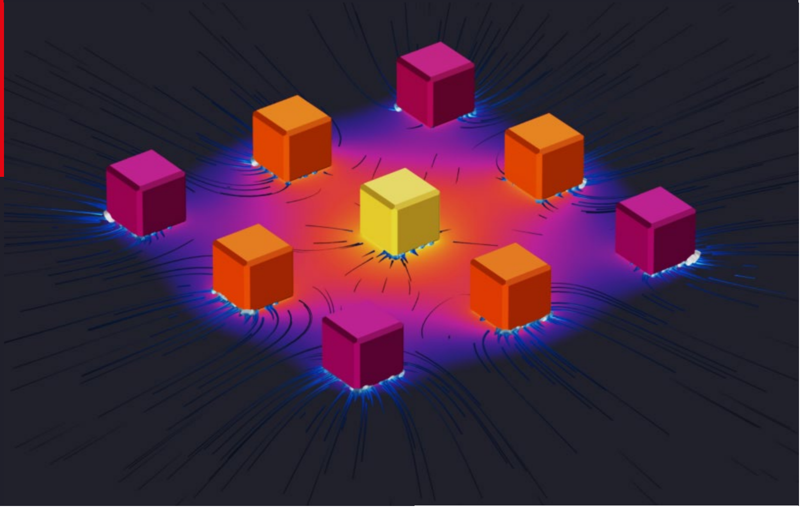
Heat and Mass Transfer
ME-341



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École
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fédérale
de Lausanne

Spring Semester

Previously

- ☑ Introduction to Heat Exchangers
- ☑ The Problem of the Heat Transfer Coefficient

Learning Objectives:

- ☑ Understand the concept and possible design of heat exchangers

Heat Transfer Mechanisms and Transport Laws

Conduction

- Planar and radial systems
- With and without heat sources
- Steady-state and Transient

Convection

- Forced convection (External and Internal)
- Free convection
- Boiling and Condensation

- Thermal resistances/Thermal circuits/Heat transfer coefficient U
- Dimensionless numbers (Re, Nu, Pr, Bi, Ra, Gr)
- Characteristic dimensions & reference temperature
- Mass/Momentum/Energy conservation equations

$$\frac{1}{\alpha}\frac{\partial T}{\partial t} = \nabla^2 T + \frac{\dot{q}}{k}$$

Conduction →Only Diffusion
(Closed system)

$$\rho c_p\left(\frac{\partial T}{\partial t} + \vec{u}\cdot\nabla T\right) = k\nabla^2 T + \dot{q}$$

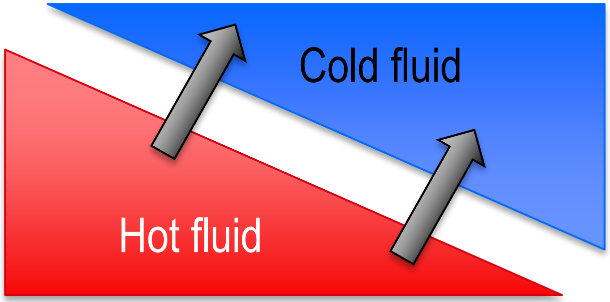
Convection → Diffusion + Advection
(Open system)

Design a Heat Exchanger

Introduction to Heat Exchangers

Air-conditioning &
Waste-heat recovery

Power production
(boilers/condensers)

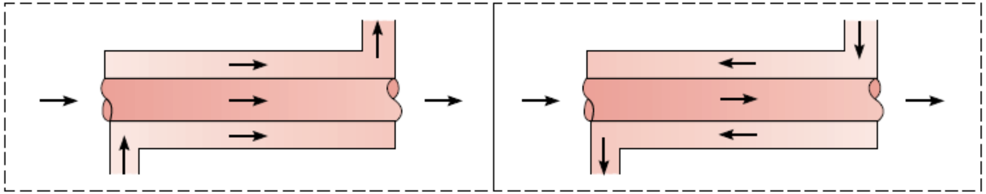


Natural Systems

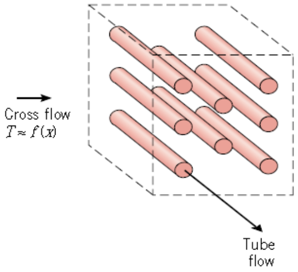
Chemical
Processing

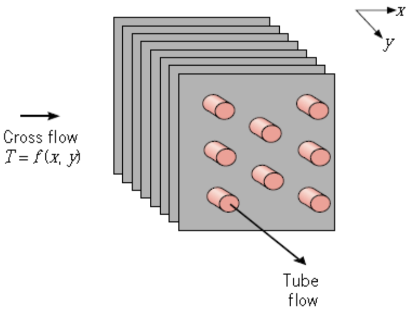
Introduction to Heat Exchangers

A. Concentric Flow

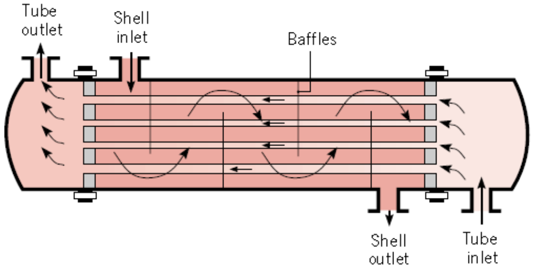


B. Cross-Flow



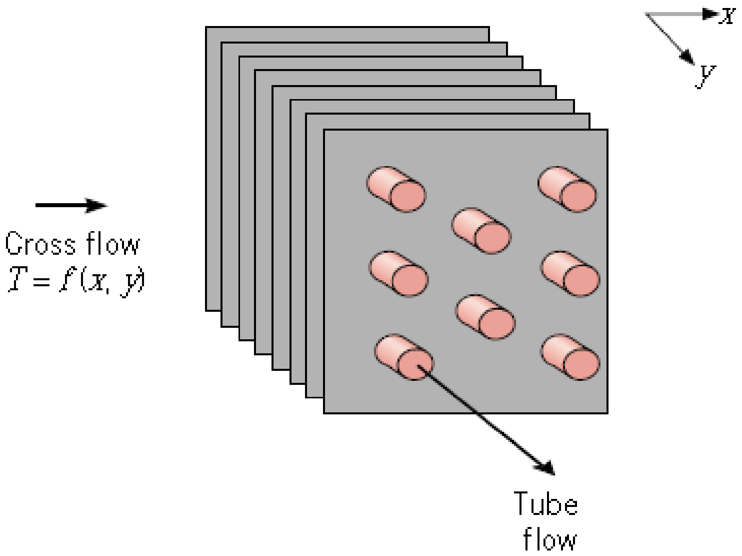


C. Shell-and-Tube



Overall Heat Transfer Coefficient

$$Q = \frac{\Delta T}{R_{tot}} = \boldsymbol{U} A \Delta T \Rightarrow U \equiv \frac{1}{R_{tot} A} \Rightarrow \frac{1}{UA} \equiv R_{tot}$$



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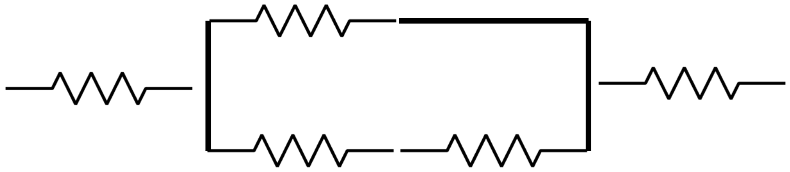

Note: we will discuss the expression for ΔT later

Overall Heat Transfer Coefficient

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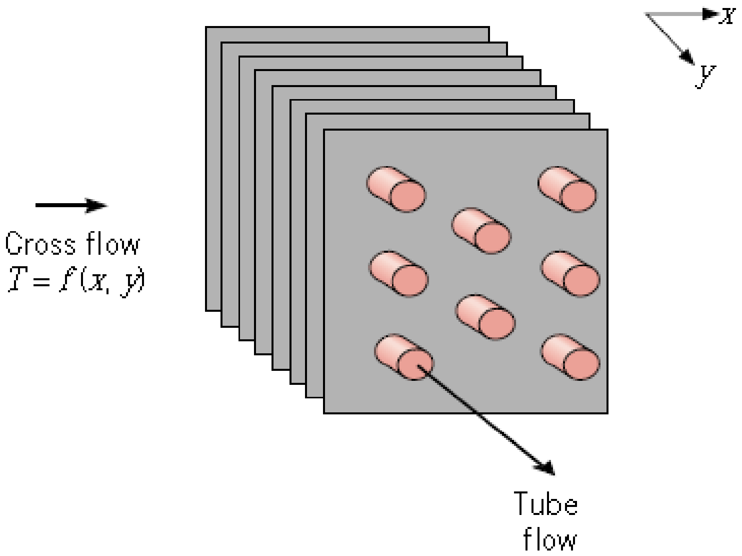


- Conduction thermal resistances
  - Planar/radial conduction
- Convection thermal resistances
  - Internal and external
- Array of Fins
- Fouling

Overall Heat Transfer Coefficient

Conduction & Convection Thermal Resistances

Planar wall

$$R_{th,cond} = \frac{L}{kA} \quad [\,K/W\,]$$

$$R_{th,conv} = \frac{1}{hA} \quad [\,K/W\,]$$

Radial System

$$R_{th,cond-cyl} = \frac{ln(r_2/r_1)}{2\pi Lk} \quad [\,K/W\,]$$

$$R_{th,conv} = \frac{1}{h2\pi rL} \quad [\,K/W\,]$$

General methodology for calculating the convection coefficient

0. Identify the type of convection (Forced/External, Forced/Internal, Free, Boiling/Condensation)

1. Recognize the flow geometry (plate, cylinder, inner/outer flow etc.) [GEOM]

2. Specify the appropriate reference temperature and evaluate the pertinent fluid properties at that temperature $\boldsymbol{T_f}$
   - External convection over a plate/cylinder: $\boldsymbol{T_f = (T_s + T_\infty)/2}$
   - External convection over a bank of tubes: $\boldsymbol{T_f = (T_{in} + T_{out})/2}$
   - Internal convection: $\boldsymbol{T_f = (T_{m,i} + T_{m,o})/2}$
   - Free convection: $\boldsymbol{T_f = (T_s + T_\infty)/2}$
   - Boiling: a) film pool boiling $\boldsymbol{T_l = T_{sat}, T_v = T_f = (T_s + T_{sat})/2}$; b) all other types of boiling $\boldsymbol{T_l = T_v = T_{sat}}$
   - Condensation (film): $\boldsymbol{T_v = T_{sat}, T_l = T_f = (T_s + T_{sat})/2}$

Note: If the necessary temperatures are unknown, we can use T_∞, T_{in} or T_{sat} to estimate all of the fluid properties. Once we obtain T_s, T_{out}, $T_{m,o}$ we need to check whether it was reasonable.

3. Calculate the Reynolds number (be careful to use the right characteristic dimension - x, L, D – and velocity - u_m, u_∞) or the Gr&Ra numbers. Determine the flow conditions (laminar/turbulent) [FLOW]

4. Decide whether a local or surface average coefficient is required [Loc/Ave]

5. Calculate Pr or get it from the table [Pr]

6. Select the appropriate correlation, determine Nu and the convection coefficient or directly h [Nu, h]

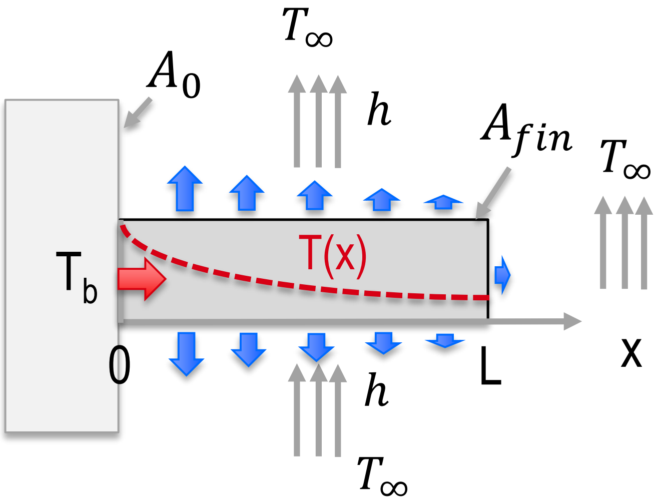
Overall Heat Transfer Coefficient

*Fins – Efficiency and Resistance (**1 Fin**)*

If T_b is the temperature of the fin base:

$$\eta_f \equiv \frac{Q_f}{Q_{f,max}} = \frac{Q_f}{hA_f(T_b - T_\infty)} = \frac{Q_f}{hA_f\theta_b}$$

$$\boldsymbol{R_f \equiv \frac{(T_b - T_\infty)}{Q_f} = \frac{1}{hA_f\eta_f}}$$

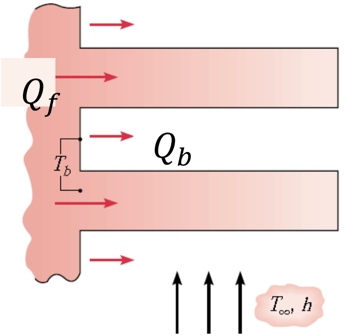


e.g. for infinite fin we get: $R_f \equiv \frac{\theta_b}{Q_f} = \frac{\theta_b}{M} = \frac{1}{\sqrt{hPkA_c}}$

Overall Heat Transfer Coefficient

*Fins – Efficiency and Resistance (**Array of Fins**)*

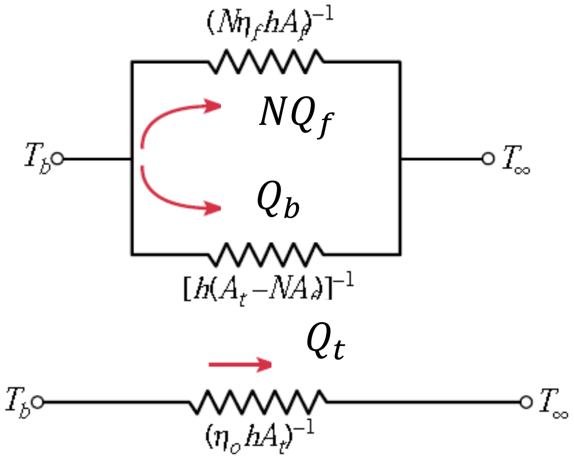
$$\eta_o \equiv \frac{Q_t}{Q_{t,max}} = \frac{Q_t}{hA_t(T_0 - T_\infty)} = \frac{NQ_f + Q_b}{h\left(NA_f + A_b\right)(T_0 - T_\infty)}$$



$$\eta_o \equiv 1 - \frac{NA_f}{A_t}(1 - \boldsymbol{\eta_f})$$ *Overall efficiency*

Single fin efficiency

$$R_o \equiv \frac{(T_0 - T_\infty)}{Q_t} = \frac{1}{\eta_o h A_t}$$



This lecture

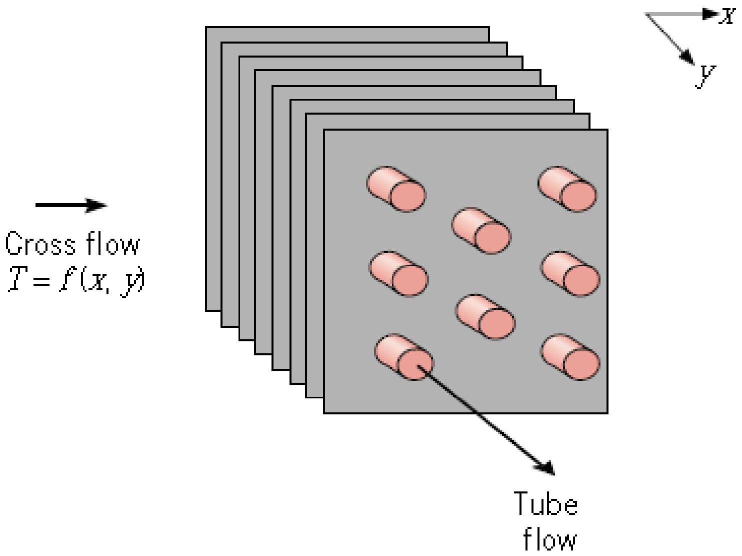
- ❑ Fouling
- ❑ The overall heat transfer coefficient

Learning Objectives:

- ❑ Calculate the overall heat transfer coefficient

Overall Heat Transfer Coefficient

$$Q = \frac{\Delta T}{R_{tot}} = \boldsymbol{U} A \Delta T \quad \Rightarrow \quad U \equiv \frac{1}{R_{tot} A} \quad \Rightarrow \quad \frac{1}{UA} \equiv R_{tot}$$



- Conduction thermal resistances
  - Planar/radial conduction
- Convection thermal resistances
  - Internal and external
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- Fouling

Overall Heat Transfer Coefficient

Fouling

Dramatic increase in thermal resistance due to poor conduction through the scaling layer

➡ Introduce a *fouling* resistance per unit area (fouling factor) R_f''

This lecture

- [x] Fouling
- [ ] The overall heat transfer coefficient

Learning Objectives:

- [ ] Calculate the overall heat transfer coefficient

Overall Heat Transfer Coefficient

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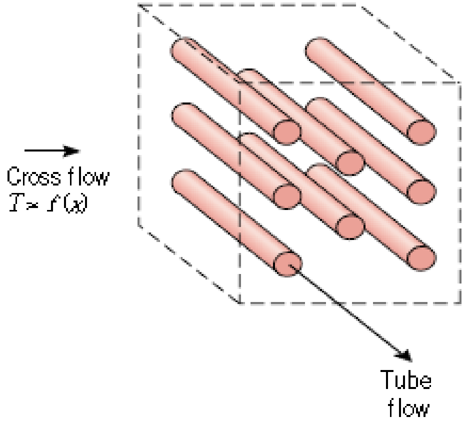
Example 1a: Without fouling



Overall Heat Transfer Coefficient

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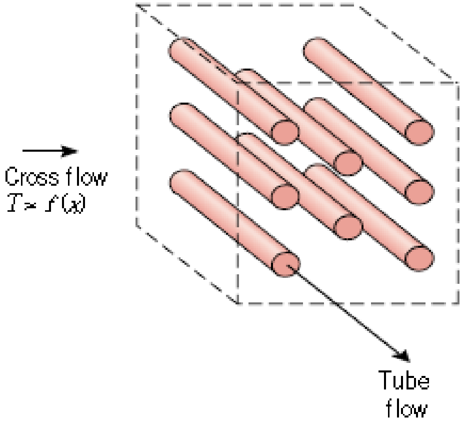
Example 1a: Without fouling



$$\frac{1}{UA} = R_{conv,out} + R_{cond} + R_{conv,in}$$

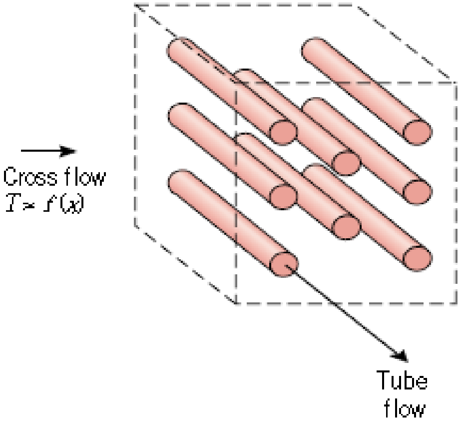
$$\frac{1}{UA} = \frac{1}{h_{out} A_{out}} + \frac{\ln(r_{out}/r_{in})}{2\pi k L} + \frac{1}{h_{in} A_{in}}$$

h_{out} forced convection external flow
h_{in} forced convection internal flow

Overall Heat Transfer Coefficient

$$Q = \frac{\Delta T}{R_{tot}} = \boldsymbol{U} A \Delta T \quad \Rightarrow \quad U \equiv \frac{1}{R_{tot} A} \quad \Rightarrow \quad \frac{1}{UA} \equiv R_{tot}$$

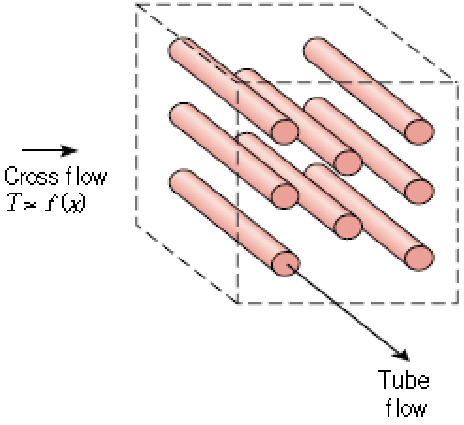
Example 1a: With fouling



Overall Heat Transfer Coefficient

$$Q = \frac{\Delta T}{R_{tot}} = \boldsymbol{U} A \Delta T \quad \Rightarrow \quad U \equiv \frac{1}{R_{tot} A} \quad \Rightarrow \quad \frac{1}{UA} \equiv R_{tot}$$

Example 1a: With fouling



$$\frac{1}{UA} = R_{conv,out} + R_{foul,out} + R_{cond} + R_{foul,in} + R_{conv,in}$$

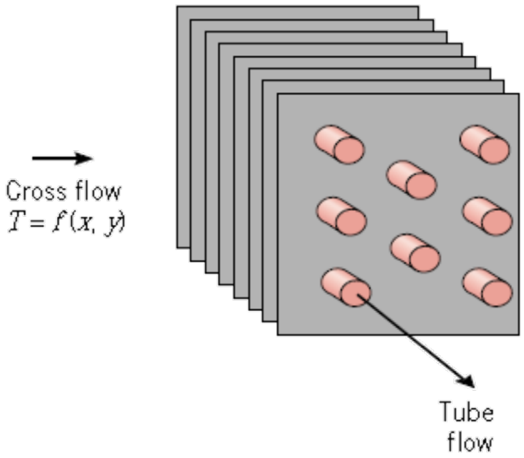
$$R_f'' \equiv A R_{foul} \quad \Rightarrow \quad R_{foul} = \frac{R_f''}{A}$$

$$\frac{1}{UA} = \frac{1}{h_{out} A_{out}} + \frac{R_{f,o}''}{A_{out}} + \frac{\ln(r_{out}/r_{in})}{2\pi k L} + \frac{R_{f,i}''}{A_{in}} + \frac{1}{h_{in} A_{in}}$$

Overall Heat Transfer Coefficient

$$Q = \frac{\Delta T}{R_{tot}} = \boldsymbol{U} A \Delta T \quad \Rightarrow \quad U \equiv \frac{1}{R_{tot} A} \quad \Rightarrow \quad \frac{1}{UA} \equiv R_{tot}$$

Example 2a: Finned, without fouling



Overall Heat Transfer Coefficient

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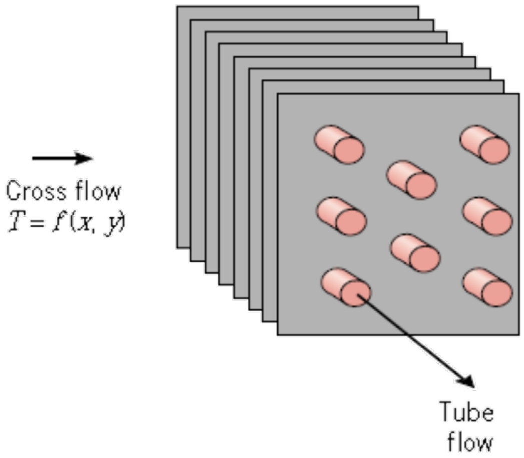


$$\frac{1}{UA} = R_{conv,out} + R_{cond} + R_{conv,in}$$

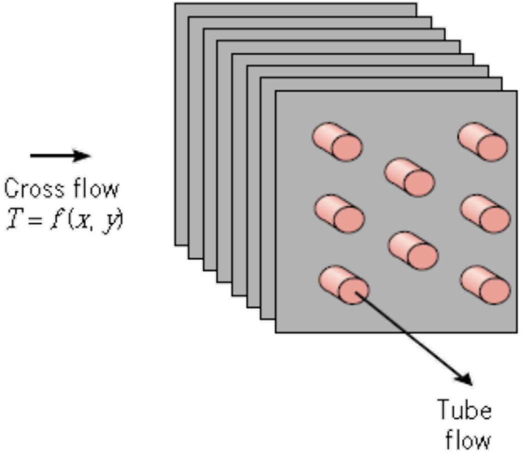
$$R_{conv,out} = R_{fin\ array} \equiv \frac{(T_0 - T_\infty)}{Q_t} = \frac{1}{\eta_o h A_{total}}$$

$$\frac{1}{UA} = \frac{1}{\eta_o h_{out} A_{out}} + \frac{\ln(r_{out}/r_{in})}{2\pi k L} + \frac{1}{h_{in} A_{in}}$$

Overall Heat Transfer Coefficient

$$Q = \frac{\Delta T}{R_{tot}} = \boldsymbol{U} A \Delta T \quad \Rightarrow \quad U \equiv \frac{1}{R_{tot} A} \quad \Rightarrow \quad \frac{1}{UA} \equiv R_{tot}$$

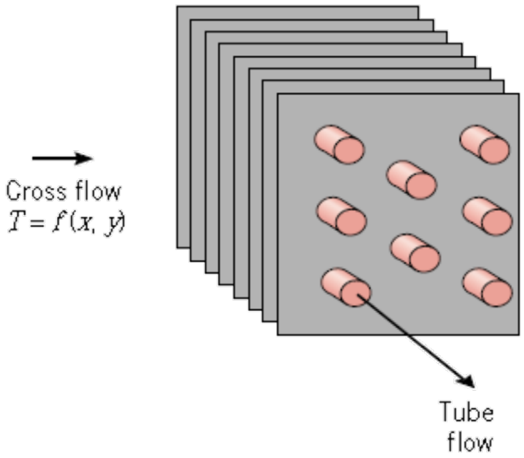
Example 2a: Finned, with fouling



Overall Heat Transfer Coefficient

$$Q = \frac{\Delta T}{R_{tot}} = \boldsymbol{U} A \Delta T \quad \Rightarrow \quad U \equiv \frac{1}{R_{tot} A} \quad \Rightarrow \quad \frac{1}{UA} \equiv R_{tot}$$

Example 2a: Finned, with fouling



$$\frac{1}{UA} = R_{conv,out} + R_{foul,out} + R_{cond} + R_{foul,in} + R_{conv,in}$$

$$R_{overall\ fin\ array} \equiv \frac{(T_0 - T_\infty)}{Q_t} = \frac{1}{\eta_o h A_t} = \frac{1}{h(\eta_o A_t)} = \frac{1}{hA'_t}$$

$$R_{foul} = \frac{R''_f}{A'_t} = \frac{R''_f}{\eta_o A_t}$$

$$\frac{1}{UA} = \frac{1}{\eta_o h_{out} A_{out}} + \frac{R''_{f,o}}{\eta_o A_{out}} + \frac{\ln(r_{out}/r_{in})}{2\pi k L} + \frac{R''_{f,i}}{A_{in}} + \frac{1}{h_{in} A_{in}}$$

Overall Heat Transfer Coefficient

$$Q = \frac{\Delta T}{R_{tot}} = \boldsymbol{U} A \Delta T \quad \Rightarrow \quad U \equiv \frac{1}{R_{tot} A} \quad \Rightarrow \quad \frac{1}{UA} \equiv R_{tot}$$

In the most general case we could have fins present also on the inner side:

$$\frac{1}{UA} = \frac{1}{\eta_{o,out} h_{out} A_{out}} + \frac{R''_{f,o}}{\eta_{o,out} A_{out}} + R_{cond} + \frac{R''_{f,i}}{\eta_{o,in} A_{in}} + \frac{1}{\eta_{o,in} h_{in} A_{in}}$$

Account for all the layers of conduction!!

Overall Heat Transfer Coefficient

$$Q = \frac{\Delta T}{R_{tot}} = \boldsymbol{U} A \Delta T \quad \Rightarrow \quad U \equiv \frac{1}{R_{tot} A} \quad \Rightarrow \quad \frac{1}{UA} \equiv R_{tot}$$

Note:

$$\frac{\mathbf{1}}{\boldsymbol{UA}} \equiv \boldsymbol{R_{tot}} = \frac{1}{U_{in} A_{in}} = \frac{1}{U_{out} A_{out}} = \frac{1}{U_{cold} A_{cold}} = \frac{1}{U_{hot} A_{hot}}$$

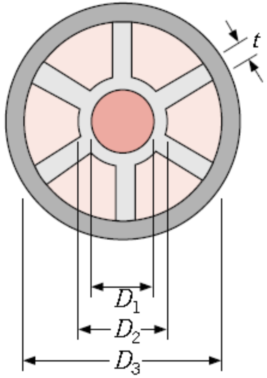
$$U_{in} \equiv \frac{1}{R_{tot} A_{in}} \neq U_{out} \equiv \frac{1}{R_{tot} A_{out}} \qquad U_{cold} \equiv \frac{1}{R_{tot} A_{cold}} \neq U_{hot} \equiv \frac{1}{R_{tot} A_{hot}}$$

Calculate first the total heat transfer resistance R_{tot} then multiply by a specific area to determine the overall heat transfer coefficient with respect to a certain surface.

Note: Do not calculate U starting from R''_{tot} because it is likely you will mess up the areas (you can do it but you have to be extra careful as each term of the specific resistance will be normalized by a different area…)

Overall Heat Transfer Coefficient – Example

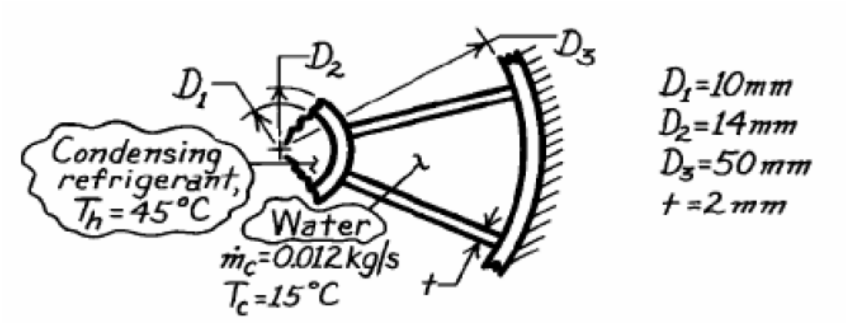
A novel design for a condenser consists of a tube of thermal conductivity 200 W/m · K with longitudinal fins snugly fitted into a larger tube. Condensing refrigerant at 45°C flows axially through the inner tube, while water at a flow rate of 0.012 kg/s passes through the six channels around the inner tube. The pertinent diameters are D_1 = 10 mm, D_2 = 14 mm, and D_3 = 50 mm, while the fin thickness is t = 2 mm. Assume that the convection coefficient associated with the condensing refrigerant is extremely large.



Determine the heat removal rate per unit tube length in a section of the tube for which the water is at 15°C.

Assumptions:

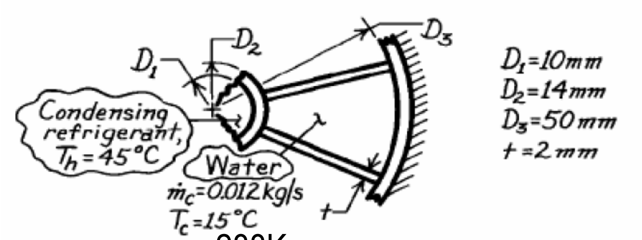
- **Negligible thermal resistance for internal convection**
- **Assume fully developed flow**
- **Assume perfect insulation on the outside of the entire structure**



Hint:

- Determine the equivalent hydraulic diameter for a given radial compartment

Overall Heat Transfer Coefficient – Example 2



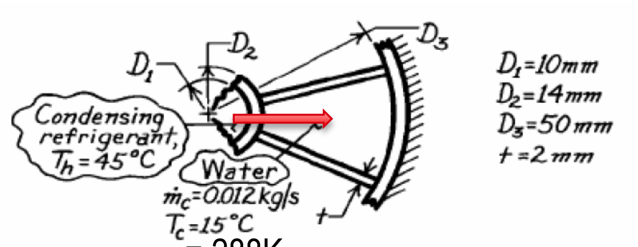
= 288K

PROPERTIES: *Table A-6*, Water ($\overline{T}_c$ = 15°C = 288 K): ρ = 1000 kg/m^3, k = 0.595 W/m·K, $\nu = \mu/\rho$ = 1138 × 10^{-6} N·s/m^2/1000 kg/m^3 = 1.138 × 10^{-6} m^2/s, Pr = 8.06; Tube fins (given): k = 200 W/m·K.

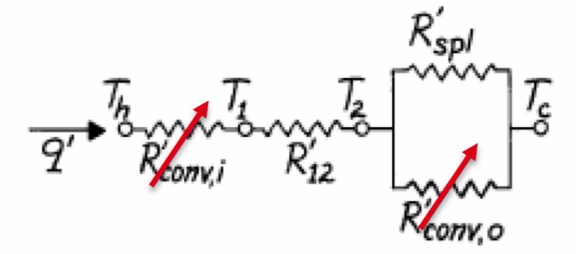
TABLE A.6 Thermophysical Properties of Saturated Water[a]

Temperature, T (K)	Pressure, p (bars)[b]	Specific Volume (m³/kg) $v_f \cdot 10^3$	v_g	Heat of Vaporization, h_{fg} (kJ/kg)	Specific Heat (kJ/kg · K) $c_{p,f}$	$c_{p,g}$	Viscosity (N · s/m²) $\mu_f \cdot 10^6$	$\mu_g \cdot 10^6$	Thermal Conductivity (W/m · K) $k_f \cdot 10^3$	$k_g \cdot 10^3$	Prandtl Number Pr_f	Pr_g	Surface Tension, $\sigma_f \cdot 10^3$ (N/m)	Expansion Coefficient, $\beta_f \cdot 10^6$ (K⁻¹)	Temperature, T (K)
273.15	0.00611	1.000	206.3	2502	4.217	1.854	1750	8.02	569	18.2	12.99	0.815	75.5	−68.05	273.15
275	0.00697	1.000	181.7	2497	4.211	1.855	1652	8.09	574	18.3	12.22	0.817	75.3	−32.74	275
280	0.00990	1.000	130.4	2485	4.198	1.858	1422	8.29	582	18.6	10.26	0.825	74.8	46.04	280
285	0.01387	1.000	99.4	2473	4.189	1.861	1225	8.49	590	18.9	8.81	0.833	74.3	114.1	285
290	0.01917	1.001	69.7	2461	4.184	1.864	1080	8.69	598	19.3	7.56	0.841	73.7	174.0	290
295	0.02617	1.002	51.94	2449	4.181	1.868	959	8.89	606	19.5	6.62	0.849	72.7	227.5	295
300	0.03531	1.003	39.13	2438	4.179	1.872	855	9.09	613	19.6	5.83	0.857	71.7	276.1	300
305	0.04712	1.005	29.74	2426	4.178	1.877	769	9.29	620	20.1	5.20	0.865	70.9	320.6	305
310	0.06221	1.007	22.93	2414	4.178	1.882	695	9.49	628	20.4	4.62	0.873	70.0	361.9	310
315	0.08132	1.009	17.82	2402	4.179	1.888	631	9.69	634	20.7	4.16	0.883	69.2	400.4	315
320	0.1053	1.011	13.98	2390	4.180	1.895	577	9.89	640	21.0	3.77	0.894	68.3	436.7	320
325	0.1351	1.013	11.06	2378	4.182	1.903	528	10.09	645	21.3	3.42	0.901	67.5	471.2	325
330	0.1719	1.016	8.82	2366	4.184	1.911	489	10.29	650	21.7	3.15	0.908	66.6	504.0	330
335	0.2167	1.018	7.09	2354	4.186	1.920	453	10.49	656	22.0	2.88	0.916	65.8	535.5	335
340	0.2713	1.021	5.74	2342	4.188	1.930	420	10.69	660	22.3	2.66	0.925	64.9	566.0	340
345	0.3372	1.024	4.683	2329	4.191	1.941	389	10.89	668	22.6	2.45	0.933	64.1	595.4	345
350	0.4163	1.027	3.846	2317	4.195	1.954	365	11.09	668	23.0	2.29	0.942	63.2	624.2	350
355	0.5100	1.030	3.180	2304	4.199	1.968	343	11.29	671	23.3	2.14	0.951	62.3	652.3	355
360	0.6209	1.034	2.645	2291	4.203	1.983	324	11.49	674	23.7	2.02	0.960	61.4	697.9	360
365	0.7514	1.038	2.212	2278	4.209	1.999	306	11.69	677	24.1	1.91	0.969	60.5	707.1	365
370	0.9040	1.041	1.861	2265	4.214	2.017	289	11.89	679	24.5	1.80	0.978	59.5	728.7	370
373.15	1.0133	1.044	1.679	2257	4.217	2.029	279	12.02	680	24.8	1.76	0.984	58.9	750.1	373.15
375	1.0815	1.045	1.574	2252	4.220	2.036	274	12.09	681	24.9	1.70	0.987	58.6	761	375
380	1.2869	1.049	1.337	2239	4.226	2.057	260	12.29	683	25.4	1.61	0.999	57.6	788	380
385	1.5233	1.053	1.142	2225	4.232	2.080	248	12.49	685	25.8	1.53	1.004	56.6	814	385
390	1.794	1.058	0.980	2212	4.239	2.104	237	12.69	686	26.3	1.47	1.013	55.6	841	390
400	2.455	1.067	0.731	2183	4.256	2.158	217	13.05	688	27.2	1.34	1.033	53.6	896	400
410	3.302	1.077	0.553	2153	4.278	2.221	200	13.42	688	28.2	1.24	1.054	51.5	952	410
420	4.370	1.088	0.425	2123	4.302	2.291	185	13.79	688	29.8	1.16	1.075	49.4	1010	420
430	5.699	1.099	0.331	2091	4.331	2.369	173	14.14	685	30.4	1.09	1.10	47.2		430

Overall Heat Transfer Coefficient – Example 2



= 288K



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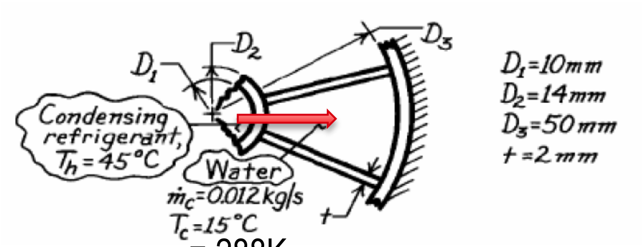
$$q' = UA'(T_h - T_c)$$

$$\frac{1}{UA'} = R'_h + R'_w + R'_c = R'_w + \frac{1}{(\eta_o h A')_c}$$

$$R'_w = \frac{\ln(D_2/D_1)}{2\pi k} = \frac{\ln(14/10)}{2\pi(200\ \mathrm{W/m\cdot K})} = 2.678\times10^{-4}\ \mathrm{m\cdot K/W}.$$

Overall Heat Transfer Coefficient – Example 2



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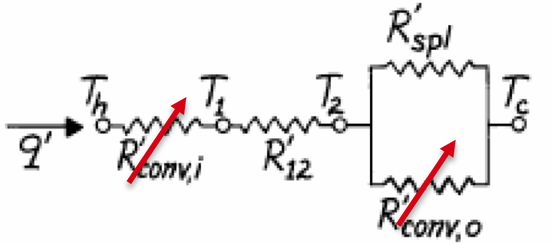

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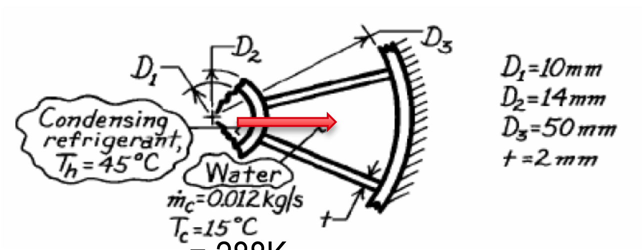
Determine the convection coefficient h

Convection
- Forced Convection
  - Internal Forced Convection
    - Flow conditions?

Laminar	$Re_D < 2300$ *Entry length ends at:* $x_{fd} = 0.05\, D Re_D$ *Friction Factor :* $4C_f = f = 64 Re_D^{-1}$	*Constant Surface Heat Flux:* q_s'' $Nu_D = 4.36$ With properties estimated at $T_f = T_m = \frac{T_{m,o}+T_{m,i}}{2}$ *Constant Surface Temperature:* T_s (i.e. $q_s''(x)$ not constant) $Nu_D = 3.66$ With properties estimated at $T_f = T_m = \frac{T_{m,o}+T_{m,i}}{2}$
Turbulent	$Re_D > 2300$ *Entry length ends at:* $10D < x_{fd} < 60D$ *Friction Factor. :* $f = (0.790 ln Re_D - 1.64)^{-2}$ $3000 < Re_D \leq 5 \cdot 10^6$	For all B.C.s: $Re_D > 10000$ $0.7 \leq Pr \leq 160$ $Nu_D = 0.023 Re_D^{4/5} Pr^n$ Where $n = 0.4$ for heating, $n = 0.3$ for cooling OR[2] $3000 < Re_D \leq 5 \cdot 10^6$ $0.5 \leq Pr \leq 3000$ $Nu_D = \frac{(f/8)(Re_D - 1000)Pr}{1 + 12.7(f/8)^{1/2}(Pr^{2/3} - 1)}$ For both properties estimated at $T_f = T_m = \frac{T_{m,o}+T_{m,i}}{2}$

Overall Heat Transfer Coefficient – Example 2



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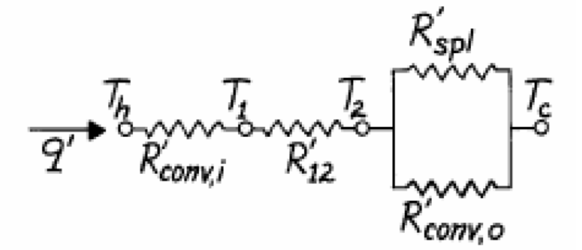


To estimate the thermal resistance on the water side (c), first evaluate the convection coefficient. The hydraulic diameter for a passage, where A_c is the cross-sectional area of the passage is

$$D_{h,c} = \frac{4A_c}{P} = \frac{4\left[\pi\left(D_3^2 - D_2^2\right)/4 - 6(D_3 - D_2)t/2\right]/6}{(\pi D_2 - 6t)/6 + (\pi D_3 - 6t)/6 + 2(D_3 - D_2)/2}$$

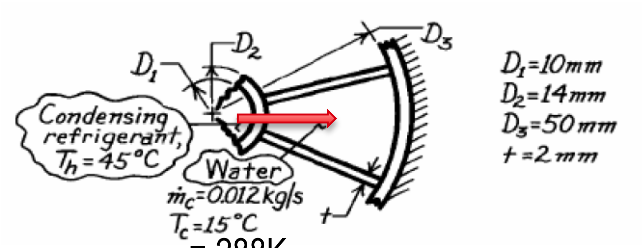
$$D_{h,c} = \frac{4\left[\pi\left(50^2 - 14^2\right)/4 - 6(50-14)\right]\times 10^{-6}\,\text{m}^2/6}{\left[(14\pi - 6\times 2)/6 + (50\pi - 6\times 2)/6 + (50-14)\right]\times 10^{-3}\,\text{m}}$$

$$D_{h,c} = \frac{4\times 2.656\times 10^{-4}\,\text{m}^2}{6.551\times 10^{-2}\,\text{m}} = 0.01622\,\text{m}.$$

$$Re_{D,c} = \frac{\left[(0.012\ \text{kg/s}/6)/\left(1000\ \text{kg/m}^3 \times 2.656\times 10^{-4}\,\text{m}^2\right)\right]\times 0.01622\text{m}}{1.138\times 10^{-6}\,\text{m}^2/\text{s}} = 107$$

$Re_D < 2300$ Laminar Flow

Overall Heat Transfer Coefficient – Example 2



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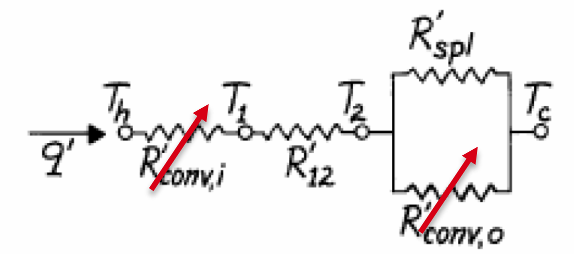

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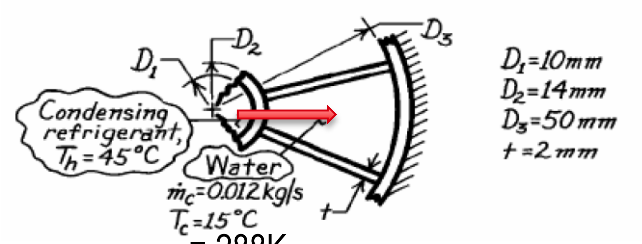
Determine the convection coefficient h

Convection
- ➡ Forced Convection
  - ➡ Internal Forced Convection
    - ➡ Laminar
      - ➡ BC?

Laminar	$Re_D < 2300$ *Entry length ends at:* $x_{fd} = 0.05\, D Re_D$ *Friction Factor :* $4C_f = f = 64 Re_D^{-1}$	*Constant Surface Heat Flux:* q_s'' $Nu_D = 4.36$ With properties estimated at $T_f = T_m = \frac{T_{m,o}+T_{m,i}}{2}$
		Constant Surface Temperature: T_s (i.e. $q_s''(x)$ not constant) $Nu_D = 3.66$ With properties estimated at $T_f = T_m = \frac{T_{m,o}+T_{m,i}}{2}$

Overall Heat Transfer Coefficient – Example 2



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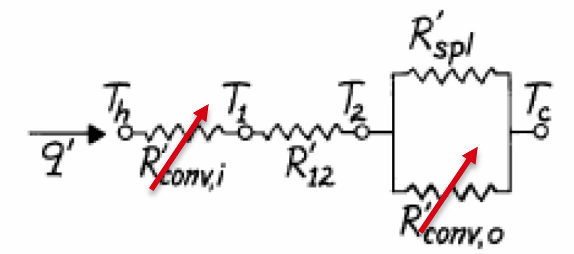

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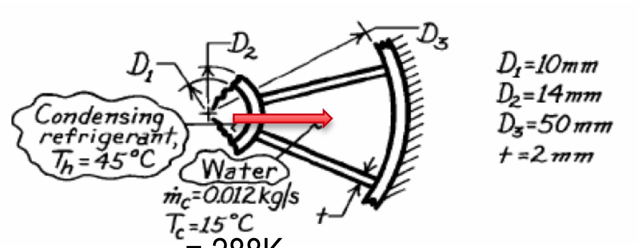
Determine the convection coefficient h

Convection
- Forced Convection
  - Internal Forced Convection
    - Laminar
      - Constant T (condensation)

Laminar	$Re_D < 2300$ Entry length ends at: $x_{fd} = 0.05\ D Re_D$ Friction Factor : $4C_f = f = 64 Re_D^{-1}$	Constant Surface Heat Flux: q_s'' $Nu_D = 4.36$ With properties estimated at $T_f = T_m = \frac{T_{m,o}+T_{m,i}}{2}$
		Constant Surface Temperature: T_s (i.e. $q_s''(x)$ not constant) $Nu_D = 3.66$ With properties estimated at $T_f = T_m = \frac{T_{m,o}+T_{m,i}}{2}$

Overall Heat Transfer Coefficient – Example 2



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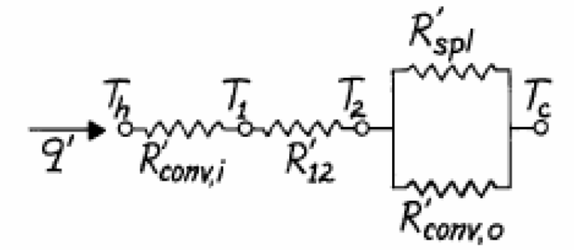

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$$D_{h,c} = \frac{4\left[\pi\left(50^2 - 14^2\right)/4 - 6(50-14)\right]\times 10^{-6}\,\mathrm{m}^2/6}{\left[(14\pi - 6\times 2)/6 + (50\pi - 6\times 2)/6 + (50-14)\right]\times 10^{-3}\,\mathrm{m}}$$

$$D_{h,c} = \frac{4\times 2.656\times 10^{-4}\,\mathrm{m}^2}{6.551\times 10^{-2}\,\mathrm{m}} = 0.01622\,\mathrm{m}.$$

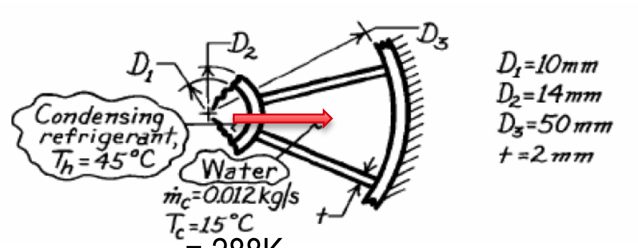
$$Re_{D,c} = \frac{\left[(0.012\ \mathrm{kg/s}/6)/\left(1000\ \mathrm{kg/m^3}\times 2.656\times 10^{-4}\,\mathrm{m}^2\right)\right]\times 0.01622\mathrm{m}}{1.138\times 10^{-6}\,\mathrm{m}^2/\mathrm{s}} = 107$$

Re_D < 2300 Laminar Flow + constant Twall

$$Nu_{D,c} = \frac{h_c D_{h,c}}{k} = 3.66$$

$$h_c = 3.66\times 0.595\ \mathrm{W/m\cdot K}/0.01622 = 134\ \mathrm{W/m^2\cdot K}.$$

Overall Heat Transfer Coefficient – Example 2



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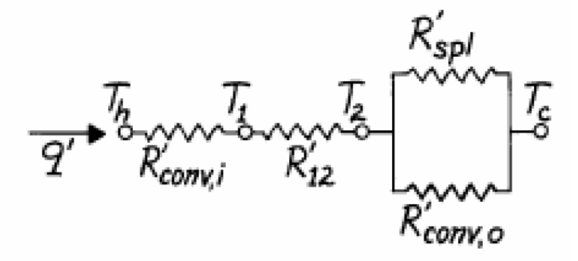
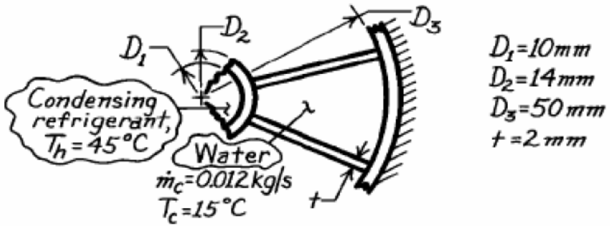
Determine the fin BC



$$q' = UA'(T_h - T_c)$$

$$\frac{1}{UA'} = R'_h + R'_w + R'_c = R'_w + \frac{1}{(\eta_o hA')_c}$$

Overall Heat Transfer Coefficient



Fins - Equations

Case	Tip Condition ($x = L$)	Temperature Distribution θ/θ_b	Fin Heat Transfer Rate Q_f
A	Convection heat transfer: $h\theta(L) = -k d\theta/dx\|_{x=L}$	$\dfrac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + (h/mk)\sinh mL}$ (3.70)	$M\dfrac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL}$ (3.72)
B	Adiabatic $d\theta/dx\|_{x=L} = 0$	$\dfrac{\cosh m(L-x)}{\cosh mL}$ (3.75)	$M \tanh mL$ (3.76)
C	Prescribed temperature: $\theta(L) = \theta_L$	$\dfrac{(\theta_L/\theta_b)\sinh mx + \sinh m(L-x)}{\sinh mL}$ (3.77)	$M\dfrac{(\cosh mL - \theta_L/\theta_b)}{\sinh mL}$ (3.78)
D	Infinite fin ($L \rightarrow \infty$): $\theta(L) = 0$	e^{-mx} (3.79)	M (3.80)

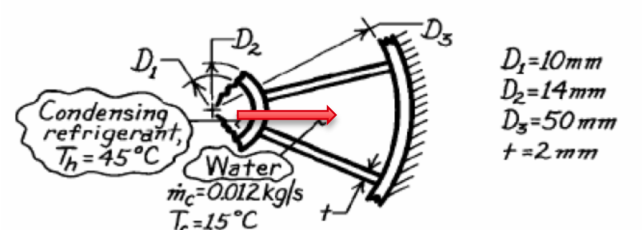
$\theta \equiv T - T_\infty$ $\qquad m^2 \equiv hP/kA_c$

$\theta_b = \theta(0) = T_b - T_\infty$ $\qquad M \equiv \sqrt{hPkA_c}\theta_b$

*to retrieve the notation of L&L remember that ξ=x/L so $m(L-x) = mL(1-\xi)$

Also in L&L $\theta = {T-T_\infty}/{T_b - T_\infty}$ and thus it is equal to $\frac{\theta}{\theta_b}$ in the notation of this table

Overall Heat Transfer Coefficient – Example 2



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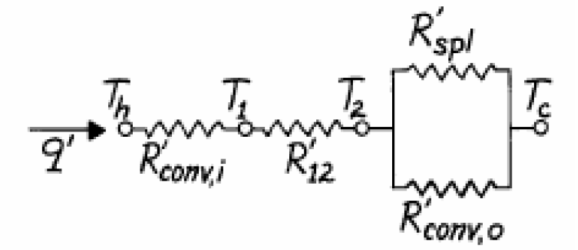

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$$q' = UA'(T_h - T_c)$$

$$\frac{1}{UA'} = R'_h + R'_w + R'_c = R'_w + \frac{1}{(\eta_o h A')_c}$$

Determine the fin BC

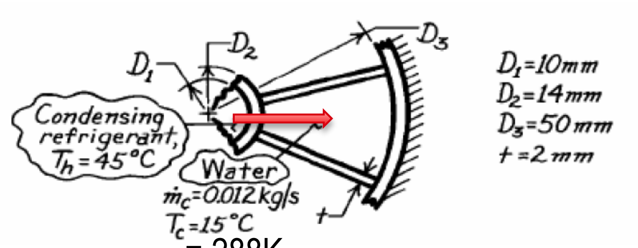
- Adiabatic Tip
  - Single fin efficiency

$$\eta_f = \frac{\tanh(mL)}{mL}$$

$$m = (2h_c/kt)^{1/2} = \left[\left(2\times 134\ \mathrm{W/m^2\cdot K}\right)/\left(200\ \mathrm{W/m\cdot K}\times 0.002\mathrm{m}\right)\right]^{1/2} = 25.88\,\mathrm{m}^{-1}$$

$$\eta_f = \frac{\tanh\left(25.88\,\mathrm{m}^{-1}\times 0.018\mathrm{m}\right)}{25.88\,\mathrm{m}^{-1}\times 0.018\mathrm{m}} = \frac{0.4348}{0.4658} = 0.934.$$

Overall Heat Transfer Coefficient – Example 2



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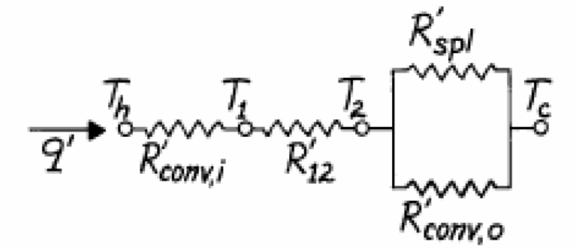

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$$q' = UA'(T_h - T_c)$$

$$\frac{1}{UA'} = R'_h + R'_w + R'_c = R'_w + \frac{1}{(\eta_o h A')_c}$$

Determine the fin BC

- Adiabatic Tip
  - Single fin efficiency
    - Array efficiency

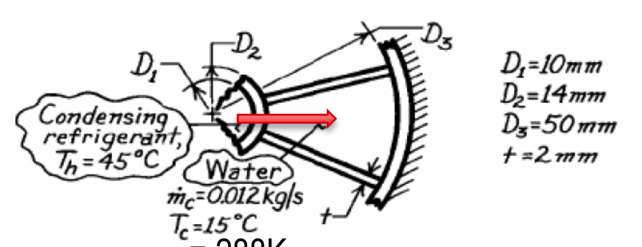
$$\eta_o = 1 - \frac{A_{f,c}}{A_c}(1 - \eta_f)$$

$$\eta_o = 1 - \frac{6(D_3 - D_2)}{6(D_3 - D_2) + (\pi D_2 - 6t)}[1 - \eta_f]$$

$$\eta_o = 1 - \frac{6(50-14)}{6(50-14) + (14\pi - 6\times 2)}(1 - 0.934) = 0.943$$

$$\frac{1}{\eta_o h A'_c} = \frac{1}{0.943 \times 134\ W/m^2 \cdot K[6(50-14) + (14\pi - 6\times 2)]\times 10^{-3} m} = 3.22\times 10^{-2} m \cdot K/W$$

Overall Heat Transfer Coefficient – Example 2



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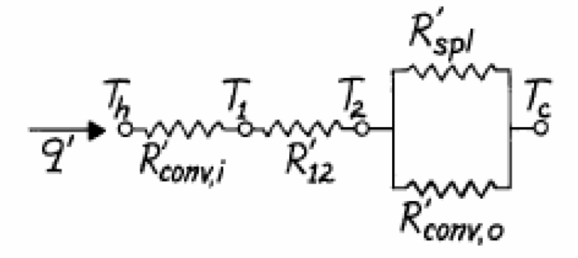

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$$q' = UA'(T_h - T_c)$$

$$\frac{1}{UA'} = R'_h + R'_w + R'_c = R'_w + \frac{1}{(\eta_o hA')_c}$$

$$q' = \frac{T_h - T_c}{R'_w + 1/(\eta_o hA')_c}$$

$$q' = \frac{(45-15)\,K}{2.678\times10^{-4}\,m\cdot K/W + 3.22\times10^{-2}\,m\cdot K/W} = 924\ W/m.$$

This lecture

- [x] Fouling
- [x] The overall heat transfer coefficient

Learning Objectives:

- [x] Calculate the overall heat transfer coefficient

Next Lecture

- ❑ Heat Exchanger
  - ❑ Parallel Flow Designs – Total Heat Transfer
  - ❑ Counter-flow Design – Total Heat Transfer

Learning Objectives:

- ❑ Calculate the total heat transfer for parallel & counter flow HE

Supplementary Slides