Heat and Mass Transfer ME-341

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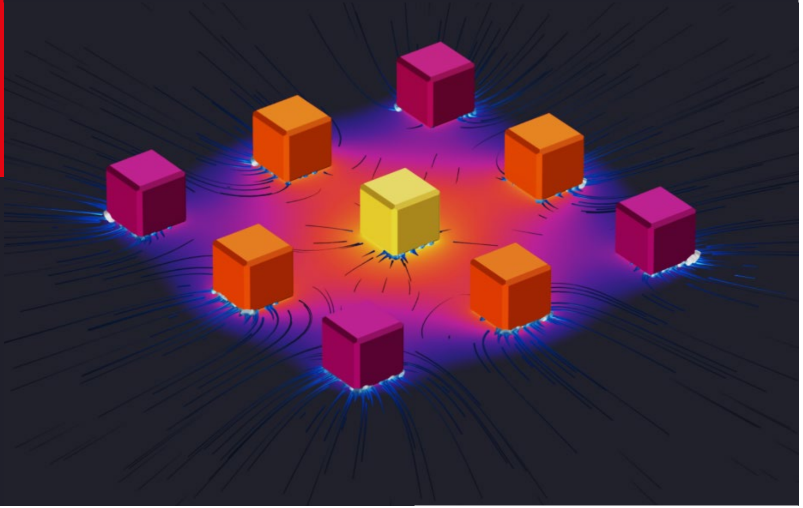


École polytechnique fédérale de Lausanne

Spring Semester

Previously

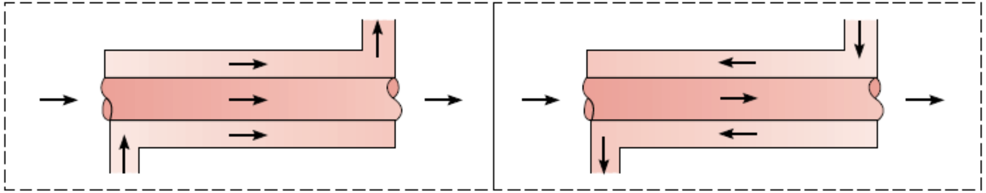
- ☑ Heat Exchangers Types
- ☑ Overall Heat Transfer Coefficient
- ☑ Parallel/Counter Flow Design
  - ☑ Temperature Profile and Total Heat Transfer

Learning Objectives:

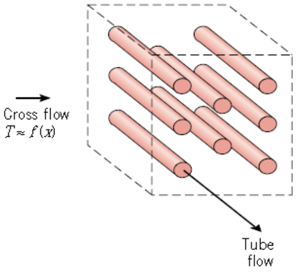
- ☑ Calculate the overall heat transfer coefficient
- ☑ Calculate the total heat transfer for parallel flow HE
- ☑ Calculate the total heat transfer for counter flow HE

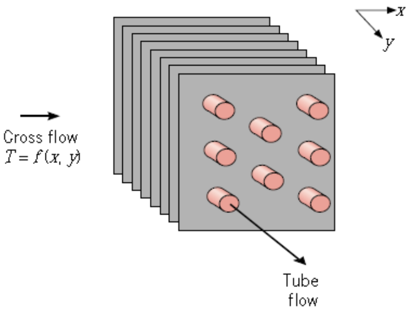
Introduction to Heat Exchangers

A. Concentric Flow

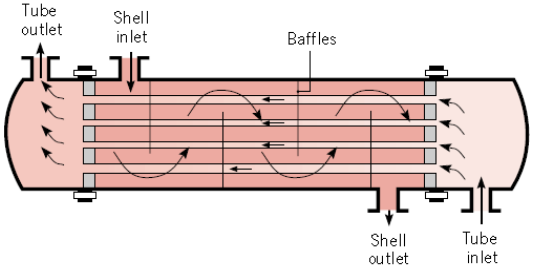


B. Cross-Flow





C. Shell-and-Tube



Overall Heat Transfer Coefficient

$$Q = \frac{\Delta T}{R_{tot}} = \boldsymbol{U} A \Delta T \quad \Rightarrow \quad U \equiv \frac{1}{R_{tot} A} \quad \Rightarrow \quad \frac{1}{UA} \equiv R_{tot}$$

In the most general case we could have fins present also on the inner side:

$$\frac{1}{UA} = \frac{1}{\eta_{o,out} h_{out} A_{out}} + \frac{R''_{f,o}}{\eta_{o,out} A_{out}} + R_{cond} + \frac{R''_{f,i}}{\eta_{o,in} A_{in}} + \frac{1}{\eta_{o,in} h_{in} A_{in}}$$

Includes all the layers of conduction!!

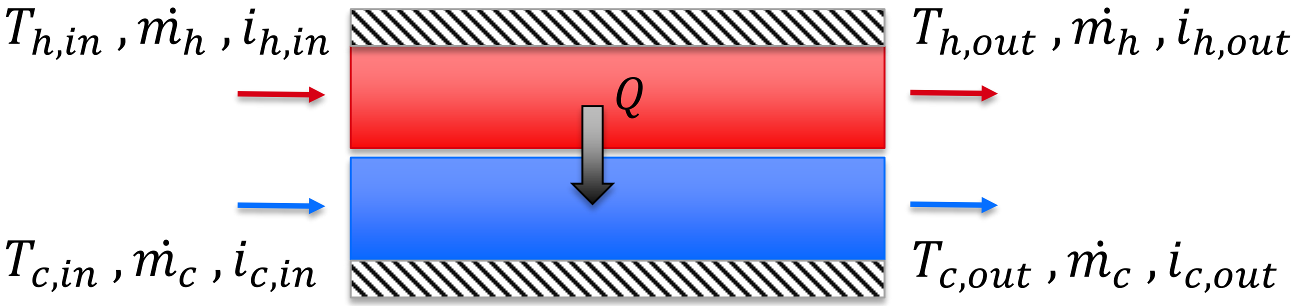
$$U_{in} \equiv \frac{1}{R_{tot} A_{in}} \neq U_{out} \equiv \frac{1}{R_{tot} A_{out}}$$

Parallel Flow Heat Exchanger

Energy balance

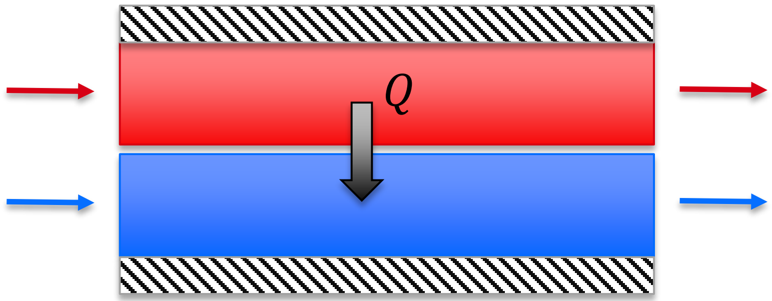
$\boldsymbol{i} = \boldsymbol{enthalphy}$

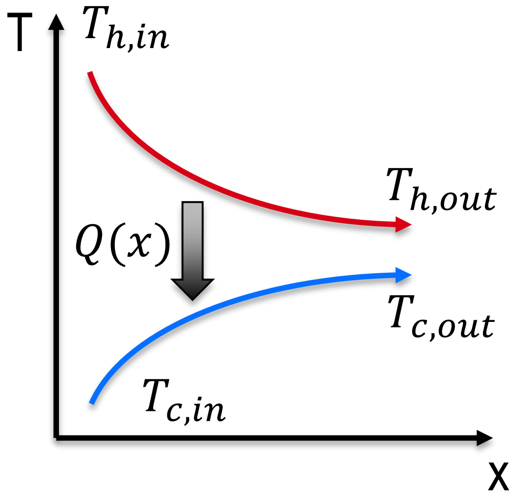
Not to confuse with convection coefficient



Parallel Flow Heat Exchanger

Temperature Profile

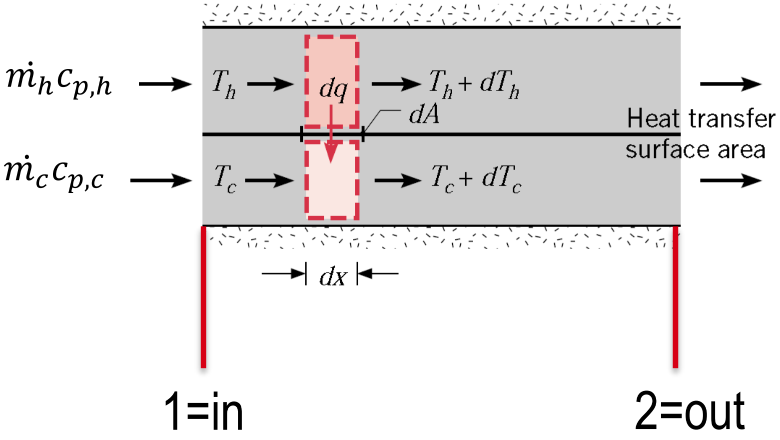




➡ What is an appropriate ΔT_m?

Parallel Flow heat Exchanger

Temperature Profile and Total Heat Transfer



$$Q = \text{UA}\frac{\Delta T_2 - \Delta T_1}{ln\left(\Delta T_2 / \Delta T_1\right)} = UA\Delta T_m$$

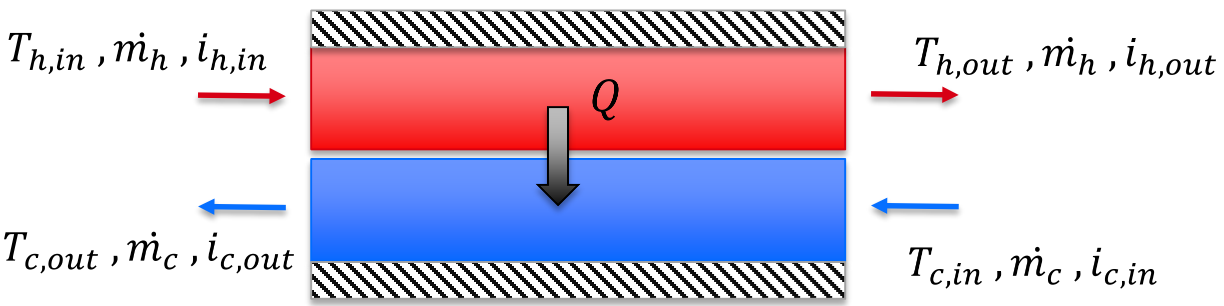
$$\Delta T_m = \frac{\Delta T_2 - \Delta T_1}{ln\left(\Delta T_2 / \Delta T_1\right)}$$

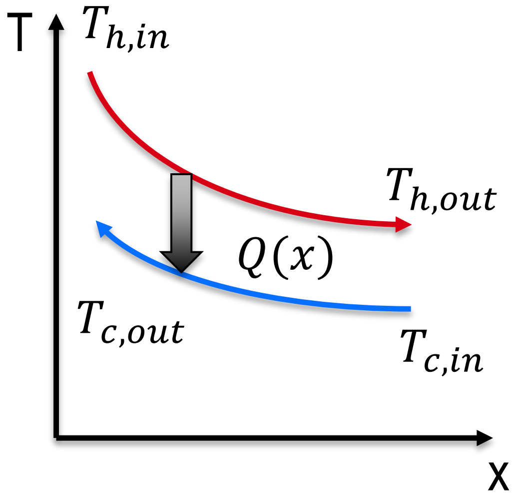
$$\Delta T_1 = \left(T_{h,1} - T_{c,1}\right) = \left(T_{h,in} - T_{c,in}\right)$$

$$\Delta T_2 = \left(T_{h,2} - T_{c,2}\right) = \left(T_{h,out} - T_{c,out}\right)$$

Counter Flow Heat Exchanger

Temperature Profile

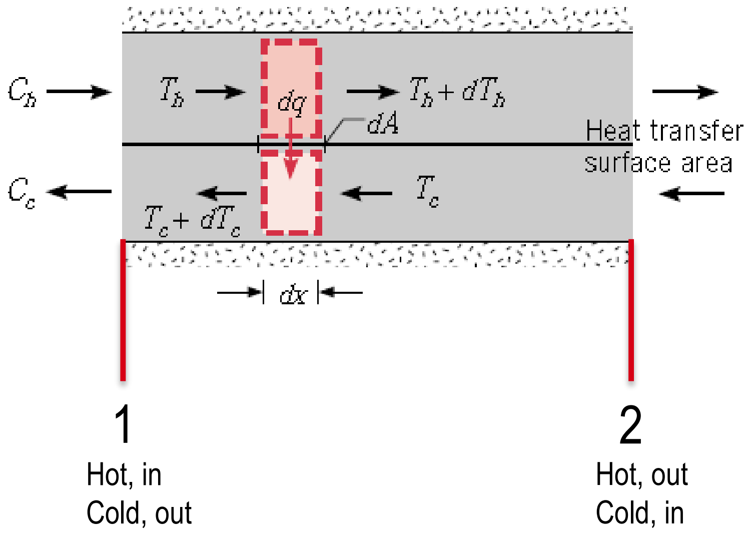




What is an appropriate ΔT_m?

Counter Flow heat Exchanger

Temperature Profile and Total Heat Transfer



$$Q = UA \frac{\Delta T_2 - \Delta T_1}{ln\left(\Delta T_2 / \Delta T_1\right)} = UA\Delta T_m$$

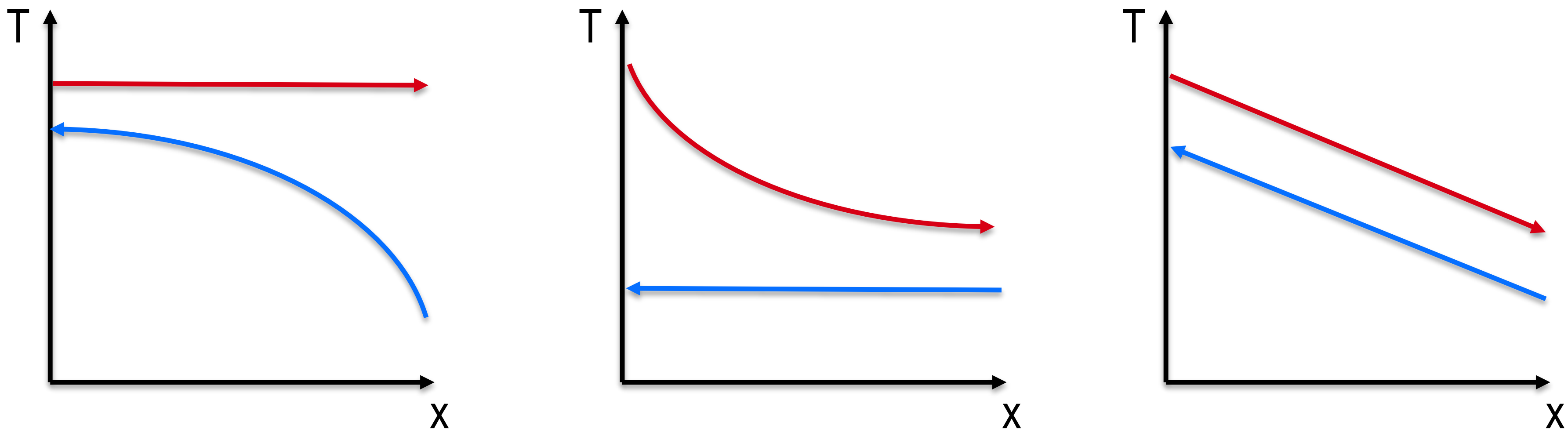
$$\Delta T_m = \frac{\Delta T_2 - \Delta T_1}{ln\left(\Delta T_2 / \Delta T_1\right)}$$

$$\Delta T_1 = \left(T_{h,1} - T_{c,1}\right) = \left(T_{h,in} - T_{c,out}\right)$$

$$\Delta T_2 = \left(T_{h,2} - T_{c,2}\right) = \left(T_{h,out} - T_{c,in}\right)$$

Special Operating Conditions

Representations Based on Counter Flow



- Condensing vapor on the hot-side
- $C_h \gg C_c$

- Evaporating liquid on the cold side
- $C_h \ll C_c$

- $C_h \sim C_c$

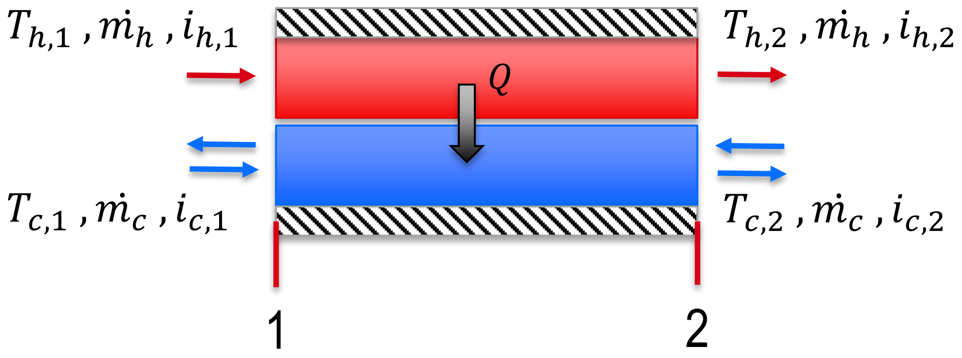
This Lecture

- ❑ Heat Exchanger Analysis/Design/Performance Calculation
  - ❑ Effectiveness-NTU method

Learning Objectives:

- ❑ Identify the design parameter for a heat exchanger
- ❑ Analyze the performance of a heat exchanger

Effectiveness-NTU Method



$$Q = -Q_h = -C_h\left(T_{h,out} - T_{h,in}\right)$$

$$Q = Q_c = C_c\left(T_{c,out} - T_{c,in}\right)$$

$$\Delta T_m = \frac{\Delta T_2 - \Delta T_1}{ln\left({}^{\Delta T_2}/_{\Delta T_1}\right)}$$

$$Q = \mathrm{UA}\frac{\Delta T_2 - \Delta T_1}{ln\left({}^{\Delta T_2}/_{\Delta T_1}\right)} = UA\Delta T_m$$

How do we design the heat exchanger if we do NOT know all the four temperatures ?
What about shell-tube heat exchangers where there are various flow configurations?

Effectiveness-NTU Method

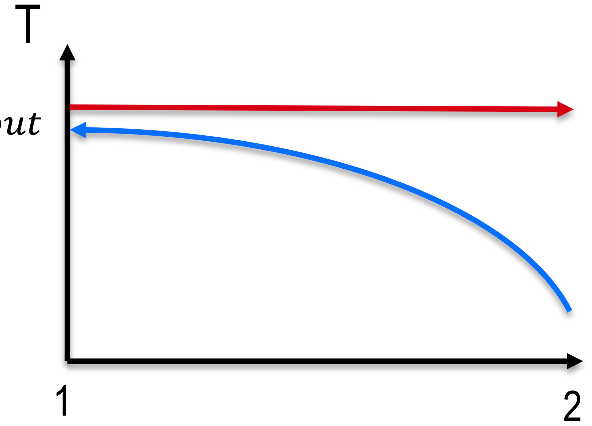
We want to **determine the highest heat transfer** in the heat exchanger. We observe that:

- The maximum Q will be: $Q_{max} = C_{?}\Delta T_{max} = C_{?}(T_{max} - T_{min})$

L→∞

$\Delta T_1 \rightarrow 0$

$T_{h,in} \sim T_{c,out}$



$C_h \gg C_c$

$$Q_{max} = C_c(T_{c,out} - T_{c,in}) = C_c(T_{h,in} - T_{c,in})$$

- The maximum ΔT occurs on the flow side with $C_{min} = \min(C_c, C_h)$

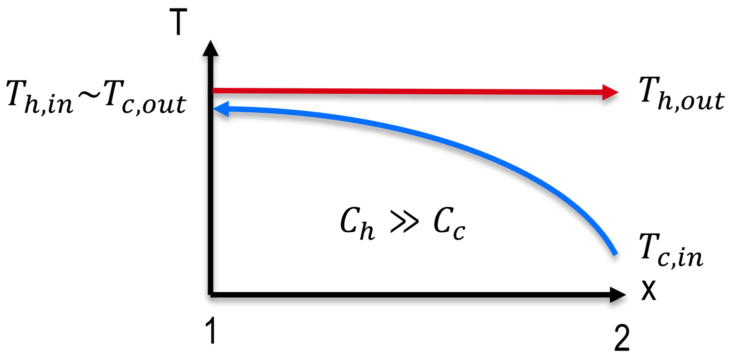
$$dq = -dq_h = -C_h dT_h$$
$$dq = dq_c = C_c dT_c$$

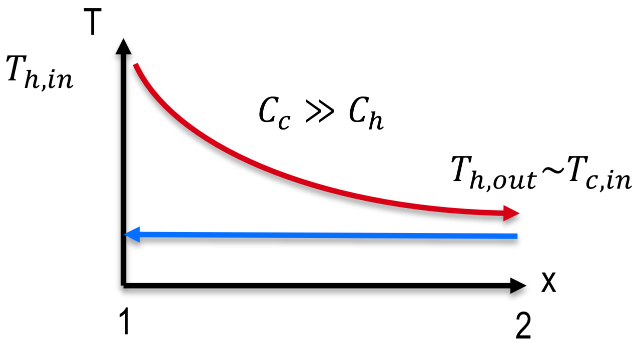
If $C_c \gg C_h$ then $|dT_c| \ll |dT_h|$ and viceversa.

Effectiveness-NTU Method

We want to **determine the highest heat transfer** in the heat exchanger. We observe that:

- The maximum Q will be: $\boldsymbol{Q_{max} = C_{min}\Delta T_{max}} = C_{min}(T_{max} - T_{min})$





$$Q_{max} = C_c\left(T_{c,out} - T_{c,in}\right) = C_c\left(T_{h,in} - T_{c,in}\right)$$

$$Q_{max} = C_h\left(T_{h,in} - T_{h,out}\right) = C_h\left(T_{h,in} - T_{c,in}\right)$$

Effectiveness-NTU Method

We want to determine the highest heat transfer for a given system.

The maximum Q will be: $\boldsymbol{Q_{max} = C_{min}\Delta T_{max} = C_{min}(T_{h,in} - T_{c,in})}$

Effectiveness $\varepsilon \equiv \dfrac{Q}{Q_{max}}$

$$\varepsilon = \frac{C_h(T_{h,in} - T_{h,out})}{C_{min}(T_{h,in} - T_{c,in})}$$

$$\varepsilon = \frac{C_c(T_{c,out} - T_{c,in})}{C_{min}(T_{h,in} - T_{c,in})}$$

$0 < \varepsilon < 1$

$Q = \varepsilon C_{min}(T_{h,in} - T_{c,in})$ If we know $\varepsilon, T_{h,in}, T_{c,in}$ we can determine Q

Effectiveness-NTU Method

We now observe that:

$$\varepsilon = \frac{Q}{Q_{max}} = \frac{UA\Delta T_{lm}}{C_{min}\left(T_{h,in} - T_{c,in}\right)} = NTU\,\frac{\Delta T_{lm}}{\left(T_{h,in} - T_{c,in}\right)}$$

Where we have defined the **number of heat transfer units NTU**, as: $\boldsymbol{NTU \equiv \frac{UA}{C_{min}}}$

More generally, it can be shown that: $\varepsilon = (NTU, \frac{C_{min}}{C_{max}})$

Therefore the heat transfer in the heat exchanger can be determined once the physical properties ($C_{c,h}$), the heat exchanger design (UA) and the inlet temperatures of the working fluids are known.

Effectiveness-NTU Method

Parallel-flow Heat Exchanger

If $C_{min} = C_h$:

$$\varepsilon = \frac{(T_{h,in} - T_{h,out})}{(T_{h,in} - T_{c,in})}$$

And also we have:

$$\frac{C_{\min}}{C_{\max}} = \frac{\dot{m}_h c_{p,h}}{\dot{m}_c c_{p,c}} = \frac{T_{c,o} - T_{c,i}}{T_{h,i} - T_{h,o}}$$

$$\Rightarrow T_{c,out} = \frac{C_{min}}{C_{max}}(T_{h,in} - T_{h,out}) + T_{c,in}$$

Last week we derived:

$$\ln\left(\frac{\Delta T_2}{\Delta T_1}\right) = -UA\left(\frac{1}{C_h} + \frac{1}{C_c}\right) \Rightarrow \ln\left(\frac{T_{h,o} - T_{c,o}}{T_{h,i} - T_{c,i}}\right) = -\frac{UA}{C_{\min}}\left(1 + \frac{C_{\min}}{C_{\max}}\right)$$

$$\Rightarrow \frac{T_{h,o} - T_{c,o}}{T_{h,i} - T_{c,i}} = \exp\left[-\mathrm{NTU}\left(1 + \frac{C_{\min}}{C_{\max}}\right)\right]$$

Effectiveness-NTU Method

Parallel-flow Heat Exchanger

If $C_{min} = C_h$: $\varepsilon = \frac{(T_{h,in} - T_{h,out})}{(T_{h,in} - T_{c,in})}$

And also we have: $\frac{C_{\min}}{C_{\max}} = \frac{\dot{m}_h c_{p,h}}{\dot{m}_c c_{p,c}} = \frac{T_{c,o} - T_{c,i}}{T_{h,i} - T_{h,o}}$

$$\Rightarrow T_{c,out} = \frac{C_{min}}{C_{max}}(T_{h,in} - T_{h,out}) + T_{c,in}$$

$$\frac{T_{h,o} - T_{c,o}}{T_{h,i} - T_{c,i}} = \exp\left[-\mathrm{NTU}\left(1 + \frac{C_{\min}}{C_{\max}}\right)\right]$$

$$\frac{T_{h,o} - T_{c,o}}{T_{h,i} - T_{c,i}} = \frac{T_{h,o} - T_{h,i} + T_{h,i} - T_{c,o}}{T_{h,i} - T_{c,i}} = \frac{(T_{h,o} - T_{h,i}) + (T_{h,i} - T_{c,i}) - (C_{\min}/C_{\max})(T_{h,i} - T_{h,o})}{T_{h,i} - T_{c,i}}$$

$-\boldsymbol{\varepsilon}$

Effectiveness-NTU Method

Parallel-flow Heat Exchanger

$$\frac{T_{h,o} - T_{c,o}}{T_{h,i} - T_{c,i}} = \exp\left[-\mathrm{NTU}\left(1 + \frac{C_{\min}}{C_{\max}}\right)\right]$$

$$\frac{T_{h,o} - T_{c,o}}{T_{h,i} - T_{c,i}} = 1 - \varepsilon\left(1 + \frac{C_{\min}}{C_{\max}}\right)$$

$$\varepsilon = \frac{1 - \exp\{-\mathrm{NTU}[1 + (C_{\min}/C_{\max})]\}}{1 + (C_{\min}/C_{\max})}$$

This expression is general and it applies to any parallel heat exchanger irrespective of the fluid which has the minimum thermal capacity.

Similar expressions of $\varepsilon = \left(NTU, \frac{C_{min}}{C_{max}}\right)$ have been derived for other heat exchanger configurations and are summarized in the next table.

Effectiv

$$C_r \equiv C_{min}/C_{max}$$

Table 11.3 Heat Exchanger Effectiveness Relations [5]

Flow Arrangement	Relation	
Concentric tube		
Parallel flow	$\varepsilon = \dfrac{1-\exp\left[-NTU(1+C_r)\right]}{1+C_r}$	(11.28a)
Counterflow	$\varepsilon = \dfrac{1-\exp\left[-NTU(1-C_r)\right]}{1-C_r\exp\left[-NTU(1-C_r)\right]}$ $(C_r < 1)$	
	$\varepsilon = \dfrac{NTU}{1+NTU}$ $(C_r = 1)$	(11.29a)
Shell-and-tube		
One shell pass (2, 4, . . . tube passes)	$\varepsilon_1 = 2\left\{1 + C_r + (1+C_r^2)^{1/2} \times \dfrac{1+\exp\left[-(NTU)_1(1+C_r^2)^{1/2}\right]}{1-\exp\left[-(NTU)_1(1+C_r^2)^{1/2}\right]}\right\}^{-1}$	(11.30a)
n Shell passes ($2n$, $4n$, . . . tube passes)	$\varepsilon = \left[\left(\dfrac{1-\varepsilon_1 C_r}{1-\varepsilon_1}\right)^n - 1\right]\left[\left(\dfrac{1-\varepsilon_1 C_r}{1-\varepsilon_1}\right)^n - C_r\right]^{-1}$	(11.31a)
Cross-flow (single pass)		
Both fluids unmixed	$\varepsilon = 1 - \exp\left[\left(\dfrac{1}{C_r}\right)(NTU)^{0.22}\left\{\exp\left[-C_r(NTU)^{0.78}\right]-1\right\}\right]$	(11.32)
C_{max} (mixed), C_{min} (unmixed)	$\varepsilon = \left(\dfrac{1}{C_r}\right)\left(1-\exp\left\{-C_r\left[1-\exp(-NTU)\right]\right\}\right)$	(11.33a)
C_{min} (mixed), C_{max} (unmixed)	$\varepsilon = 1-\exp\left(-C_r^{-1}\left\{1-\exp\left[-C_r(NTU)\right]\right\}\right)$	(11.34a)
All exchangers ($C_r = 0$)	$\varepsilon = 1-\exp(-NTU)$	(11.35a)

PHASE CHANGE!
($C_r = 0$)

Effectiv

Table 11.4 Heat Exchanger NTU Relations

Flow Arrangement	Relation	
Concentric tube		
Parallel flow	$\mathrm{NTU} = -\dfrac{\ln[1-\varepsilon(1+C_r)]}{1+C_r}$	(11.28b)
Counterflow	$\mathrm{NTU} = \dfrac{1}{C_r - 1}\ln\left(\dfrac{\varepsilon - 1}{\varepsilon C_r - 1}\right) \quad (C_r < 1)$ $\mathrm{NTU} = \dfrac{\varepsilon}{1-\varepsilon} \quad (C_r = 1)$	(11.29b)
Shell-and-tube		
One shell pass (2, 4, . . . tube passes)	$(\mathrm{NTU})_1 = -(1+C_r^2)^{-1/2}\ln\left(\dfrac{E-1}{E+1}\right)$	(11.30b)
	$E = \dfrac{2/\varepsilon_1 - (1+C_r)}{(1+C_r^2)^{1/2}}$	(11.30c)
n Shell passes (2n, 4n, . . . tube passes)	Use Equations 11.30b and 11.30c with $\varepsilon_1 = \dfrac{F-1}{F-C_r} \quad F = \left(\dfrac{\varepsilon C_r - 1}{\varepsilon - 1}\right)^{1/n} \quad \mathrm{NTU} = n(\mathrm{NTU})_1$	(11.31b, c, d)
Cross-flow (single pass)		
C_{max} (mixed), C_{min} (unmixed)	$\mathrm{NTU} = -\ln\left[1 + \left(\dfrac{1}{C_r}\right)\ln(1-\varepsilon C_r)\right]$	(11.33b)
C_{min} (mixed), C_{max} (unmixed)	$\mathrm{NTU} = -\left(\dfrac{1}{C_r}\right)\ln[C_r\ln(1-\varepsilon)+1]$	(11.34b)
All exchangers ($C_r = 0$)	$\mathrm{NTU} = -\ln(1-\varepsilon)$	(11.35b)

$$C_r \equiv C_{min}/C_{max}$$

Effectiveness-NTU Method

$$C_r \equiv C_{min}/C_{max}$$

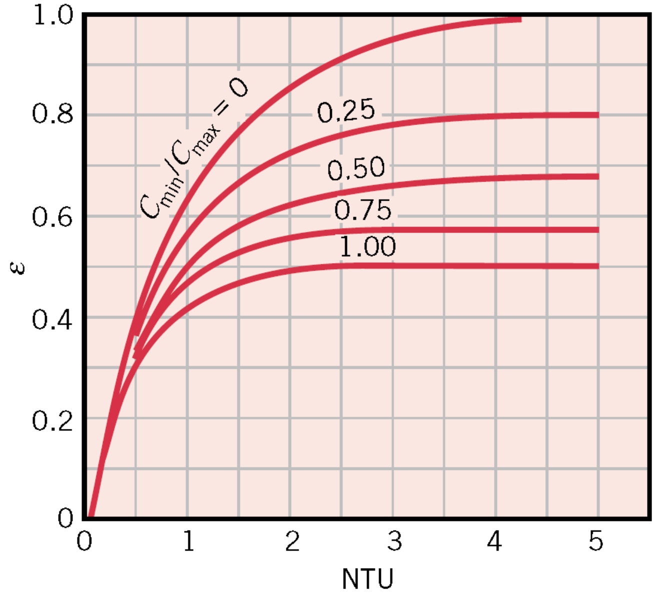


FIGURE 11.10 Effectiveness of a parallel-flow heat exchanger (Equation 11.28).

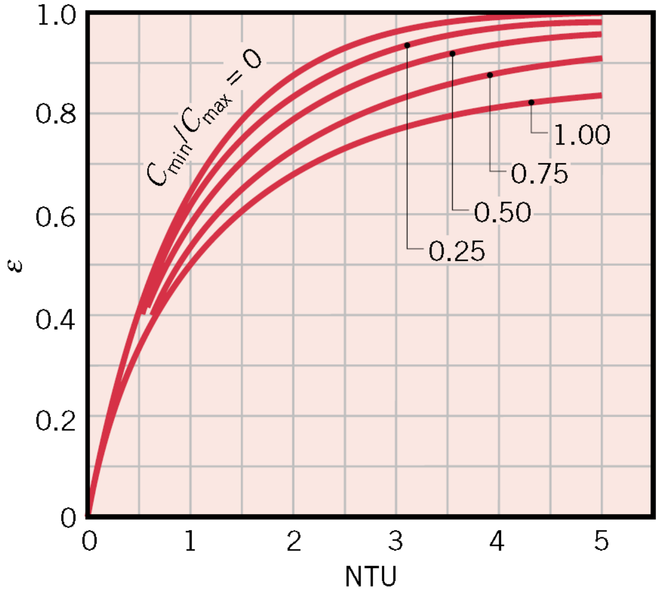


FIGURE 11.11 Effectiveness of a counterflow heat exchanger (Equation 11.29).

Effectiveness-NTU Method

$$C_r \equiv C_{min}/C_{max}$$

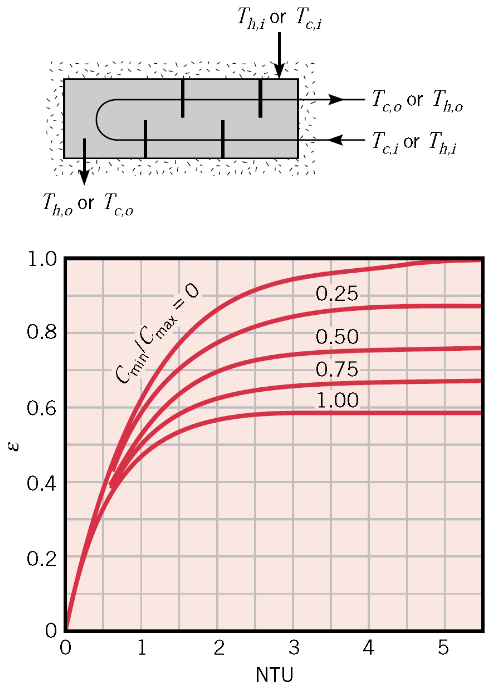


FIGURE 11.12 Effectiveness of a shell-and-tube heat exchanger with one shell and any multiple of two tube passes (two, four, etc. tube passes) (Equation 11.30).

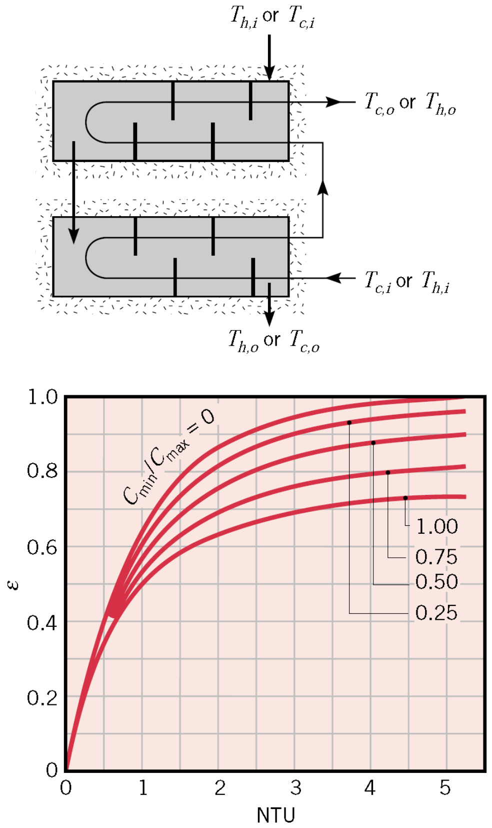


FIGURE 11.13 Effectiveness of a shell-and-tube heat exchanger with two shell passes and any multiple of four tube passes (four, eight, etc. tube passes) (Equation 11.31 with $n = 2$).

Effectiveness-NTU Method

$$C_r \equiv C_{min}/C_{max}$$

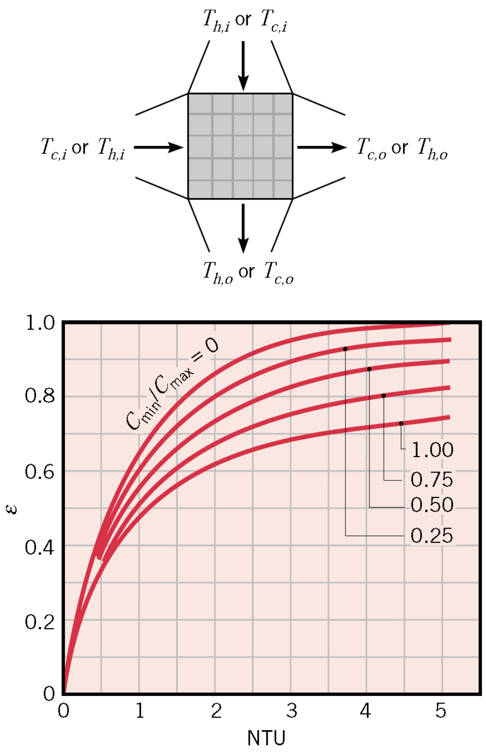


FIGURE 11.14 Effectiveness of a single-pass, cross-flow heat exchanger with both fluids unmixed (Equation 11.32).

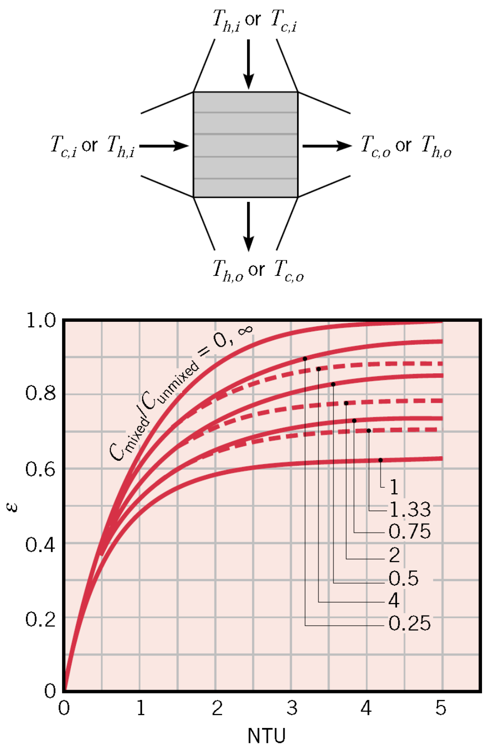


FIGURE 11.15 Effectiveness of a single-pass, cross-flow heat exchanger with one fluid mixed and the other unmixed (Equations 11.33, 11.34).

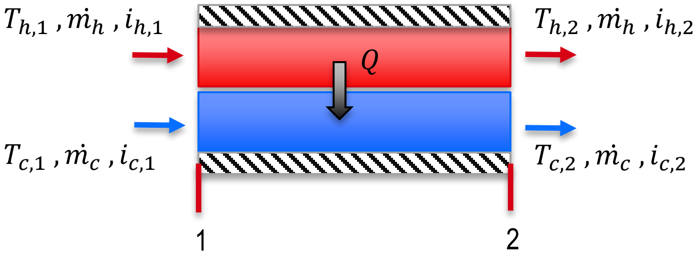
This Lecture

- ☑ Heat Exchanger Analysis/Design/Performance Calculation
  - ❑ Effectiveness-NTU method

Learning Objectives:

- ☑ Identify the design parameter for a heat exchanger
- ☑ Analyze the performance of a heat exchanger

Heat Exchanger Problems



Design problem:

All input and output temperatures are given, as well as flow rates. We need to determine the appropriate heat exchanger area.

➡ *Calculate ε and C_r, then NTU then A*

Performance analysis problem:

We have a heat exchanger of prescribed dimensions, we know the inlet conditions and we need to determine the outlet conditions and the overall heat transfer.

➡ *Calculate NTU and C_r and then determine ε*

Next Lecture

- ❑ Design and performance Analysis of Heat Exchangers

Learning Objectives:

- ❑ Identify the design parameter for a heat exchanger
- ❑ Analyze the performance of a heat exchanger