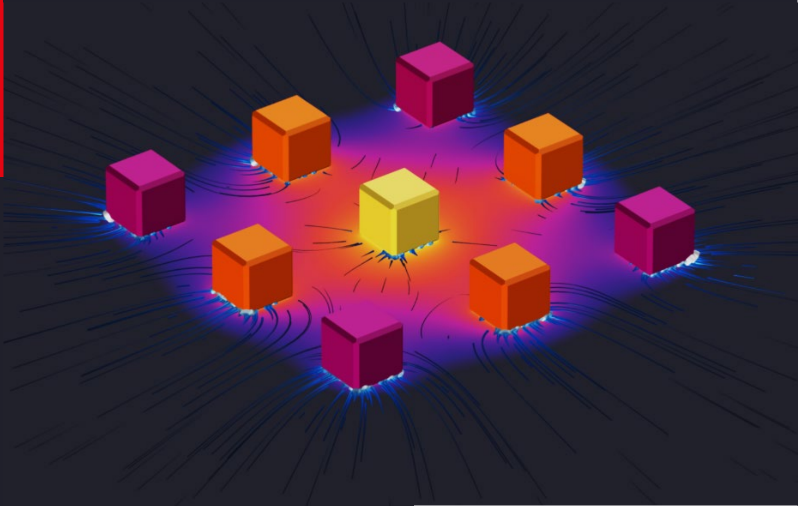
Heat and Mass Transfer ME-341

Instructor: Giulia Tagliabue



École polytechnique fédérale de Lausanne

Spring Semester

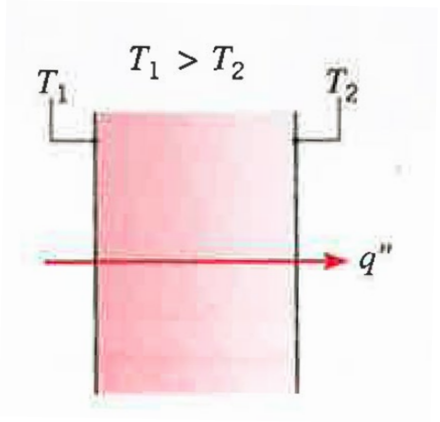
Indicative Feedback!

Until Now

- ☑ Heat Diffusion and Boundary Conditions (W1L2-3)
  - ☑ Steady State Heat Diffusion Equation
    - ☑ Without Heat sources (W1L3-4; W2L1)
      - ☑ Thermal Resistance & Overall Heat Transfer Coefficient
      - ☑ Bi number
      - ☑ Thermal Circuits
    - ☑ WITH Heat Sources (W2L2-3)
    - ☑ Fins and Fin Arrays (W3L1-3)
  - ☑ Transient Heat Diffusion (W4L1-3)

Transport Laws

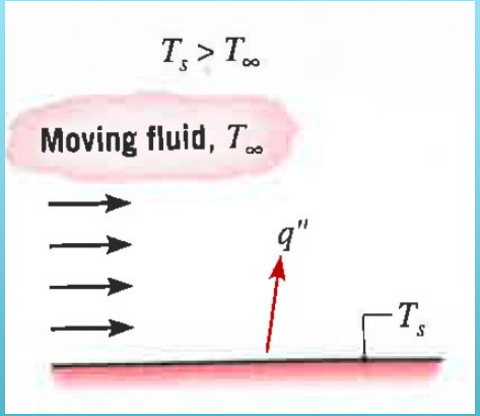
Conduction



Fourier's Law

$$q'' = -k\frac{dT}{dx}$$

Convection



Newton's Law

$$q'' = \bar{h}\,(T_s - T_\infty)$$

Until now it was only a boundary condition and h was given, now we want to calculate it.

Introduction to Convection

Convection refers to the heat transfer between a **solid** and a **fluid in motion** when they are at **different temperatures**.

Introduction to Convection

Convection refers to the heat transfer between a **solid** and a **fluid in motion** when they are at **different temperatures**.

Give an example of convection (3/4 words)

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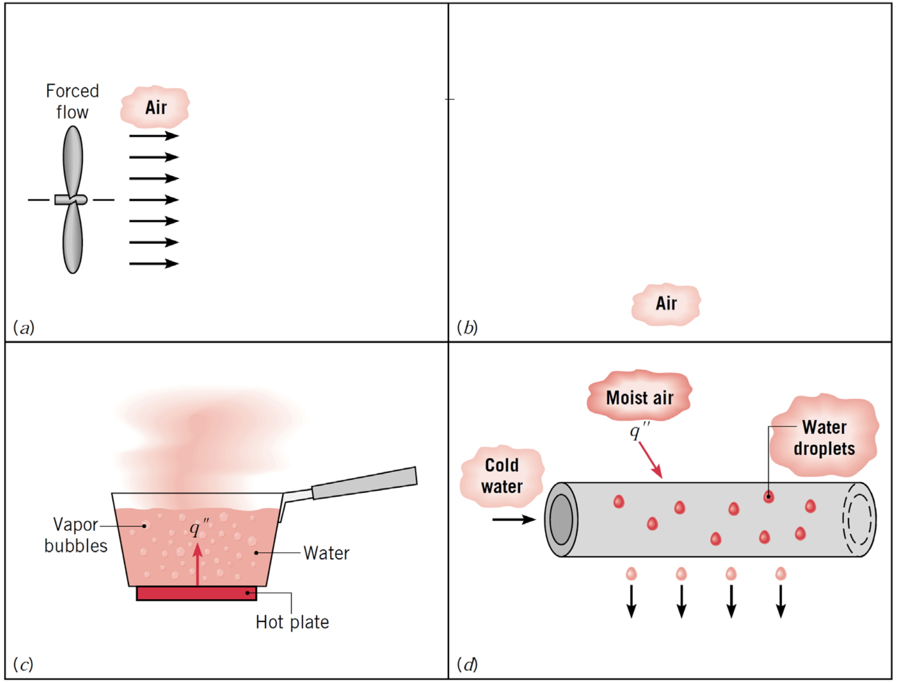
Introduction to Convection

1. Forced Convection

2. Natural (Free) Convection

3. Boiling

4. Condensation



During the next weeks we will study these four convection processes

Introduction to Convection

Convection refers to the heat transfer between a **solid** and a **fluid in motion** when they are at **different temperatures**.

During convection heat is transferred through both **diffusion** (random molecular motion) and **advection** (macroscopic mass transport)

➡ Let's first revise some fundamental concepts of fluid dynamics

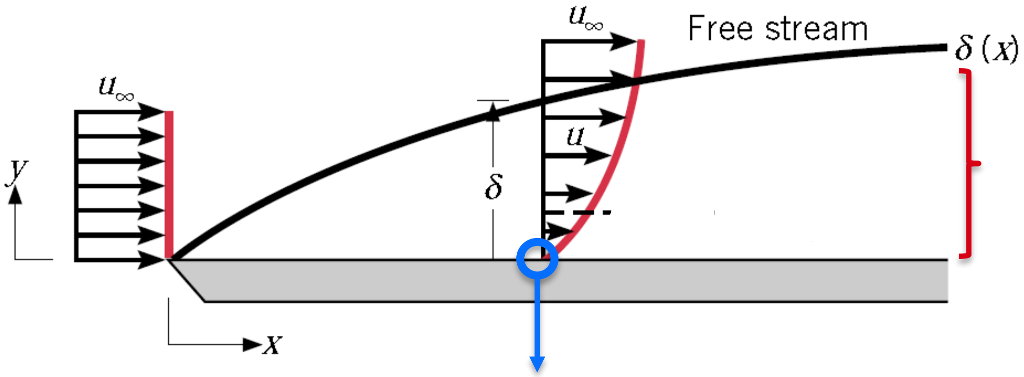
This Lecture

- RECAP of Fluid Dynamics and the velocity boundary layer
  - Laminar and Turbulent flow
  - Viscous and inviscid flow shear stress and friction coefficient
  - Velocity boundary layer equations
    - Mass and momentum conservation equations
    - Dimensionless momentum conservation equation and Re number
    - Velocity boundary layer over an horizontal flat plate (laminar flow)
- Newton's law and the convection coefficient
  - The thermal boundary layer and Nu number
  - The problem of convection

Learning Objectives:

- Calculate the velocity boundary layer thickness and friction coefficient
- Understand the thermal boundary layer concept
- Understand the challenges of forced convection

RECAP of Fluid Dynamics: velocity boundary layer



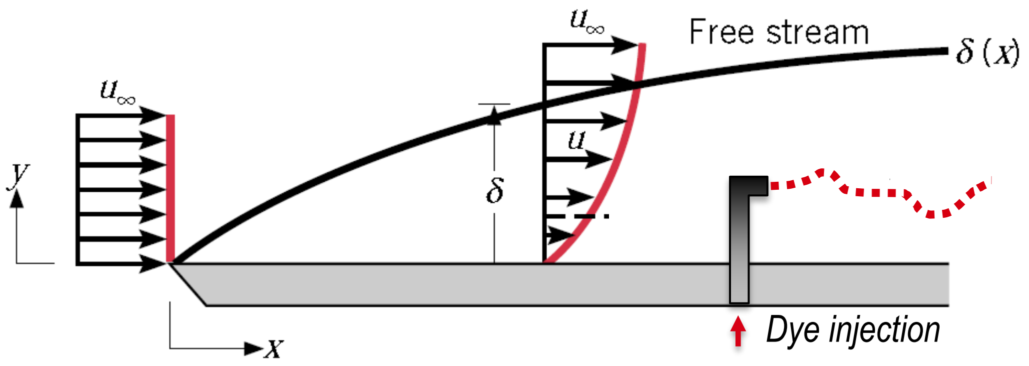
$u(\delta) = 0.99u_\infty$

Velocity Boundary Layer

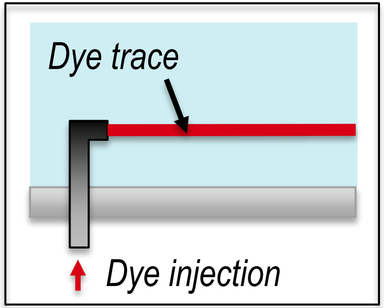
$u(0) = 0$

No-slip condition

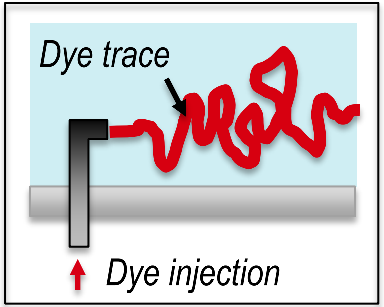
RECAP of Fluid Dynamics: laminar and turbulent flow



Laminar



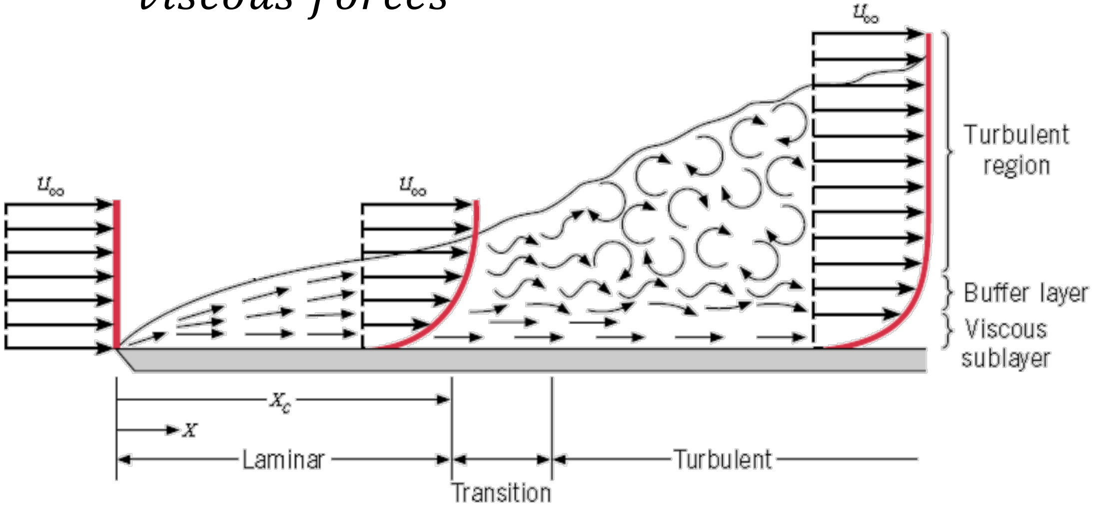
Turbulent



How does the dye trace looks like? Is it straight? Wavy?

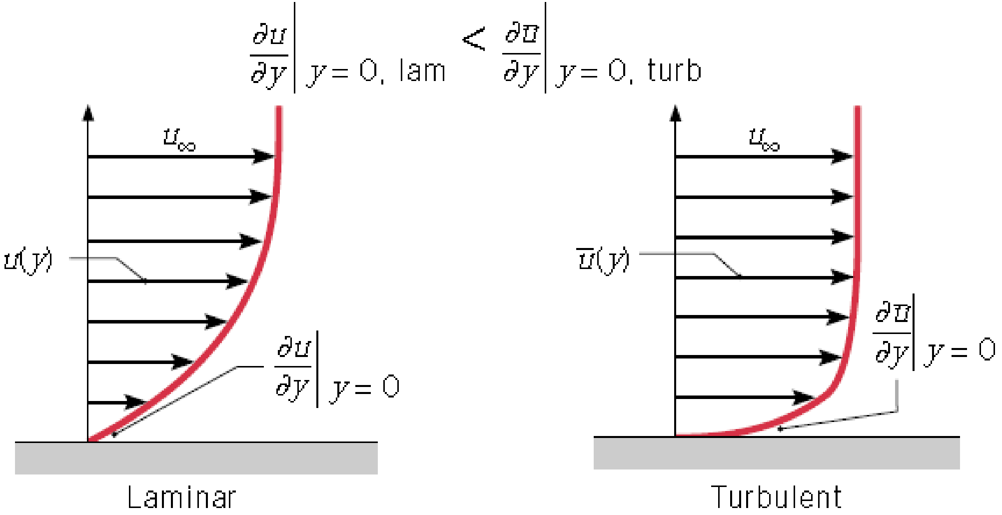
RECAP of Fluid Dynamics: laminar and turbulent flow

Reynolds number : $Re = \dfrac{inertia\ forces}{viscous\ forces}$



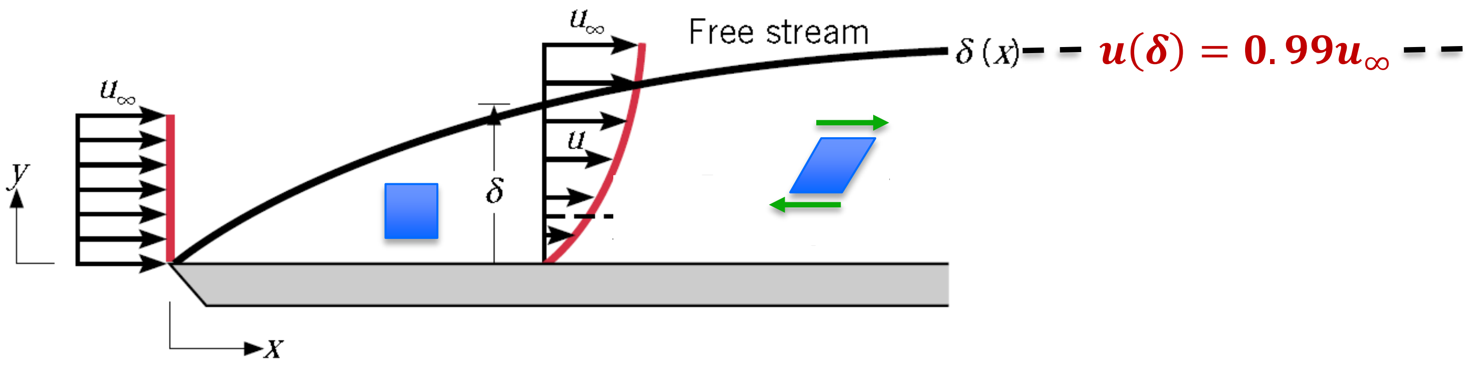
The transition occurs at a critical Reynolds number, Re_{cr}, that depends on the geometry

RECAP of Fluid Dynamics: laminar and turbulent flow

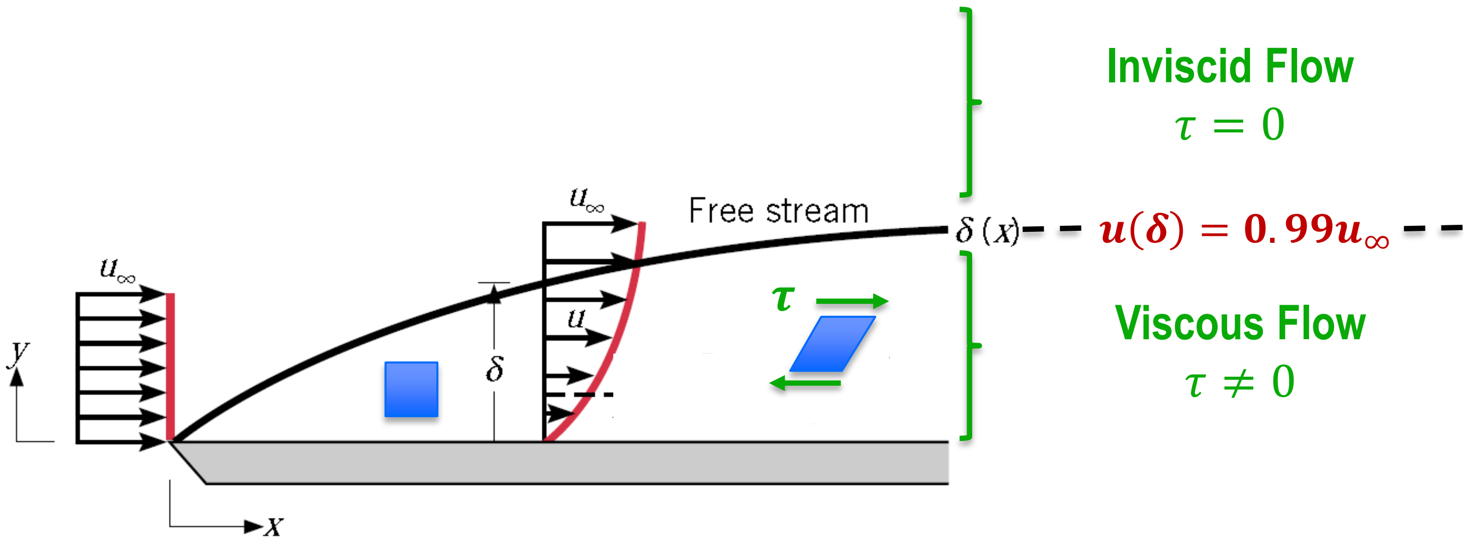


Turbulence “flattens” the velocity profile in the boundary layer

RECAP of Fluid Dynamics: viscous and inviscid flow, shear stress and friction coefficient



RECAP of Fluid Dynamics: viscous and inviscid flow, shear stress and friction coefficient

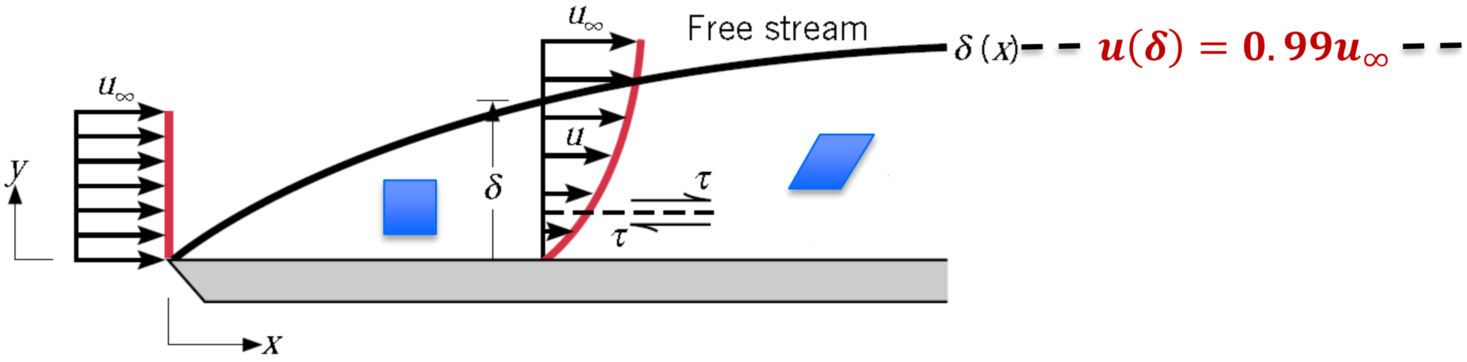


Shear stress τ = friction force per unit area

Newtonian fluids: $\tau(\bar{y}) = \mu \left.\frac{\partial u}{\partial y}\right|_{y=\bar{y}} \left[{}^{N}/_{m^2}\right]$ where $\mu \left[\frac{Ns}{m^2}\right] = dynamic\ viscosity = \rho[\frac{kg}{m^3}] \cdot \nu[\frac{m^2}{s}]$

At the wall ($y = 0$): $\tau(0) = \tau_w = C_f \frac{\rho u_\infty^2}{2}$ where $C_f = friction\ coefficient$

RECAP of Fluid Dynamics: Velocity Boundary layer equations



To write the equation of the velocity boundary layer we must write the mass and momentum conservation on a control volume

RECAP of Fluid Dynamics: Velocity Boundary layer equations

Conservation of Mass (Continuity Eqn.)

the sum of the masses entering and exiting the control volume must be zero

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0$$

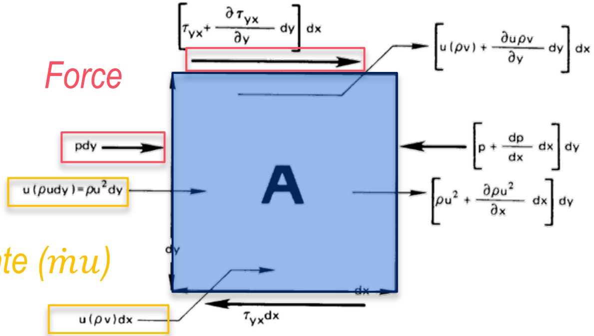
For an incompressible fluid (i.e. liquids) :

$$\nabla \cdot \bar{u} = 0$$

RECAP of Fluid Dynamics: Velocity Boundary layer equations

Conservation of Momentum (Navier-Stokes Eqns.)

the sum of all forces acting on a control volume must equal the net rate at which momentum leaves the control volume



Assumptions:

- Incompressible fluid
- 2D flow
- $\partial u/\partial x \ll \partial v/\partial y$
- $v \ll u$
- $p = p(x)$
- ρ~const
- μ~const

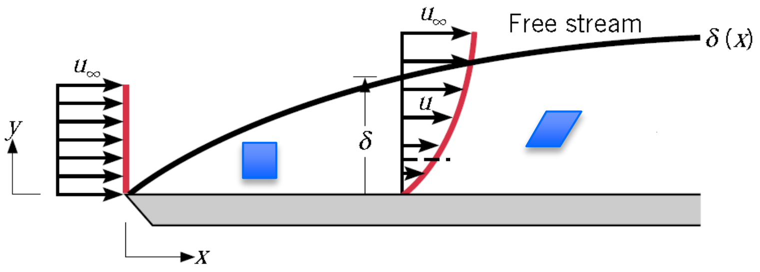
Newtonian fluids: $$\tau_{yx} = \mu \frac{\partial u}{\partial y}$$

$$\mu \left[N \cdot \frac{s}{m^2}\right] = (dynamic) viscosity$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2}$$

Inertial Part *Viscous Part*

RECAP of Fluid Dynamics: Velocity Boundary layer equations



Navier-Stokes equations and dimensionless variables:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2}$$

$$L = characteristic\ length\ [m]$$

$$x^* = \frac{x}{L} \qquad y^* = \frac{y}{L}$$

$$u^* = \frac{u}{u_\infty} \qquad v^* = \frac{v}{u_\infty} \qquad p^* = \frac{p}{\rho u_\infty^2}$$

Dimensionless Navier-Stokes equations and Re number:

$$u^*\frac{\partial u^*}{\partial x^*} + v^*\frac{\partial u^*}{\partial y^*} = -\frac{\partial p^*}{\partial x^*} + \frac{1}{\boldsymbol{Re_L}}\frac{\partial^2 u^*}{\partial {y^*}^2}$$

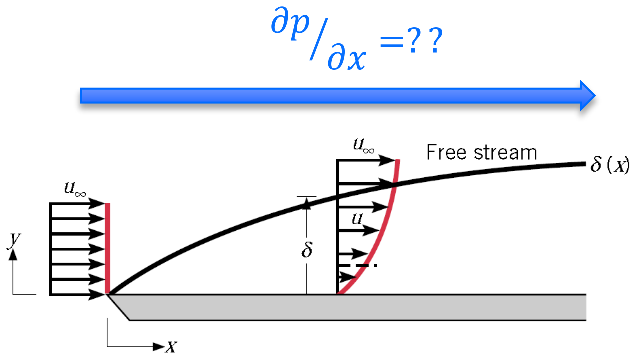
Determines the flow condition

$$Re = \frac{inertia\ forces}{viscous\ forces} = \frac{\rho u_\infty L}{\mu} = \frac{u_\infty L}{\nu}$$

$$\nu = \frac{\mu}{\rho} = kinematic\ viscosity[m^2/s]$$

RECAP of Fluid Dynamics: Velocity Boundary layer equations (LAMINAR FLOW)

Horizontal plate immersed in a flowing fluid
Re < $5 \cdot 10^5$ (laminar flow)



Navier-Stokes equations

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2}$$

The pressure gradient can be obtained writing the Bernoulli equation for the free flow above the boundary layer.

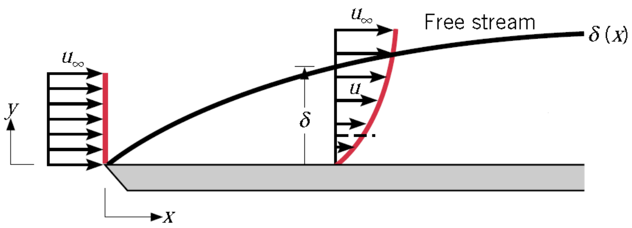
$$\frac{p}{\rho} + \frac{u_\infty^2}{2} = constant \quad \Rightarrow \quad \frac{1}{\rho}\frac{dp}{dx} = -u_\infty \frac{\partial u_\infty}{\partial x} = 0$$

$$\Rightarrow \quad u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2}$$

Solve to find the velocity profile

RECAP of Fluid Dynamics: Velocity Boundary layer equations (LAMINAR FLOW)

$$\partial p / \partial x = 0$$



$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2}$$

streamline ψ such that:

$$u = -\left.\frac{\partial \psi}{\partial y}\right|_x \qquad v = -\left.\frac{\partial \psi}{\partial x}\right|_y$$

$$\eta \equiv y\sqrt{u_\infty/\nu x}$$

$$f(\eta) \equiv \frac{\psi}{u_\infty\sqrt{\nu x/u_\infty}}$$

$$2\frac{d^3 f}{\partial \eta^3} + f\frac{d^2 f}{\partial \eta^2} = 0$$

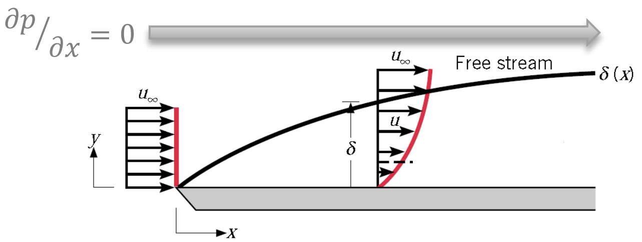
Numerical solution

Velocity profile $\frac{u}{u_\infty} = \frac{df}{d\eta}$

Viscous stress at the wall

$$\tau_w = \mu\left.\frac{\partial u}{\partial u}\right|_{y=0} = \mu u_\infty \frac{\sqrt{u_\infty}}{\sqrt{\nu x}}\left.\frac{d^2 f}{\partial \eta^2}\right|_{\eta=0}$$

RECAP of Fluid Dynamics: Velocity Boundary layer equations (LAMINAR FLOW)



Numerical solution of $$2\frac{d^3 f}{\partial \eta^3} + f\frac{d^2 f}{\partial \eta^2} = 0$$

$y\sqrt{u_\infty/\nu x}$		u/u_∞	$v\sqrt{x/\nu u_\infty}$	
η	$f(\eta)$	$f'(\eta)$	$(\eta f' - f)/2$	$f''(\eta)$
0.00	0.00000	0.00000	0.00000	0.33206
0.20	0.00664	0.06641	0.00332	0.33199
0.40	0.02656	0.13277	0.01322	0.33147
0.60	0.05974	0.19894	0.02981	0.33008
0.80	0.10611	0.26471	0.05283	0.32739
1.00	0.16557	0.32979	0.08211	0.32301
2.00	0.65003	0.62977	0.30476	0.26675
3.00	1.39682	0.84605	0.57067	0.16136
4.00	2.30576	0.95552	0.75816	0.06424
4.918	[illegible]	*0.99000*	0.83344	0.01837
6.00	4.27964	0.99898	0.85712	0.00240
8.00	6.27923	1.00000^-	0.86039	0.00001

- *Boundary layer thickness:*

$$\delta(x):\ \frac{u(y=\delta)}{u_\infty} = 0.99 = \frac{df}{d\eta} = f' \quad \Rightarrow \quad 4.92 = \eta = \delta\sqrt{u_\infty/\nu x}$$

$$\Rightarrow \quad \frac{\delta}{x} = \frac{4.92}{\sqrt{Re_x}} \qquad Re_x = \frac{u_\infty x}{\nu}$$

- *Tangential stress at the wall (τ_w) and friction coefficient (C_f):*

$$\tau_w = \mu u_\infty \frac{\sqrt{u_\infty}}{\sqrt{\nu x}} f''\Big|_{\eta=0} \quad \Rightarrow \quad \tau_w = 0.332\frac{\mu u_\infty}{x}\sqrt{Re_x}$$

$$C_f \equiv \frac{\tau_w}{\rho u_\infty^2/2} \quad \Rightarrow \quad C_f = \frac{0.664}{\sqrt{Re_x}}$$

RECAP of Fluid Dynamics: Velocity Boundary layer equations (LAMINAR FLOW)

What is the thickness of the BL at x=5cm if water at 12C flows over a long plate with u_∞ = 5m/s ?

TABLE A.6 Thermophysical Properties of Saturated Water[a]

Tempera-ture, T (K)	Pressure, p (bars)[b]	Specific Volume (m^3/kg)		Heat of Vapor-ization, h_{fg} (kJ/kg)	Specific Heat (kJ/kg · K)		Viscosity ($N \cdot s/m^2$)	
		$v_f \cdot 10^3$	v_g		$c_{p,f}$	$c_{p,g}$	$\mu_f \cdot 10^6$	$\mu_g \cdot 10^6$
273.15	0.00611	1.000	206.3	2502	4.217	1.854	1750	8.02
275	0.00697	1.000	181.7	2497	4.211	1.855	1652	8.09
280	0.00990	1.000	130.4	2485	4.198	1.858	1422	8.29
285	0.01387	1.000	99.4	2473	4.189	1.861	1225	8.49

This Lecture

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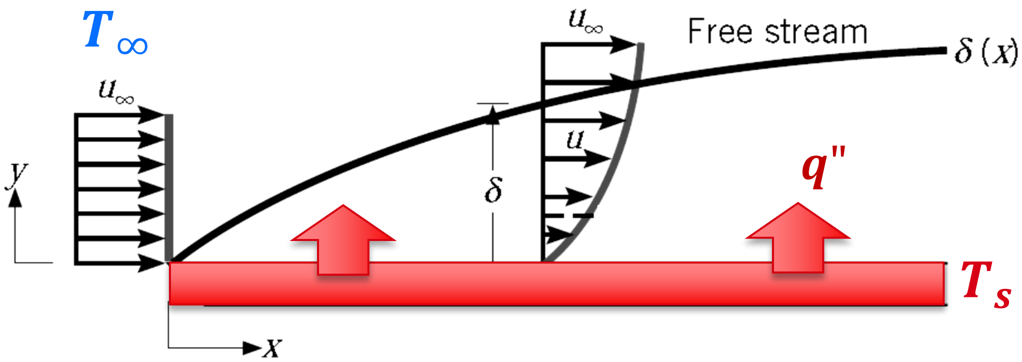
  - ❑ Laminar and Turbulent flow
  - ❑ Viscous and inviscid flow shear stress and friction coefficient
  - ❑ Velocity boundary layer equations
    - ▪ Mass and momentum conservation equations
    - ▪ Dimensionless momentum conservation equation and Re number
    - ▪ Velocity boundary layer over an horizontal flat plate (laminar flow)
- ❑ Newton’s law and the convection coefficient
  - ❑ The thermal boundary layer and Nu number
  - ❑ The problem of convection

Learning Objectives:

- ❑ Calculate the velocity boundary layer thickness and friction coefficient
- ❑ Understand the thermal boundary layer concept
- ❑ Understand the challenges of forced convection

Newton's law and the convection coefficient



$T_s > T_\infty$

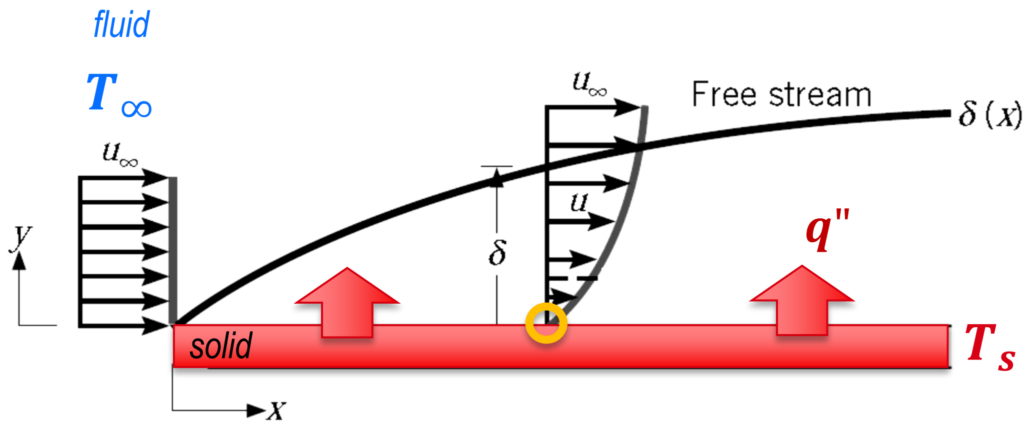
Newton's law: the heat transfer is proportional to the temperature difference between the wall and the unperturbed fluid

$$q'' = h\,(T_s - T_\infty)\ [W/m^2] \quad \text{or} \quad Q = hA\,(T_s - T_\infty)\ [W]$$

where $h = convection\ coefficient$

Where does h come from?

Newton's law and the convection coefficient



$$T_s > T_\infty$$

At the wall the velocity is zero (no advection, only diffusion) thus we can use **Fourier's law**:

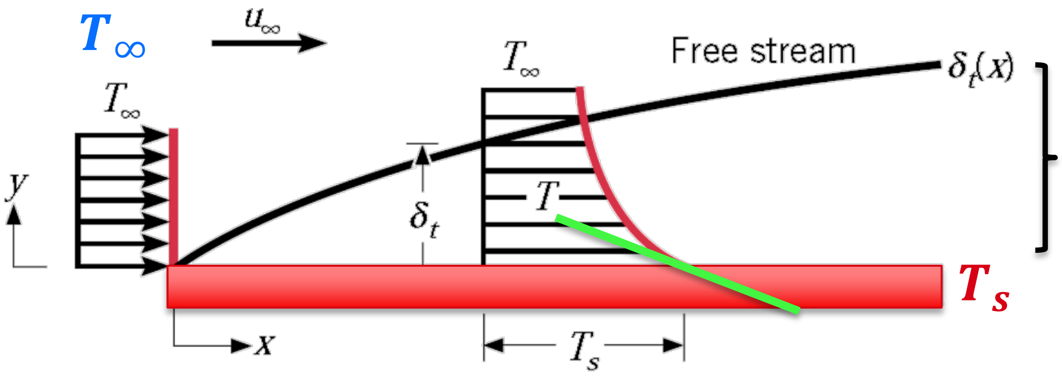
$$q''(0) = -\mathbf{k_f} \left.\frac{\partial T}{\partial y}\right|_{y=0}$$

k_f = thermal conductivity of the **fluid**

$$\Rightarrow \quad h = \frac{q''(0)}{(T_s - T_\infty)} = \frac{-k_f \left.\frac{\partial T}{\partial y}\right|_{y=0}}{(T_s - T_\infty)}$$

The thermal boundary layer and Nu number

$$T_s > T_\infty$$



$$\frac{T_s - T(\delta_t)}{T_s - T_\infty} = 0.99$$

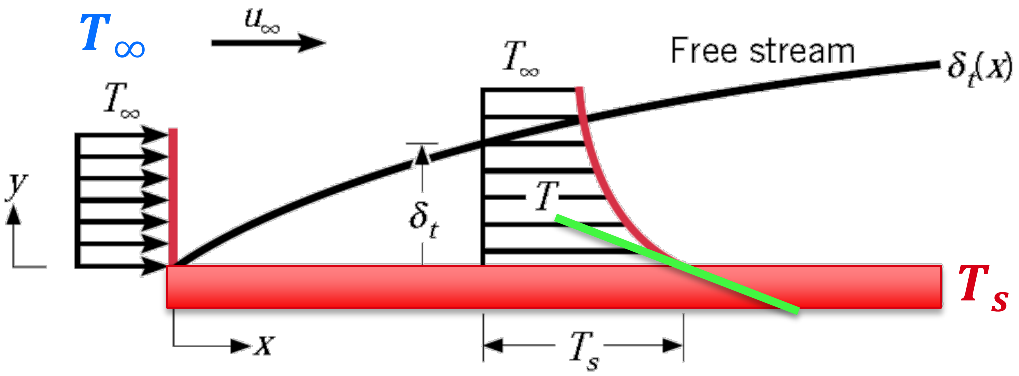
Thermal Boundary Layer

$$h = \frac{-k_f \left.\frac{\partial T}{\partial y}\right|_{y=0}}{(T_s - T_\infty)} \Rightarrow \frac{hL_c}{k_f} = \left.\frac{\partial\left(\frac{T_s - T}{T_s - T_\infty}\right)}{\partial\left(\frac{y}{L_c}\right)}\right|_{y/L_c=0} = Nu_L$$

Nusselt number

L_c is a characteristic dimension of the problem (i.e. length of the plate)

The problem of convection



$$T_s > T_\infty$$

$$h = \frac{-k_f \left.\frac{\partial T}{\partial y}\right|_{y=0}}{(T_s - T_\infty)}$$

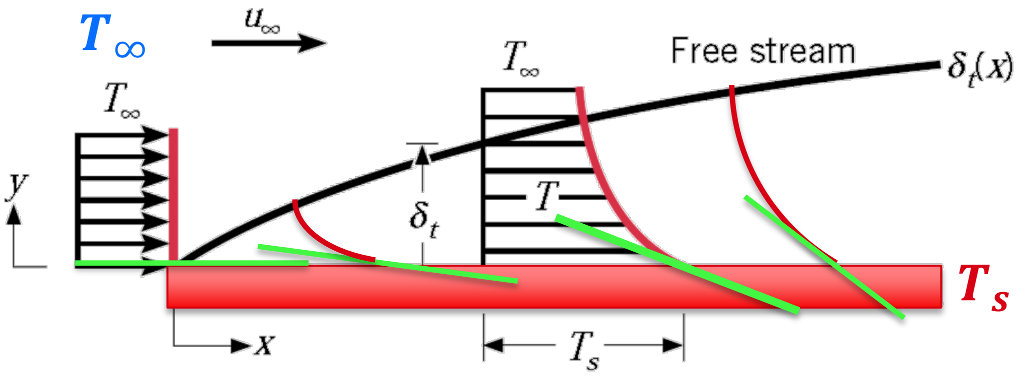
To determine h we need to know the temperature gradient at the wall. This in turn depends on the flow conditions.

To obtain T(x,y), we must solve the coupled momentum (Navier-Stokes) and energy (heat diffusion) equations.

Analytical solutions, however, exist only for simple cases.

For all other cases we rely on **empirical correlations** or numerical simulations.

The problem of convection



$$T_s > T_\infty$$

$$h = \frac{-k_f \left.\frac{\partial T}{\partial y}\right|_{y=0}}{(T_s - T_\infty)}$$

The temperature gradient at the wall changes as the temperature boundary layer develops. Therefore **the convection coefficient varies spatially**.

Local convection coefficient *Average convection coefficient*

$$Q = (T_s - T_\infty)\int_{A_s} h dA_s = \bar{h} A_s (T_s - T_\infty)$$

$$\bar{h} = \frac{1}{A_s}\int_{A_s} h dA_s$$

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Next Lecture

- The thermal boundary layer equations
- Comparison of velocity and thermal boundary layer, Pr number
- Dimensionless numbers and the problem of the boundary layer physical properties

Learning Objectives:

- Understand the thermal boundary layer concept and equations
- Understand and calculate the dimensionless numbers (Re, Nu, Pr)