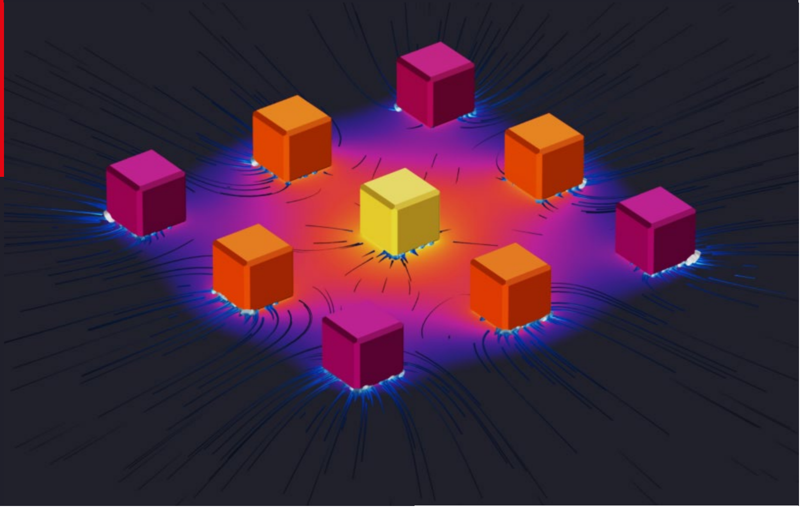
Heat and Mass Transfer
ME-341

Instructor: Giulia Tagliabue



École
polytechnique
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de Lausanne

Spring Semester

Previously

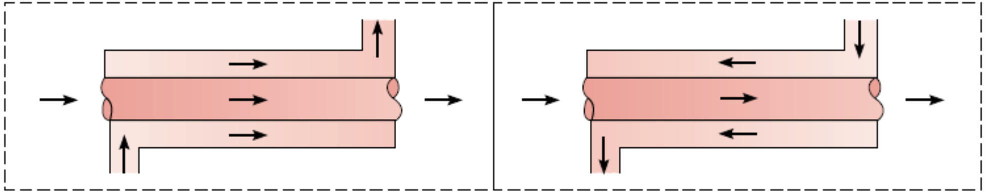
- ☑ Introduction to Heat Exchangers
- ☑ Overall Heat Transfer Coefficient

Learning Objectives:

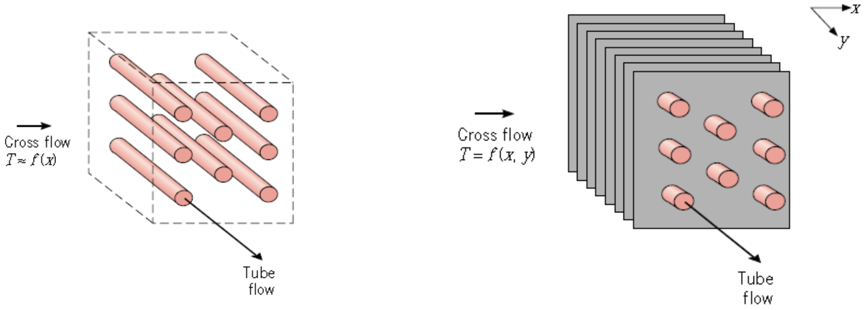
- ☑ Understand the concept and possible design of heat exchangers
- ☑ Calculate the overall heat transfer coefficient

Introduction to Heat Exchangers

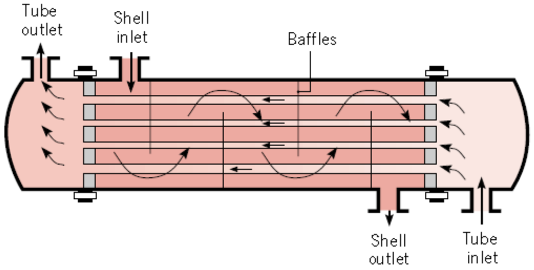
A. Concentric Flow



B. Cross-Flow

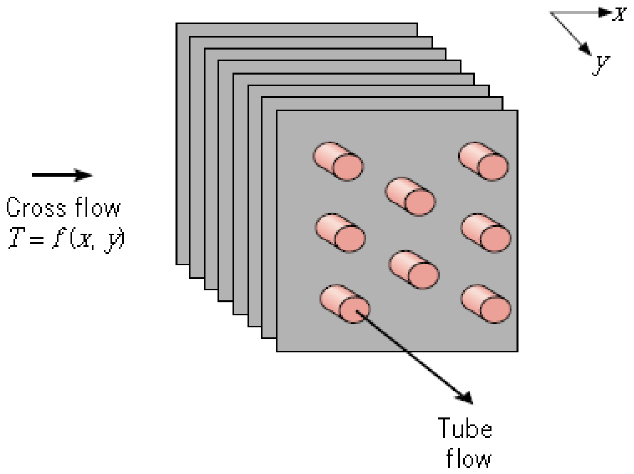


C. Shell-and-Tube



Overall Heat Transfer Coefficient

$$Q = \frac{\Delta T}{R_{tot}} = \boldsymbol{U} A \Delta T \quad \Rightarrow \quad U \equiv \frac{1}{R_{tot} A} \quad \Rightarrow \quad \frac{1}{UA} \equiv R_{tot}$$



- Conduction thermal resistances
  - Planar/radial conduction
- Convection thermal resistances
  - Internal and external
- Array of Fins
- Fouling

Overall Heat Transfer Coefficient

Conduction & Convection Thermal Resistances

Planar wall

$$R_{th,cond} = \frac{L}{kA} \quad [\text{ K/W }]$$

$$R_{th,conv} = \frac{1}{hA} \quad [\text{ K/W }]$$

Radial System

$$R_{th,cond-cyl} = \frac{ln(r_2/r_1)}{2\pi Lk} \quad [\text{ K/W }]$$

$$R_{th,conv} = \frac{1}{h2\pi rL} \quad [\text{ K/W }]$$

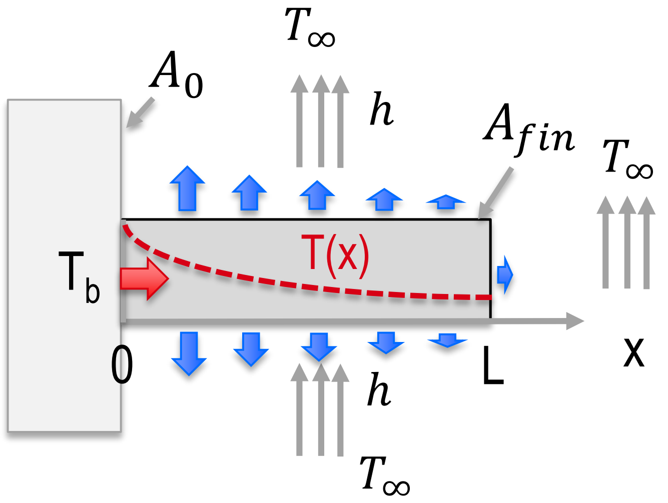
Overall Heat Transfer Coefficient

*Fins – Efficiency and Resistance (**1 Fin**)*

If T_b is the temperature of the fin base:

$$\eta_f \equiv \frac{Q_f}{Q_{f,max}} = \frac{Q_f}{hA_f(T_b - T_\infty)} = \frac{Q_f}{hA_f\theta_b}$$

$$\boldsymbol{R_f \equiv \frac{(T_b - T_\infty)}{Q_f} = \frac{1}{hA_f\eta_f}}$$

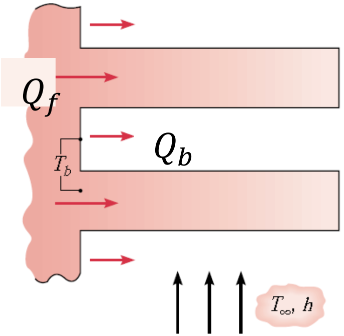


e.g. for infinite fin we get: $R_f \equiv \frac{\theta_b}{Q_f} = \frac{\theta_b}{M} = \frac{1}{\sqrt{hPkA_c}}$

Overall Heat Transfer Coefficient

*Fins – Efficiency and Resistance (**Array of Fins**)*

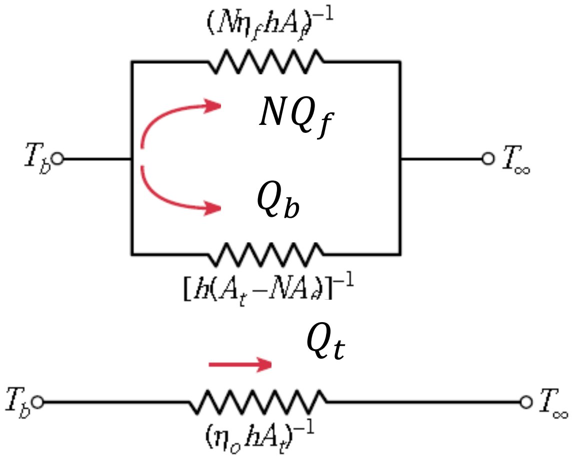
$$\eta_o \equiv \frac{Q_t}{Q_{t,max}} = \frac{Q_t}{hA_t(T_0 - T_\infty)} = \frac{NQ_f + Q_b}{h\left(NA_f + A_b\right)(T_0 - T_\infty)}$$



$$\eta_o \equiv 1 - \frac{NA_f}{A_t}(1 - \boldsymbol{\eta_f})$$ *Overall efficiency*

Single fin efficiency

$$R_o \equiv \frac{(T_0 - T_\infty)}{Q_t} = \frac{1}{\eta_o h A_t}$$



Overall Heat Transfer Coefficient

Fouling

Dramatic increase in thermal resistance due to poor conduction through the scaling layer

- Introduce a *fouling* resistance per unit area (fouling factor) R_f''

- $R_f'' \equiv AR_{foul}$ ➡ $R_{foul} = \frac{R_f''}{A}$

Overall Heat Transfer Coefficient

$$Q = \frac{\Delta T}{R_{tot}} = \boldsymbol{U} A \Delta T \quad \Rightarrow \quad U \equiv \frac{1}{R_{tot} A} \quad \Rightarrow \quad \frac{1}{UA} \equiv R_{tot}$$

In the most general case we could have fins present also on the inner side:

$$\frac{1}{UA} = \frac{1}{\eta_{o,out} h_{out} A_{out}} + \frac{R''_{f,o}}{\eta_{o,out} A_{out}} + \underbrace{R_{cond}} + \frac{R''_{f,i}}{\eta_{o,in} A_{in}} + \frac{1}{\eta_{o,in} h_{in} A_{in}}$$

Includes all the layers of conduction!!

$$U_{in} \equiv \frac{1}{R_{tot} A_{in}} \neq U_{out} \equiv \frac{1}{R_{tot} A_{out}}$$

This Lecture

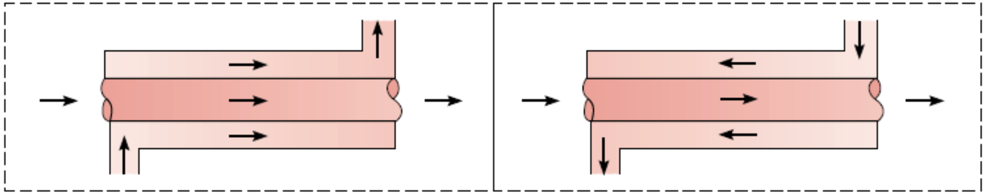
- ❑ Parallel Flow Design
  - ❑ Temperature Profile and Total Heat Transfer
- ❑ Counter-flow Design

Learning Objectives:

- ❑ Calculate the total heat transfer for parallel flow HE
- ❑ Calculate the total heat transfer for counter flow HE

Introduction to Heat Exchangers

A. Concentric Flow



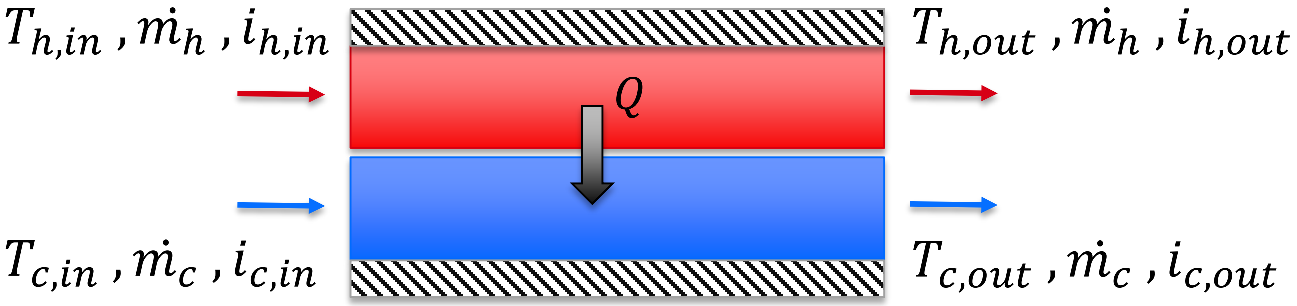
Parallel Flow Counter Flow

Parallel Flow Heat Exchanger

Energy balance

$\boldsymbol{i} \quad = \boldsymbol{enthalphy}$

We use this anomalous symbol to avoid confusion with convection coefficient



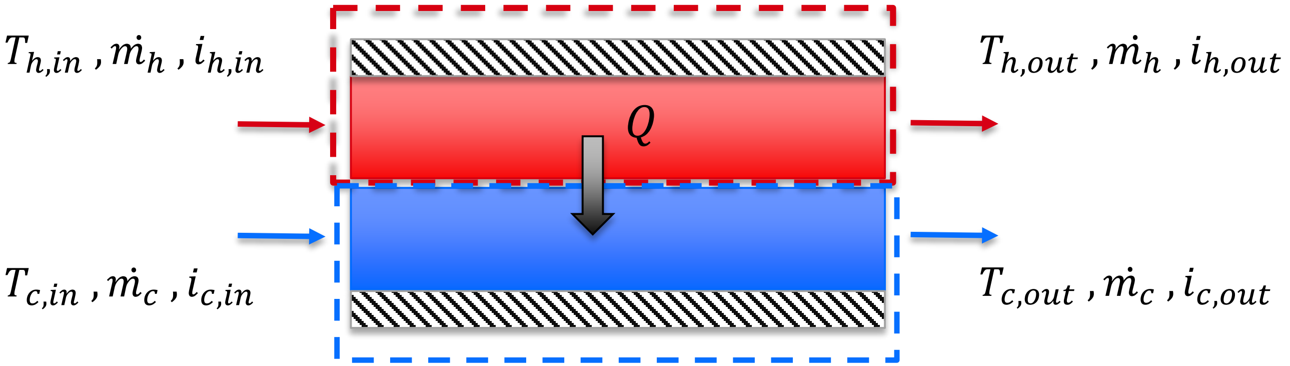
Parallel Flow Heat Exchanger

Energy balance $Q = Q_c = -Q_h$

$\boldsymbol{i} = \boldsymbol{enthalphy}$

We use this anomalous symbol to avoid confusion with convection coefficient

$$Q_h = \dot{m}_h\left(i_{h,out} - i_{h,in}\right) = \dot{m}_h c_{p,h}\left(T_{h,out} - T_{h,in}\right) < 0$$



$$Q_c = \dot{m}_c\left(i_{c,out} - i_{c,in}\right) = \dot{m}_c c_{p,c}\left(T_{c,out} - T_{c,in}\right) > 0$$

Incompressible fluid
True if no phase change

Average temperature on that cross-section

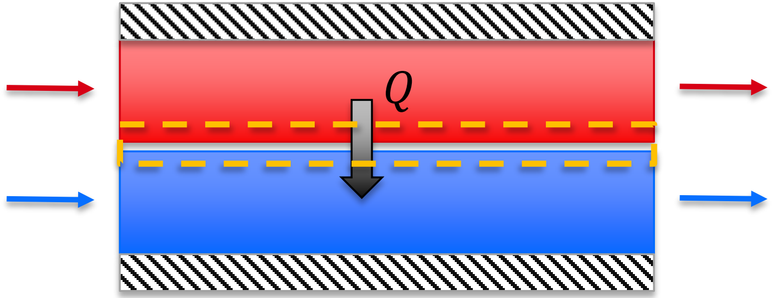
Parallel Flow Heat Exchanger

Energy balance

$$i \quad = enthalphy$$

We use this anomalous symbol to avoid confusion with convection coefficient

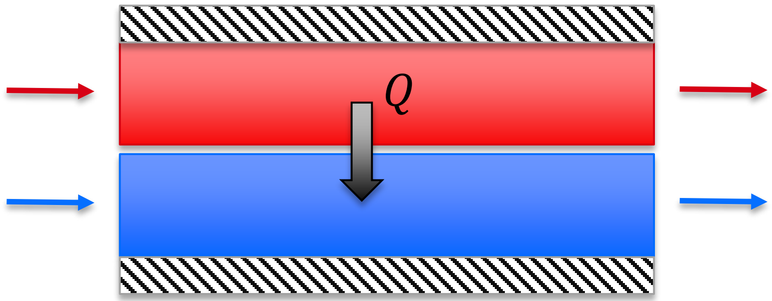
$$Q = UA\Delta T_m$$

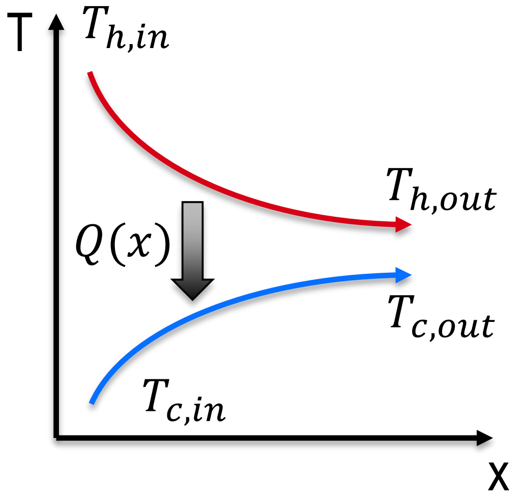


- What is an appropriate ΔT_m?
- It is related to the temperature profile

Parallel Flow Heat Exchanger

Temperature Profile



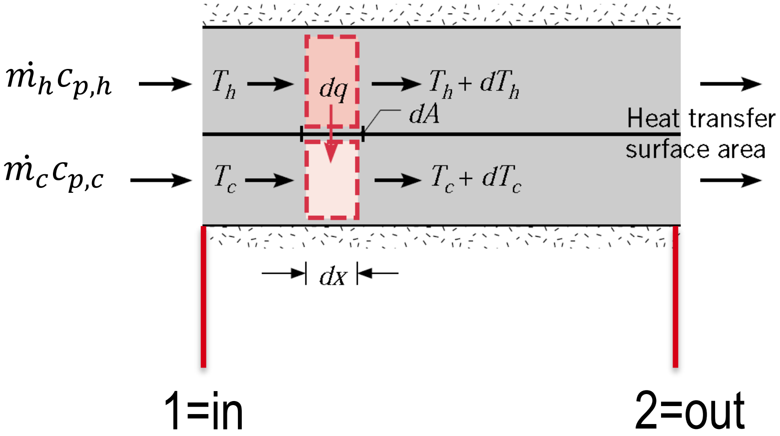


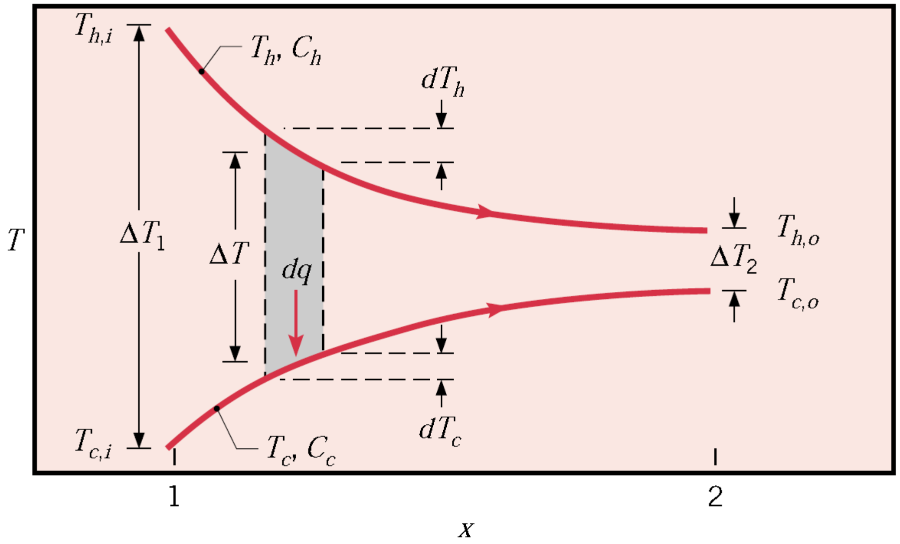
- What is an appropriate ΔT_m?

- Let's write a local energy balance, similar to what we did for internal forced convection

Parallel Flow heat Exchanger

Temperature Profile

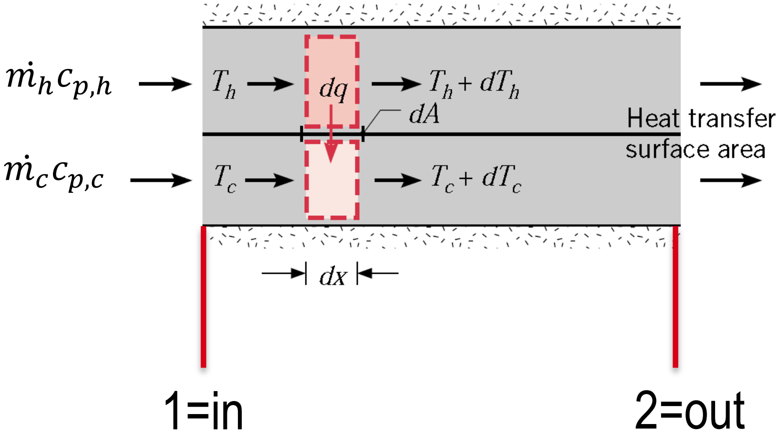




Need to write a differential equation for ΔT

Parallel Flow heat Exchanger

Temperature Profile



1. The heat exchanger is insulated from its surroundings, in which case the only heat exchange is between the hot and cold fluids.
2. Axial conduction along the tubes is negligible.
3. Potential and kinetic energy changes are negligible.
4. The fluid specific heats are constant.
5. The overall heat transfer coefficient is constant.

$$dq = \dot{m} \ (i_{x+dx} - i_x)$$

$$dq = \dot{m}_c c_{p,c}\, dT_c \equiv C_c\, dT_c \quad \text{Heat gain cold side}$$

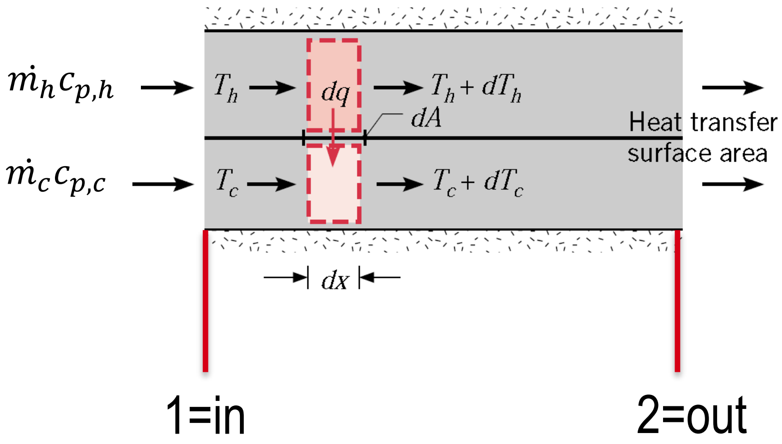
$$= -\dot{m}_h c_{p,h}\, dT_h \equiv -C_h\, dT_h \quad \text{Heat loss hot side}$$

Also we can write:

$$\Delta T = T_h - T_c \Rightarrow dq = U \Delta T\, dA$$

Parallel Flow heat Exchanger

Temperature Profile



$$\Delta T = T_h - T_c \quad \Rightarrow \quad d(\Delta T) = dT_h - dT_c$$

Where: $dT_h = -\dfrac{dq}{C_h}$ $\quad dT_c = \dfrac{dq}{C_c}$

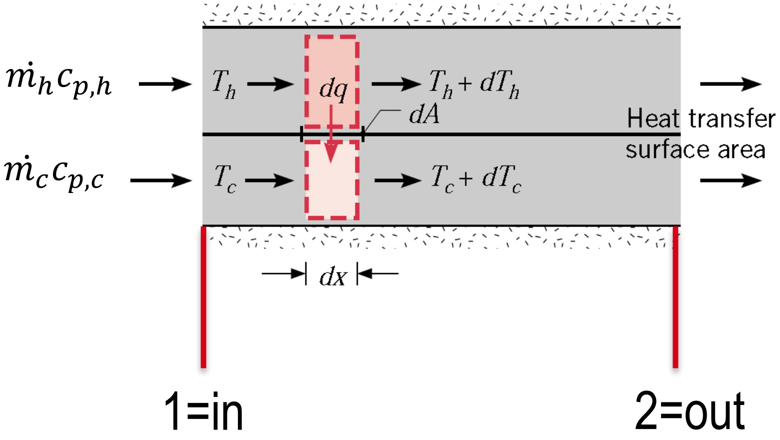
$$\Rightarrow \quad d(\Delta T) = -dq\left(\frac{1}{C_h} + \frac{1}{C_c}\right)$$

$$\Rightarrow \quad d\Delta T = -\mathrm{U}\Delta T \mathrm{dA}\left(\frac{1}{C_h} + \frac{1}{C_c}\right)$$

$$\Rightarrow \quad \int_1^2 \frac{d(\Delta T)}{\Delta T} = -U\left(\frac{1}{C_h} + \frac{1}{C_c}\right)\int_1^2 dA$$

Parallel Flow heat Exchanger

Temperature Profile



$$\int_1^2 \frac{d(\Delta T)}{\Delta T} = -U\left(\frac{1}{C_h} + \frac{1}{C_c}\right)\int_1^2 dA$$

$$\Rightarrow \ln\left(\frac{\Delta T_2}{\Delta T_1}\right) = -UA\left(\frac{1}{C_h} + \frac{1}{C_c}\right)$$

$$Q = -Q_h = -\dot{m}_h c_{p,h}(T_{h,1} - T_{h,2}) = C_h(T_{h,1} - T_{h,2})$$

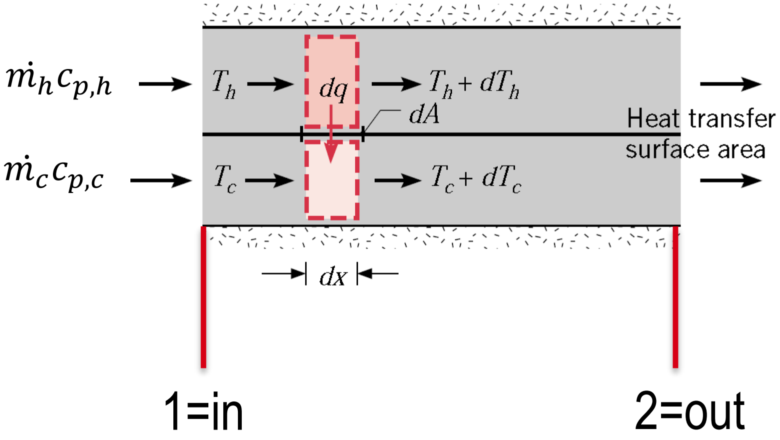
$$Q = Q_c = \dot{m}_c c_{p,c}(T_{c,2} - T_{c,1}) = C_h(T_{c,2} - T_{c,1})$$

$$C_h = Q/(T_{h,1} - T_{h,2}) \qquad C_c = Q/(T_{c,2} - T_{c,1})$$

$$\Rightarrow ln\left(\frac{\Delta T_2}{\Delta T_1}\right) = -\mathrm{UA}\left(\frac{(T_{h,1} - T_{h,2})}{Q} + \frac{(T_{c,2} - T_{c,1})}{Q}\right)$$

Parallel Flow heat Exchanger

Temperature Profile and Total Heat Transfer



$$ln\left(\frac{\Delta T_2}{\Delta T_1}\right) = -\mathrm{UA}\left(\frac{(T_{h,1} - T_{c,1})}{Q} + \frac{(T_{c,2} - T_{h,2})}{Q}\right)$$

$$ln\left(\frac{\Delta T_2}{\Delta T_1}\right) = -\mathrm{UA}\left(\frac{\Delta T_1}{Q} - \frac{\Delta T_2}{Q}\right)$$

$$Q = \mathrm{UA}\frac{\Delta T_2 - \Delta T_1}{ln\left({}^{\Delta T_2}/_{\Delta T_1}\right)} = UA\Delta T_m$$

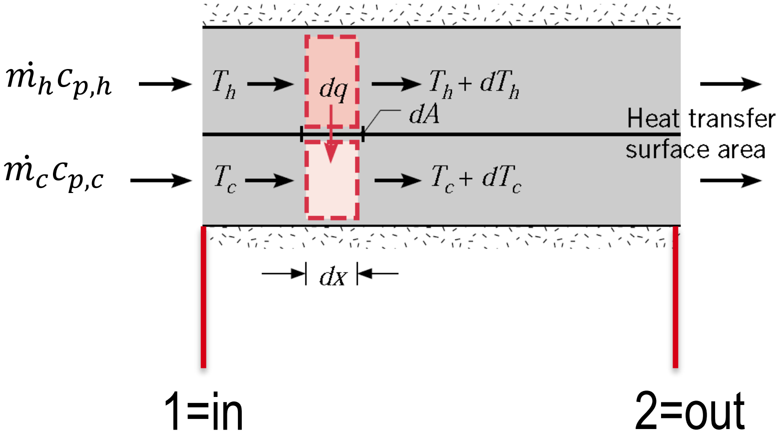
$$\Delta T_m = \frac{\Delta T_2 - \Delta T_1}{ln\left({}^{\Delta T_2}/_{\Delta T_1}\right)}$$

$$\Delta T_1 = (T_{h,1} - T_{c,1})$$

$$\Delta T_2 = (T_{h,2} - T_{c,2})$$

Parallel Flow heat Exchanger

Temperature Profile and Total Heat Transfer



$$Q = \mathrm{UA}\frac{\Delta T_2 - \Delta T_1}{ln\left(\Delta T_2/\Delta T_1\right)} = UA\Delta T_m$$

$$\Delta T_m = \frac{\Delta T_2 - \Delta T_1}{ln\left(\Delta T_2/\Delta T_1\right)}$$

$$\Delta T_1 = \left(T_{h,1} - T_{c,1}\right) = \left(T_{h,in} - T_{c,in}\right)$$

$$\Delta T_2 = \left(T_{h,2} - T_{c,2}\right) = \left(T_{h,out} - T_{c,out}\right)$$

This Lecture

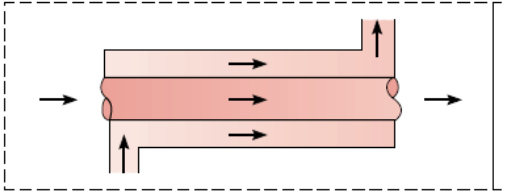
- [x] Parallel Flow Design
    - [x] Temperature Profile and Total Heat Transfer
- [ ] Counter-flow Design

Learning Objectives:

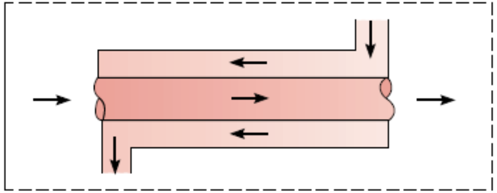
- [x] Calculate the total heat transfer for parallel flow HE
- [ ] Calculate the total heat transfer for counter flow HE

Introduction to Heat Exchangers

A. Concentric Flow



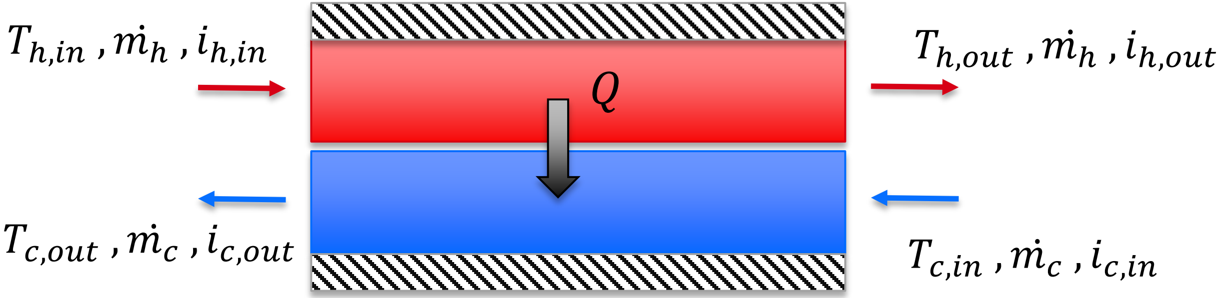
Parallel Flow

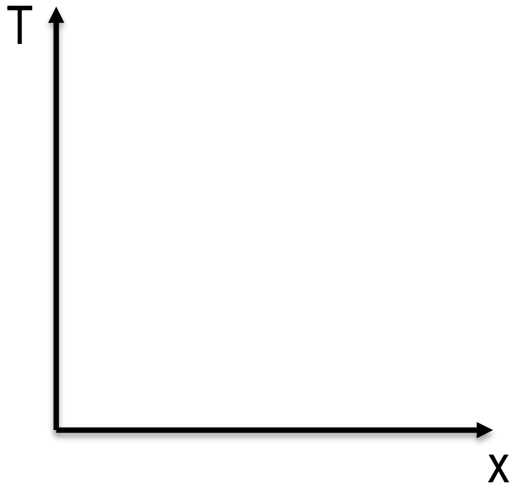


Counter Flow

Counter Flow Heat Exchanger

Temperature Profile

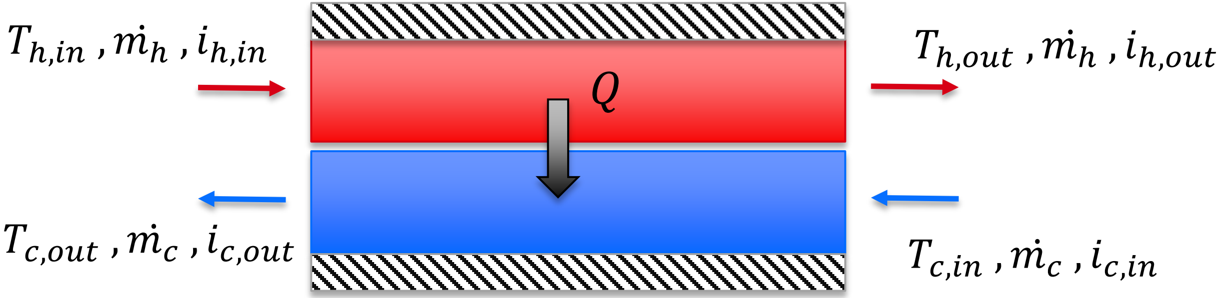


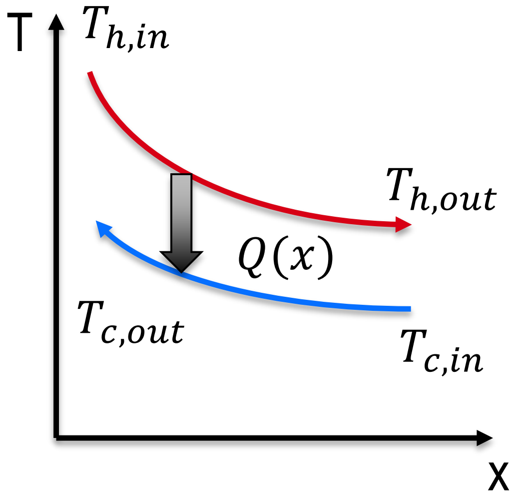


➡ What is an appropriate ΔT_m?

Counter Flow Heat Exchanger

Temperature Profile

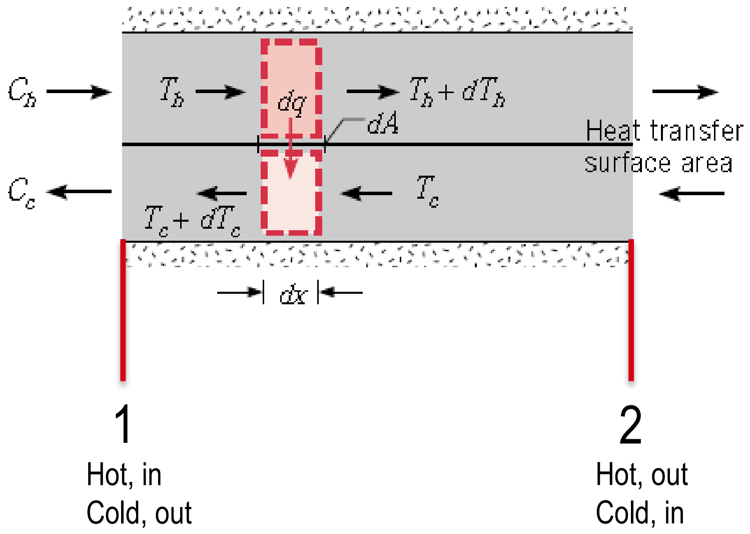




➡ What is an appropriate ΔT_m?

Counter Flow heat Exchanger

Temperature Profile and Total Heat Transfer



$$Q = \text{UA}\frac{\Delta T_2 - \Delta T_1}{ln\left(\Delta T_2 / \Delta T_1\right)} = UA\Delta T_m$$

$$\Delta T_m = \frac{\Delta T_2 - \Delta T_1}{ln\left(\Delta T_2 / \Delta T_1\right)}$$

$$\Delta T_1 = \left(T_{h,1} - T_{c,1}\right) = \left(T_{h,in} - T_{c,out}\right)$$

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This Lecture

- ☑ Parallel Flow Design
  - ☑ Temperature Profile and Total Heat Transfer
- ☑ Counter-flow Design

Learning Objectives:

- ☑ Calculate the total heat transfer for parallel flow HE
- ☑ Calculate the total heat transfer for counter flow HE

Next Lecture

- ❑ Special Operating Conditions
- ❑ Exercises

Learning Objectives:

- ❑ Recognize specific conditions