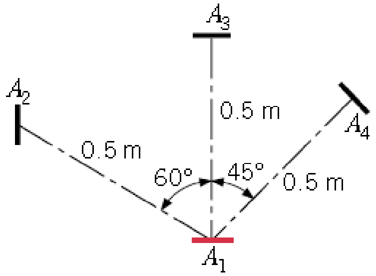
$\lambda_1 = 0.3\ \mu m$, $T = 5800$ K: $\lambda_1 T = 1740\ \mu m \cdot K$, $F_{(0\to\lambda_1)} = 0.0335$

$\lambda_2 = 2.5\ \mu m$, $T = 5800$ K: $\lambda_2 T = 14{,}500\ \mu m \cdot K$, $F_{(0\to\lambda_2)} = 0.9664$

Supplementary Slides

Radiation Intensity *I* and Emissive Power *E* : Example

A small surface of area $A_1 = 10^{-3}\ m^2$ is known to emit diffusely, and from measurements the total intensity associated with emission in the normal direction is $I_n = 7000\ W/m^2 \cdot sr$.



Radiation emitted from the surface is intercepted by three other surfaces of area $A_2 = A_3 = A_4 = 10^{-3}\ m^2$, which are 0.5 m from A_1 and are oriented as shown. What is the intensity associated with emission in each of the three directions? What are the solid angles subtended by the three surfaces when viewed from A_1? What is the rate at which radiation emitted by A_1 is intercepted by the three surfaces?

Assumptions:

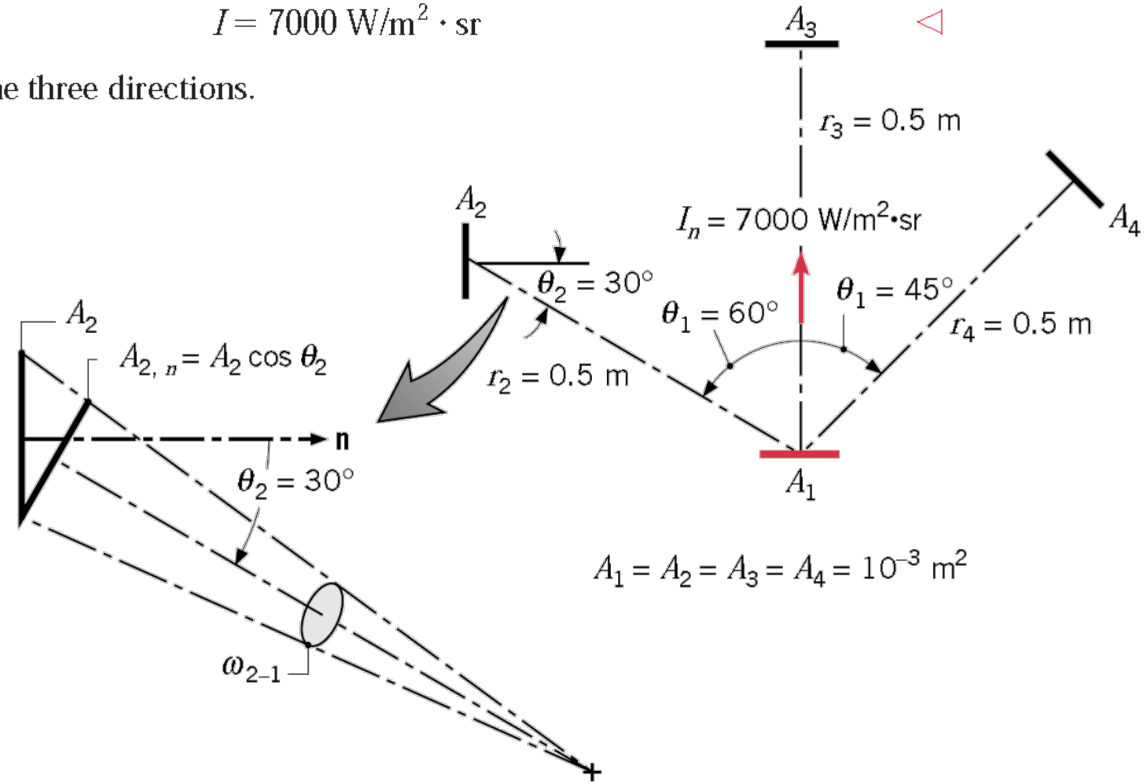
1. Surface A_1 emits diffusely.
2. A_1, A_2, A_3, and A_4 may be approximated as differential surfaces, $(A_j/r_j^2) \ll 1$.

Radiation Intensity *I* and Emissive Power *E* : Example

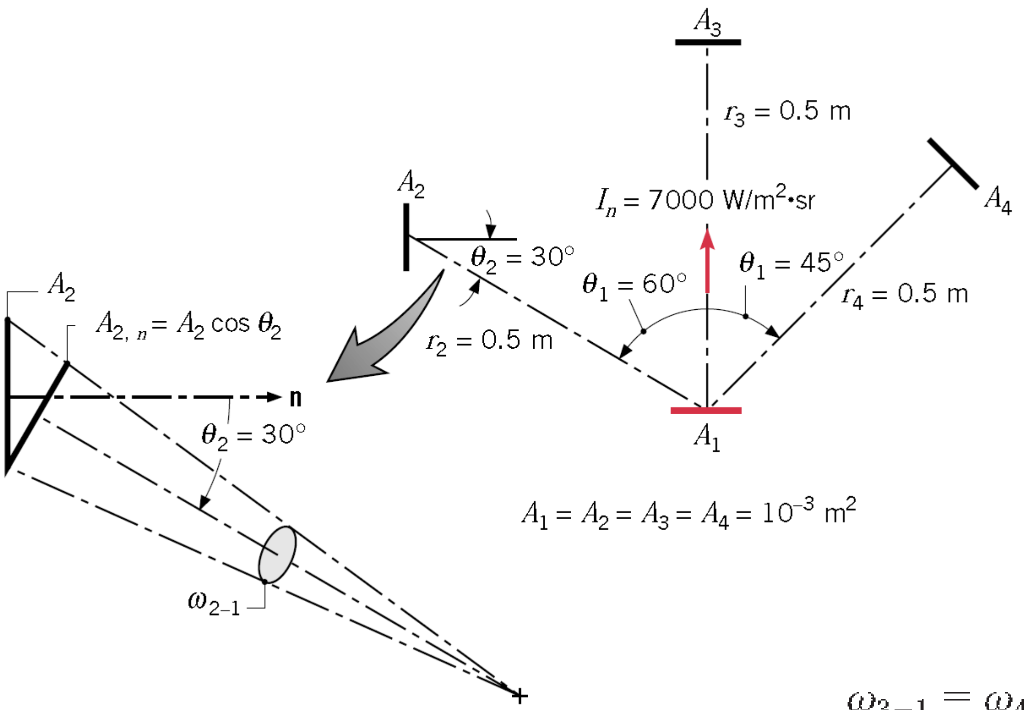
1. From the definition of a diffuse emitter, we know that the intensity of the emitted radiation is independent of direction. Hence

$$I = 7000\ \mathrm{W/m^2 \cdot sr} \quad \triangleleft$$

for each of the three directions.



Radiation Intensity *I* and Emissive Power *E* : Example

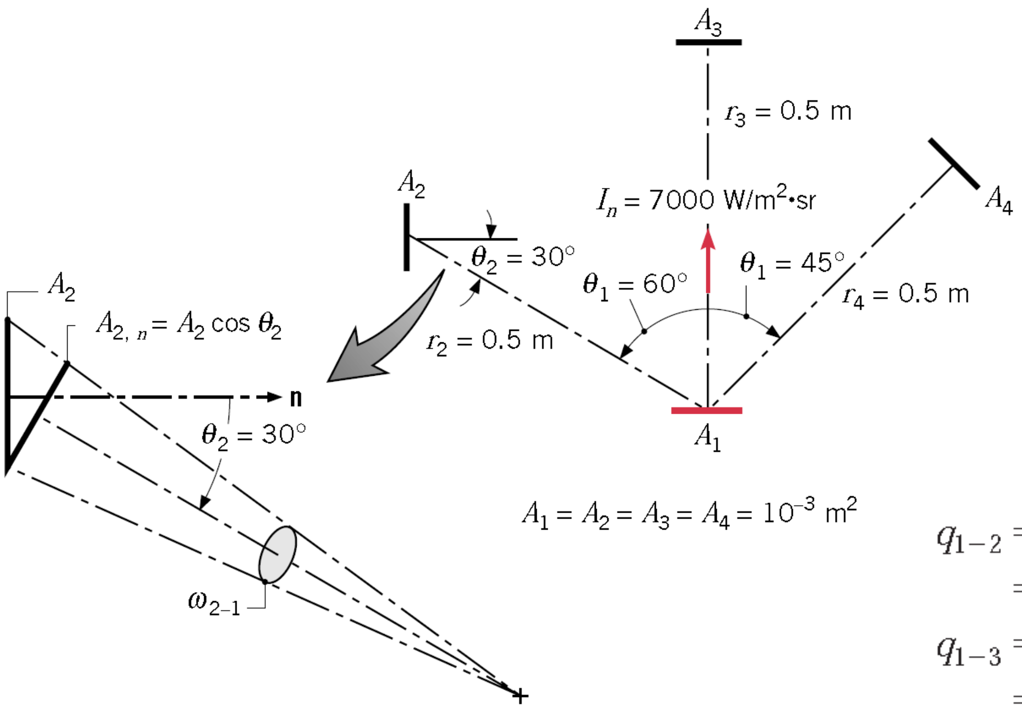


$$d\omega \equiv \frac{dA_n}{r^2}$$

$$\omega_{3-1} = \omega_{4-1} = \frac{A_3}{r^2} = \frac{10^{-3}\ \mathrm{m}^2}{(0.5\ \mathrm{m})^2} = 4.00 \times 10^{-3}\ \mathrm{sr}$$

$$\omega_{2-1} = \frac{A_2 \cos\theta_2}{r^2} = \frac{10^{-3}\ \mathrm{m}^2 \times \cos 30°}{(0.5\ \mathrm{m})^2} = 3.46 \times 10^{-3}\ \mathrm{sr}$$

Radiation Intensity *I* and Emissive Power *E* : Example



$$dq_\lambda = I_{\lambda,e}(\lambda, \theta, \phi)\, dA_1 \cos\theta \, d\omega$$

$$dq_{1\to j} = I_1 A_1 cos\theta_1 d\omega_j$$

$$\begin{aligned} q_{1-2} &= 7000\ \mathrm{W/m^2 \cdot sr}\ (10^{-3}\ \mathrm{m^2} \times \cos 60°)\, 3.46 \times 10^{-3}\ \mathrm{sr} \\ &= 12.1 \times 10^{-3}\ \mathrm{W} \\ q_{1-3} &= 7000\ \mathrm{W/m^2 \cdot sr}\ (10^{-3}\ \mathrm{m^2} \times \cos 0°)\, 4.00 \times 10^{-3}\ \mathrm{sr} \\ &= 28.0 \times 10^{-3}\ \mathrm{W} \\ q_{1-4} &= 7000\ \mathrm{W/m^2 \cdot sr}\ (10^{-3}\ \mathrm{m^2} \times \cos 45°)\, 4.00 \times 10^{-3}\ \mathrm{sr} \\ &= 19.8 \times 10^{-3}\ \mathrm{W} \end{aligned}$$