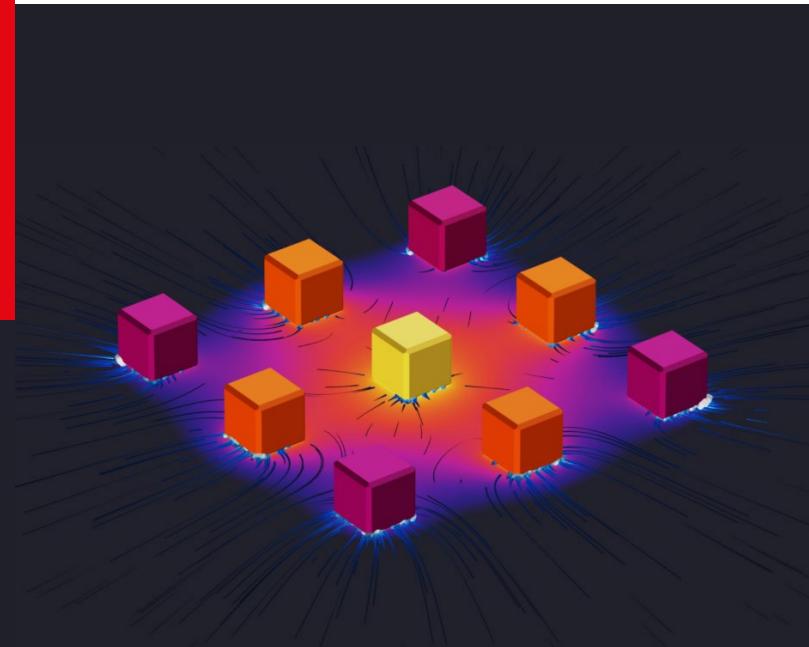


# Heat and Mass Transfer

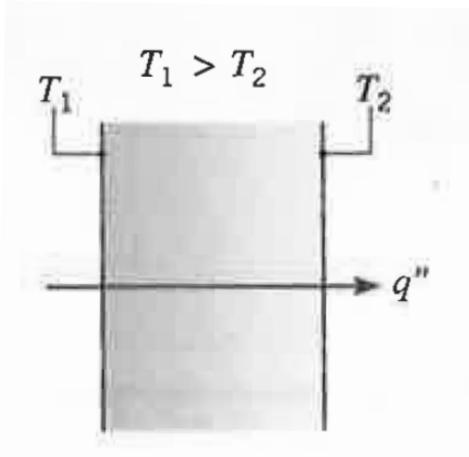
## ME-341

*Instructor:* Giulia Tagliabue

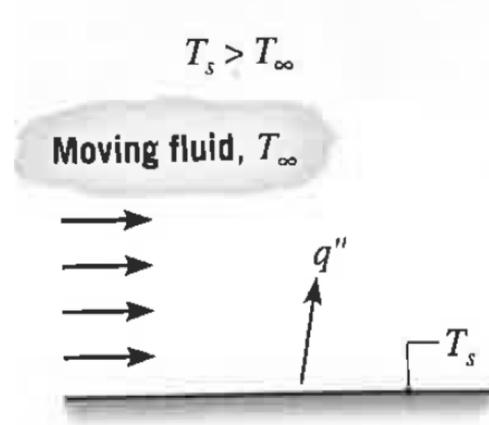


# Heat Transfer Mechanisms

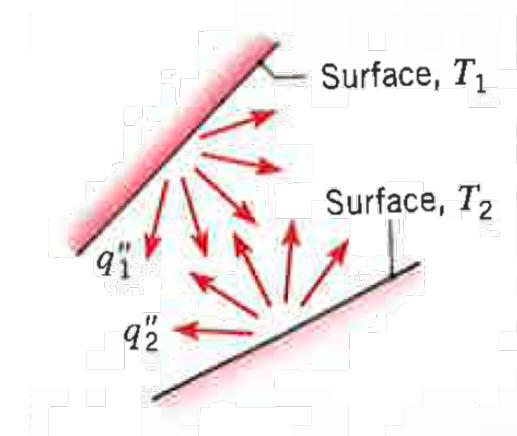
## Conduction



## Convection



## Radiation



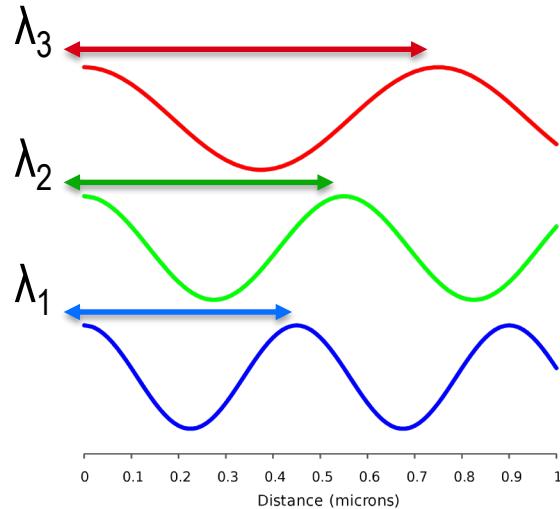
Involves mass transport

Involve physical contact

# Electromagnetic Radiation

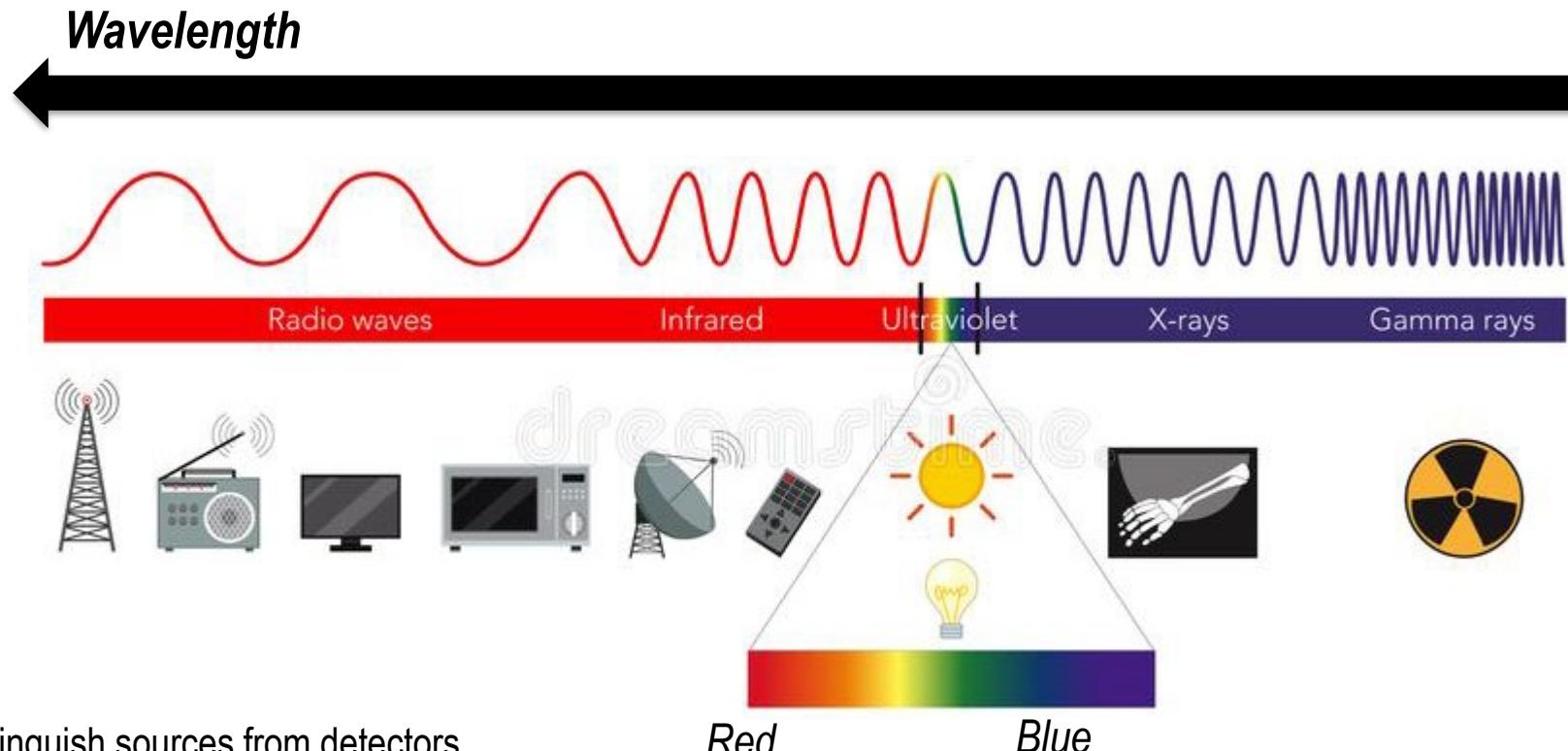


$\lambda = \text{wavelength}$



$$c = \lambda v$$

# Electromagnetic Radiation



# Thermal Radiation

High T



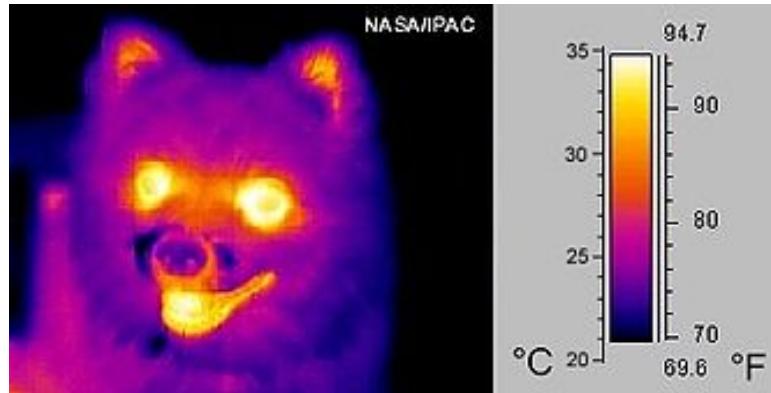
*Very Bright Visible Light Emission*

Medium T



*Visible Light Emission*

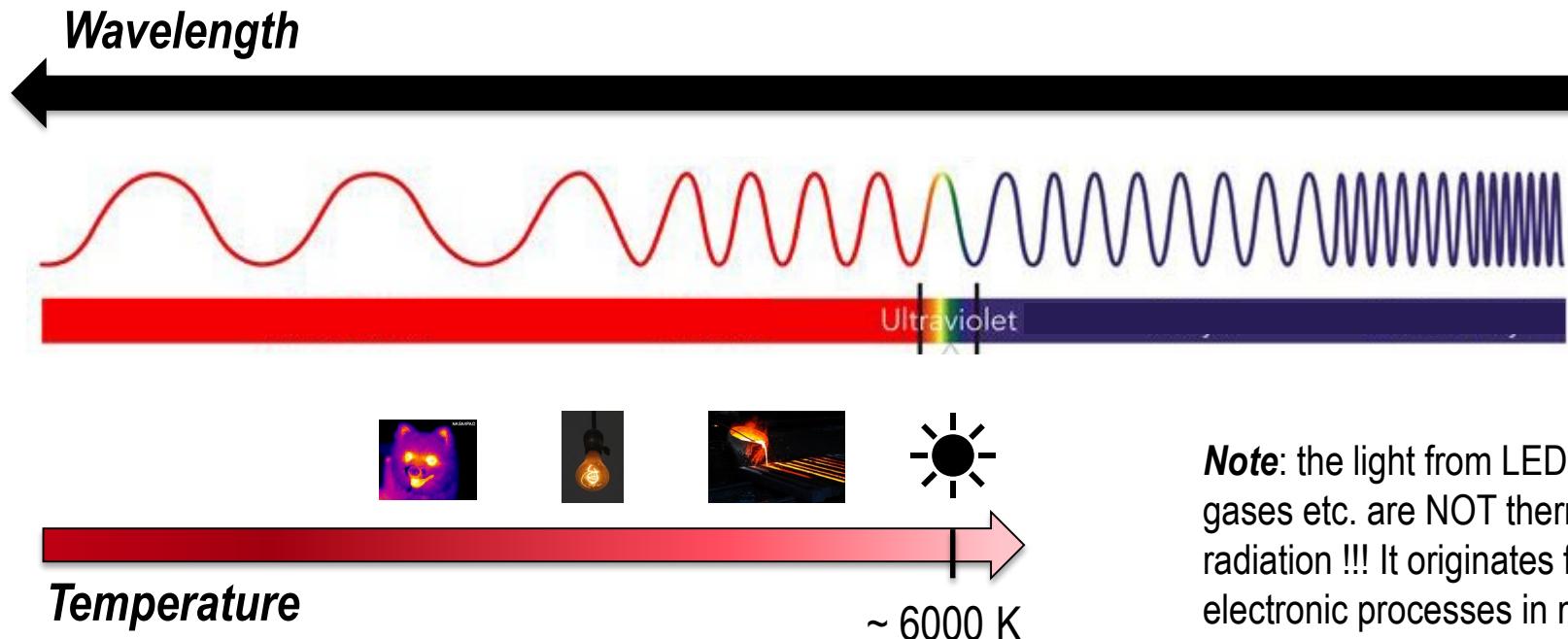
Low T



*Infrared Light Emission (needs IR camera)*

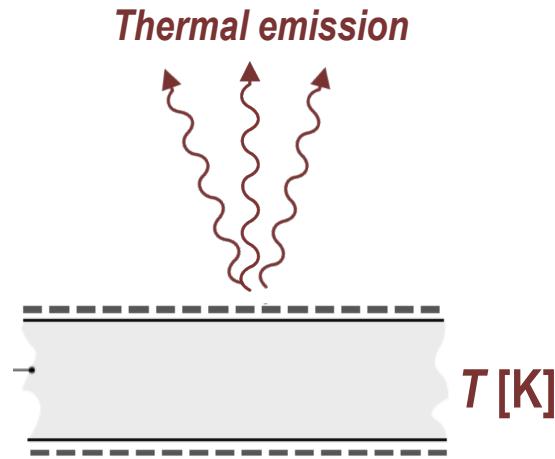
What is the relationship between the temperature of an object and its **emission** of electromagnetic waves (intensity and wavelength)?

# Thermal Radiation



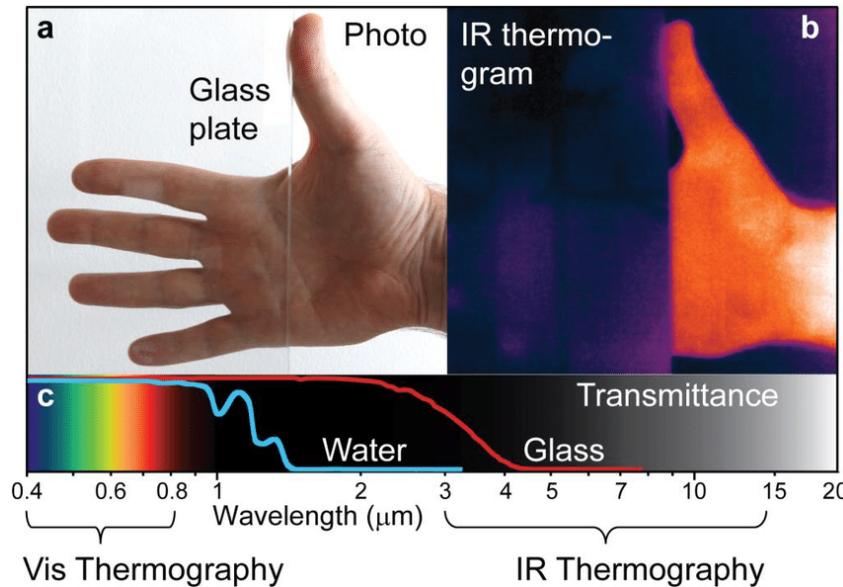
**Note:** the light from LEDs, from gases etc. are NOT thermal radiation !!! It originates from electronic processes in materials.

# Thermal Radiation



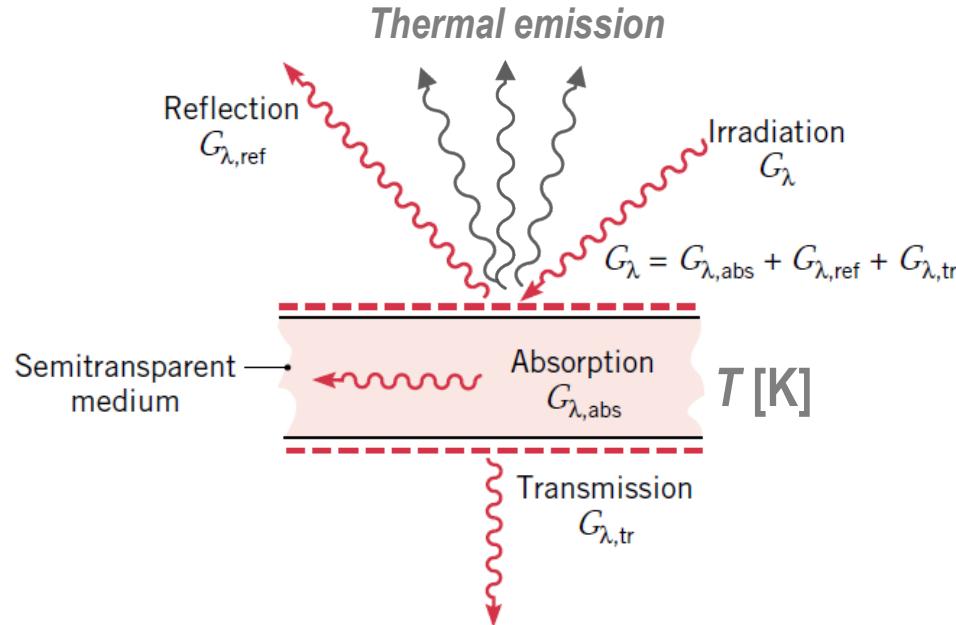
Temperature and material properties determine how an object EMITS thermal radiation.

# Thermal Radiation



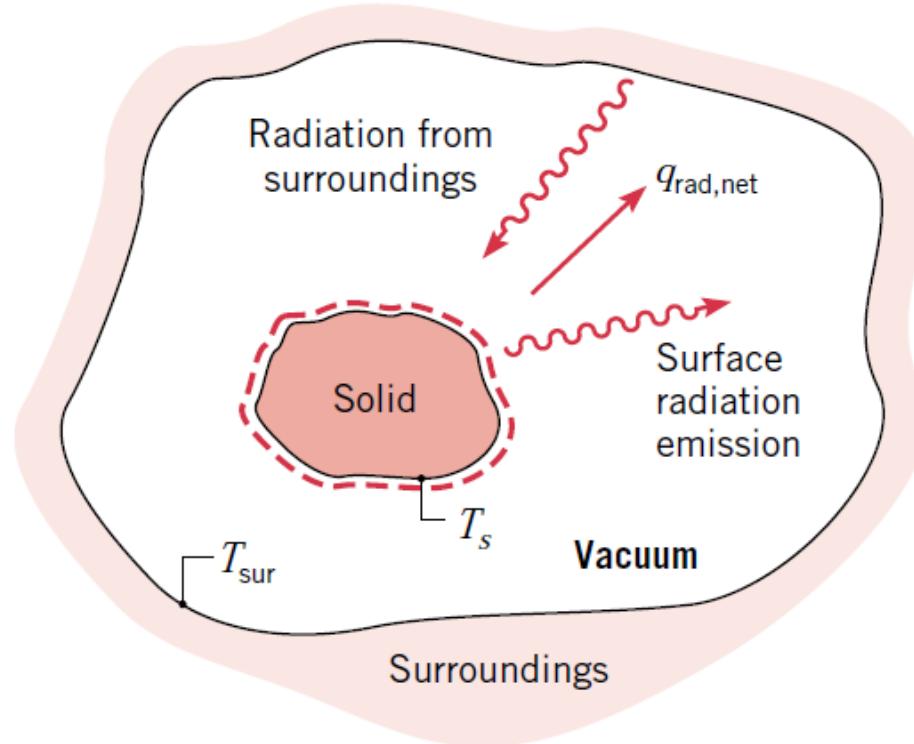
How does thermal radiation interacts with matter?

# Thermal Radiation



**Wavelength and material properties determine how thermal radiation interacts with objects.**

# Radiative Heat Transfer



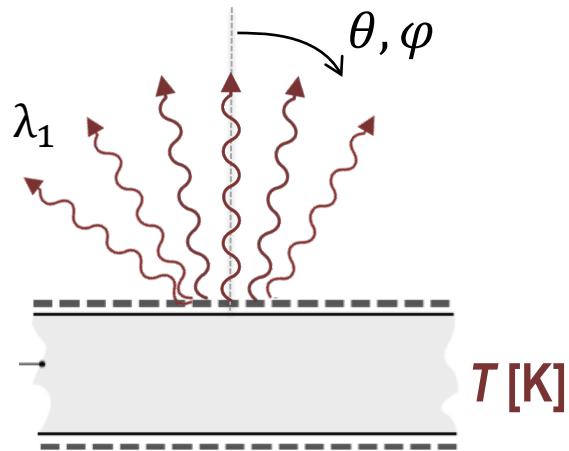
# This lecture

- Emission of Thermal Radiation
  - Spatial distribution and Diffuse Emitter
  - Spectral distribution
  - Stefan-Boltzmann and Wien's laws

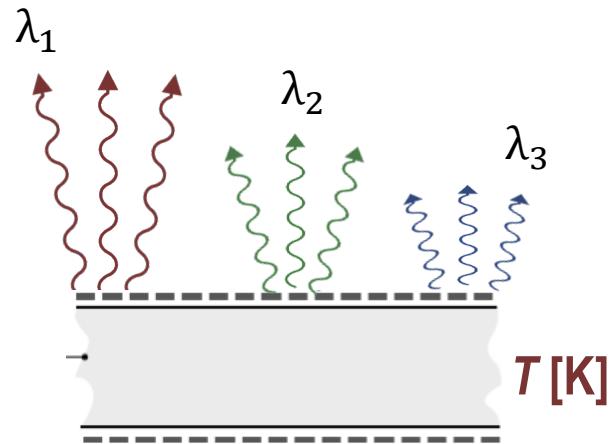
## Learning Objective:

- Understand emission of thermal radiation
- Quantify the emission of thermal radiation

# Emission of Thermal Radiation

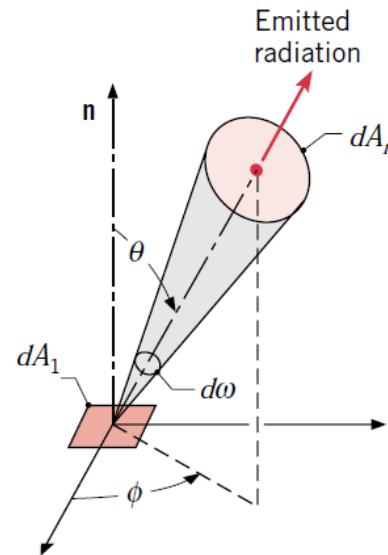
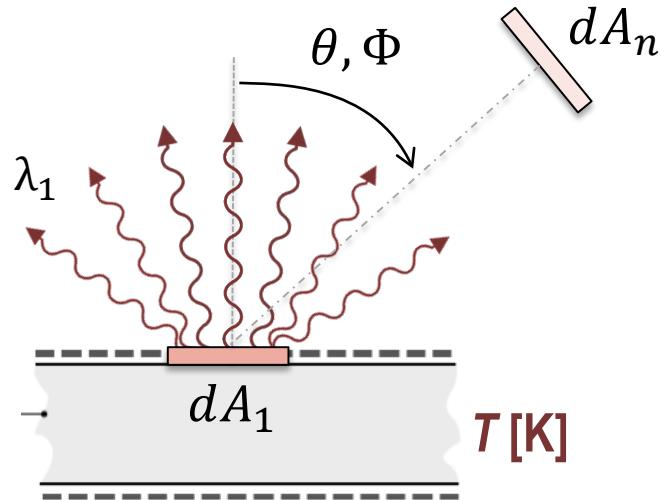


*Spatial distribution*



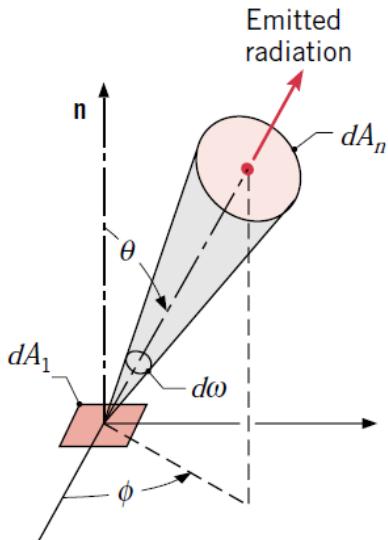
*Spectral distribution*

# Emission of Thermal Radiation: Spatial Distribution



If the surface  $dA_1$  emits energy at a rate  $dQ$  [W],  
how much of this radiation reaches the area  $dA_n$   
located at an angle  $\theta, \Phi$  with respect to the normal to  $dA_1$ ?

# Emission of Thermal Radiation: Spatial Distribution



$$I_{\lambda,e}(\lambda, \theta, \Phi, T) = \text{spectral intensity}$$

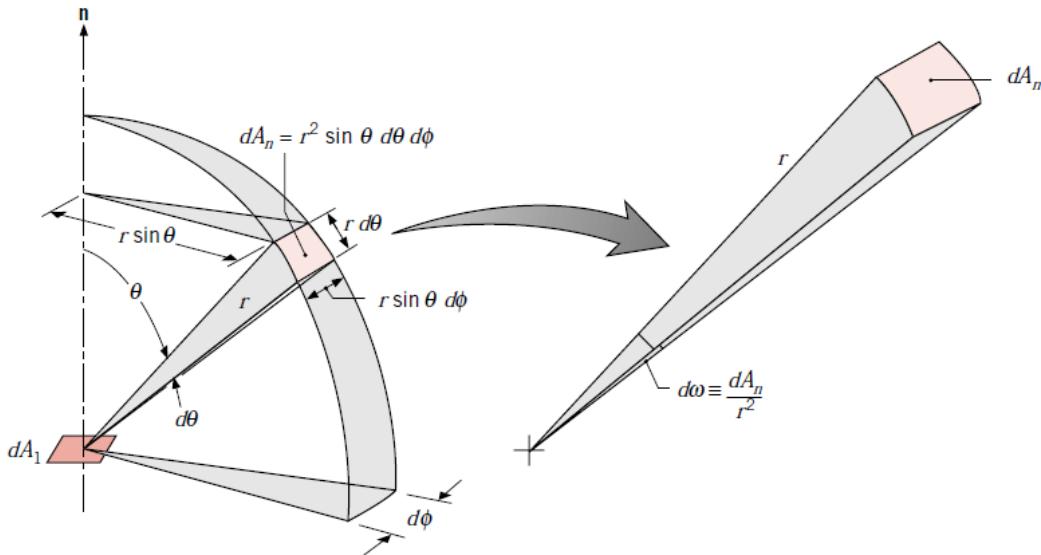
rate at which energy is emitted at wavelength  $\lambda$  and along the direction  $(\Phi, \theta)$

- per unit area of the emitting surface **normal to this direction** =  $dA_1 \cos\theta$
- per unit solid angle about this direction =  $d\omega$
- per unit wavelength interval about  $\lambda$  =  $d\lambda$

$$\rightarrow I_{\lambda,e}(\lambda, \theta, \Phi) = \frac{dQ}{dA_1 \cos\theta d\omega d\lambda}$$

Note: for convenience, we avoid to explicitly indicate the  $T$  dependence

# Emission of Thermal Radiation: Spatial Distribution



$$I_{\lambda,e}(\lambda, \theta, \Phi) = \frac{dQ}{dA_1 \cos \theta d\omega d\lambda}$$

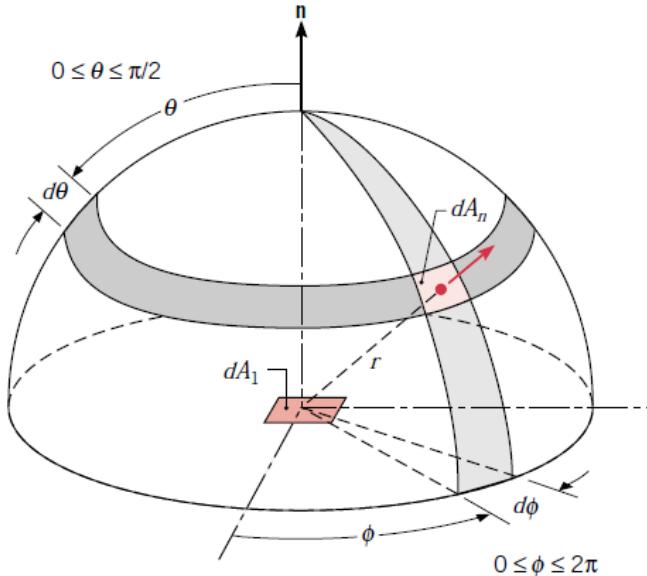
$$dA_n = r^2 \sin \theta d\theta d\Phi$$

$$\rightarrow d\omega \equiv \frac{dA_n}{r^2} = \sin \theta d\theta d\Phi$$

$$\rightarrow I_{\lambda,e}(\lambda, \theta, \Phi) = \frac{dQ/d\lambda}{dA_1 \cos \theta (\sin \theta d\Phi d\theta)} = \frac{dQ_\lambda/dA_1}{\cos \theta \sin \theta d\Phi d\theta} = \frac{dq_\lambda''}{\cos \theta \sin \theta d\Phi d\theta}$$

# Emission of Thermal Radiation: Spatial Distribution

$$dq_{\lambda}'' = I_{\lambda,e}(\lambda, \theta, \Phi) \cos\theta \sin\theta d\Phi d\theta$$



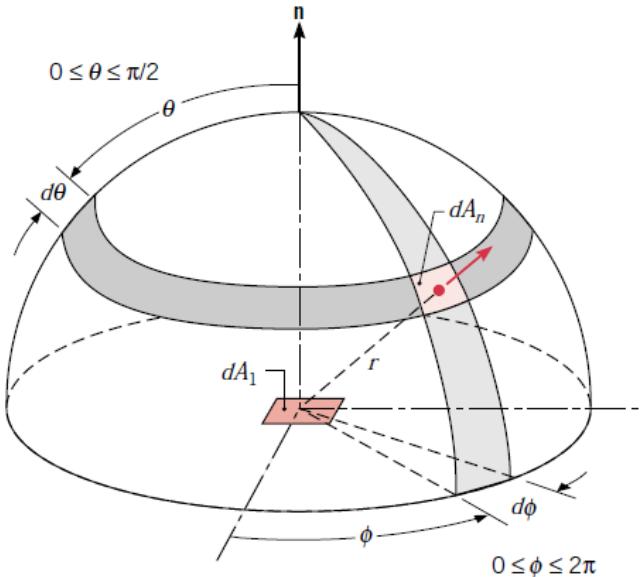
*Spectral (hemispherical) emissive power*  $\left[ \frac{W}{m^2 \mu m} \right]$ :

$$E_{\lambda}(\lambda) = q_{\lambda}''(\lambda) = \int_0^{2\pi} \int_0^{\pi/2} I_{\lambda,e}(\lambda, \theta, \Phi) \cos\theta \sin\theta d\Phi d\theta$$

*Total (hemispherical) emissive power*  $E \left[ \frac{W}{m^2} \right]$ :

$$E = \int_0^{\infty} E_{\lambda}(\lambda) d\lambda = \int_0^{\infty} \int_0^{2\pi} \int_0^{\pi/2} I_{\lambda,e}(\lambda, \theta, \Phi) \cos\theta \sin\theta d\Phi d\theta d\lambda$$

# Emission of Thermal Radiation: Diffuse Emitter



We define a **diffuse emitter** as a surface that emits radiation at the same rate irrespective of the emission direction  $(\theta, \Phi)$ :

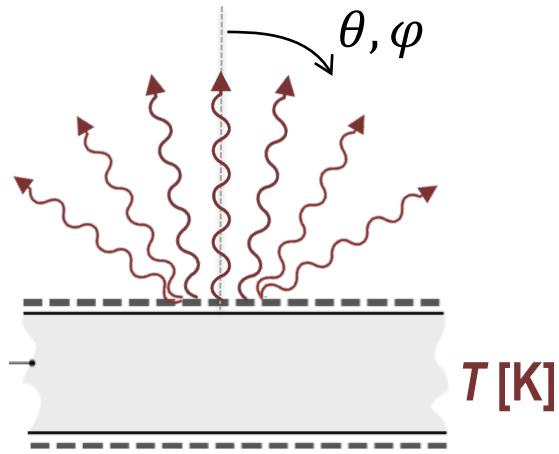
$$I_{\lambda,e}(\lambda, \theta, \Phi) = I_{\lambda,e}(\lambda)$$

$$E_{\lambda}(\lambda) = I_{\lambda,e}(\lambda) \int_0^{2\pi} \int_0^{\pi/2} \cos\theta \sin\theta d\Phi d\theta = \pi I_{\lambda,e}(\lambda)$$

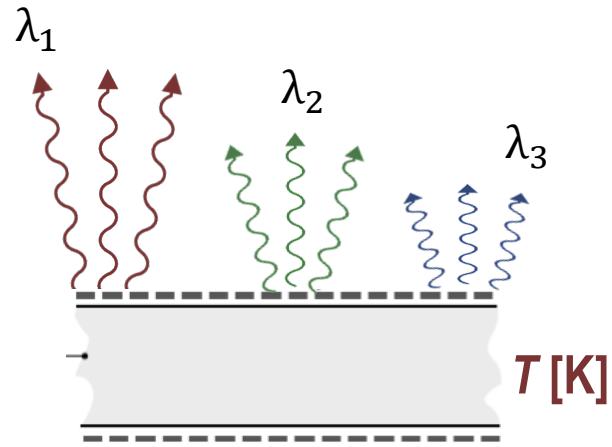
$$E = \pi \int_0^{\infty} I_{\lambda,e}(\lambda) d\lambda = \pi I_e$$

$$I_e = \int_0^{\infty} I_{\lambda,e}(\lambda) d\lambda = \text{total intensity of emitted radiation}$$

# Emission of Thermal Radiation



$T$  [K]



$T$  [K]

$$E(T) = \int_0^{\infty} \int_0^{2\pi} \int_0^{\pi/2} I_{\lambda,e}(\lambda, \theta, \Phi, T) \cos\theta \sin\theta d\Phi d\theta d\lambda$$

$$E(T) = \pi \int_0^{\infty} I_{\lambda,e}(\lambda, T) d\lambda \quad (\text{diffuse emitter})$$

**Spectral distribution**

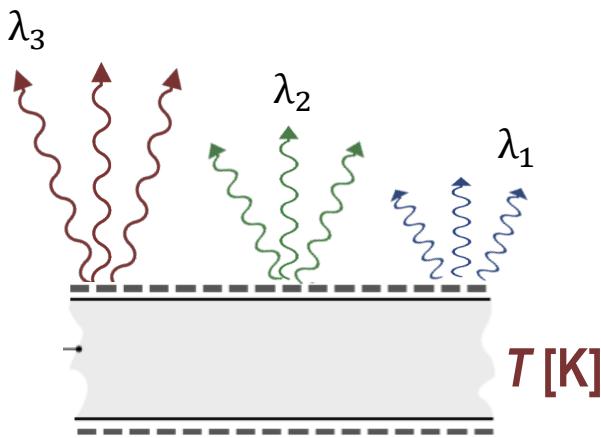
# This lecture

- Emission of Thermal Radiation
  - Spatial distribution and Diffuse Emitter
  - Spectral distribution
  - Stefan-Boltzmann and Wien's laws

## Learning Objective:

- Understand emission of thermal radiation
- Quantify the emission of thermal radiation

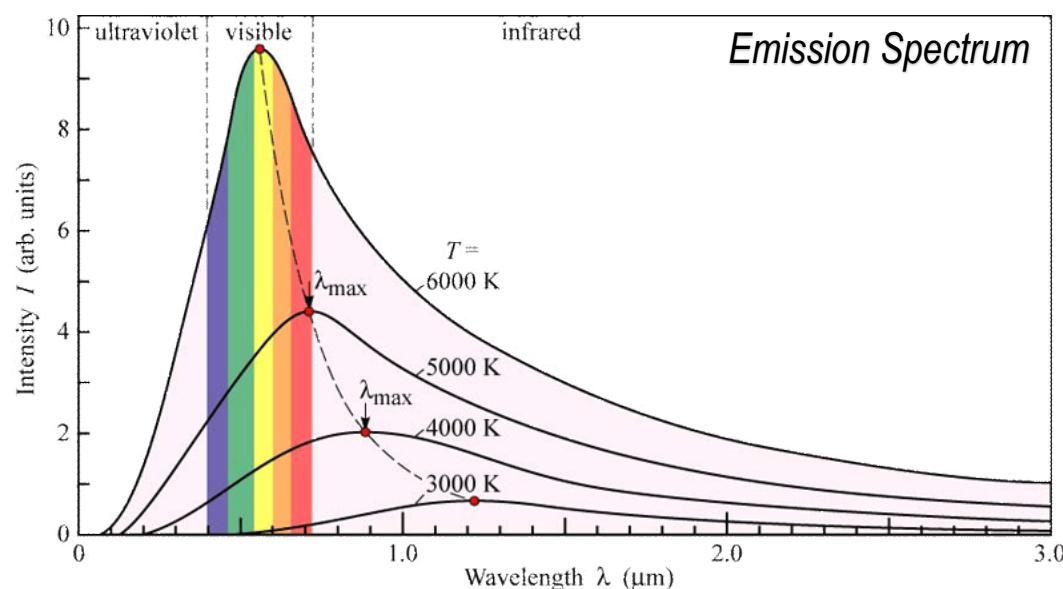
# Emission of Thermal Radiation: Spectral Distribution



Thermal radiation consists of electromagnetic waves with many different wavelengths.

The **temperature** of the object determines the *spectrum* of emitted thermal radiation.

# Emission of Thermal Radiation: Spectral Distribution



Spectral intensity of a *black-body*\*  $\left[ \frac{W}{m^2 \text{sr } \mu\text{m}} \right]$ :

$$I_{\lambda,b}(\lambda, T) = \frac{2hc_0^2}{\lambda^5 [\exp(hc_0/\lambda kT) - 1]}$$

$h = 6.626 \times 10^{-34} \text{ J} \cdot \text{s}$  Planck constant

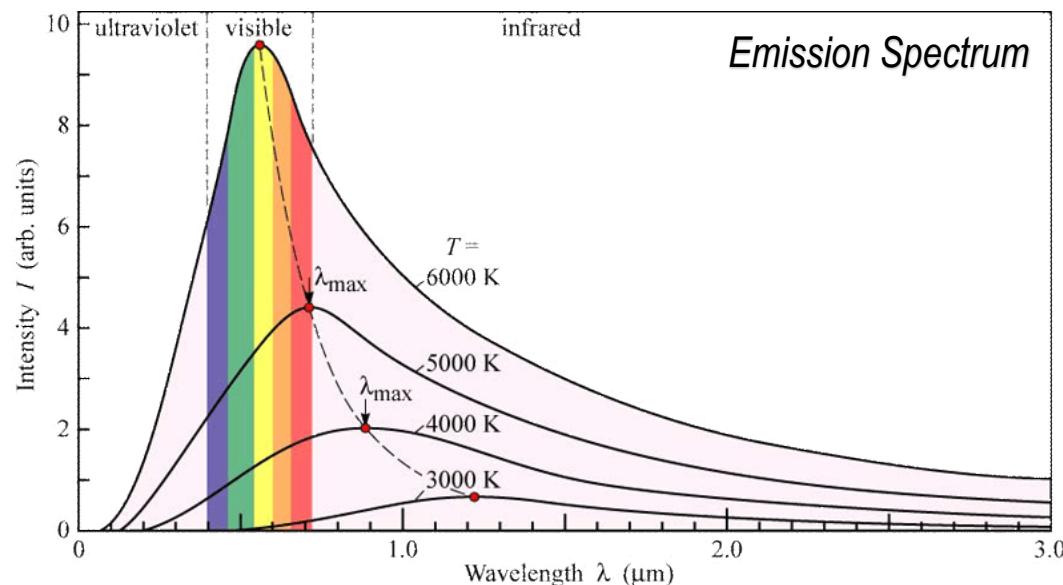
$k = 1.381 \times 10^{-23} \text{ J/K}$  Boltzmann constant

$c_0 = 2.998 \times 10^8 \text{ m/s}$  Speed of light

**Temperatures MUST BE in Kelvin: T[K]**

\*a black-body is a **diffuse emitter** that absorbs all energy that impinges on it. We discuss this in detail later

# Emission of Thermal Radiation: Spectral Distribution



**Stefan-Boltzmann Law (total emissive power)**

$$E_b(T) = \pi \int_0^{\infty} I_{\lambda,b}(\lambda, T) d\lambda = \sigma T^4$$

$T$  = absolute temperature [K]

$$\sigma = 5.670367 \cdot 10^{-8} \text{ W/m}^2\text{K}^4$$

**Total intensity of thermal emission**

$$I_b = \frac{E_b}{\pi}$$

**Temperatures MUST BE in Kelvin:  $T[\text{K}]$**

# Emission of Thermal Radiation: Spectral Distribution

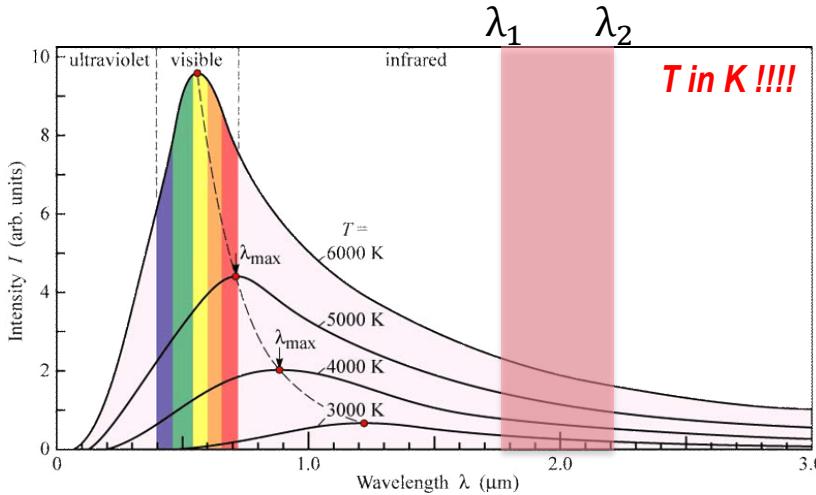


TABLE 12.1 Blackbody Radiation Functions

$\lambda T$ ( $\mu\text{m} \cdot \text{K}$ )	$F_{(0 \rightarrow \lambda)}$	$I_{\lambda, b}(\lambda, T) / \sigma T^5$ ( $\mu\text{m} \cdot \text{K} \cdot \text{sr})^{-1}$	$\frac{I_{\lambda, b}(\lambda, T)}{I_{\lambda, b}(\lambda_{\max}, T)}$
200	0.000000	$0.375034 \times 10^{-27}$	0.000000
400	0.000000	$0.490335 \times 10^{-13}$	0.000000
600	0.000000	$0.104046 \times 10^{-8}$	0.000014
800	0.000016	$0.991126 \times 10^{-7}$	0.001372
1,000	0.000321	$0.118505 \times 10^{-5}$	0.016406
1,200	0.002134	$0.523927 \times 10^{-5}$	0.072534
1,400	0.007790	$0.134411 \times 10^{-4}$	0.186082
1,600	0.019718	0.249130	0.344904
1,800	0.039341	0.375568	0.519949

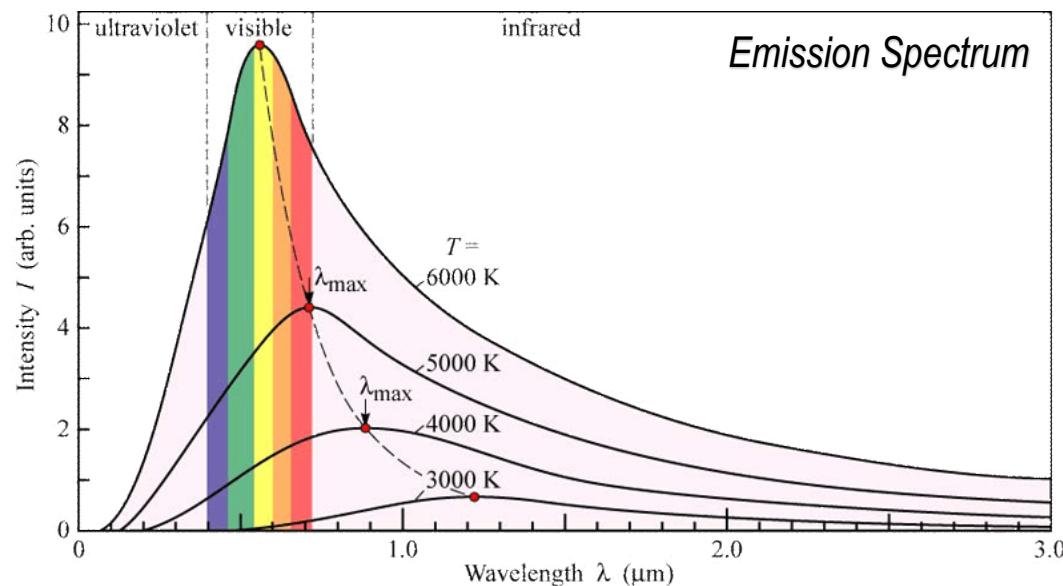
Often we want to know what is the emissive power within a limited wavelength range  $[\lambda_1, \lambda_2]$ . Therefore we define:

$$F_{(0 \rightarrow \lambda)} \equiv \frac{\int_0^\lambda E_{\lambda, b} d\lambda}{\int_0^\infty E_{\lambda, b} d\lambda} = \frac{\int_0^\lambda E_{\lambda, b} d\lambda}{\sigma T^4} = \int_0^{\lambda T} \frac{E_{\lambda, b}}{\sigma T^5} d(\lambda T) = f(\lambda T)$$

The values of  $F$  are tabulated. Hence within a given wavelength range we have:

$$F_{(\lambda_1 \rightarrow \lambda_2)} = \frac{\int_0^{\lambda_2} E_{\lambda, b} d\lambda - \int_0^{\lambda_1} E_{\lambda, b} d\lambda}{\sigma T^4} = F_{(0 \rightarrow \lambda_2)} - F_{(0 \rightarrow \lambda_1)}$$

# Emission of Thermal Radiation: Spectral Distribution

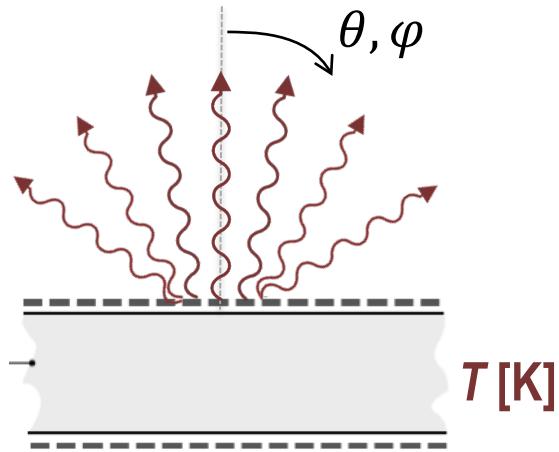


**Wien's Law (peak emission)**

$$(\lambda T)_{e_{\lambda=\max}} = 2898 \mu\text{m} \cdot \text{K}$$

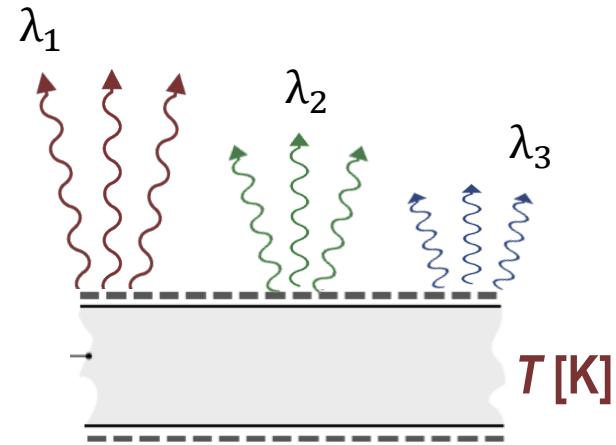
Temperatures MUST BE in Kelvin:  $T[\text{K}]$

# Emission of Thermal Radiation (Black-body)



$$I_{\lambda,e}(\lambda, \theta, \Phi, T) = I_{\lambda,b}(\lambda, T)$$

$$E_b(T) = \sigma T^4$$



$$I_{\lambda,b}(\lambda, T) = \frac{2hc_0^2}{\lambda^5 [\exp(hc_0/\lambda kT) - 1]}$$

# This lecture

- Emission of Thermal Radiation
  - Spatial distribution and Diffuse Emitter
  - Spectral distribution
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## Learning Objective:

- Understand emission of thermal radiation
- Quantify the emission of thermal radiation

# Next Lecture

- Interaction of Thermal Radiation with Matter
  - Absorptivity, Reflectivity and Transmissivity
  - Irradiation and Radiosity
- Black-body
- Real surfaces: Emissivity, Diffuse & Gray Surfaces, Kirchoff's Laws