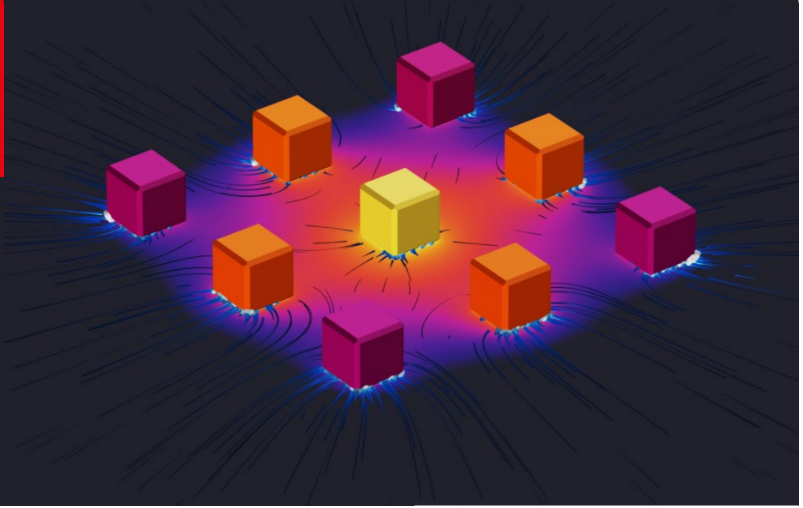
EPFL

Heat and Mass Transfer ME-341

Instructor: Giulia Tagliabue

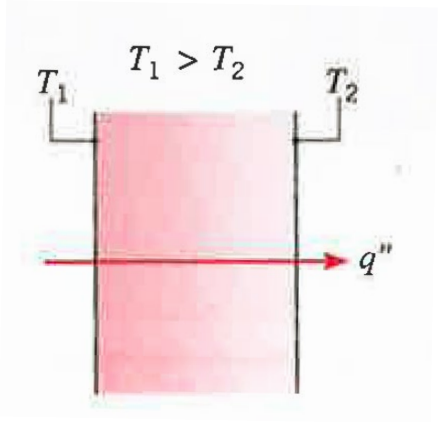


École polytechnique fédérale de Lausanne

Spring Semester

Transport Laws

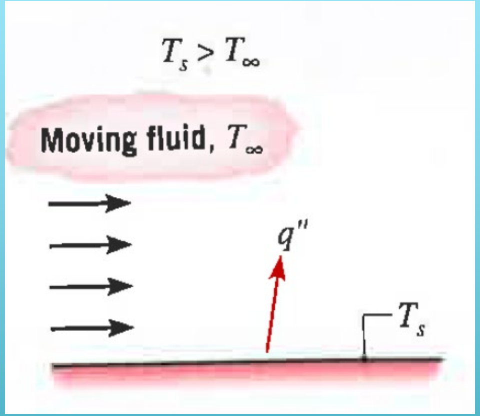
Conduction



Fourier's Law

$$q'' = -k\frac{dT}{dx}$$

Convection



Newton's Law

$$q'' = \bar{h}\,(T_s - T_\infty)$$

Until now it was only a boundary condition and h was given, now we want to calculate it.

Introduction to Convection

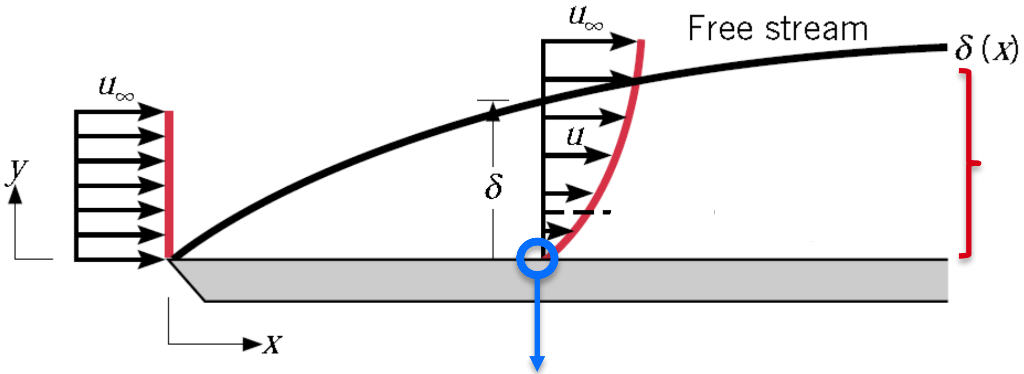
Convection refers to the heat transfer between a **solid** and a **fluid in motion**

when they are at **different temperatures**.

During convection heat is transferred through

both **diffusion** (random molecular motion) and **advection** (macroscopic mass transport)

RECAP of Fluid Dynamics: velocity boundary layer



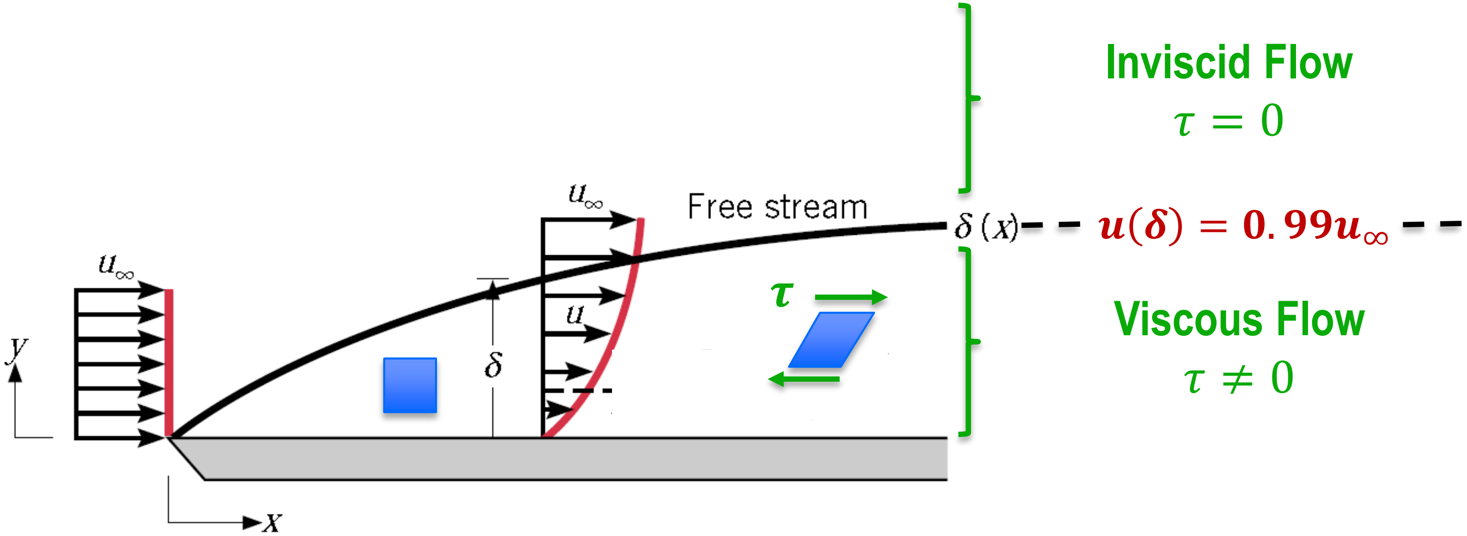
$$u(\delta) = 0.99u_\infty$$

Velocity Boundary Layer

$$u(0) = 0$$

No-slip condition

RECAP of Fluid Dynamics: viscous and inviscid flow, shear stress and friction coefficient

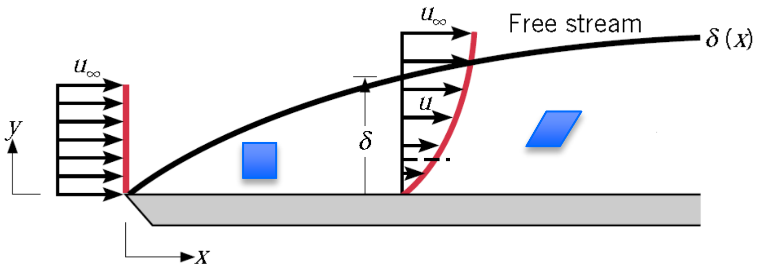


Shear stress τ = friction force per unit area

Newtonian fluids: $\tau(\bar{y}) = \mu \left.\frac{\partial u}{\partial y}\right|_{y=\bar{y}} \left[{}^{N}/_{m^2}\right]$ where $\mu \left[\frac{Ns}{m^2}\right] = dynamic\ viscosity = \rho[\frac{kg}{m^3}] \cdot \nu[\frac{m^2}{s}]$

At the wall ($y = 0$): $\tau(0) = \tau_w = C_f \frac{\rho u_\infty^2}{2}$ where $C_f = friction\ coefficient$

RECAP of Fluid Dynamics: Velocity Boundary layer equations



Navier-Stokes equations and dimensionless variables:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2}$$

$$L = characteristic\ length\ [m]$$

$$x^* = \frac{x}{L} \qquad y^* = \frac{y}{L}$$

$$u^* = \frac{u}{u_\infty} \qquad v^* = \frac{v}{u_\infty} \qquad p^* = \frac{p}{\rho u_\infty^2}$$

Dimensionless Navier-Stokes equations and Re number:

$$u^*\frac{\partial u^*}{\partial x^*} + v^*\frac{\partial u^*}{\partial y^*} = -\frac{\partial p^*}{\partial x^*} + \frac{1}{\boldsymbol{Re_L}}\frac{\partial^2 u^*}{\partial {y^*}^2}$$

Determines the flow condition (laminar/turbulent)

$$Re = \frac{inertia\ forces}{viscous\ forces} = \frac{\rho u_\infty L}{\mu} = \frac{u_\infty L}{\nu}$$

$$\nu = \frac{\mu}{\rho} = kinematic\ viscosity[{m^2}/{s}]$$

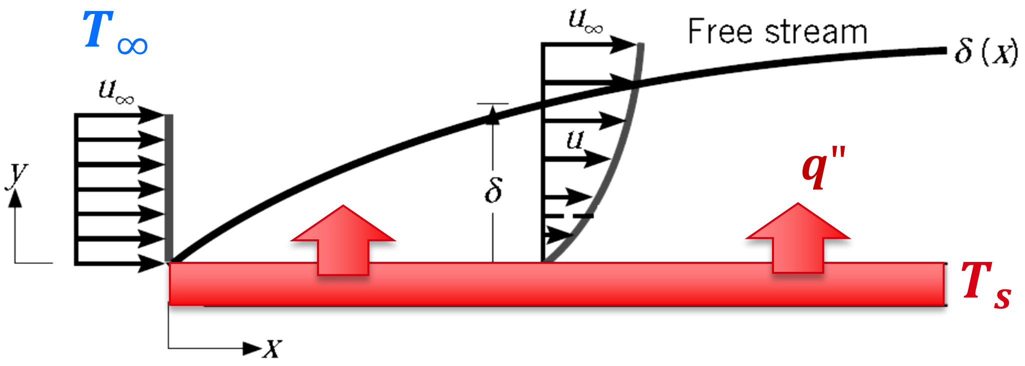
This Lecture

- ❑ The thermal boundary layer equations
- ❑ Comparison of velocity and thermal boundary layer, Pr number

Learning Objectives:

- ❑ Understand the thermal boundary layer concept and equations
- ❑ Understand and calculate the dimensionless numbers (Re, Nu, Pr)

Newton's law and the convection coefficient



$T_s > T_\infty$

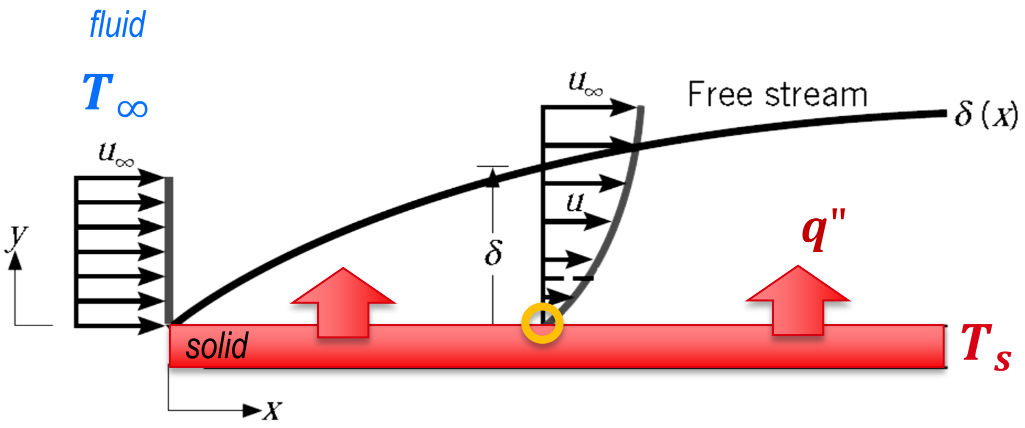
Newton's law: the heat transfer is proportional to the temperature difference between the wall and the unperturbed fluid

$$q'' = h\,(T_s - T_\infty)\;[W/m^2] \quad \text{or} \quad Q = hA\,(T_s - T_\infty)\;[W]$$

where $h = convection\ coefficient$

Where does h come from?

Newton's law and the convection coefficient



$$T_s > T_\infty$$

At the wall the velocity is zero (no advection, only diffusion) thus we can use **Fourier's law**:

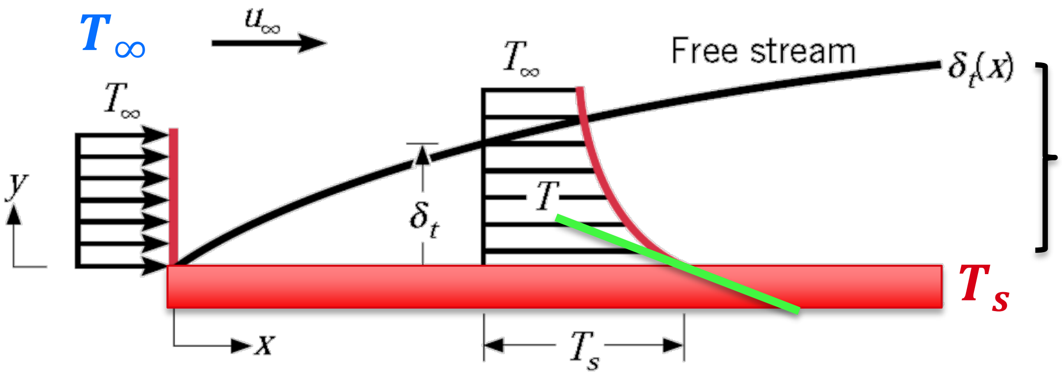
$$q''(0) = -k_f \left.\frac{\partial T}{\partial y}\right|_{y=0}$$

k_f = thermal conductivity of the **fluid**

$$\Rightarrow \quad h = \frac{q''(0)}{(T_s - T_\infty)} = \frac{-k_f \left.\frac{\partial T}{\partial y}\right|_{y=0}}{(T_s - T_\infty)}$$

The thermal boundary layer and Nu number

$$T_s > T_\infty$$



$$\frac{T_s - T(\delta_t)}{T_s - T_\infty} = 0.99$$

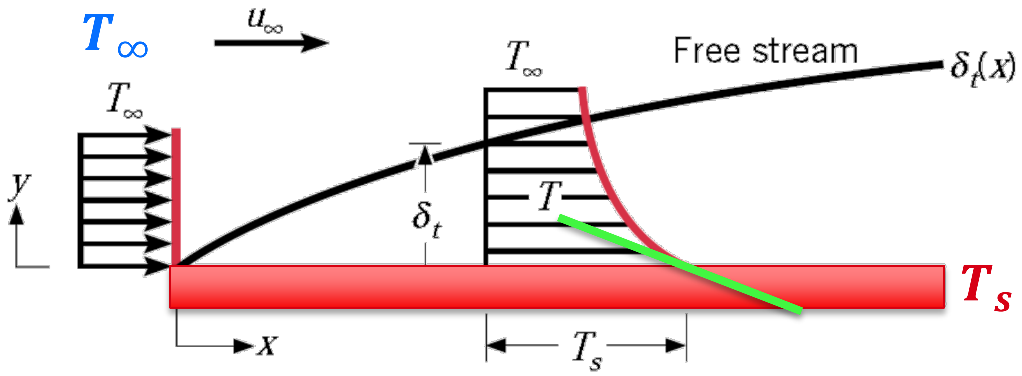
Thermal Boundary Layer

$$h = \frac{-k_f \left.\frac{\partial T}{\partial y}\right|_{y=0}}{(T_s - T_\infty)} \quad \Rightarrow \quad \frac{h L_c}{k_f} = \left.\frac{\partial\left(\frac{T_s - T}{T_s - T_\infty}\right)}{\partial\left(\frac{y}{L_c}\right)}\right|_{y/L_c=0} = Nu_L$$

Nusselt number

L_c is a characteristic dimension of the problem (i.e. length of the plate)

The problem of convection



$$T_s > T_\infty$$

$$h = \frac{-k_f \left.\frac{\partial T}{\partial y}\right|_{y=0}}{(T_s - T_\infty)}$$

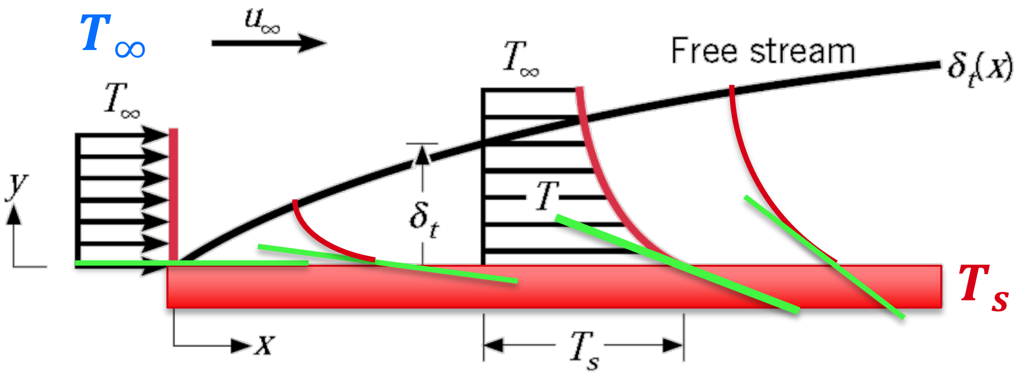
To determine h we need to know the temperature gradient at the wall. This in turn depends on the flow conditions.

To obtain T(x,y), we must solve the coupled momentum (Navier-Stokes) and energy (heat diffusion) equations.

Analytical solutions, however, exist only for simple cases.

For all other cases we rely on **empirical correlations** or numerical simulations.

The problem of convection



$$T_s > T_\infty$$

$$h = \frac{-k_f \left.\frac{\partial T}{\partial y}\right|_{y=0}}{(T_s - T_\infty)}$$

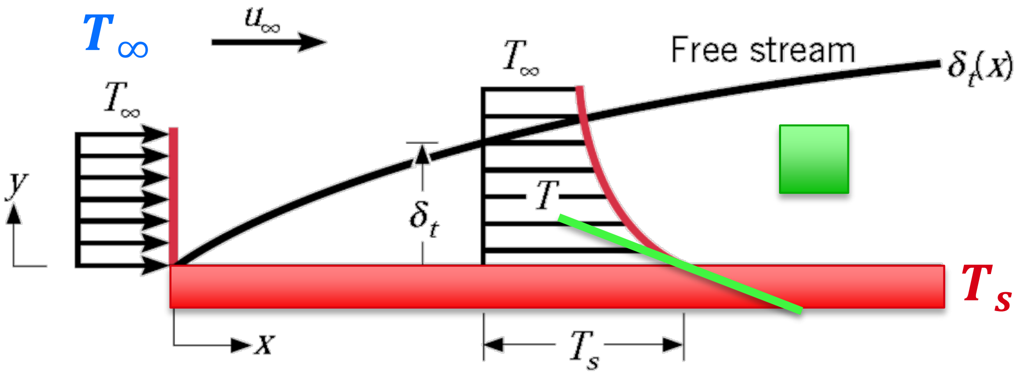
The temperature gradient at the wall changes as the temperature boundary layer develops. Therefore **the convection coefficient varies spatially**.

Local convection coefficient   *Average convection coefficient*

$$Q = (T_s - T_\infty)\int_{A_s} h dA_s = \bar{h}A_s(T_s - T_\infty)$$

$$\bar{h} = \frac{1}{A_s}\int_{A_s} h dA_s$$

The thermal boundary layer equations



$$T_s > T_\infty$$

$$h = \frac{-k_f \left.\frac{\partial T}{\partial y}\right|_{y=0}}{(T_s - T_\infty)}$$

If we determine T(x,y), we know $\left.\frac{\partial T}{\partial y}\right|_{y=0}$ and we can obtain the convection coefficient

- Conservation of mass ✓
- Conservation of momentum ✓

Velocity boundary layer

- Conservation of energy → ***Thermal boundary layer***

This Lecture

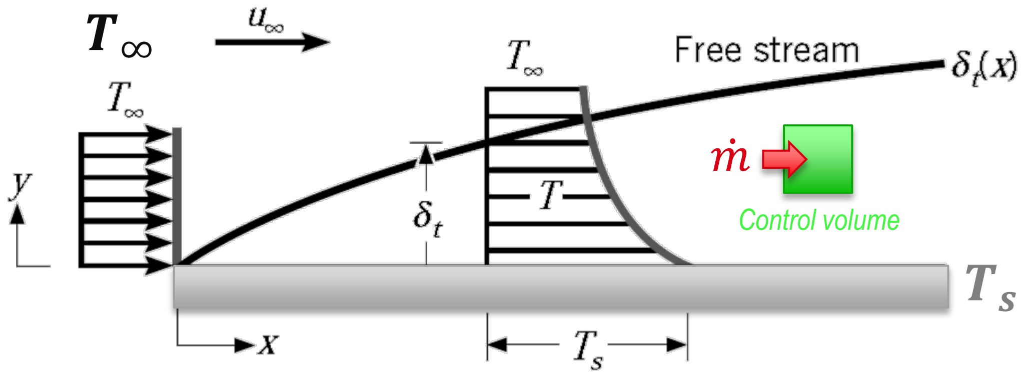
- ❑ The thermal boundary layer equations
- ❑ Comparison of velocity and thermal boundary layer, Pr number

Learning Objectives:

- ❑ Understand the thermal boundary layer concept and equations
- ❑ Understand and calculate the dimensionless numbers (Re, Nu, Pr)

The thermal boundary layer equations: energy equations for open systems

$$T_s > T_\infty$$



Closed system (W1L2):

$$\dot{U} = Q - \dot{W} + \dot{E}_{gen}$$

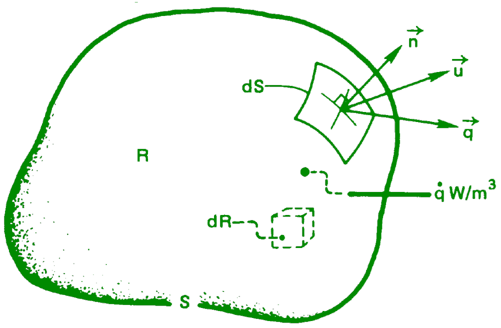
Open system:

$$\dot{U} = \dot{m}\left(u + pv + \frac{1}{2}V^2 + gz\right)_{in} - \dot{m}\left(u + pv + \frac{1}{2}V^2 + gz\right)_{out} + \dot{Q} - \dot{W} + \dot{E}_{gen}$$

where $u = specific\ internal\ energy;\ v = specific\ volume = 1/\rho$

The thermal boundary layer equations: energy equations for open systems

Non-isothermal S!



The control volume does not move *No machine*

$$\dot{U} = \dot{m}\left(u + pv + \cancel{\frac{1}{2}V^2} + \cancel{gz}\right)_{in} - \dot{m}\left(u + pv + \cancel{\frac{1}{2}V^2} + \cancel{gz}\right)_{out} + \dot{Q} - \cancel{W} + \dot{E}_{gen}$$

$$\dot{U} = \dot{m}\,(h)_{in} - \dot{m}\,(h)_{out} + \dot{Q} + \dot{E}_{gen}$$

$$\dot{U} = -(\rho\vec{u}\cdot\vec{n})(h) + \dot{Q} + \dot{E}_{gen}$$

$\vec{u} = velocity; h = specific\ enthalpy$

For the terms $\dot{U}$, $\dot{Q}, \dot{E}_{gen}$ we have the same expressions as in the closed system case

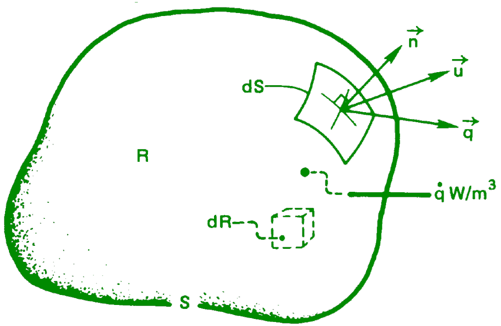
$$\int_V \frac{\partial\rho u}{\partial t}dR = -\int_S (\rho h\vec{u})\cdot(\vec{n}dS) - \int_S (-k\,\nabla T)\cdot(\vec{n}dS) + \int_V \dot{q}dR$$

Assumption 1: isotropic material
Assumption 2: potential and kinetic energy changes are negligible

The thermal boundary layer equations: energy equations for open systems

Non-isothermal S!

$$\int_V \frac{\partial \rho u}{\partial t} dR = -\int_S (\rho h \vec{u}) \cdot (\vec{n} dS) - \int_S (-k \nabla T) \cdot (\vec{n} dS) + \int_V \dot{q} dR$$



Assumption 1: isotropic material
Assumption 2: potential and kinetic energy changes are negligible

Assumption 3: we neglect the effect of pressure changes *dp* on enthalpy, internal energy and density

$$\hat{h} = \hat{u} + p/\rho, \text{ and that } d\hat{h} = c_p\, dT + (\partial \hat{h}/\partial p)_T\, dp$$

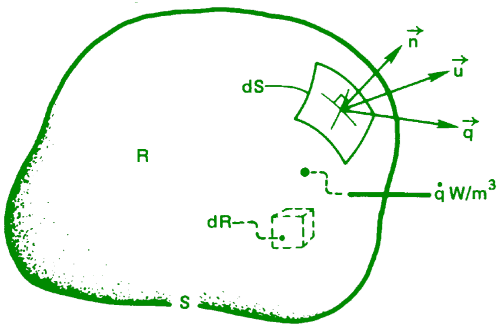
Assumption 4: density changes result only from temperature changes so they are small and the fluid behaves as if incompressible $\nabla \cdot \vec{u} = 0$

Assumption 5: all material parameters are temperature independent (k, μ)

Assumption 6: viscous stresses do not dissipate enough energy to warm the fluid

The thermal boundary layer equations: energy equations for open systems

Non-isothermal S!



$$\int_V \frac{\partial \rho u}{\partial t} dR = -\int_S (\rho h \vec{u}) \cdot (\vec{n} dS) - \int_S (-k\, \nabla T) \cdot (\vec{n} dS) + \int_V \dot{q}\, dR$$

$$\int_R \left(\rho \frac{\partial \hat{u}}{\partial t} + \rho \nabla \cdot (\vec{u}\, \hat{h}) - \nabla \cdot k \nabla T - \dot{q} \right) dR = 0$$

(Gauss' law)

$$\rho \left(\frac{\partial \hat{u}}{\partial t} + \underbrace{\nabla \cdot (\vec{u}\hat{h})}_{= \vec{u} \cdot \nabla \hat{h} + \hat{h}\, \nabla \cdot \vec{u}} \right) - k \nabla^2 T - \dot{q} = 0$$

continuity

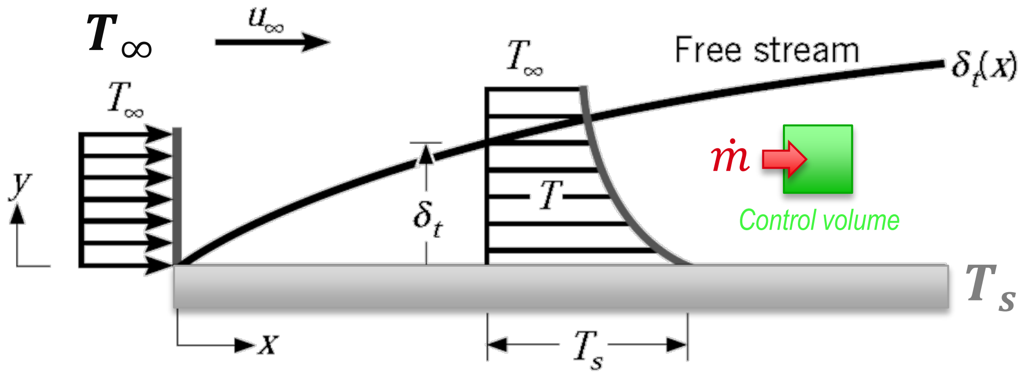
Assumption 1-6 (see slide 36)

Because of assumption 3: $d\hat{u} = d\hat{h} - d\left(\frac{p}{\rho}\right) \approx d\hat{h}$ $\quad d\hat{h} \approx c_p\, dT,$

$$\rho c_p \left(\frac{\partial T}{\partial t} + \vec{u} \cdot \nabla T \right) = k \nabla^2 T + \dot{q}$$

The thermal boundary layer equations: energy equations for open systems

$$T_s > T_\infty$$



Closed system (W1L2):

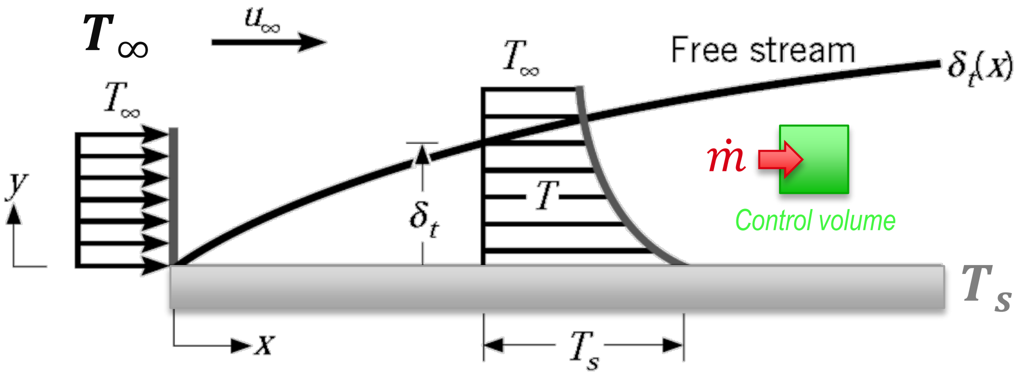
$$\rho c \frac{\partial T}{\partial t} = k\nabla^2 T + \dot{q}$$

Open system:

$$\underbrace{\rho c_p \left(\frac{\partial T}{\partial t}\right.}_{\text{Energy storage}} + \underbrace{\left.\vec{\boldsymbol{u}} \cdot \boldsymbol{\nabla T}\right)}_{\text{Enthalpy advection}} = \underbrace{k\nabla^2 T}_{\text{Heat diffusion}} + \underbrace{\dot{q}}_{\text{Energy generation}}$$

The thermal boundary layer equations: energy equations for open systems

$$T_s > T_\infty$$



Closed system (W1L2):

$$\rho c \frac{\partial T}{\partial t} = k\nabla^2 T + \dot{q}$$

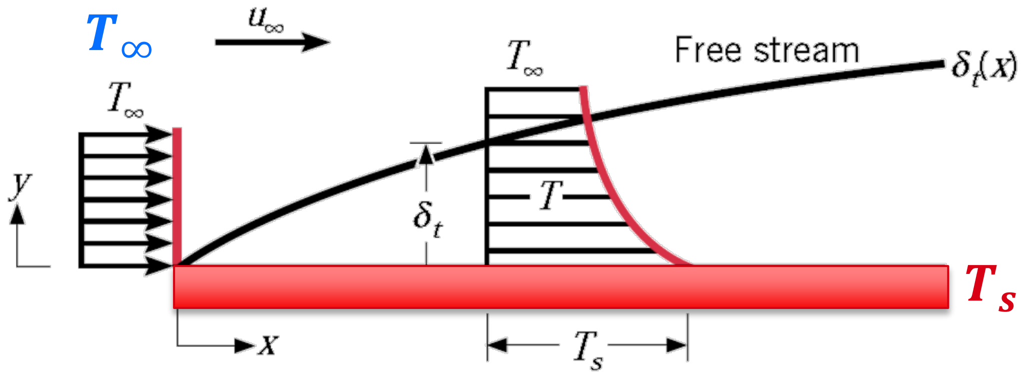
Open system:

Steady-state

No heat sources in the fluid

$$\rho c_p\left(\underbrace{\cancel{\frac{\partial T}{\partial t}}}_{\text{Energy storage}} + \underbrace{\vec{\boldsymbol{u}}\cdot\boldsymbol{\nabla T}}_{\text{Enthalpy advection}}\right) = \underbrace{\overbrace{k\nabla^2 T}^{\frac{\partial T}{\partial x} \ll \frac{\partial T}{\partial y}}}_{\text{Heat diffusion}} + \underbrace{\cancel{\dot{q}}}_{\text{Energy generation}}$$

The thermal boundary layer equations



$$T_s > T_\infty$$

$$h = \frac{-k_f \left.\frac{\partial T}{\partial y}\right|_{y=0}}{(T_s - T_\infty)}$$

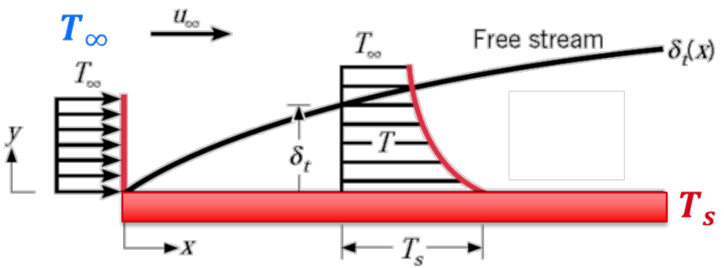
- ❑ Conservation of mass $\qquad \dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} = 0$
- ❑ Conservation of momentum $\qquad u\dfrac{\partial u}{\partial x} + v\dfrac{\partial u}{\partial y} = -\dfrac{1}{\rho}\dfrac{\partial p}{\partial x} + \nu\dfrac{\partial^2 u}{\partial y^2}$
- ❑ Conservation of energy $\qquad u\dfrac{\partial T}{\partial x} + v\dfrac{\partial T}{\partial y} = \alpha\dfrac{\partial^2 T}{\partial y^2}$

Velocity boundary layer

The geometry and flow characteristics are critical for convective heat transfer

Thermal boundary layer

The thermal boundary layer equations: Prandtl number (Pr)



$L = characteristic\ length\ [m]$ $\quad x^* = \dfrac{x}{L}$ $\quad y^* = \dfrac{y}{L}$ $\quad u^* = \dfrac{u}{u_\infty}$ $\quad v^* = \dfrac{v}{u_\infty}$ $\quad p^* = \dfrac{p}{\rho u_\infty^2}$ $\quad T^* = \dfrac{T - T_s}{T_\infty - T_s}$

Dimensionless Navier-Stokes equations:

$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = -\frac{\partial p^*}{\partial x^*} + \frac{1}{\boldsymbol{Re_L}} \frac{\partial^2 u^*}{\partial {y^*}^2}$$

$$Re = \frac{inertia\ forces}{viscous\ forces} = \frac{\rho u_\infty L}{\mu} = \frac{u_\infty L}{\nu}$$

Dimensionless energy conservation equation:

$$u^* \frac{\partial T^*}{\partial x^*} + v^* \frac{\partial T^*}{\partial y^*} = \frac{1}{\boldsymbol{Re_L Pr}} \frac{\partial^2 T^*}{\partial {y^*}^2}$$

$$Pr = \frac{kinematic\ (momentum)\ diffusivity}{heat\ diffusivity} = \frac{\nu}{\alpha_f}$$

This Lecture

- [x] The thermal boundary layer equations
- [ ] Comparison of velocity and thermal boundary layer, Pr number

Learning Objectives:

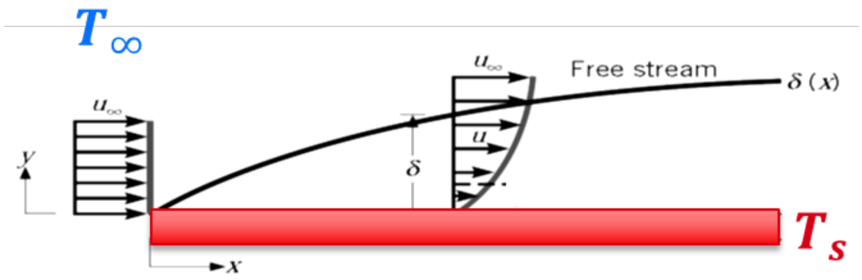
- [ ] Understand the thermal boundary layer concept and equations
- [ ] Understand and calculate the dimensionless numbers (Re, Nu, Pr)

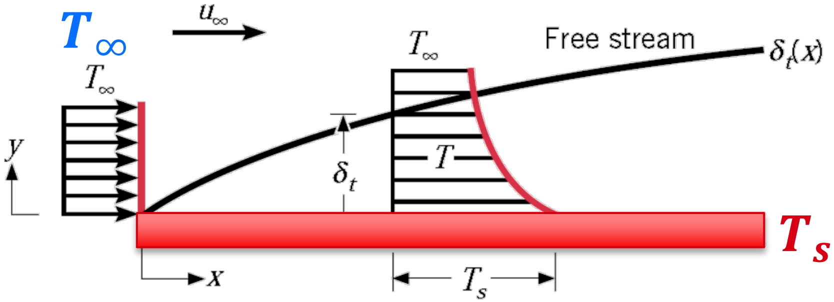
Comparison of velocity and thermal boundary layers, Pr number

Velocity boundary layer $\boldsymbol{\delta(x)}$

$$\boldsymbol{\delta(x) \neq \delta_t(x)}$$

Thermal boundary layer $\boldsymbol{\delta_t(x)}$





Dimensionless Navier-Stokes equations and Re number:

$$u^*\frac{\partial u^*}{\partial x^*} + v^*\frac{\partial u^*}{\partial y^*} = -\frac{\partial p^*}{\partial x^*} + \frac{1}{\boldsymbol{Re_L}}\frac{\partial^2 u^*}{\partial y^{*2}}$$

Dimensionless energy conservation equation and Pr number:

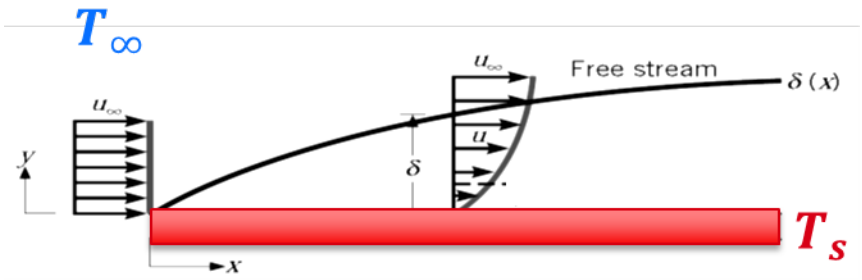
$$u^*\frac{\partial T^*}{\partial x^*} + v^*\frac{\partial T^*}{\partial y^*} = \frac{1}{\boldsymbol{Re_L Pr}}\frac{\partial^2 T^*}{\partial y^{*2}}$$

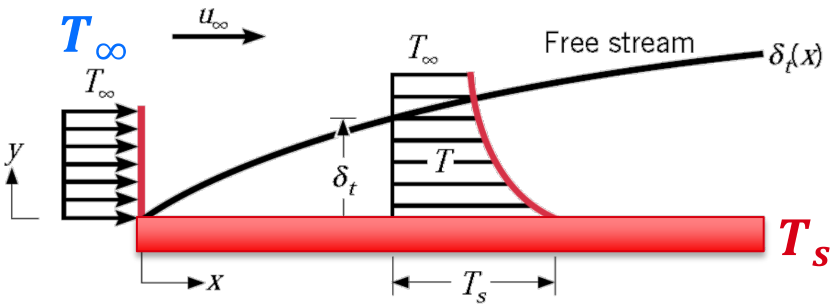
Prandtl number: $$Pr = \frac{momentum\ diffusivity}{heat\ diffusivity} = \frac{\nu}{\alpha_f} = \frac{\mu c_{p,f}}{k_f} = \frac{\delta}{\delta_t}$$

BEWARE! These are the FLUID properties, not the solid ones

The thermal boundary layer equations: Prandtl number (Pr)

Velocity boundary layer $\boldsymbol{\delta(x)}$ $\qquad \boldsymbol{\delta(x) \neq \delta_t(x)}$ $\qquad$ *Thermal boundary layer* $\boldsymbol{\delta_t(x)}$





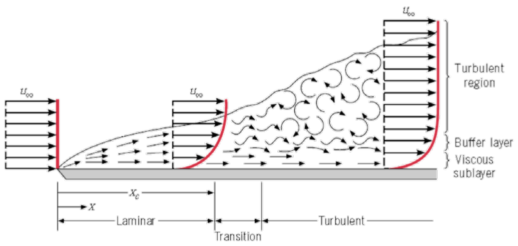
For $Pr \ll 1$

the thermal BL $\boldsymbol{\delta_t(x)}$ is thinner or thicker than the velocity BL $\boldsymbol{\delta(x)}$?

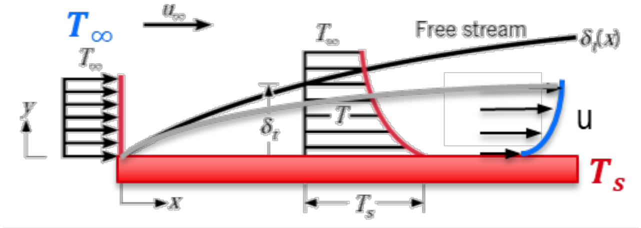
$Pr \ll 1$ | heat diffuses very quickly relative to momentum | a small temperature gradient is sufficient to sustain a large heat flux | The thermal boundary layer is much LARGER than the velocity one | $\delta(x) \ll \delta_t(x)$

Dimensionless numbers

$$Re_L = \frac{inertia\ forces}{viscous\ forces} = \frac{\rho u_\infty L}{\mu} = \frac{u_\infty L}{\nu}$$



$$Pr = \frac{momentum\ diffusivity}{heat\ diffusivity} = \frac{\nu_f}{\alpha_f} = \frac{\mu c_{p,f}}{k_f} = \frac{\delta}{\delta_t}$$



$$Nu_L = \frac{hL_c}{k_f} = \left.\frac{\partial\left(\frac{T_s - T}{T_s - T_\infty}\right)}{\partial\left(\frac{y}{L_c}\right)}\right|_{y/L_c=0}$$

If we can calculate Nu then we can obtain the convection coefficient

This Lecture

- ☑ The thermal boundary layer equations
- ☑ Comparison of velocity and thermal boundary layer, Pr number

Learning Objectives:

- ☑ Understand the thermal boundary layer concept and equations
- ❑ Understand and calculate the dimensionless numbers (Re, Nu, Pr)

Next Lecture

- ❑ Forced convection coefficient for laminar flow over a flat horizontal plate (local and average)
- ❑ Correlations for the forced convection coefficient of:
  a. Flat plate in parallel flow under different flow and heating conditions
  b. External flow on a cylinder
  c. External flow on a bank of tubes
- ❑ General methodology for calculating the convection coefficient

Learning Objectives:

- ❑ Calculate the convection coefficient for various geometries in forced convection