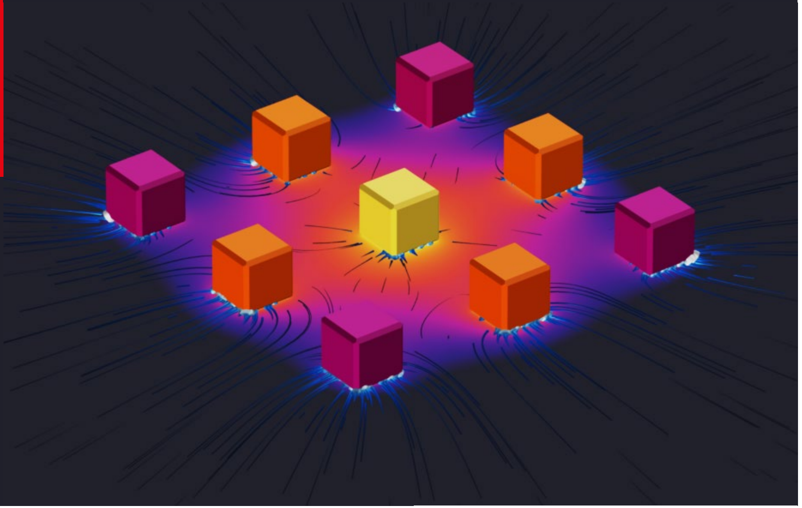
Heat and Mass Transfer
ME-341

Instructor: Giulia Tagliabue



Spring Semester

École
polytechnique
fédérale
de Lausanne

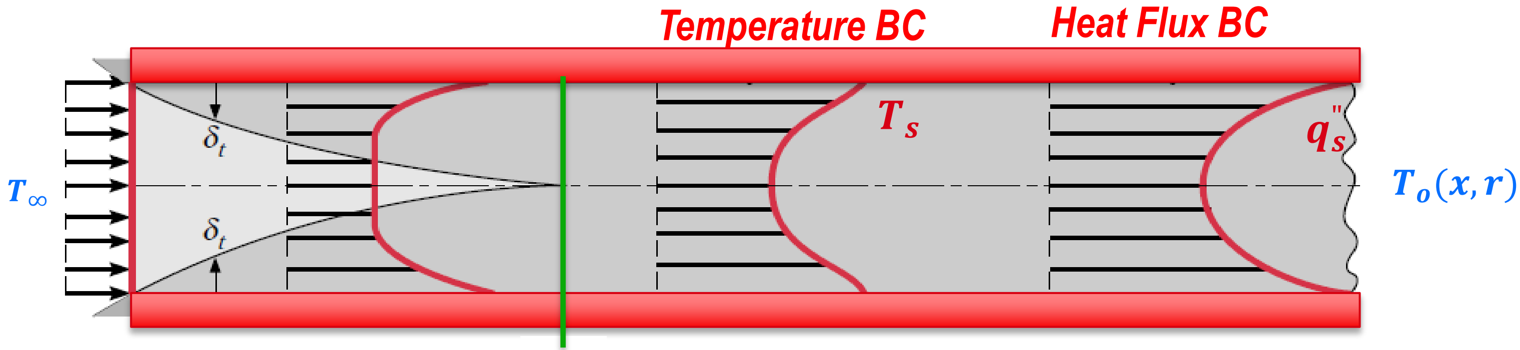
Previously

- ❑ Internal Flows
  - ✓ Fluid-dynamic aspects (velocity profile and pressure)
  - ✓ Thermal aspects and fully developed region

Learning Objectives:

- ✓ Understand the critical aspects of flows in pipes
- ✓ Understand critical aspects of heat transfer in pipes

Internal Flows: Thermal Aspects

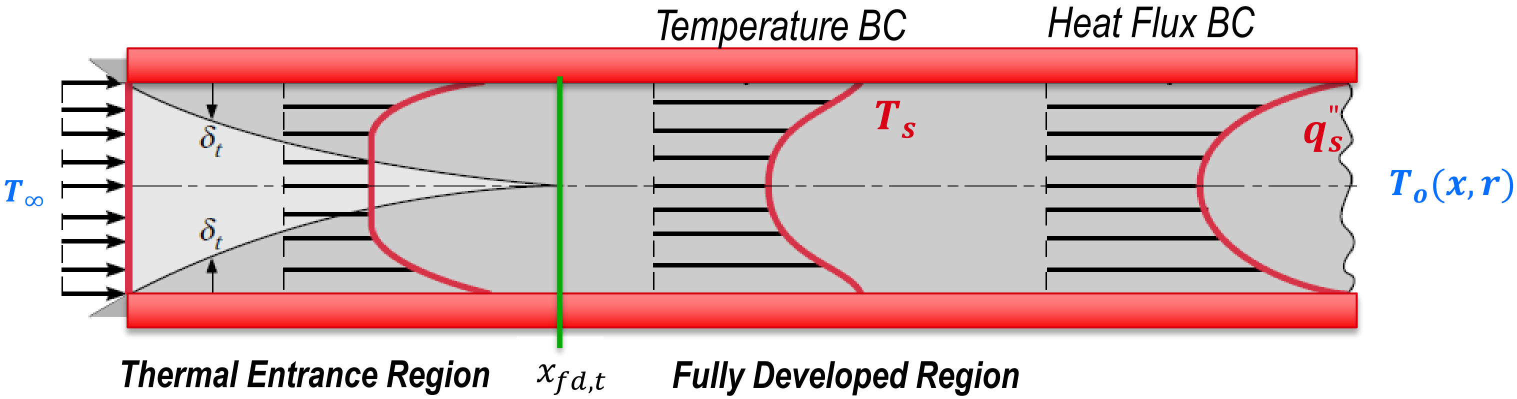


We thus define a dimensionless temperature: $$\theta = \frac{T_s(x) - T(r,x)}{T_s(x) - T_m(x)}$$ $T_m(x)$ mean temperature $T_m = \frac{2}{u_m r_o^2}\int_0^{r_o} uTr dr$

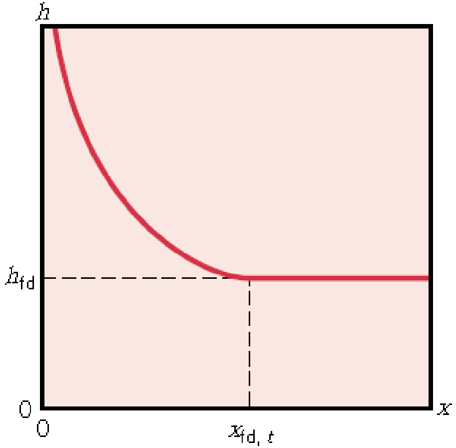
And the fully developed condition can be defined as: $$\left.\frac{\partial \theta}{\partial x}\right|_{x=x_{fd,t}} = \left.\frac{\partial}{\partial x}\left[\frac{T_s(x) - T(r,x)}{T_s(x) - T_m(x)}\right]\right|_{x=x_{fd,t}} = 0$$

the temperature profiles are SIMILAR.

Internal Flows: Thermal Aspects

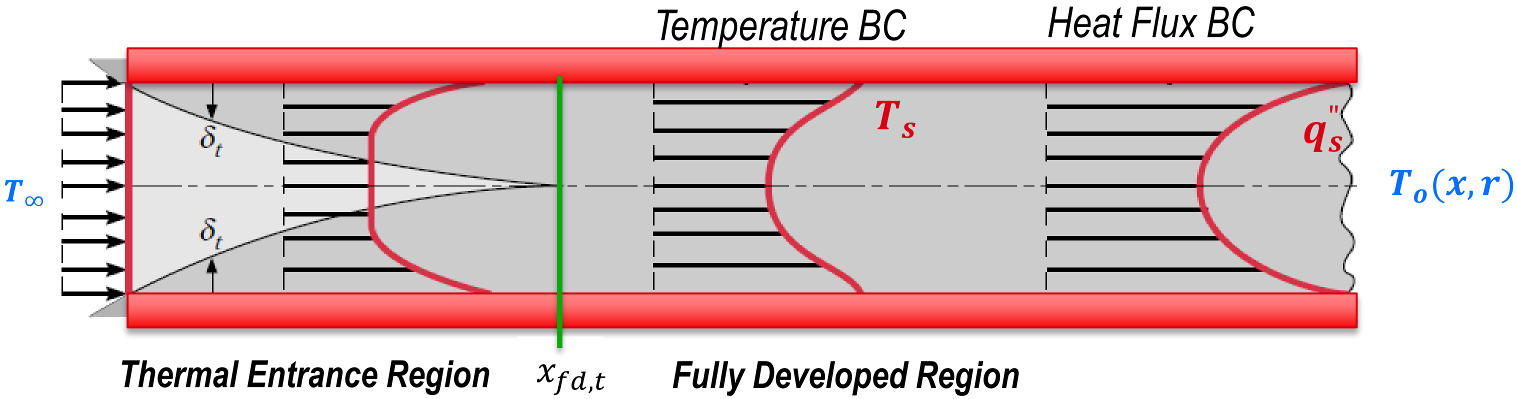


In the entrance region h DECREASES



In the fully developed region h is CONSTANT

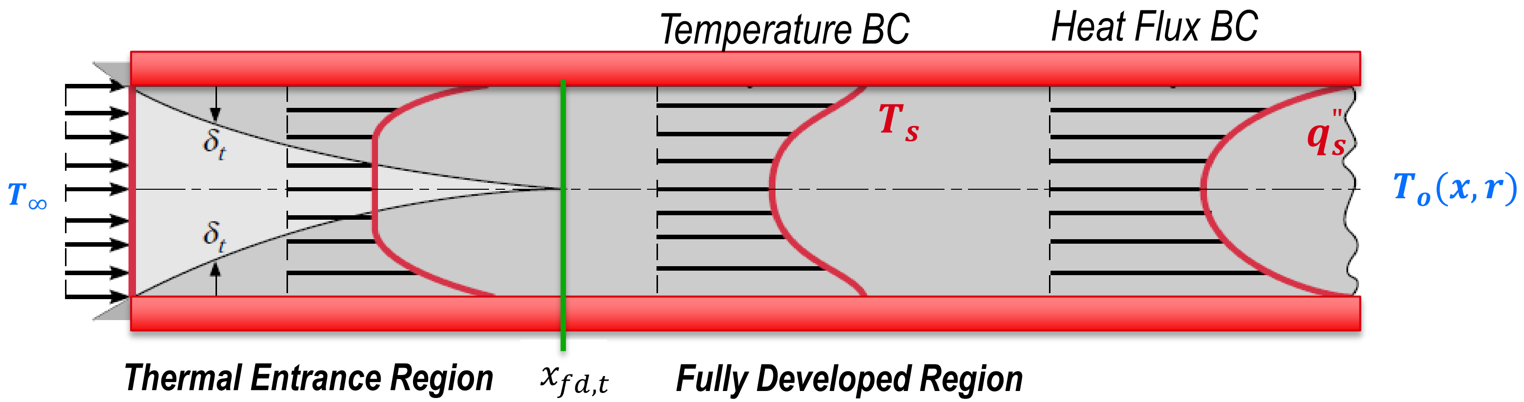
Internal Flows: Thermal Aspects



$$\left.\frac{\partial \theta}{\partial x}\right|_{x=x_{fd,t}} = \frac{(T_s - T_m)\,{}^{\partial}\!/_{\partial x}(T_s - T) - (T_s - T)\,{}^{\partial}\!/_{\partial x}(T_s - T_m)}{(T_s - T_m)^2} \equiv 0$$

$$\frac{\frac{\partial T_s}{\partial x} - \frac{\partial T}{\partial x}}{(T_s - T_m)} - \frac{(T_s - T)}{(T_s - T_m)^2}\left(\frac{\partial T_s}{\partial x} - \frac{\partial T_m}{\partial x}\right) = 0$$

Internal Flows: Thermal Aspects



$$\left.\frac{\partial\theta}{\partial x}\right|_{x=x_{fd,t}} \equiv 0 \quad \Rightarrow \quad \left.\frac{\partial T}{\partial x}\right|_{x=x_{fd,t}} = \left.\frac{dT_s}{dx}\right|_{x=x_{fd,t}} - \left.\frac{T_s - T}{T_s - T_m}\frac{dT_s}{dx}\right|_{x=x_{fd,t}} + \left.\frac{T_s - T}{T_s - T_m}\frac{dT_m}{dx}\right|_{x=x_{fd,t}}$$

The fully developed condition establishes a precise relationship between $\frac{\partial T}{\partial x}, \frac{dT_s}{dx}, \frac{dT_m}{dx}$

We will use this relationship to simplify the equations and determine the temperature profile $T(r,x)$

Forced Convection

Convection refers to the heat transfer between a **solid** and a **fluid in motion** when they are at **different temperatures**.

FLUID DYNAMICS

Mass conservation → Continuity equation
Momentum conservation → Navier-Stokes equations

Flow condition (Laminar/turbulent) → $\boldsymbol{Re}$

Velocity profile: $\vec{\boldsymbol{u}}(\boldsymbol{x},\boldsymbol{y})$

- Shear stress τ_w
- Friction coefficient C_f
- Friction factor f

HEAT TRANSFER

Energy conservation → 1st Law of Thermodynamics

Boundary Conditions (Heat flux/Temperature)
$\boldsymbol{Pr}$

Heat transfer includes advection!

Temperature profile: $\boldsymbol{T}(\boldsymbol{x},\boldsymbol{y})$

No slip condition $u(x,0) = 0$

$$Q_{conv} = Q_{cond,wall}$$

Transport Laws (Newton/Fourier)

$$h(T_s - T_\infty) = -k_f \left.\frac{\partial \boldsymbol{T}}{\partial \boldsymbol{y}}\right|_{y=0}$$

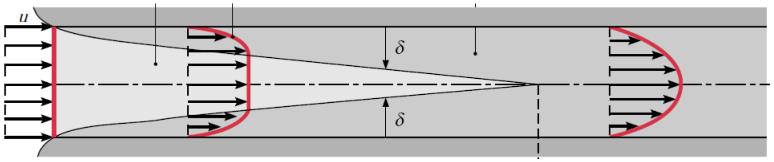
$\boldsymbol{Nu}$

Internal Forced Convection

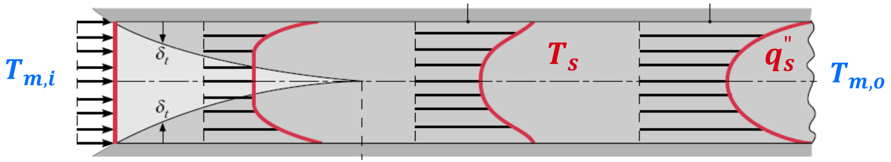
FLUID DYNAMICS
Find the *velocity profile*: $\vec{u}(r,x)$

HEAT TRANSFER
Find the temperature profile: $\mathrm{T}(r,x)$

Velocity Profile



Temperature Profile



In the fully developed region $\partial u/\partial x = 0$

$$\frac{u(r)}{u_m} = 2\left[1 - \left(\frac{r}{r_o}\right)^2\right]$$

$$u_m = -\frac{r_o^2}{8\mu}\frac{dp}{dx}$$

$\vec{u}(r)$

$$T_m = \frac{2}{u_m r_o^2}\int_0^{r_o} uTr\,dr$$

$$\theta = \frac{T_s(x) - T(r,x)}{T_s(x) - T_m(x)}$$

In the fully developed region $\partial\theta/\partial x = 0$

$$\left.\frac{\partial T}{\partial x}\right|_{x=x_{fd,t}} = \left.\frac{dT_s}{dx}\right|_{x=x_{fd,t}} - \left.\frac{T_s - T}{T_s - T_m}\frac{dT_s}{dx}\right|_{x=x_{fd,t}} + \left.\frac{T_s - T}{T_s - T_m}\frac{dT_m}{dx}\right|_{x=x_{fd,t}}$$

$$Q_{conv} = Q_{cond,wall} \qquad h(T_s - T_m) = -k_f\left.\frac{\partial T}{\partial r}\right|_{r=0}$$

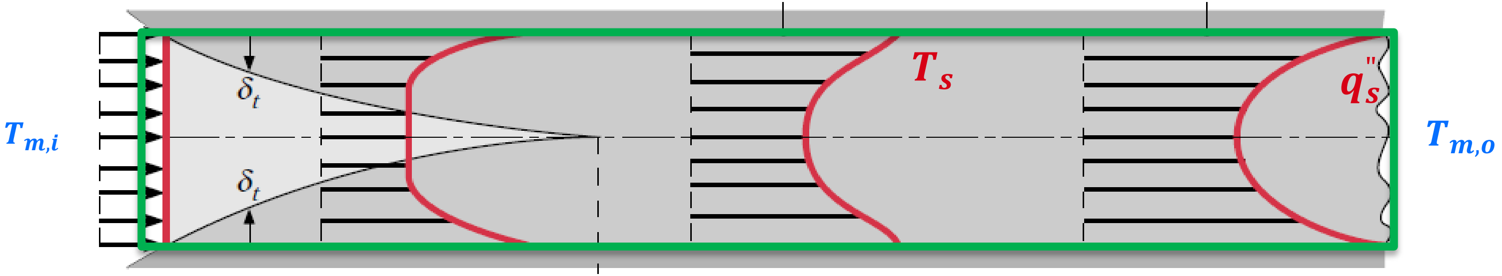
This Lecture

- ❑ Temperature and heat flow for internal flows
- ❑ Convection coefficient for laminar flow in circular tubes

Learning Objectives:

- ❑ Calculate the heat transfer coefficient for flow in pipes under different geometrical and flow conditions

Internal Flows: Thermal Aspects

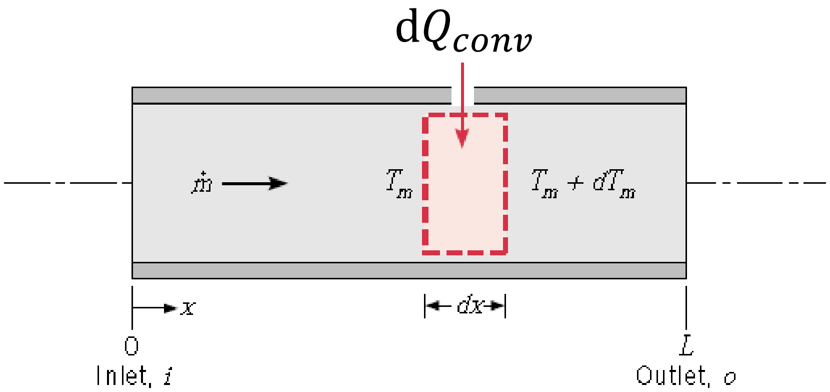


We can write a global energy balance on the entire pipe:

$$Q_{conv} = \dot{m}c_p(T_{m,o} - T_{m,i})$$

If we determine $T_m(x)$ we can calculate the total amount of heat transferred via convection.
We need to write a local energy balance.

Internal Flows: Energy Balance



Assumptions:

- Negligible viscous dissipation
- Incompressible flow
- Negligible heat transfer along x

We can write a local energy balance: $\dot{m}c_p[(T_m + dT_m) - T_m] = dQ_{conv}$

Where:

$$dQ_{conv} = q_s'' P dx$$

$$q_s'' = h(T_s - T_m)$$

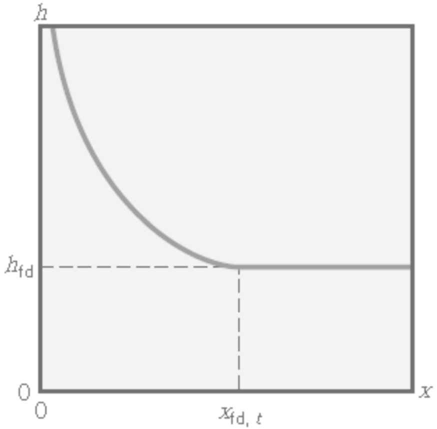
$$\frac{dT_m}{dx} = \frac{q_s'' P}{\dot{m}c_p} = \frac{P}{\dot{m}c_p} h(T_s - T_m)$$

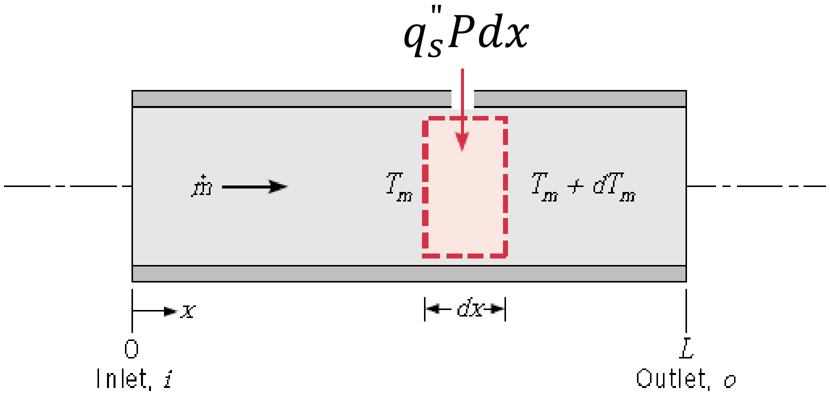
For a constant pipe radius

$$P = \pi D$$

To solve the differential equation we have to apply the BCs.

Internal Flows: Energy Balance – Constant Heat Flux BC q_s''

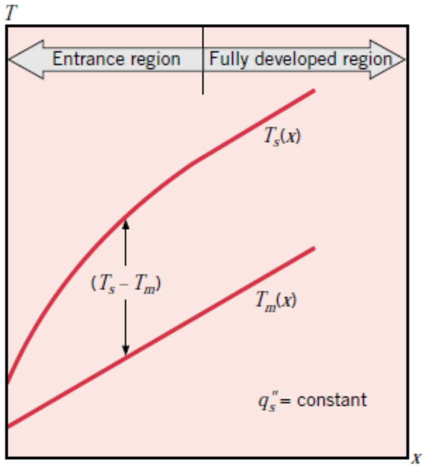




Assumptions:

- Negligible viscous dissipation
- Incompressible flow
- Negligible heat transfer along x

h convection coefficient INSIDE the pipe



For a constant heat flux q_s'' we have: $\frac{dT_m}{dx} = \frac{q_s''P}{\dot{m}c_p}$ ➡ $T_m(x) = \frac{q_s''P}{\dot{m}c_p}x + T_{m,i}$

So $T_m(x)$ increases linearly. What about $T_s(x)$?

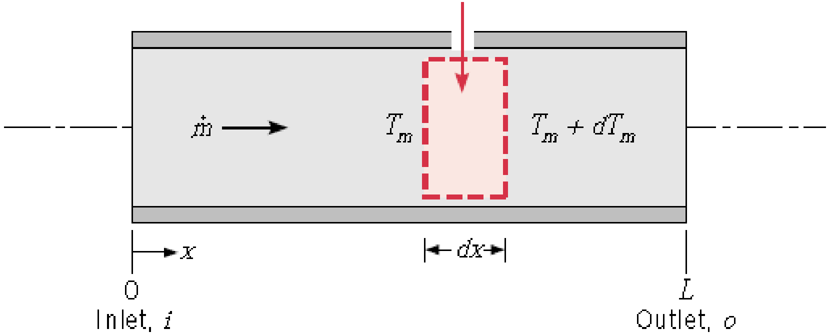
$$(T_s - T_m) = \frac{q_s''}{h}$$

➡ *Entrance region*:
$(T_s - T_m)$ increases because h decreases

➡ *Fully developed region*:
$(T_s - T_m)$ is constant because h is constant

Internal Flows: Energy Balance – Constant Temperature BC T_s

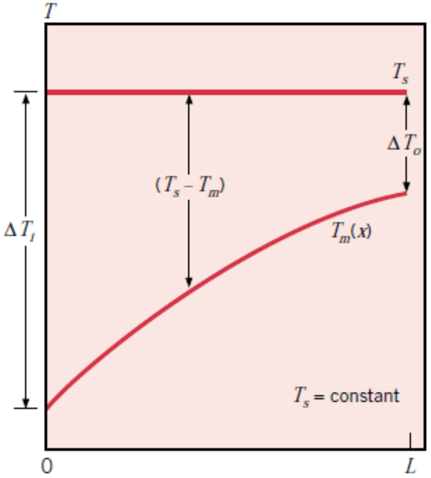
$$q_s'' P dx = h(T_s - T_m) P dx$$



Assumptions:

- Negligible viscous dissipation
- Incompressible flow
- Negligible heat transfer along x

h convection coefficient INSIDE the pipe



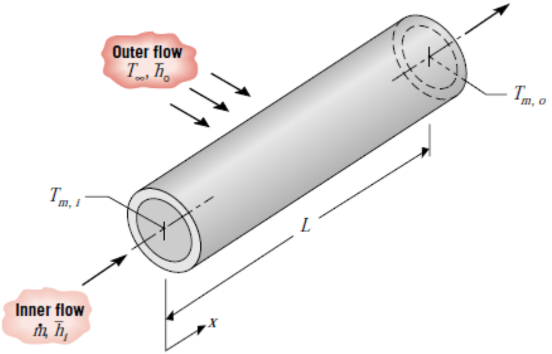
For a constant surface temperature T_s we have: $\dfrac{dT_m}{dx} = \dfrac{P}{\dot{m}c_p} h(T_s - T_m)$ $\quad \Delta T = (T_s - T_m)$

$$\Rightarrow \quad -\frac{d\Delta T}{dx} = \frac{P}{\dot{m}c_p} h \Delta T \quad \Rightarrow \quad \frac{d\Delta T}{\Delta T} = -\frac{P}{\dot{m}c_p} h dx \quad \Rightarrow \quad ln\frac{\Delta T_x}{\Delta T_i} = -\frac{Px}{\dot{m}c_p} \boxed{\frac{1}{x}\int_0^x h dx} \; \bar{h}$$

$$\Rightarrow \quad \frac{T_s - T_{m,o}}{T_s - T_{m,i}} = exp\left(-\frac{PL\bar{h}}{\dot{m}c_p}\right)$$

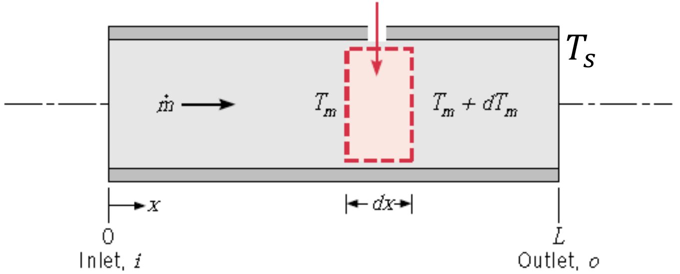
$\Rightarrow$ $(T_s - T_m)$ decreases exponentially

Internal Flows: Energy Balance – Constant Temperature BC T_s



$T_\infty, \overline{h_{out}}$

$$h(T_s - T_m)Pdx = h_{out}(T_\infty - T_s)Pdx$$



Often the surface temperature is maintained constant with forced external convection, hence using a fluid with known T_∞

h convection coefficient INSIDE the pipe

h_{out} convection coefficient OUTSIDE the pipe

Constant surface temperature T_s

$$\frac{T_s - T_{m,o}}{T_s - T_{m,i}} = exp\left(-\frac{\bar{h}A}{\dot{m}c_p}\right)$$

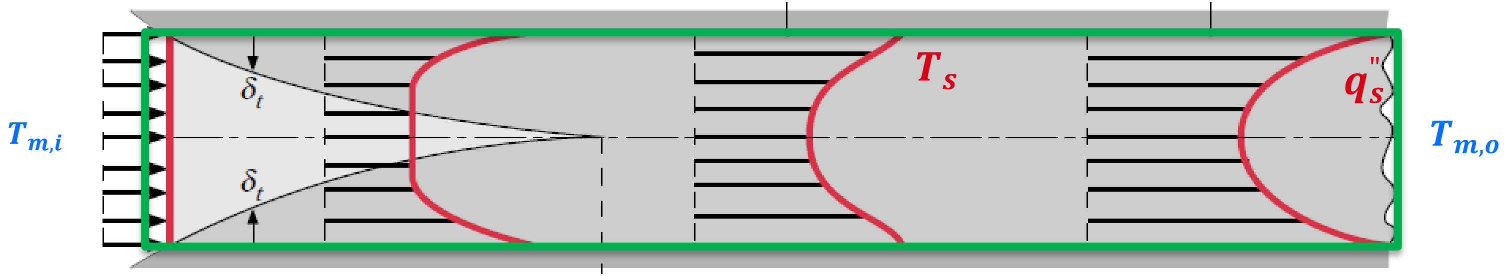
$$\frac{T_s - T_{m,o}}{T_s - T_{m,i}} = exp\left(-\frac{1}{\dot{m}c_p R_{conv,in}}\right)$$

Constant external fluid temperature T_∞

$$\frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = \exp\left(-\frac{1}{\dot{m}c_p R_{tot}}\right) = exp\left(-\frac{\bar{U}A}{\dot{m}c_p}\right)$$

$$R_{tot} = R_{conv,o} + R_{cond} + R_{conv,in} = \frac{1}{A_o h_{out}} + \frac{ln\, {}^{r_o}/_{r_i}}{2\pi Lk} + \frac{1}{A_{in} h}$$

Internal Flows: Total Heat Transfer



Constant Heat Flux:

$$Q_{conv} = q_s'' P dx$$

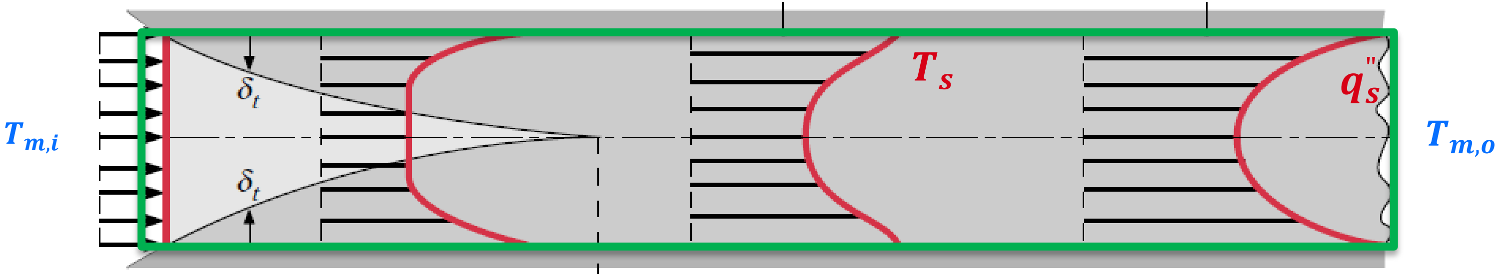
Constant Surface Temperature:

$$Q_{conv} = \dot{m}c_p\left(T_{m,o} - T_{m,i}\right) = -\dot{m}c_p(\Delta T_o - \Delta T_i) = -\dot{m}c_p ln\frac{\Delta T_o}{\Delta T_i}\frac{\Delta T_o - \Delta T_i}{ln\left((\Delta T_o)/(\Delta T_i)\right)}$$

Slide 13: $ln\frac{\Delta T_o}{\Delta T_i} = -\frac{PL\bar{h}}{\dot{m}c_p} = -\frac{A\bar{h}}{\dot{m}c_p}$ $\qquad \Delta T_{lm} \equiv \frac{\Delta T_o - \Delta T_i}{ln\left((\Delta T_o)/(\Delta T_i)\right)}$

$$Q_{conv} = \bar{h}A\Delta T_{lm} \qquad Q_{conv} = \bar{U}A\Delta T_{lm}$$

Internal Flows: Convection Coefficient



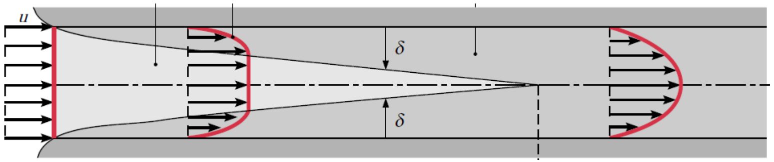
What about the convection coefficient ?

Internal Forced Convection

FLUID DYNAMICS
Find the *velocity profile*: $\vec{u}(r,x)$

HEAT TRANSFER
Find the temperature profile: $\mathrm{T}(r,x)$

Velocity Profile



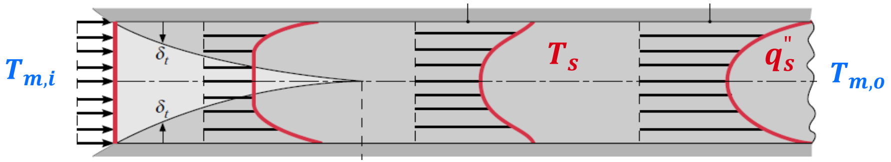
In the fully developed region $\partial u/\partial x = 0$

$$\frac{u(r)}{u_m} = 2\left[1 - \left(\frac{r}{r_o}\right)^2\right]$$

$$u_m = -\frac{r_o^2}{8\mu}\frac{dp}{dx}$$

$\vec{u}(r)$

Temperature Profile



$$\theta = \frac{T_s(x) - T(r,x)}{T_s(x) - T_m(x)}$$

In the fully developed region $\partial\theta/\partial x = 0$

- Constant surface heat flux $\quad T_m(x) = \frac{q_s'' P}{\dot{m}c_p}x + T_{m,i} \quad Q_{conv} = q_s'' PL$
- Constant surface temperature $\quad \frac{T_s - T_{m,o}}{T_s - T_{m,i}} = exp\left(-\frac{\bar{h}A}{\dot{m}c_p}\right) \quad Q_{conv} = \bar{h}A\Delta T_{lm}$

$$Q_{conv} = Q_{cond,wall} \qquad h(T_s - T_m) = -k_f \left.\frac{\partial T}{\partial r}\right|_{r=0}$$

We need the temperature profile!

This Lecture

- ✔ Temperature and heat flow for internal flows
- ❑ Convection coefficient for laminar flow in circular tubes

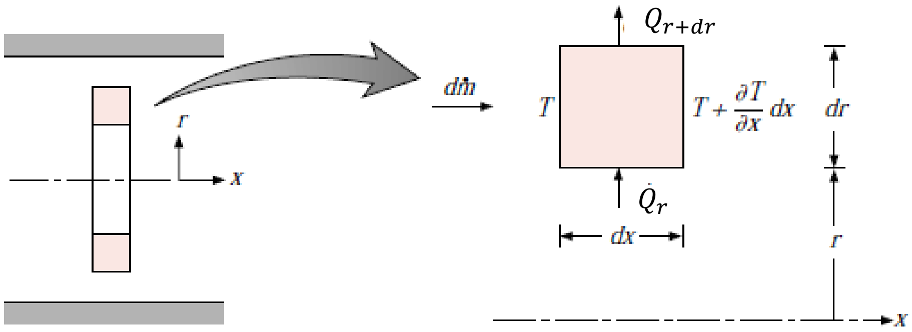
Learning Objectives:

- ❑ Calculate the heat transfer coefficient for flow in pipes under different geometrical and flow conditions

Convection Coefficient for Laminar Flow in Circular Tubes

We already know the velocity profile, $\vec{u}(r,x)$, now we have to write the energy balance to find $\mathrm{T}(r,x)$*

ANNULAR control volume



Assumptions:

- Negligible viscous dissipation
- Incompressible flow
- Negligible heat transfer along x
- **Fully developed flow ($x>x_{fd,t}$)**
- **Laminar flow ($Re_D<2300$)**

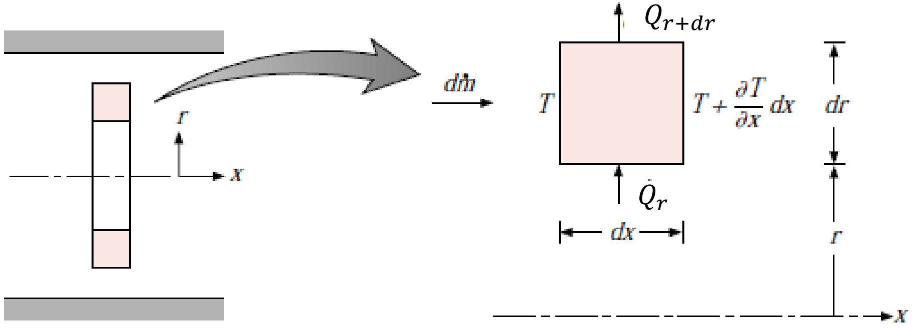
From the velocity profile calculations, we know that there is no radial component of the velocity: $\vec{u}(r,x)=u(r)$

➡ There is no advection along the radial direction ➡ **Heat is exchanged by DIFFUSION along r (Fourier law)**

Convection Coefficient for Laminar Flow in Circular Tubes

We already know the velocity profile, $\vec{u}(r,x)$, now we have to write the energy balance to find $\mathrm{T}(r,x)$*

ANNULAR control volume



Assumptions:

- Negligible viscous dissipation
- Incompressible flow
- Negligible heat transfer along x
- **Fully developed flow ($x>x_{fd,t}$)**
- **Laminar flow ($Re_D<2300$)**

1st Law of thermodynamics (Open system):

$$\frac{\partial U}{\partial t} = d\dot{m}\left(u + pv + \frac{1}{2}V^2 + gz\right)_{in} - d\dot{m}\left(u + pv + \frac{1}{2}V^2 + gz\right)_{out} + Q - \dot{W} + \dot{E}_{gen}$$

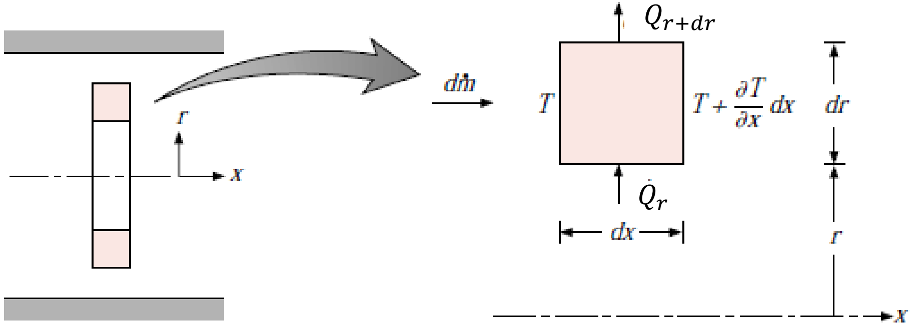
Heat exchanged by DIFFUSION along r

As in W5L1-1h, slide 34, we assume: $\Delta(u + pv) = \Delta h \approx c_p \Delta T$

Convection Coefficient for Laminar Flow in Circular Tubes

We already know the velocity profile, $\vec{u}(r,x)$, now we have to write the energy balance to find $\mathrm{T}(r,x)$*

ANNULAR control volume



Assumptions:

- Negligible viscous dissipation
- Incompressible flow
- Negligible heat transfer along x
- **Fully developed flow ($x>x_{fd,t}$)**
- **Laminar flow ($Re_D<2300$)**

1st Law of thermodynamics (Open system):

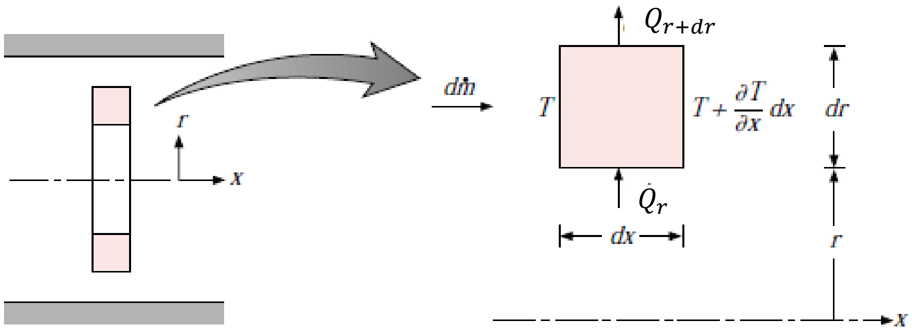
$$0 = d\dot{m}\, c_p \left(T - \left(T + \frac{\partial T}{\partial x} dx \right) \right) + Q_r - Q_{r+dr}$$

Where: $d\dot{m} = \rho(2\pi r dr)u(r)$ $\quad Q_r = -k_f(2\pi r dx)\dfrac{\partial T}{\partial r}$ *(Fourier Law)* $\quad Q_{r+dr} = Q_r + \dfrac{\partial Q_r}{\partial r} dr$

Convection Coefficient for Laminar Flow in Circular Tubes

We already know the velocity profile, $\vec{u}(r,x)$, now we have to write the energy balance to find $\mathrm{T}(r,x)$*

ANNULAR control volume



Assumptions:

- Negligible viscous dissipation
- Incompressible flow
- Negligible heat transfer along x
- **Fully developed flow ($x>x_{fd,t}$)**
- **Laminar flow ($Re_D<2300$)**

1st Law of thermodynamics (Open system):

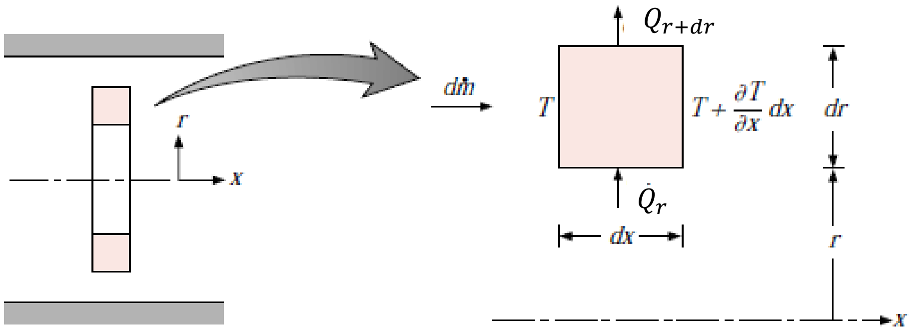
$$0 = \rho(2\pi r dr)u(r)c_p\left(\frac{\partial T}{\partial x}dx\right) + \frac{\partial Q_r}{\partial r}dr = \rho(2\pi r)u(r)c_p\frac{\partial T}{\partial x}dx + \frac{\partial}{\partial r}\left(-k_f(2\pi r dx)\frac{\partial T}{\partial r}\right)$$

$$\rho r u(r)c_p\frac{\partial T}{\partial x} = k_f\frac{\partial}{\partial r}\left(\mathrm{r}\frac{\partial T}{\partial r}\right)$$

Convection Coefficient for Laminar Flow in Circular Tubes

We already know the velocity profile, $\vec{u}(r,x)$, now we have to write the energy balance to find $\mathrm{T}(r,x)$*

ANNULAR control volume



Assumptions:

- Negligible viscous dissipation
- Incompressible flow
- Negligible heat transfer along x
- **Fully developed flow ($x>x_{fd,t}$)**
- **Laminar flow ($Re_D<2300$)**

1st Law of thermodynamics (Open system):

$$u\frac{\partial T}{\partial x} = \frac{k_f}{\rho c_p}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) \Rightarrow u\frac{\partial T}{\partial x} = \frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)$$

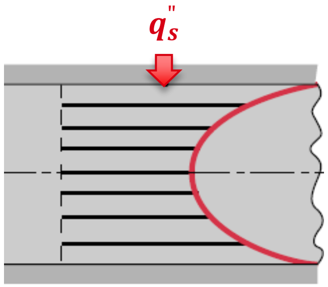
We now have to apply the BCs at $r = r_0$ and we have two possibilities:

A. Constant heat flux
B. Constant surface temperature

Convection Coefficient for Laminar Flow in Circular Tubes

A. *Constant heat flux*

Let's make a few observation that help us relate $T(r,x)$ with $T_m(x)$, which we calculated previously.



1. In every point the heat flux can be written as: $q_s'' = h(T_s(x) - T_m(x))$

In W6L1-1h, slide 23 we reasoned that $\frac{h}{k_f} \neq f(x) \rightarrow h \neq f(x)$

$$\Rightarrow \frac{\partial(T_s(x) - T_m(x))}{\partial x} = \frac{\partial(q_s''/h)}{\partial x} = 0 \quad \Rightarrow \frac{\partial T_s(x)}{\partial x} = \frac{\partial T_m(x)}{\partial x}$$

2. From the fully developed condition $\partial\theta/\partial x = 0$:

$$\left.\frac{\partial T}{\partial x}\right|_{x=x_{fd,t}} = \left.\frac{dT_s}{dx}\right|_{x=x_{fd,t}} - \left.\frac{T_s - T}{T_s - T_m}\frac{dT_s}{dx}\right|_{x=x_{fd,t}} + \left.\frac{T_s - T}{T_s - T_m}\frac{dT_m}{dx}\right|_{x=x_{fd,t}} \quad \Rightarrow \frac{\partial T_s(x)}{\partial x} = \frac{\partial T(r,x)}{\partial x}$$

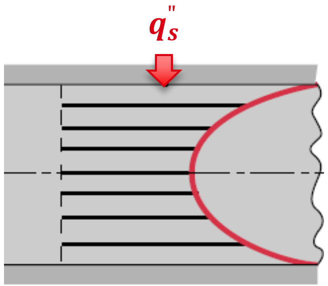
We have previously obtained: $T_m(x) = \frac{q_s'' P}{\dot{m}c_p}x + T_{m,i}$ $\Rightarrow$ $\frac{\partial T}{\partial x} = \frac{\partial T_s}{\partial x} = \frac{\partial T_m}{\partial x} = \frac{q_s'' P}{\dot{m}c_p} = constant$

$$\Rightarrow u\frac{\partial T}{\partial x} = \frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)$$

If we can treat $\frac{\partial T}{\partial x}$ as a constant factor, this becomes just a differential equation of a single variable function (r), which we can easily integrate

Convection Coefficient for Laminar Flow in Circular Tubes

A. Constant heat flux



$$u\frac{\partial T}{\partial x} = \frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)$$

$$\frac{u(r)}{u_m} = 2\left[1-\left(\frac{r}{r_o}\right)^2\right]$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = \frac{2u_m}{\alpha}\left(\frac{dT_m}{dx}\right)\left[1-\left(\frac{r}{r_0}\right)^2\right]$$

where $\frac{\partial T_m}{\partial x} = \frac{q_s'' P}{\dot{m} c_p}$

$$T(r,x) = \frac{2u_m}{\alpha}\left(\frac{dT_m}{dx}\right)\left[\frac{r^2}{4} - \frac{r^4}{16r_0^2}\right] + C_1 \ln(r) + C_2$$

Boundary Conditions:

- Finite temperature at $r = 0$ — $C_1 = 0$
- At the surface $T(r_0, x) = T_s(x)$ — $C_2 = T_s(x) - \frac{2u_m}{\alpha}\left(\frac{dT_m}{dx}\right)\left[\frac{3r_0^2}{16}\right]$

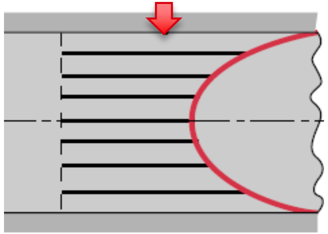
$$T(r,x) = T_s(x) - \frac{2u_m r_0^2}{\alpha}\left(\frac{dT_m}{dx}\right)\left[\frac{3}{16} + \frac{1}{16}\left(\frac{r}{r_0}\right)^4 - \frac{1}{4}\left(\frac{r}{r_0}\right)^2\right]$$

Now we use the transport laws (Newton) to obtain the convection coefficient h

Convection Coefficient for Laminar Flow in Circular Tubes

A. Constant heat flux

From Newton's law: $q_s'' = h(T_s(x) - T_m(x))$ ➡ Knowing $T(r,x)$ we derive $T_s(x) - T_m(x)$



$$T_m = \frac{2}{u_m r_o^2}\int_0^{r_o} uTr\,dr$$

where

$$T(r,x) = T_s(x) - \frac{2u_m r_0^2}{\alpha}\left(\frac{dT_m}{dx}\right)\left[\frac{3}{16} + \frac{1}{16}\left(\frac{r}{r_0}\right)^4 - \frac{1}{4}\left(\frac{r}{r_0}\right)^2\right]$$

$$\frac{u(r)}{u_m} = 2\left[1 - \left(\frac{r}{r_o}\right)^2\right]$$

➡ $T_m(x) = T_s(x) - \frac{11}{48}\frac{u_m r_0^2}{\alpha}\left(\frac{dT_m}{dx}\right)$ ➡ $T_s(x) - T_m(x) = \frac{11}{48}\frac{u_m r_0^2}{\alpha}\left(\frac{dT_m}{dx}\right)$

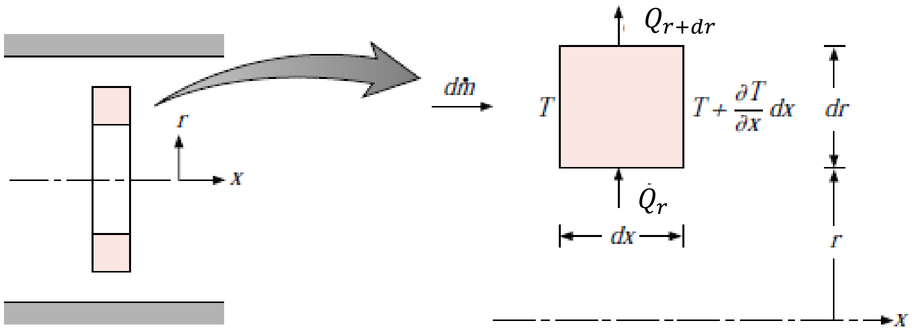
We also remember that: $\frac{dT_m}{dx} = \frac{q_s'' P}{\dot{m} c_p}$ ➡ $T_s(x) - T_m(x) = \frac{11}{48}\frac{u_m r_0^2}{\alpha}\left(\frac{q_s'' P}{\dot{m} c_p}\right)$ where $P = \pi 2 r_0$ $\dot{m} = \rho u_m \pi r_0^2$

➡ $h = \frac{q_s''}{T_s(x) - T_m(x)} = \frac{48}{11}\frac{k_f}{2r_0} = constant$ ➡ $\boldsymbol{Nu_D = \frac{hD}{k_f} = 4.36}$

Convection Coefficient for Laminar Flow in Circular Tubes

We already know the velocity profile, $\vec{u}(r,x)$, now we have to write the energy balance to find $\mathrm{T}(r,x)$*

ANNULAR control volume



Assumptions:

- Negligible viscous dissipation
- Incompressible flow
- Negligible heat transfer along x
- **Fully developed flow ($x>x_{fd,t}$)**
- **Laminar flow ($Re_D<2300$)**

1st Law of thermodynamics (Open system):

$$u\frac{\partial T}{\partial x} = \frac{k_f}{\rho c_p}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) \Rightarrow u\frac{\partial T}{\partial x} = \frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)$$

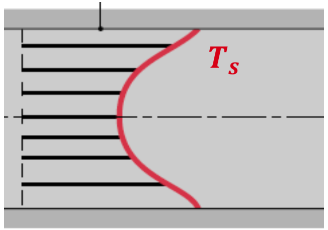
We now have to apply the BCs at $r = r_0$ and we have two possibilities:

A. Constant heat flux
B. Constant surface temperature

Convection Coefficient for Laminar Flow in Circular Tubes

B. Constant T_s

Let's make a few observation that help us relate $T(r,x)$ with $T_m(x)$, which we calculated previously.



From the fully developed condition $\partial\theta/\partial x = 0$:

$$\left.\frac{\partial T}{\partial x}\right|_{x=x_{fd,t}} = \left.\frac{dT_s}{dx}\right|_{x=x_{fd,t}} - \left.\frac{T_s - T}{T_s - T_m}\frac{dT_s}{dx}\right|_{x=x_{fd,t}} + \left.\frac{T_s - T}{T_s - T_m}\frac{dT_m}{dx}\right|_{x=x_{fd,t}}$$

So we can substitute and obtain:

$$u\frac{\partial T}{\partial x} = \frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) \Rightarrow \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = \frac{2u_m}{\alpha}\left(\frac{dT_m}{dx}\right)\left[1 - \left(\frac{r}{r_0}\right)^2\right]\frac{T_s - T}{T_s - T_m}$$

An iterative solution results in:

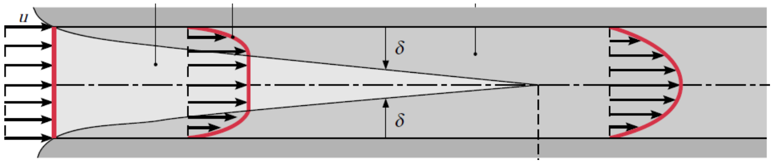
$$\boldsymbol{Nu_D = \frac{hD}{k_f} = 3.66}$$

Internal Forced Convection

FLUID DYNAMICS
Find the *velocity profile*: $\vec{u}(r,x)$

HEAT TRANSFER
Find the temperature profile: $\mathrm{T}(r,x)$

Velocity Profile



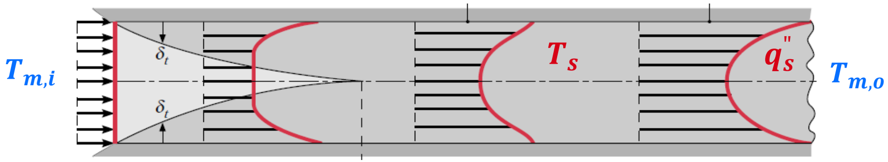
In the fully developed region $\partial u/\partial x = 0$

$$\frac{u(r)}{u_m} = 2\left[1 - \left(\frac{r}{r_o}\right)^2\right]$$

$$u_m = -\frac{r_o^2}{8\mu}\frac{dp}{dx}$$

$\vec{u}(r)$

Temperature Profile



$$\theta = \frac{T_s(x) - T(r,x)}{T_s(x) - T_m(x)}$$

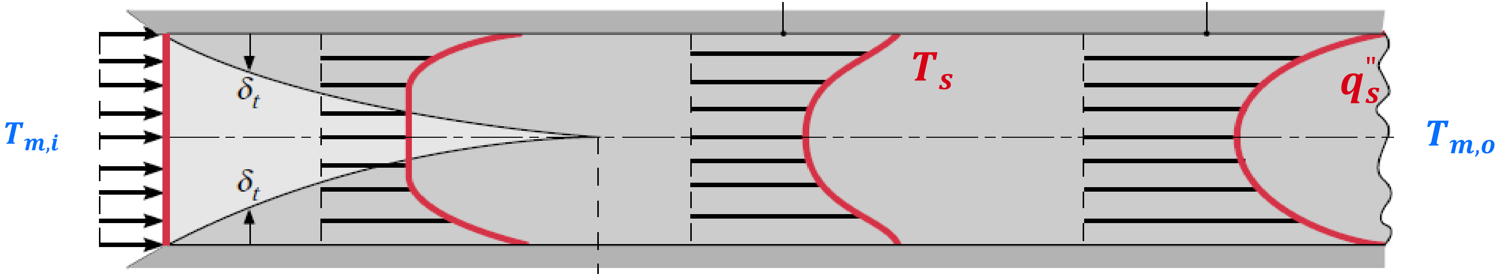
In the fully developed region $\partial\theta/\partial x = 0$

- Constant surface heat flux $\quad T_m(x) = \frac{q_s''P}{\dot{m}c_p}x + T_{m,i} \quad Q_{conv} = q_s''PL$
- Constant surface temperature $\quad \frac{T_s - T_{m,o}}{T_s - T_{m,i}} = exp\left(-\frac{\bar{h}A}{\dot{m}c_p}\right) \quad Q_{conv} = \bar{h}A\Delta T_{lm}$

$$Q_{conv} = Q_{cond,wall}$$
$$T(r,x)$$

- Constant surface heat flux $\quad \boldsymbol{Nu_D = hD/k_f = 4.36}$
- Constant surface temperature $\quad \boldsymbol{Nu_D = hD/k_f = 3.66}$

The problem of the physical properties of the boundary layer



$$\frac{hD}{k_f} = Nu_D = C$$

BL temperature: $T_f = T_m = \frac{(T_{m,i} + T_{m,o})}{2}$

Fluid physical properties	
Density ρ(T_f)	
Viscosity v(T_f)	
Thermal diffusivity $\alpha_f(T_f)$	Specific heat $c_{p,f}(T_f)$ Thermal conductivity $k_f(T_f)$

This Lecture

- ☑ Temperature and heat flow for internal flows
- ☑ Convection coefficient for laminar flow in circular tubes

Learning Objectives:

- ❑ Calculate the heat transfer coefficient for flow in pipes under different geometrical and flow conditions

Next Lecture

- ❑ Correlations for internal forced convection
  - ❑ Circular tubes (laminar and turbulent)
  - ❑ Non-circular tubes
- ❑ The entrance region

Learning Objectives:

- ❑ Calculate the heat transfer coefficient for flow in pipes under different geometrical and flow conditions