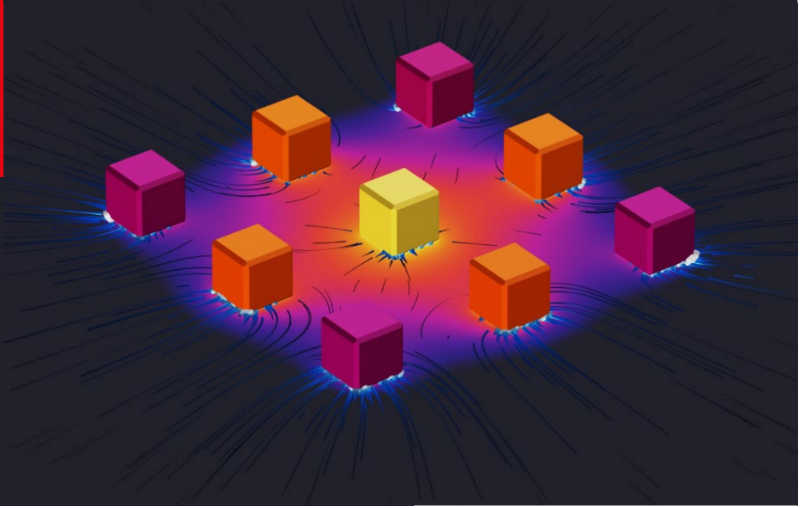
Heat and Mass Transfer
ME-341

Instructor: Giulia Tagliabue



École
polytechnique
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de Lausanne

Spring Semester

Convection Coefficient for Laminar Flow

Fully developed flow ($x>x_{fd,t}$)

Internal Forced Convection

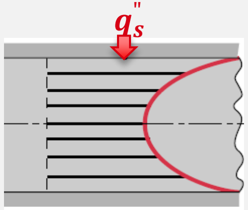
External Forced Convection

Laminar Flow condition & Film properties

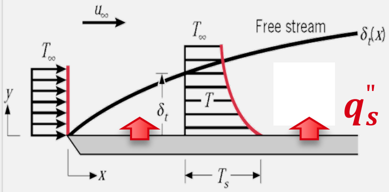
Internal: $Re_D<2300$ $\quad T_f = T_m = \frac{(T_{m,i}+T_{m,o})}{2}$

External: $Re_x<5\cdot 10^5$ $\quad T_f = \frac{(T_s+T_\infty)}{2}$

A. Constant heat flux

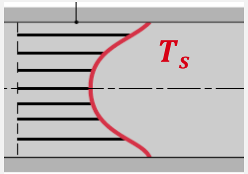


$$Nu_D = \frac{hD}{k_f} = 4.36$$

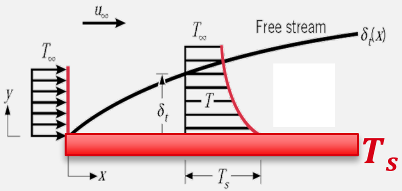


$$Nu_x = 0.435 Re_x^{1/2} Pr^{1/3}$$

B. Constant T_s



$$Nu_D = \frac{hD}{k_f} = 3.66$$



$$Nu_x = 0.332 Re_x^{1/2} Pr^{1/3}$$

$$\overline{Nu_x} = 0.664 Re_x^{1/2} Pr^{1/3}$$

This Lecture

- ❑ Correlations for internal forced convection
  - ❑ Circular tubes (laminar and turbulent)
  - ❑ Non-circular tubes
- ❑ The entrance region

Learning Objectives:

- ❑ Calculate the heat transfer coefficient for flow in pipes under different geometrical and flow conditions

Convection Coefficient for Turbulent Flow in Circular Tubes

$Re_D > 2300$

$Re_D > 10000$

$$\frac{hD}{k_f} = Nu_D = 0.023 Re_x^{4/5} Pr^n \quad \begin{cases} n = 0.4 \text{ for HEATING} \\ n = 0.3 \text{ for COOLING} \end{cases}$$

$3000 < Re_D < 5 \cdot 10^6$

$$Nu_D = \frac{(f/8)(Re_D - 1000)Pr}{1 + 12.7(f/8)^{1/2}(Pr^{2/3} - 1)}$$

$$f = (0.790 \ln Re_D - 1.64)^{-2}$$

Convection Coefficient for Non-circular Tubes

When dealing with non-circular tubes, we have to define a proper characteristic length.

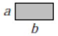
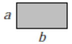
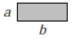
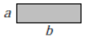
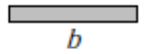
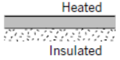
We observe that for a circular tube: $A_c = \dfrac{\pi D^2}{4} = \dfrac{(\pi D)D}{4} = \dfrac{PD}{4}$ where $\mathrm{P} = \pi D = perimeter$

For a non-circular tube we thus define:

$$D_h = \frac{4A_c}{P}$$

hydraulic diameter

TABLE 8.1 Nusselt numbers and friction factors for fully developed laminar flow in tubes of differing cross section

Cross Section	$\frac{b}{a}$	$Nu_D \equiv \frac{hD_h}{k}$ (Uniform q''_s)	$Nu_D \equiv \frac{hD_h}{k}$ (Uniform T_s)	$f\,Re_{D_h}$
	—	4.36	3.66	64
	1.0	3.61	2.98	57
	1.43	3.73	3.08	59
	2.0	4.12	3.39	62
	3.0	4.79	3.96	69
	4.0	5.33	4.44	73
	8.0	6.49	5.60	82
	∞	8.23	7.54	96
	∞	5.39	4.86	96
	—	3.11	2.49	53

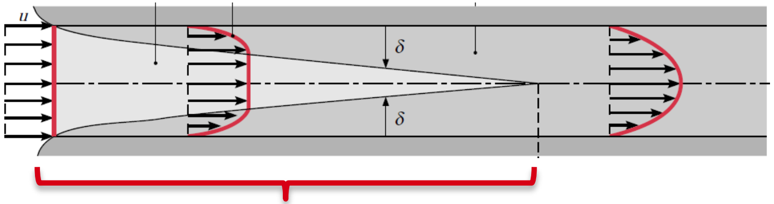
This Lecture

- ✓ Correlations for internal forced convection
  - ✓ Circular tubes (laminar and turbulent)
  - ✓ Non-circular tubes
- ❑ The entrance region

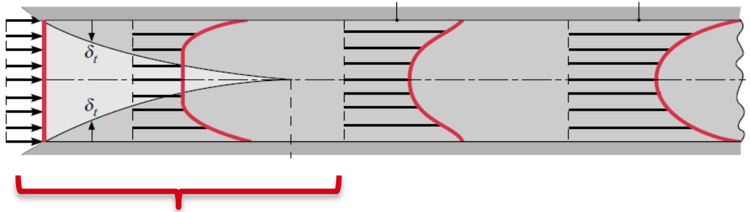
Learning Objectives:

- ✓ Calculate the heat transfer coefficient for flow in pipes under different geometrical and flow conditions

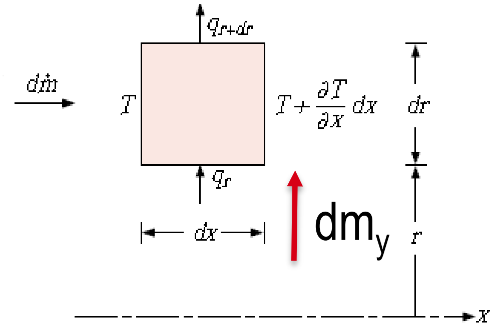
The Entrance Region



In the entrance region there is advection also in the radial direction



The temperature gradients along x cannot be simplified as done in the previous slides.



Two possible cases:

- Thermal entry length with fully developed velocity profile (Pr >> 1)
- Combined thermal and flow entry length

This Lecture

- ☑ Correlations for internal forced convection
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Learning Objectives:

- ☑ Calculate the heat transfer coefficient for flow in pipes under different geometrical and flow conditions

Example 1

Exercise 6.3

Ethylene glycol flows at $0.01kg/s$ through a $3mm$ diameter, thin-walled tube. The tube is submerged in a well-stirred water bath maintained at $25°C$. If the fluid enters the tube at $85°C$ what heat transfer rate and tube length are required for the fluid to leave at $35°C$?

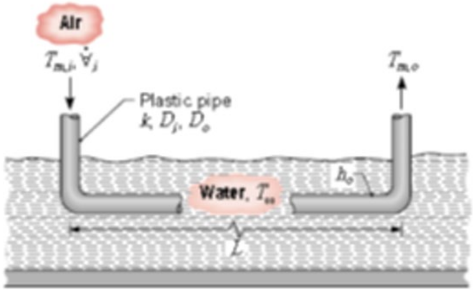
Use the following properties of ethylene glycol at $T_m = (85 + 35)/2 = 60°C = 333K$:

- $c_p = 2562J/kgK$
- $\mu = 0.522 \cdot 10^{-2} Ns/m^2$
- $k = 0.260W/mK$
- $Pr = 51.3$

Example 2

Exercise 6.5

To cool a summer home without using a vapor-compression refrigeration cycle, air is routed through a plastic pipe ($k = 0.15W/mK$, $D_i = 0.15m$, $D_o = 0.17m$) that is submerged in an adjoining body of water. The water temperature is nominally at $T_\infty = 17C$ and a convection coefficient of $h_o = 1500W/m^2K$ is maintained at the outer surface of the pipe. If air from the home enters the pipe at a temperature of $T_{m,i} = 29C$ and a volumetric flow rate of $V_i = 0.025m^3/s$, What pipe length L is needed to provide a discharge temperature of $T_{m,o} = 21°C$?



Forced Convection RECAP

Introduction to Convection

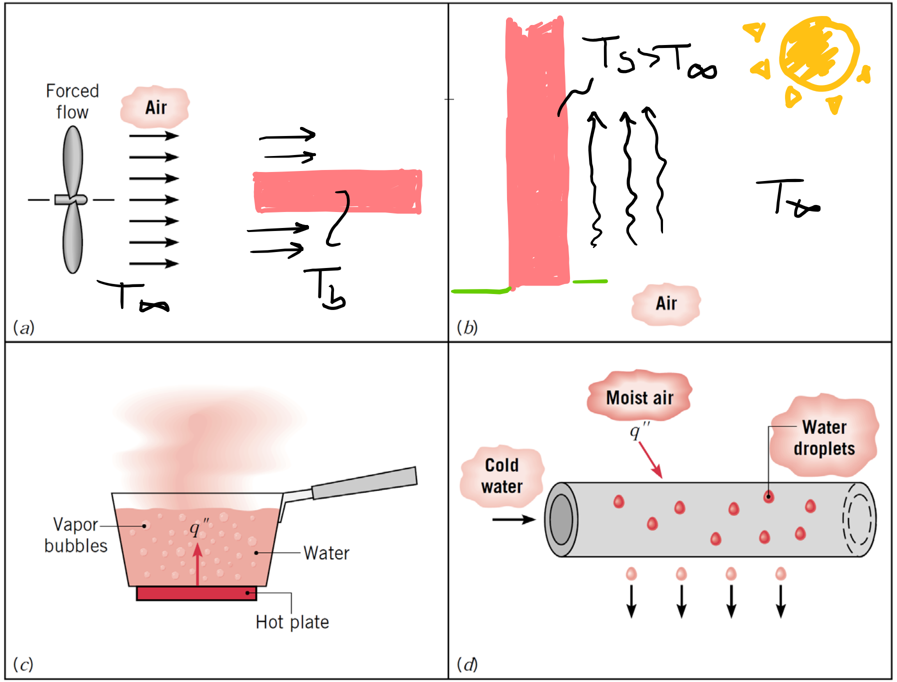
Convection refers to the heat transfer between a **solid** and a **fluid in motion** when they are at **different temperatures**.

1. Forced Convection

2. Natural (Free) Convection

3. Boiling

4. Condensation



Forced Convection

Convection refers to the heat transfer between a **solid** and a **fluid in motion** when they are at **different temperatures**.

Forced convection occurs when we can impose **the initial velocity** (or mass flow rate) and **temperature** of the fluid.

FLUID DYNAMICS

Mass conservation → Continuity equation
Momentum conservation → Navier-Stokes equations

Flow condition (Laminar/turbulent) → $\boldsymbol{Re}$

Velocity profile: $\vec{\boldsymbol{u}}(\boldsymbol{x}, \boldsymbol{y})$

- Shear stress τ_w
- Friction coefficient C_f
- Friction factor f

HEAT TRANSFER

Energy conservation → 1st Law of Thermodynamics

Boundary Conditions (Heat flux/Temperature)
$\boldsymbol{Pr}$

Heat transfer includes advection!

Temperature profile: $\boldsymbol{T}(\boldsymbol{x}, \boldsymbol{y})$

No slip condition $u(x, 0) = 0$

$$Q_{conv} = Q_{cond,wall}$$

Transport Laws (Newton/Fourier)

$$h(T_s - T_\infty) = -k_f \left.\frac{\partial \boldsymbol{T}}{\partial \boldsymbol{y}}\right|_{y=0}$$

$\boldsymbol{Nu}$

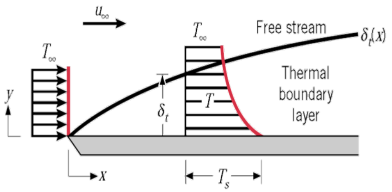
Forced Convection

Forced convection occurs when we can impose **the initial velocity** (or mass flow rate) and **temperature** of the fluid.

1. Forced Convection

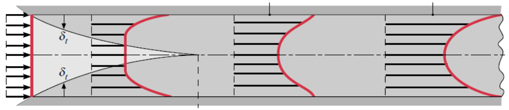
1. External Forced Convection:

- the velocity/thermal boundary layer can grow indefinitely
- there exist always and “unperturbed” flow at u_∞, T_∞



2. Internal Forced Convection

- the velocity/thermal boundary layer cannot grow indefinitely
- there exist an “entrance” region and a “fully developed” region



→ In the fully developed region the velocity profile does not change with position $\frac{\partial u}{\partial x} = 0, \exists u_m = const$

→ In the fully developed region the temperature profiles are similar $\frac{\partial \theta}{\partial x} = 0, \exists T_m(x)$

Dimensionless numbers & Forced Convection

$$Re = \frac{inertia\ forces}{viscous\ forces}$$

$$Pr = \frac{momentum\ diffusivity}{heat\ diffusivity}$$

$$Nu = \frac{convection\ heat\ transfer}{conduction\ heat\ transfer}$$

External Forced Convection

$$Re_x = \frac{\rho u_\infty x}{\mu}$$

$x = L\ for\ plate$
$x = D\ for\ cylinder$
Etc.

$$Pr = \frac{\nu_f}{\alpha_f} = \frac{\rho_f c_{p,f}}{k_f}$$

$$Nu_x = \frac{hx}{k_f}$$

Internal Forced Convection

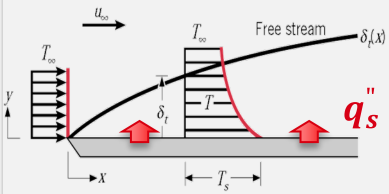
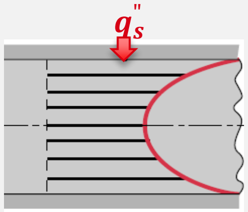
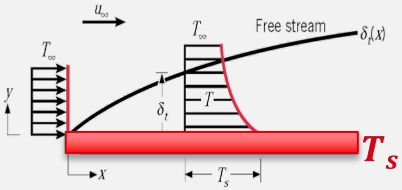
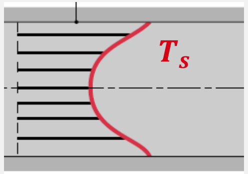
$$Re_D = \frac{\rho u_m D_h}{\mu} = \frac{4\dot{m}}{\mu \pi D_h}$$

$D_h = \frac{4A_c}{P} =$ hydraulic diameter

$$Pr = \frac{\nu_f}{\alpha_f} = \frac{\rho_f c_{p,f}}{k_f}$$

$$Nu_D = \frac{hD_h}{k_f}$$

Forced Convection: Coefficient for Laminar Flow Fully developed conditions ($x>x_{fd,t}$)

	External Forced Convection		Internal Forced Convection	
Laminar Flow condition & Film properties	$Re_x < 5 \cdot 10^5$	$T_f = \frac{(T_s + T_\infty)}{2}$	$Re_D < 2300$	$T_f = T_m = \frac{(T_{m,i} + T_{m,o})}{2}$
A. *Constant heat flux*		$Nu_x = 0.435 Re_x^{1/2} Pr^{1/3}$		$Nu_D = 4.36$
B. *Constant T_s*		$Nu_x = 0.332 Re_x^{1/2} Pr^{1/3}$ $\overline{Nu_x} = 0.664 Re_x^{1/2} Pr^{1/3}$		$Nu_D = 3.66$

Forced Convection

Forced convection occurs when we can impose the initial velocity (or mass flow rate) and temperature of the fluid.

1. Forced Convection

1. External Forced Convection:

- Horizontal plate → **Flow Condition (Re)**: Laminar vs Turbulent; **Thermal BC**: Heat Flux vs Temperature
- Cross-flow around a cylinder
- Bank of tubes

2. Internal Forced Convection

- Circular pipe
- Non-circular pipe

→ **Flow Condition (Re)**: Laminar vs Turbulent; **Thermal BC**: Heat Flux vs Temperature

General methodology for calculating the forced convection coefficient

0. Identify the type of convection (**Forced/External, Forced/Internal**, Natural, Boiling/Condensation)

1. Recognize the flow geometry (plate, cylinder, inner/outer flow etc.) **[GEOM]**

2. Specify the appropriate reference temperature and evaluate the pertinent fluid properties at that temperature $\boldsymbol{T_f}$
   - External convection over a plate/cylinder: $\boldsymbol{T_f = (T_s + T_\infty)/2}$
   - External convection over a bank of tubes: $\boldsymbol{T_f = (T_{in} + T_{out})/2}$
   - Internal convection: $\boldsymbol{T_f = (T_{m,i} + T_{m,o})/2}$

Note: If the necessary temperatures are unknown, we can use T_∞ or T_{in} to estimate the fluid properties. Once we obtain T_s, T_{out}, $T_{m,o}$ we need to check whether it was reasonable.

3. Calculate the Reynolds number (be careful to use the right characteristic dimension - x, L, D – and velocity - u_m, u_∞) and determine the flow conditions (laminar/turbulent) **[FLOW]**

4. Decide whether a local or surface average coefficient is required **[Loc/Ave]**

5. Calculate Pr or get it from the table **[Pr]**

6. Select the appropriate correlation, determine Nu and the convection coefficient **[Nu, h]**

Q&A Session

Jupyter Notebooks

Jupyter Notebooks are interactive teaching supports that combine text, images, animations and pieces of codes (Python). We have developed a Jupyter Notebook on Convection to help you revise the theory and practice the use of the correlations thanks to a number of guided exercises.

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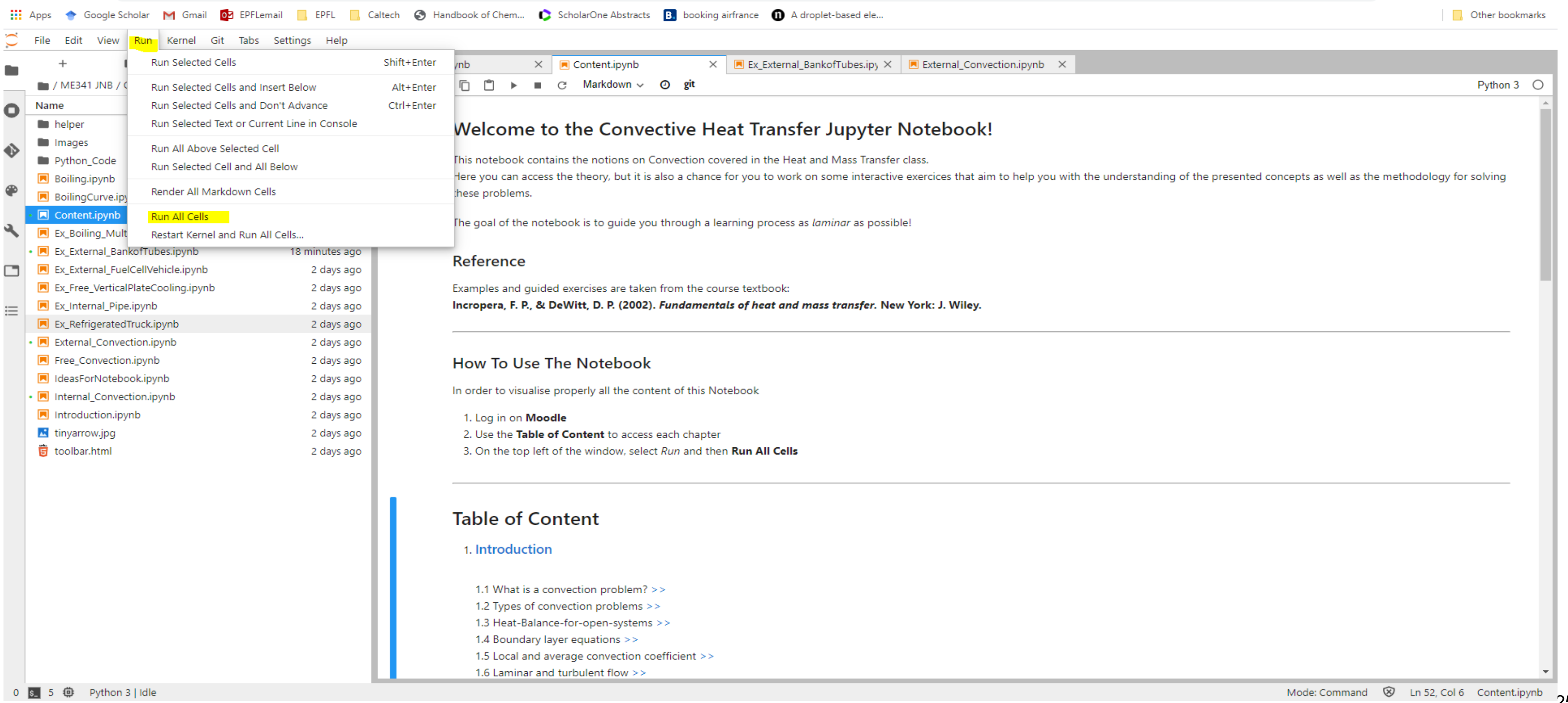
1. Log in on Moodle
2. Open a new tab and paste this link: https://go.epfl.ch/ConvectionNotebook

The first time you access the notebook, it will require some minutes to start the JupyterLab environment as the system will have to copy the files to your directory. Once the system starts, you should see a list of files on the left and a web-like page on the right (possibly with some code visible).

Jupyter Notebooks

3. On the tope-left side of the JupyterLab environment, click on “**Run**” and then on “**Run All Cells**” (repeat every time you open a new page)



Enhancement of Convection Coefficient

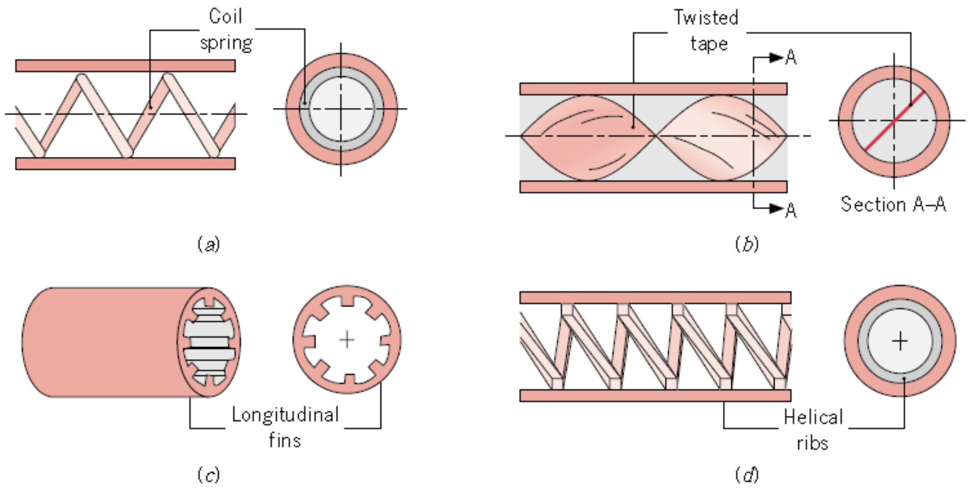
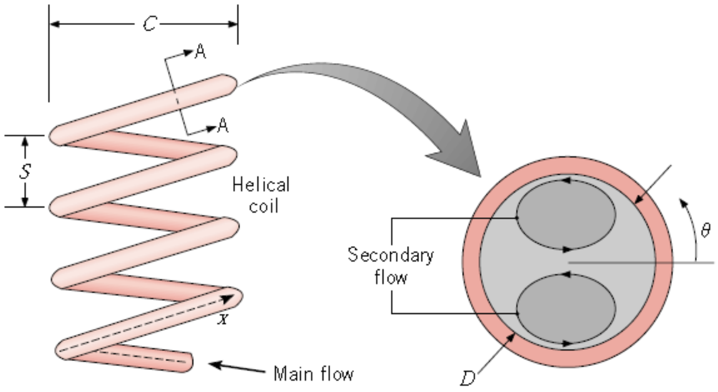


FIGURE 8.12 Internal flow heat transfer enhancement schemes: (a) longitudinal section and end view of coil-spring wire insert, (b) longitudinal section and cross-sectional view of twisted tape insert, (c) cut-away section and end view of longitudinal fins, and (d) longitudinal section and end view of helical ribs.



Coiled pipe (Nu will depend also on θ !)

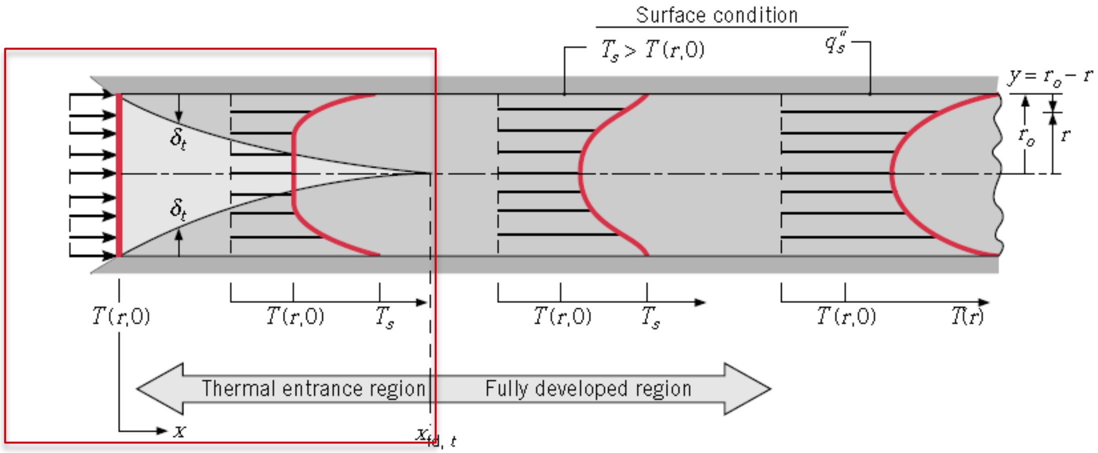
Table of Correlation for internal flow

TABLE 8.4 Summary of convection correlations for flow in a circular tube[a,b,e]

Correlation		Conditions
$f = 64/Re_D$	(8.19)	Laminar, fully developed
$Nu_D = 4.36$	(8.53)	Laminar, fully developed, uniform q''_s
$Nu_D = 3.66$	(8.55)	Laminar, fully developed, uniform T_s
$\overline{Nu}_D = 3.66 + \dfrac{0.0668(D/L)Re_D Pr}{1 + 0.04[(D/L)Re_D Pr]^{2/3}}$	(8.56)	Laminar, thermal entry (or combined entry with $Pr \gtrsim 5$), uniform T_s
or		
$\overline{Nu}_D = 1.86\left(\dfrac{Re_D Pr}{L/D}\right)^{1/3}\left(\dfrac{\mu}{\mu_s}\right)^{0.14}$	(8.57)	Laminar, combined entry, $0.6 \lesssim Pr \lesssim 5$, $0.0044 \lesssim (\mu/\mu_s) \lesssim 9.75$, uniform T_s
$f = 0.316\, Re_D^{-1/4}$	(8.20a)[c]	Turbulent, fully developed, $Re_D \lesssim 2 \times 10^4$
$f = 0.184\, Re_D^{-1/5}$	(8.20b)[c]	Turbulent, fully developed, $Re_D \gtrsim 2 \times 10^4$
or		
$f = (0.790 \ln Re_D - 1.64)^{-2}$	(8.21)[c]	Turbulent, fully developed, $3000 \lesssim Re_D \lesssim 5 \times 10^6$
$Nu_D = 0.023 Re_D^{4/5} Pr^n$	(8.60)[d]	Turbulent, fully developed, $0.6 \lesssim Pr \lesssim 160$, $Re_D \gtrsim 10{,}000$, $(L/D) \gtrsim 10$, $n = 0.4$ for $T_s > T_m$ and $n = 0.3$ for $T_s < T_m$
or		
$Nu_D = 0.027 Re_D^{4/5} Pr^{1/3}\left(\dfrac{\mu}{\mu_s}\right)^{0.14}$	(8.61)[d]	Turbulent, fully developed, $0.7 \lesssim Pr \lesssim 16{,}700$, $Re_D \gtrsim 10{,}000$, $L/D \gtrsim 10$
or		
$Nu_D = \dfrac{(f/8)(Re_D - 1000)Pr}{1 + 12.7(f/8)^{1/2}(Pr^{2/3} - 1)}$	(8.62)[d]	Turbulent, fully developed, $0.5 \lesssim Pr \lesssim 2000$, $3000 \lesssim Re_D \lesssim 5 \times 10^6$, $(L/D) \gtrsim 10$
$Nu_D = 4.82 + 0.0185(Re_D Pr)^{0.827}$	(8.64)	Liquid metals, turbulent, fully developed, uniform q''_s, $3.6 \times 10^3 \lesssim Re_D \lesssim 9.05 \times 10^5$, $10^2 \lesssim Pe_D \lesssim 10^4$
$Nu_D = 5.0 + 0.025(Re_D Pr)^{0.8}$	(8.65)	Liquid metals, turbulent, fully developed, uniform T_s, $Pe_D \gtrsim 100$

[a]The mass transfer correlations may be obtained by replacing Nu_D and Pr by Sh_D and Sc, respectively.
[b]Properties in Equations 8.53, 8.55, 8.60, 8.61, 8.62, 8.64, and 8.65 are based on T_m; properties in Equations 8.19, 8.20, and 8.21 are based on $T_f \equiv (T_s + T_m)/2$; properties in Equations 8.56 and 8.57 are based on $\overline{T}_m \equiv (T_{m,i} + T_{m,o})/2$.
[c]Equations 8.20 and 8.21 pertain to smooth tubes. For rough tubes, Equation 8.62 should be used with the results of Figure 8.3.
[d]As a first approximation, Equations 8.60, 8.61, or 8.62 may be used to evaluate the average Nusselt number $\overline{Nu}_D$ over the entire tube length, if $(L/D) \gtrsim 10$. The properties should then be evaluated at the average of the mean temperature, $\overline{T}_m \equiv (T_{m,i} + T_{m,o})/2$.
[e]For tubes of noncircular cross section, $Re_D \equiv D_h u_m/\nu$, $D_h \equiv 4A_c/P$, and $u_m = \dot{m}/\rho A_c$. Results for fully developed laminar flow are provided in Table 8.1. For turbulent flow, Equation 8.60 may be used as a first approximation.

The Entry Region



Two possible solutions:

- Thermal entry length with fully developed velocity profile (Pr >> 1 or unheated length)
- Combined thermal and flow entry length

The Entry Region

Constant Surface Temperature

Thermal entry problem

$$\overline{Nu}_D = 3.66 + \frac{0.0668(D/L)Re_D Pr}{1 + 0.04[(D/L)Re_D Pr]^{2/3}}$$

$$\begin{bmatrix} \text{thermal entrance length} \\ or \\ \text{combined entrance length with } Pr \gtrsim 5 \end{bmatrix}$$

$\overline{Nu}_D \equiv \bar{h}D/k.$

Equation 8.56

Combined entry problem

$$\overline{Nu}_D = 1.86\left(\frac{Re_D Pr}{L/D}\right)^{1/3}\left(\frac{\mu}{\mu_s}\right)^{0.14}$$

$$0.60 \lesssim Pr \lesssim 5$$

$$0.0044 \lesssim \left(\frac{\mu}{\mu_s}\right) \lesssim 9.75$$

provided $\overline{Nu}_D \geq 3.66$

Equation 8.57

These properties must be evaluated at $\bar{T}_m = (\bar{T}_{m,i} + \bar{T}_{m,o})/2$

The Entry Region