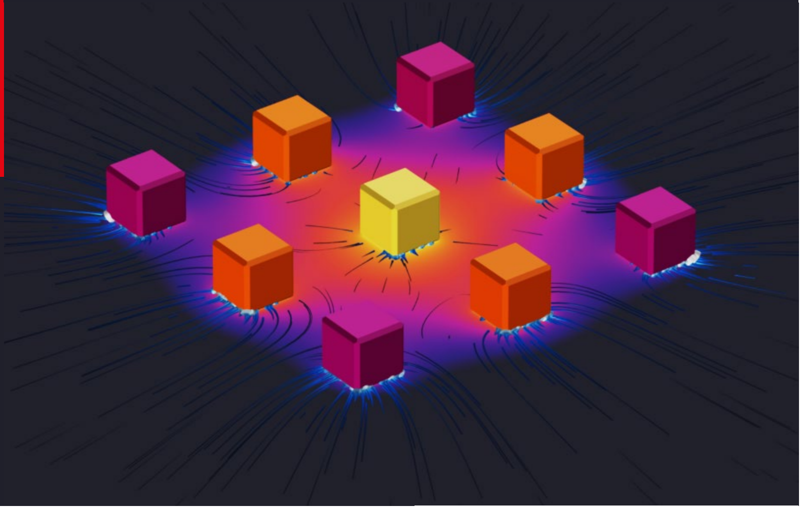
Heat and Mass Transfer
ME-341

Instructor: Giulia Tagliabue



École
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de Lausanne

Spring Semester

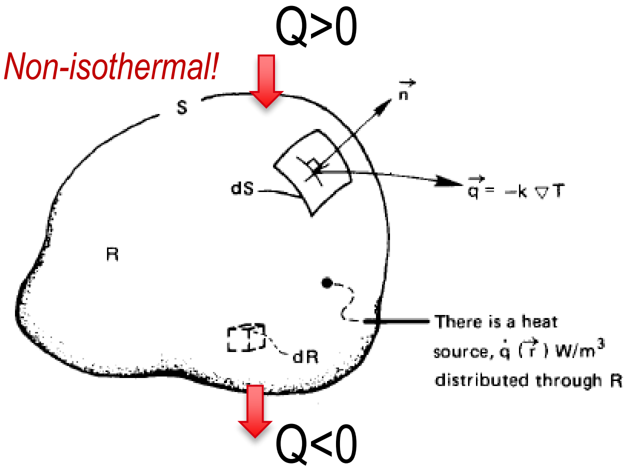
Until Now

- ☑ Heat Diffusion and Boundary Conditions (W1L2-3)
- ☑ Heat Diffusion Equation without Heat sources (W1L3-4; W2L1)
  - ☑ Thermal Resistance & Overall Heat Transfer Coefficient
  - ☑ Bi number
  - ☑ Thermal Circuits
- ☑ Heat Diffusion WITH Heat Sources (W2L2-3)

Learning Objectives:

- ☑ Solve 1D&2D steady state heat transfer problems with/without heat sources

Heat Diffusion Equation – 3D



1st law: $$0 = Q - W + \dot{E}_{gen} - mc\frac{dT}{dt}$$

Fourier law: $$0 = \int_V \left(\nabla\cdot(k\,\nabla T) + \dot{q} - \rho c\frac{\partial T}{\partial t}\right) dR$$

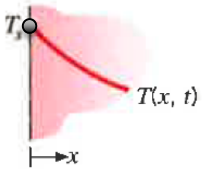
$$\boldsymbol{\nabla^2 T + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}}$$

$$\alpha = \frac{k}{\rho c} = thermal\ diffusivity\ [\frac{m^2}{s}]$$

<u>Assumption 1</u>: incompressible medium
<u>Assumption 2</u>: medium at rest (no convection)
<u>Assumption 3</u>: isotropic material
<u>Assumption 4</u>: *k* is independent of *T*

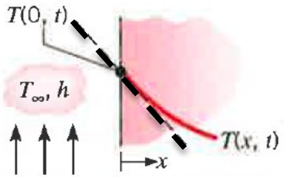
Heat Diffusion Equation – Boundary Conditions

B.C. of the 1st kind (*Dirichlet condition*):
constant surface temperature



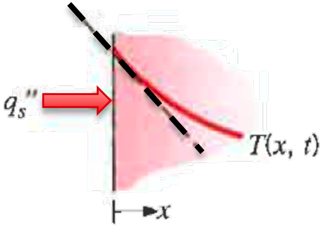
$$T(\vec{x})_{\vec{r}=\vec{r}_i} = T_w$$

B.C. of the 3rd kind (*Robin condition*):
convection surface condition

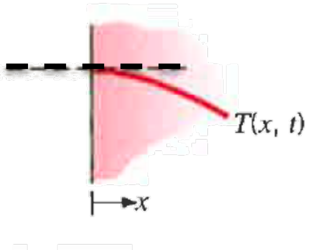


$$-k\left(\frac{\partial T}{\partial x}\right)_{x_i} = h(T(x_i, t) - T_\infty)$$

B.C. of the 2nd kind (*Neumann condition*):
known heat flux



$$-k\left(\frac{\partial T}{\partial x}\right)_{x=x_i} = q_w''$$

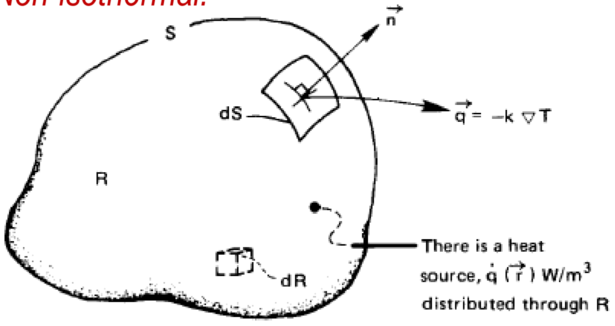


$$-k\left(\frac{\partial T}{\partial x}\right)_{x=x_i} = 0$$

(adiabatic/symmetry)

Heat Diffusion Equation – 3D

Non-isothermal!



$$\nabla^2 T + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

To solve the equation we need:

- Initial condition: $\mathrm{T}(\mathrm{t}=0) = T_i(x, y, z)$
- Boundary conditions

<u>Assumption 1</u>: incompressible medium
<u>Assumption 2</u>: medium at rest (no convection)
<u>Assumption 3</u>: isotropic material
<u>Assumption 4</u>: *k* is independent of *T*

<u>Assumption 5</u>: steady-state ($\partial/\partial t = 0$)

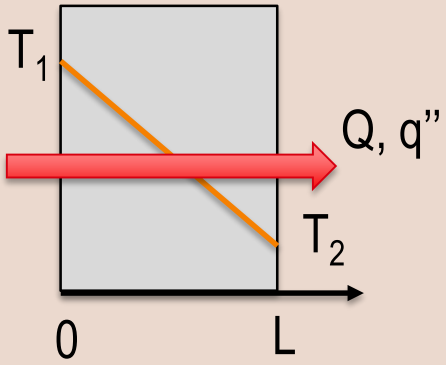
<u>Assumption 6</u>: no heat sources ($\dot{q} = 0$)

$$\boldsymbol{\nabla^2 T = 0}$$

Heat Diffusion Equation – 1D, steady-state, no-heat sources, Dirichlet's BC

Planar Wall

$$T(x) = \frac{T_2 - T_1}{L} x + T_1$$

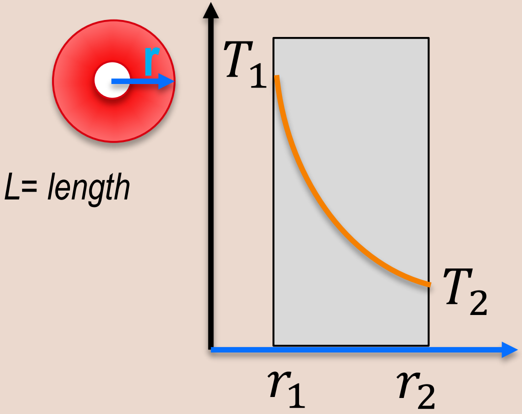


$$Q = -kA\frac{dT}{dx} = -\frac{kA}{L}(T_2 - T_1) = const$$

$$q" = Q/A = const$$

Radial System

$$T(r) = \frac{T_1 - T_2}{\ln(r_1/r_2)} \ln\left(\frac{r}{r_2}\right) + T_2$$

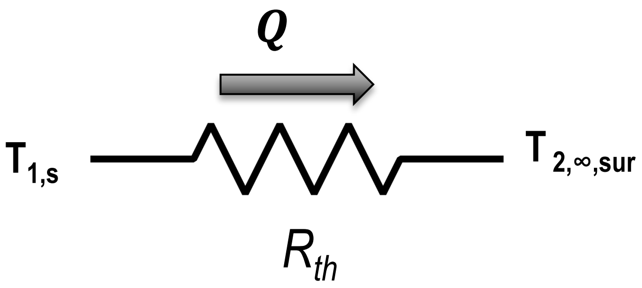


$$Q = -kA\frac{dT}{dr} = -\frac{k(2\pi \not{r} l)}{\ln(r_1/r_2)} \frac{(T_2 - T_1)}{\not{r}} = const$$

q=Q/(2πrl)=q"$(r) \neq$ *const*

Thermal Resistance and Overall Heat Transfer Coefficient

$$Q = \frac{(T_{1,s} - T_{2,\infty sur})}{R_{th}}$$

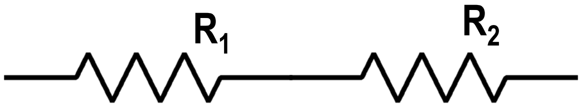


	Planar Wall	***Radial System***
Conduction:	$\frac{L}{kA}$	$\frac{ln(r_2/r_1)}{2\pi Lk}$
Convection:	$\frac{1}{hA}$	$\frac{1}{h2\pi rL}$
Radiation:	$\frac{1}{h_{rad}A}$	$\frac{1}{h_{rad}2\pi rL}$
	$R''_{th} = R_{th}A$	$R'_{th,cyl} = R_{th,cyl}L$

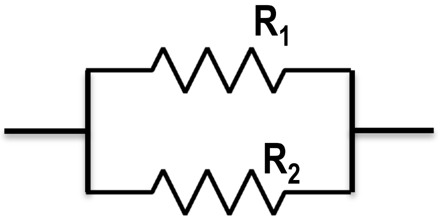
It is valid ONLY if $\dot{q} = 0$
(no heat sources)

Overall heat transfer coefficient: $U = \frac{1}{R_{th}A}$

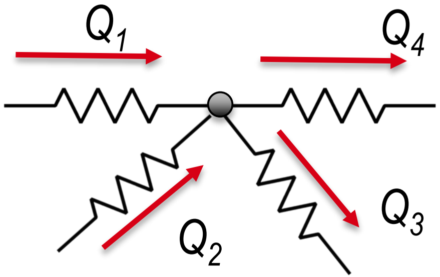
Thermal Circuits



$$R_{series} = R_1 + R_2 + .. = \sum R_i$$



$$\frac{1}{R_{parallel}} = \frac{1}{R_1} + \frac{1}{R_2} + .. = \sum \frac{1}{R_i}$$

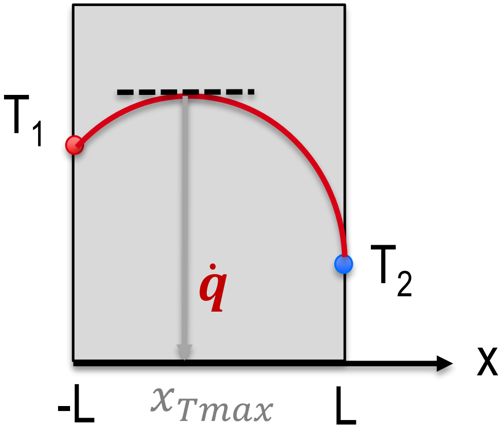


Kirchoff's 1st Law:

$$\sum Q_i = 0$$

$$Q_i > 0 \ enters$$
$$Q_i < 0 \ exits$$

Heat Diffusion Equation – 1D, steady-state, WITH heat sources



$$T(x) = \frac{\dot{q}}{2k} L^2 \left(1 - \frac{x^2}{L^2}\right) + \frac{T_2 - T_1}{2}\frac{x}{L} + \frac{T_1 + T_2}{2}$$

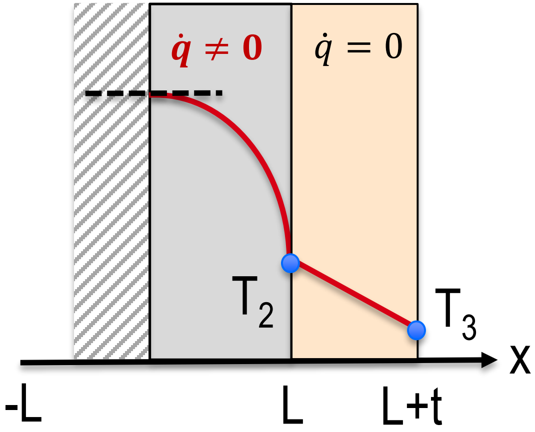
$$Q = -kA\frac{\partial T}{\partial x} = -kA\left(-\frac{\dot{q}}{k}x + \frac{T_2 - T_1}{2L}\right) = Q(x)$$

The heat transfer rate is not constant!
It depends on the position along x.

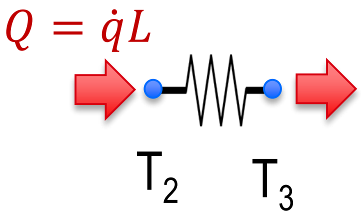
$$Q(x) \neq \frac{\Delta T}{R_{th}}$$

The electrical analogy fails!
We cannot use the thermal resistance concept in layers with heat sources.

Heat Diffusion Equation – 1D, steady-state, WITH heat sources



$$T(x) = \frac{\dot{q}}{2k} L^2 \left(1 - \frac{x^2}{L^2}\right) + \frac{T_2 - T_1}{2}\frac{x}{L} + \frac{T_1 + T_2}{2}$$



We can draw an equivalent electrical circuit only for the layers WITHOUT heat sources. Layers WITH heat sources inject into the equivalent circuit a certain Q

This Week

- Heat transfer from extended surfaces (Fins)
- Fins Performance
- Array of Fins

Learning Objectives:

- Understand the concept of fins
- Calculate the temperature profile in fins with constant cross-section
- Calculate the performance of a fin-based system

This Lecture

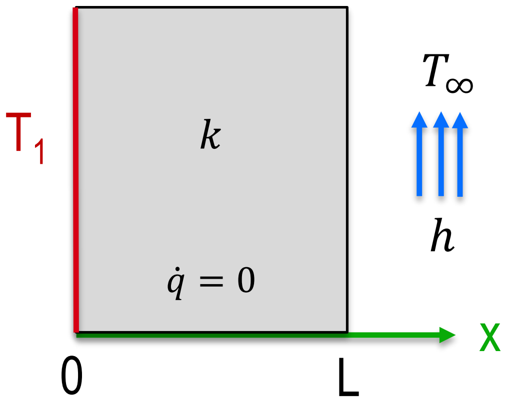
- ❑ Heat transfer from extended surfaces (Fins)

Learning Objectives:

- ❑ Understand the concept of fins

Heat Transfer from Extended Surfaces

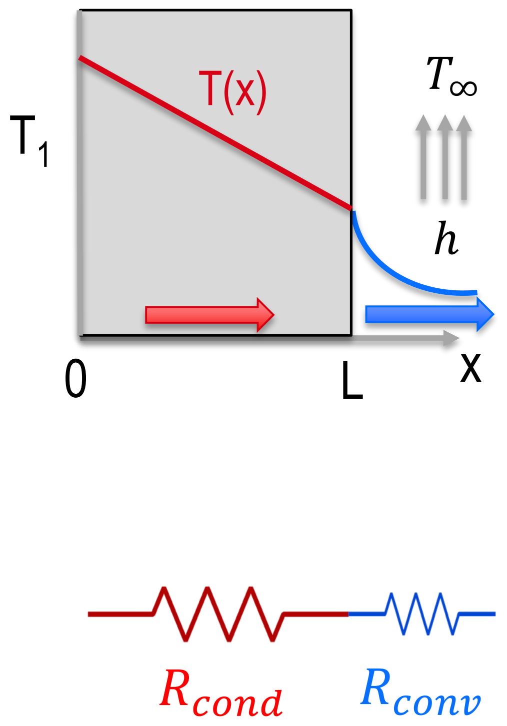
Case 1



- Identify the thermal resistances
- Indicate with an arrow the direction of the conductive heat transfer rate and the convective heat transfer rate
- Draw the electrical circuit equivalent to each problem
- Sketch T(x) in the solid

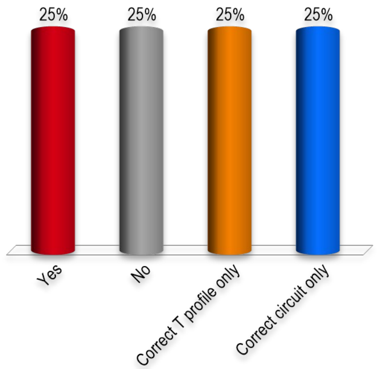
Heat Transfer from Extended Surfaces

Case 1



Did you draw this thermal circuit and T profile?

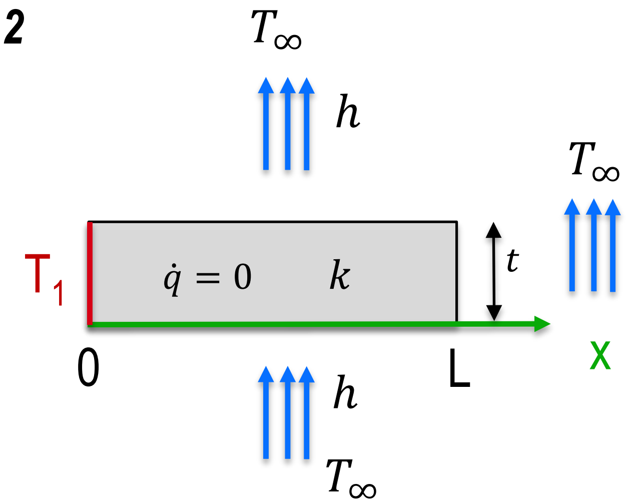
A. Yes
B. No
C. Correct T profile only
D. Correct circuit only



Heat Transfer from Extended Surfaces

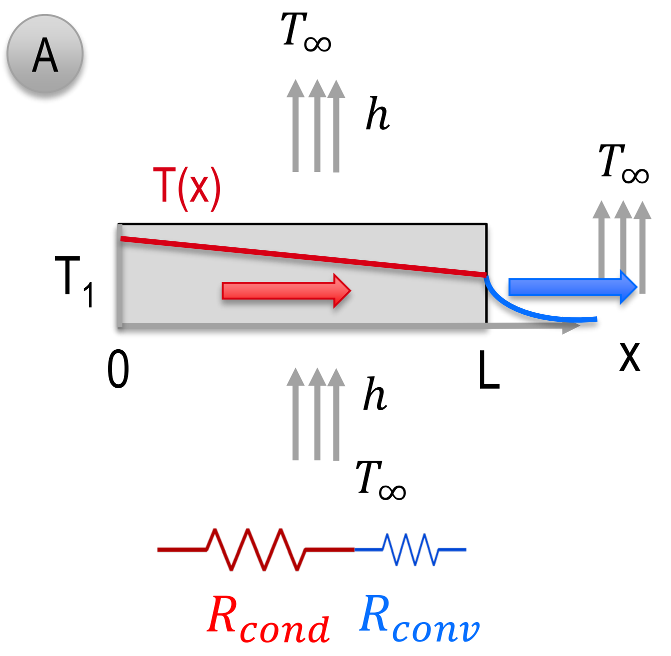
- Identify the thermal resistances
- Indicate with an arrow the direction of the conductive heat transfer rate and the convective heat transfer rate
- Draw the electrical circuit equivalent to each problem
- Sketch T(x) in the solid

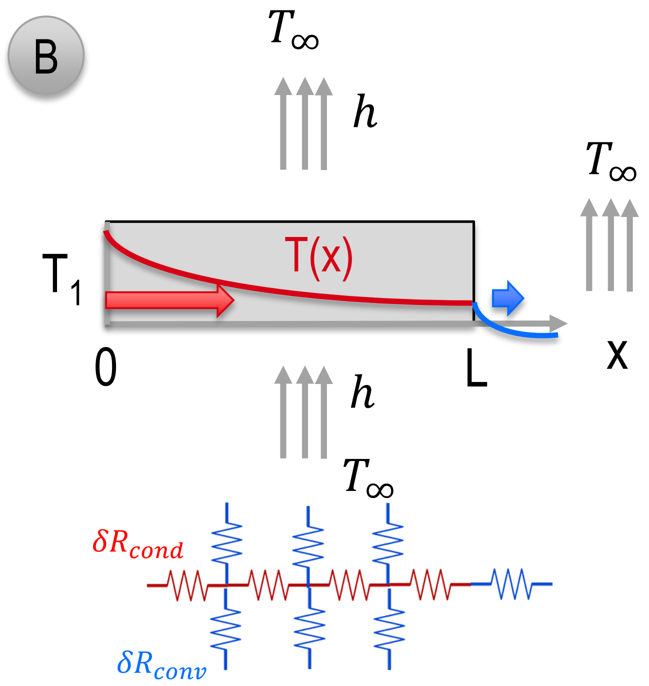
Case 2



Hypothesis: the thickness *t* of the solid is so small or the thermal conductivity *k* so high that the temperature is uniform along the y-direction, i.e. $T = T(x)$

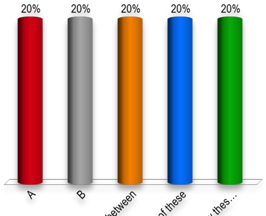
Heat Transfer from Extended Surfaces





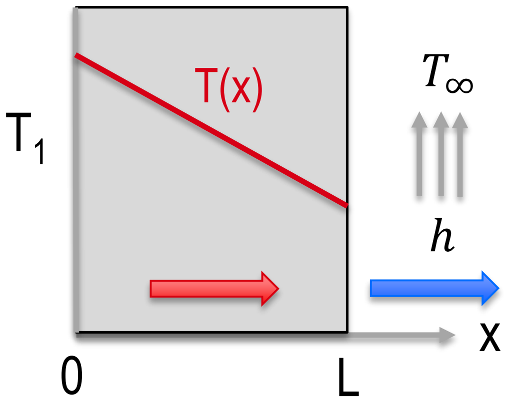
What electrical circuit and T profile did you draw?

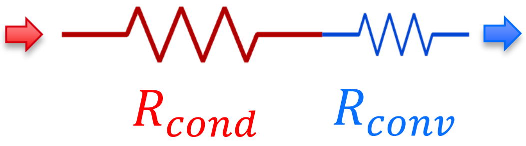
A. A
B. B
C. Something in between
D. None of these
E. I could not draw these quantities



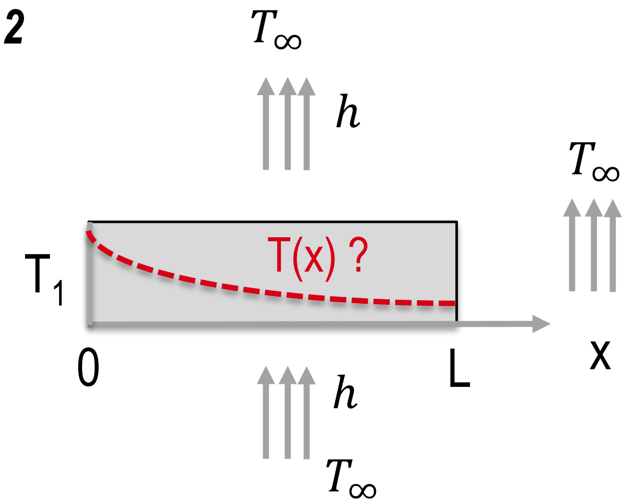
Heat Transfer from Extended Surfaces

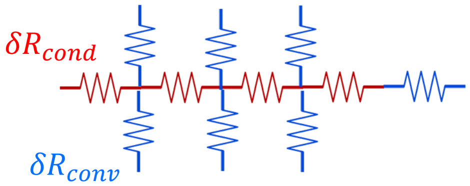
Case 1





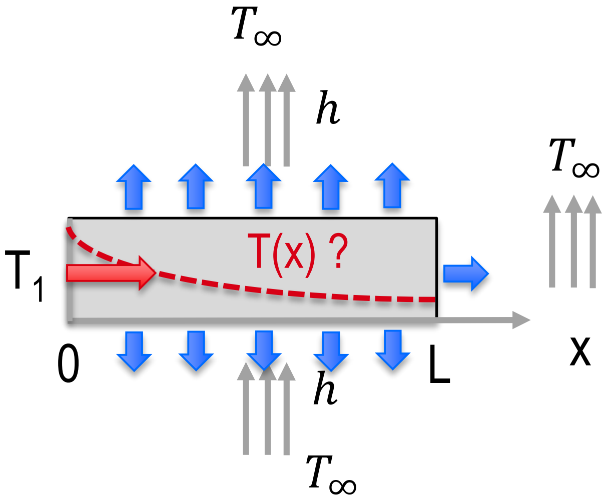
Case 2



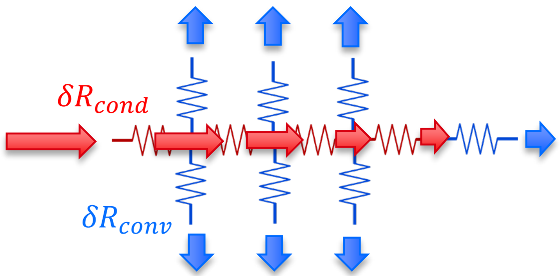


Heat Transfer from Extended Surfaces

Based on our hypothesis, the temperature of the solid is uniform along the y-direction hence at any position along x, the surface and the bulk have the same value, i.e. $T = T(x) = T_s(x)$



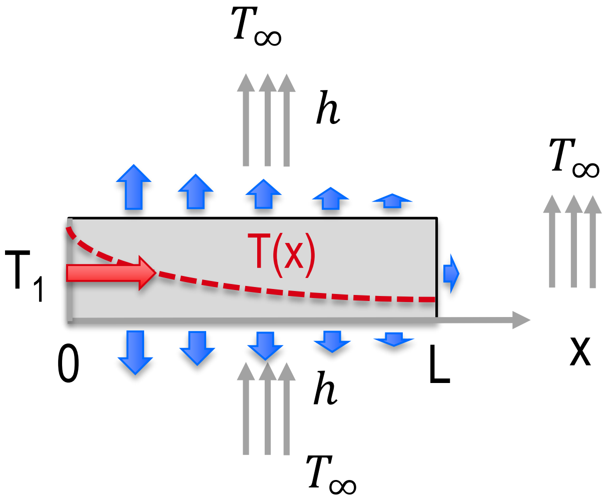
As we move along the solid, part of the heat is removed by convection. Hence the heat transfer rate by conduction along x decreases.



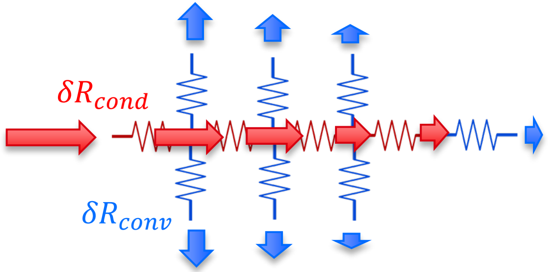
Heat Transfer from Extended Surfaces

$$q'' = \bar{h}\,(T_s(x) - T_\infty) \propto T_s(x) - T_\infty$$

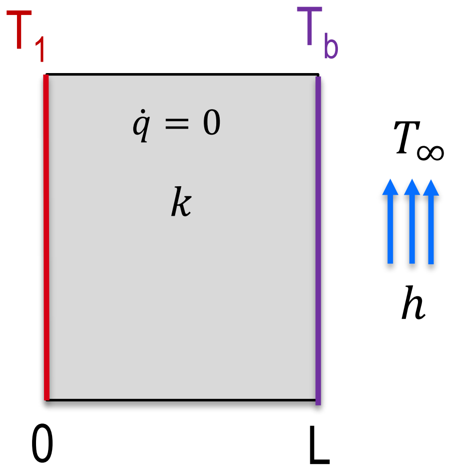
As the temperature of the solid decreases and gets more similar to the fluid temperature, T_∞, the heat flux by convection decreases



As we move along the solid, part of the heat is removed by convection. Hence the heat transfer rate by conduction along x decreases.



Heat Transfer from Extended Surfaces



$$T_1 = 600C \qquad T_\infty = 5C$$
$$h = 50W/m^2K \quad k = 0.5W/mK$$
$$H = 2m \quad W = 1m \quad L = 0.5m$$

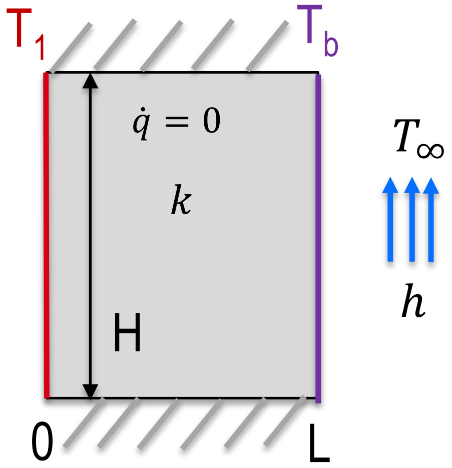
$$Q_x = 1167W < Q_{required} = 1500W$$

Verify that the total heat transfer rate Qx is lower than the required one.

What can you do to solve the problem?

Note that the physical dimensions (H,W,L) and the material (k) cannot be changed because of structural reasons.

Heat Transfer from Extended Surfaces

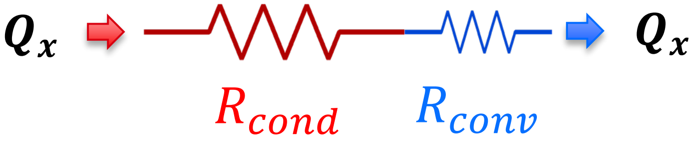


In many technical problems, T_1 and R_{cond} are fixed by the operating conditions and the mechanical requirements of a structure. Then the heat transfer rate is primarily controlled by convection. If we need to increase Q_x then we can:

(i) decrease T_∞;

(ii) decrease $R_{conv} = {}^1/_{hA}$ by increasing h or A.

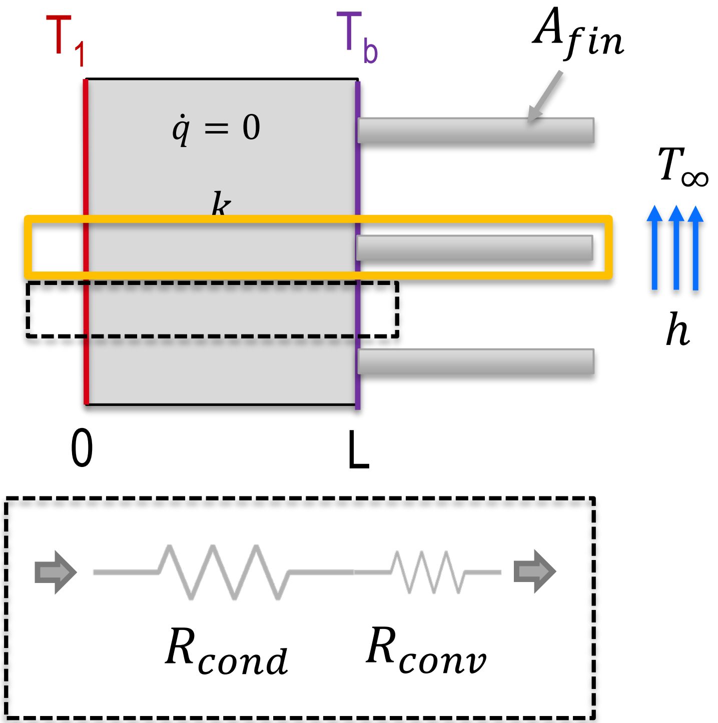
Decreasing T_∞ and increasing h can have a high energy cost so we increase A by adding a fin.

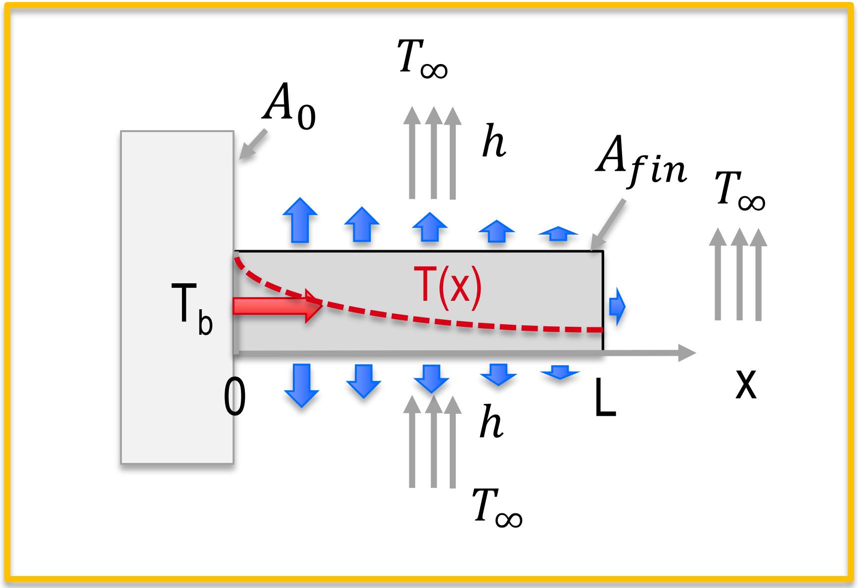


$$R_{cond} = \frac{L}{kA} \qquad R_{conv} = \frac{1}{hA}$$

$$Q_x = \frac{T_1 - T_\infty}{R_{cond} + R_{conv}}$$

Heat Transfer from Extended Surfaces





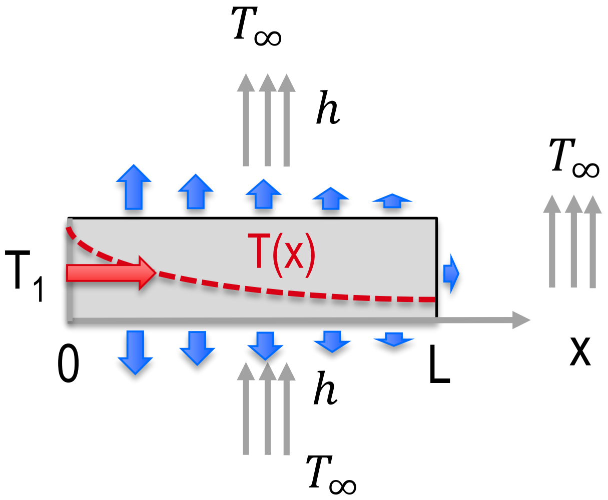
Heat Transfer from Extended Surfaces

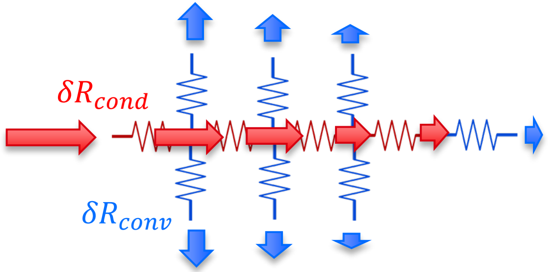
$$q'' = \bar{h}\,(T_s(x) - T_\infty) \propto T_s(x) - T_\infty$$

As the temperature of the solid decreases and gets more similar to the fluid temperature, the heat transfer rate by convection decreases

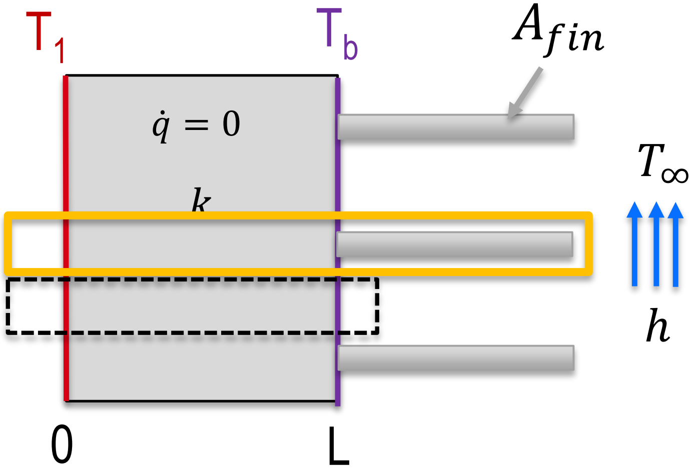
$$Q = \bar{h}A\,(T_s - T_\infty) \propto A$$

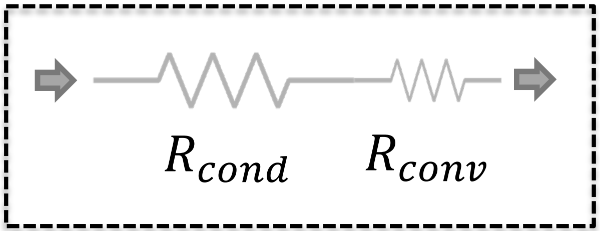
Increasing the surface area, the total heat transfer rate increases

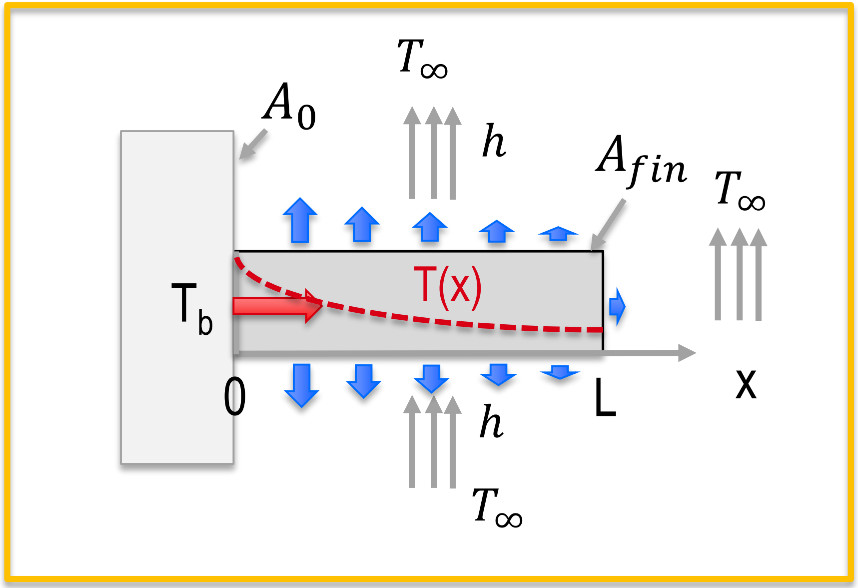




Heat Transfer from Extended Surfaces

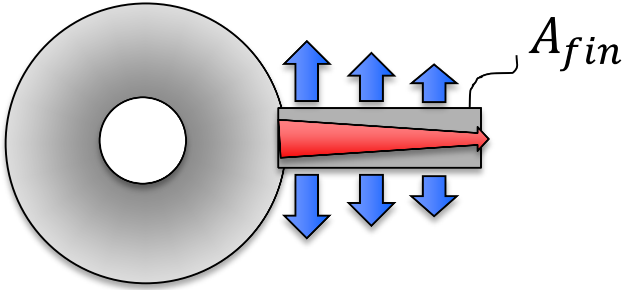


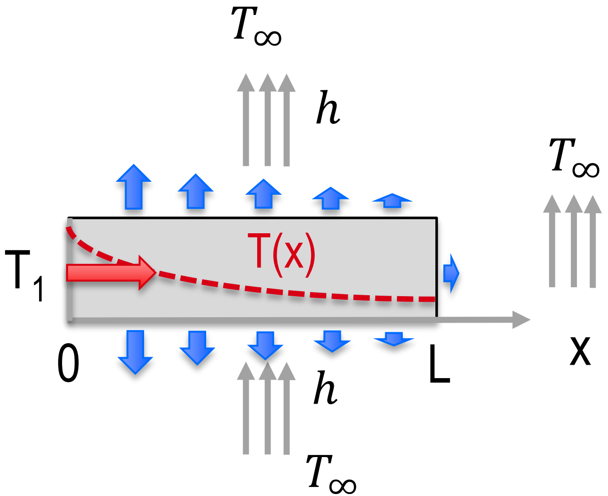


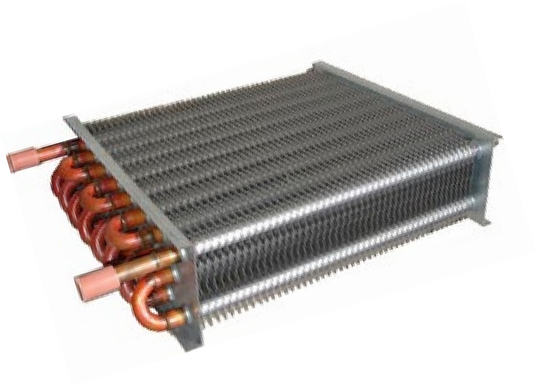


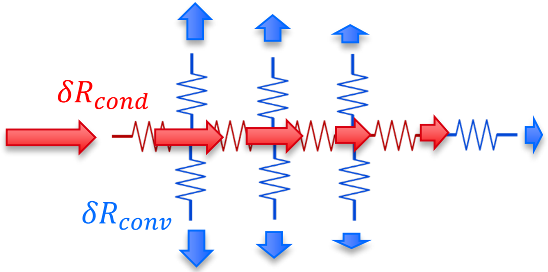
The fin has a more complex behavior than the simple planar system because heat transfer by conduction and convection occur along different directions and are interdependent. Hence a simple equivalent electrical circuit cannot be immediately identified.

Heat Transfer from Extended Surfaces





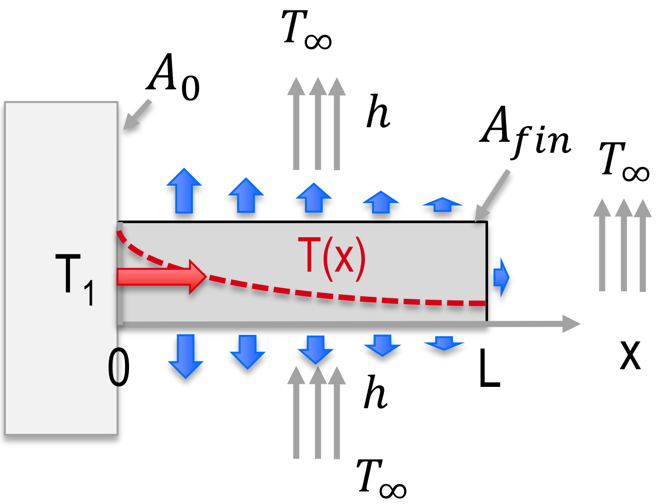




Heat Transfer from Extended Surfaces

$$q'' = \bar{h}\,(T_s(x) - T_\infty) \propto T_s(x) - T_\infty$$

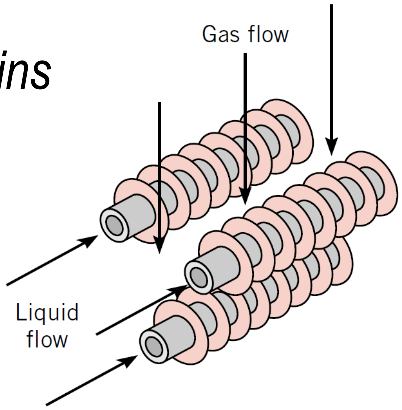
$$Q = \bar{h}A\,(T_s - T_\infty) \propto A = A_0 + A_{fin}$$

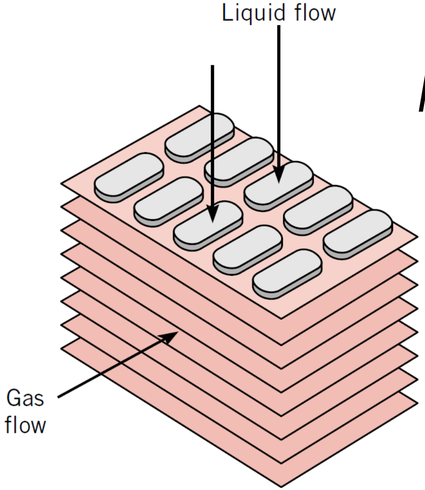


It is necessary to **optimize the fin** in order to maximize the advantage of adding extra surface area (A_{fin}) without wasting material (beyond a certain length L, the temperature difference will be so small that the heat transfer becomes negligible)

Heat Transfer from Extended Surfaces

Radial Fins



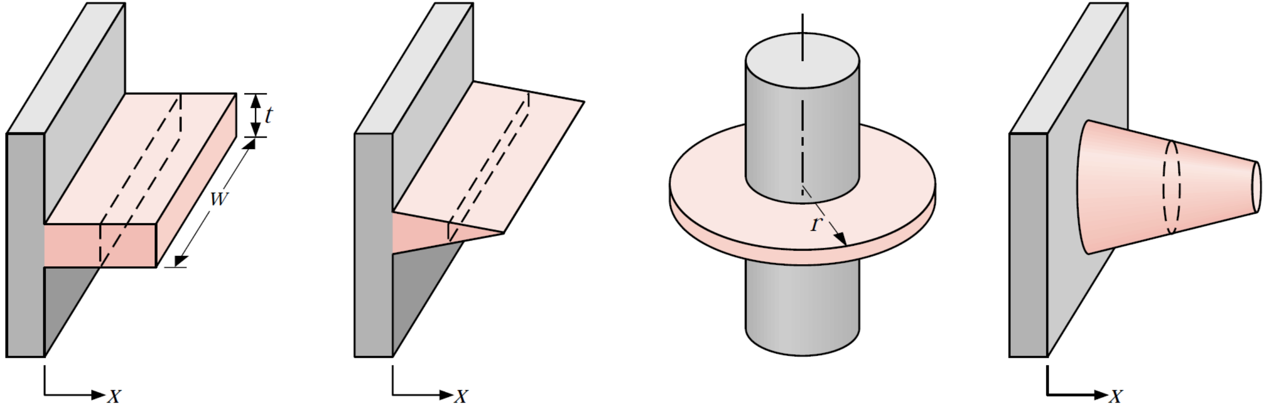


Planar Fins

It is necessary to **optimize the fin** in order to maximize the advantage of adding extra surface area (A_{fin}) without wasting material (beyond a certain length L, the temperature difference will be so small that the heat transfer becomes negligible)

Heat Transfer from Extended Surfaces

Uniform Cross-Section	Non-uniform Cross-Section



What is the temperature profile along the fin, $T = T(x)$?

This Lecture

- ☑ Heat transfer from extended surfaces (Fins)

Learning Objectives:

- ☑ Understand the concept of fins

Next lecture

- ❑ Heat transfer from extended surfaces (Fins)
  - ❑ Boundary Conditions and Temperature Profiles

Learning Objectives:

- ❑ Calculate the temperature profile in fins with constant cross-section