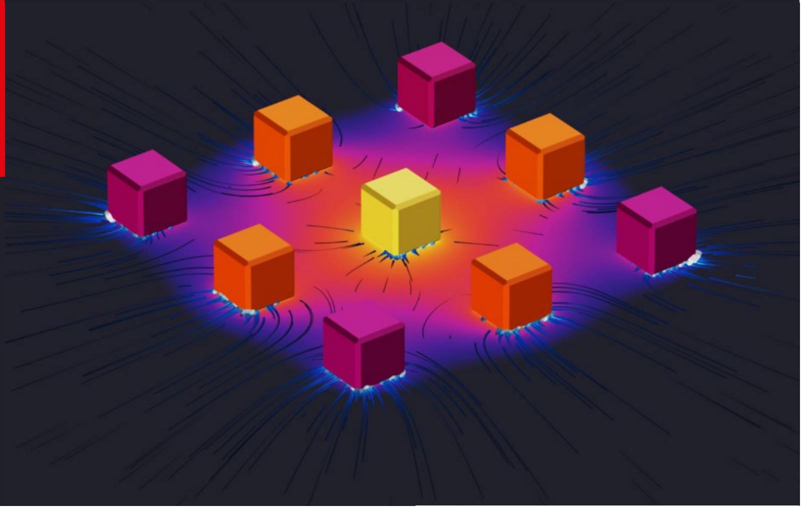
Heat and Mass Transfer
ME-341

Instructor: Giulia Tagliabue



École
polytechnique
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de Lausanne

Spring Semester

Previously

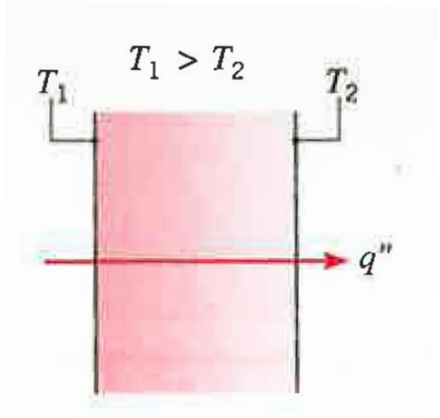
- ☑ Intro to heat transfer and Transport Laws
- ☑ Heat diffusion equation; Thermal conductivity and diffusivity
- ☑ Types of boundary conditions; Planar and cylindrical (1D) solutions
- ☑ Thermal resistance; Bi Number
- ☑ Intro to Thermal Circuits

Learning Objectives:

- ☑ Identify the types of heat transfer mechanisms
- ☑ Derive the general heat diffusion equation; Understand material parameters
- ☑ Identify the possible boundary conditions; Express them mathematically
- ☑ Calculate the temperature profile in a planar or cylindrical wall
- ☑ Calculate the thermal resistances; Calculate the Bi number
- ☑ Solve 1D problems using thermal circuits

Transport Laws

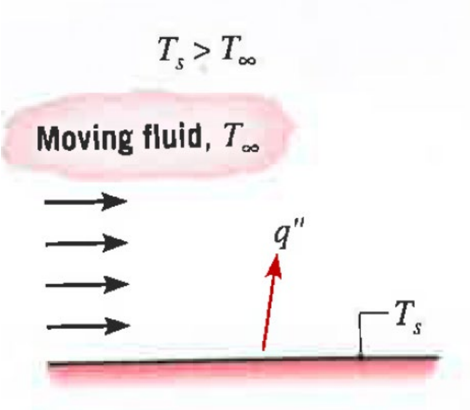
Conduction



Fourier's Law

$$q'' = -k\frac{dT}{dx}$$

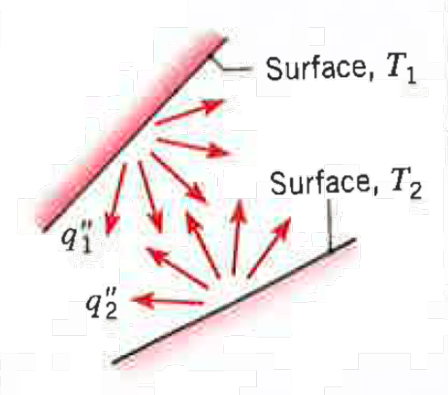
Convection



Newton's Law

$$q'' = \bar{h}\,(T_s - T_\infty)$$

Radiation

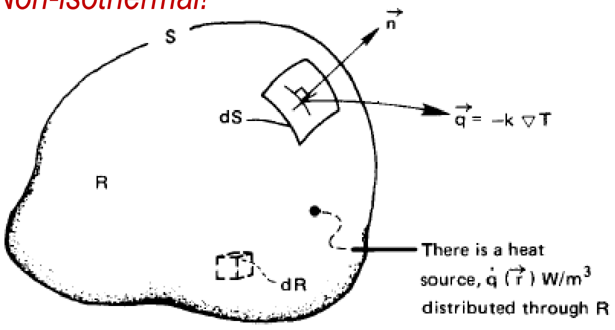


Stefan-Boltzmann Law

$$Q_{rad} = \varepsilon\sigma A_s(T^4 - T_{sur}^4)$$

Heat Diffusion Equation – 3D

Non-isothermal!



$$\nabla^2 T + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

To solve the equation we need:

- Initial condition: $\mathrm{T(t = 0)} = T_i(x, y, z)$
- **Boundary conditions**

<u>Assumption 1</u>: incompressible medium
<u>Assumption 2</u>: medium at rest (no convection)
<u>Assumption 3</u>: isotropic material
<u>Assumption 4</u>: *k* is independent of *T*

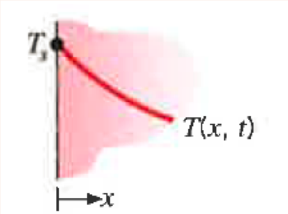
<u>Assumption 5</u>: steady-state ($\partial/\partial t = 0$)

<u>Assumption 6</u>: no heat sources ($\dot{q} = 0$)

$$\boldsymbol{\nabla^2 T = 0}$$

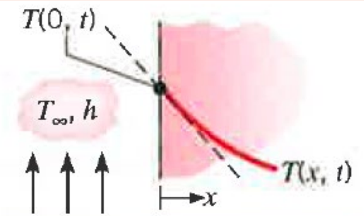
Heat Diffusion Equation – Boundary Conditions

B.C. of the 1st kind (*Dirichlet condition*):
constant surface temperature



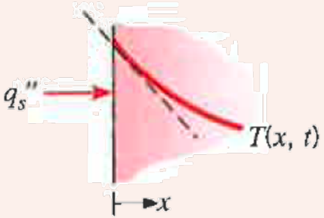
$$T(\vec{r})_{\vec{r}=\vec{r}_i} = T_w$$

B.C. of the 3rd kind (*Robin condition*):
convection surface condition



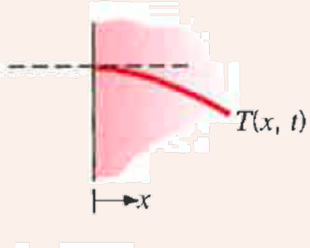
$$-k\left(\frac{\partial T}{\partial x}\right)_{x_i} = h(T(x_i, t) - T_\infty)$$

B.C. of the 2nd kind (*Neumann condition*):
known heat flux



First derivative ⟷ Heat flux

$$-k\left(\frac{\partial T}{\partial x}\right)_{x=x_i} = q_w''$$



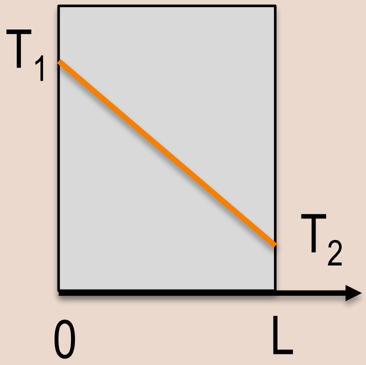
$$-k\left(\frac{\partial T}{\partial x}\right)_{x=x_i} = 0$$

(adiabatic)

Heat Diffusion Equation – 1D, steady-state, no-heat sources, Dirichlet's BC

Planar Wall

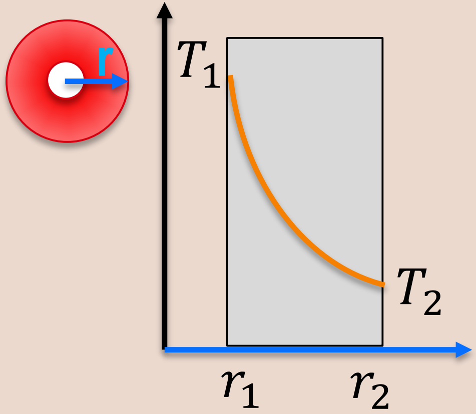
$$T(x) = \frac{T_2 - T_1}{L} x + T_1$$



Linear T !!

Radial System

$$T(r) = \frac{T_1 - T_2}{\ln(r_1/r_2)} \ln\left(\frac{r}{r_2}\right) + T_2$$

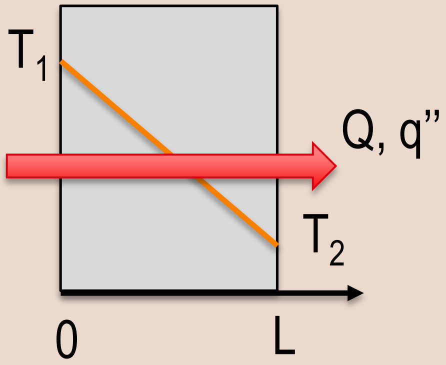


Logarithmic T!!

Heat Diffusion Equation – 1D, steady-state, no-heat sources, Dirichlet's BC

Planar Wall

$$T(x) = \frac{T_2 - T_1}{L} x + T_1$$

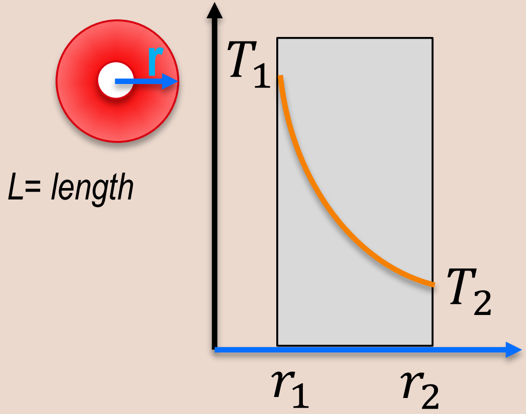


$$Q = -kA\frac{dT}{dx} = -\frac{kA}{L}(T_2 - T_1) = const$$

$$q'' = Q/A = const$$

Radial System

$$T(r) = \frac{T_1 - T_2}{\ln(r_1/r_2)} \ln\left(\frac{r}{r_2}\right) + T_2$$

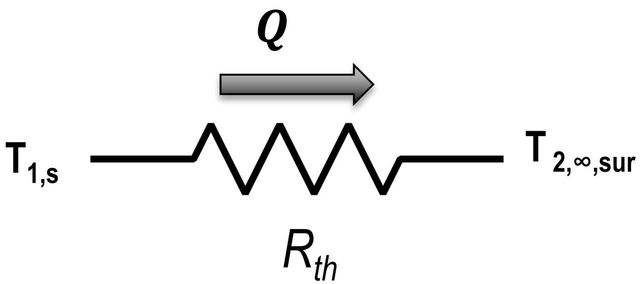


$$Q = -kA\frac{dT}{dr} = -\frac{k(2\pi \cancel{r} l)}{\ln(r_1/r_2)}\frac{(T_2 - T_1)}{\cancel{r}} = const$$

$$q = Q/(2\pi r l) = q''(r) \neq const$$

Thermal Resistance

$$Q = \frac{(T_{1,s} - T_{2,\infty sur})}{R_{th}}$$



	Planar Wall	Radial System
Conduction:	$\frac{L}{kA}$	$\frac{ln(r_2/r_1)}{2\pi Lk}$
Convection:	$\frac{1}{hA}$	$\frac{1}{h2\pi rL}$
Radiation:	$\frac{1}{h_{rad}A}$	$\frac{1}{h_{rad}2\pi rL}$

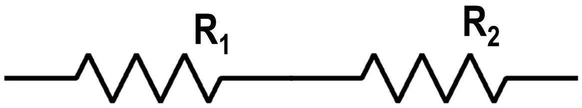
$$q" = \frac{Q}{A} = \frac{(T_{1,s} - T_{2,\infty sur})}{AR_{th}} = \frac{(T_{1,s} - T_{2,\infty sur})}{R"_{th}}$$

$$R^{"}_{th} = R_{th}A$$

$$q' = \frac{Q}{L} = \frac{(T_{1,s} - T_{2,\infty sur})}{lR_{th,cyl}} = \frac{(T_{1,s} - T_{2,\infty sur})}{R'_{th,cyl}}$$

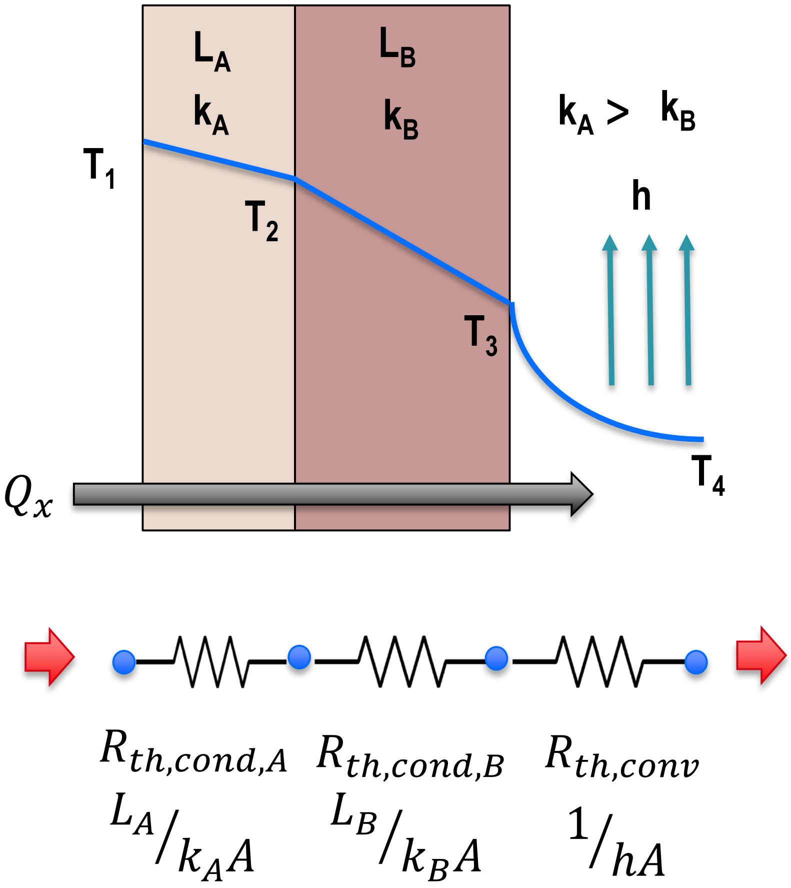
$$R'_{th,cyl} = R_{th,cyl}L$$

Thermal Resistance



$$R_{series} = R_1 + R_2 + .. = \sum R_i$$

Thermal Resistance



$A = 4m^2$

$T_1 = 900C = 1173K$
$T_4 = 10C = 283K$

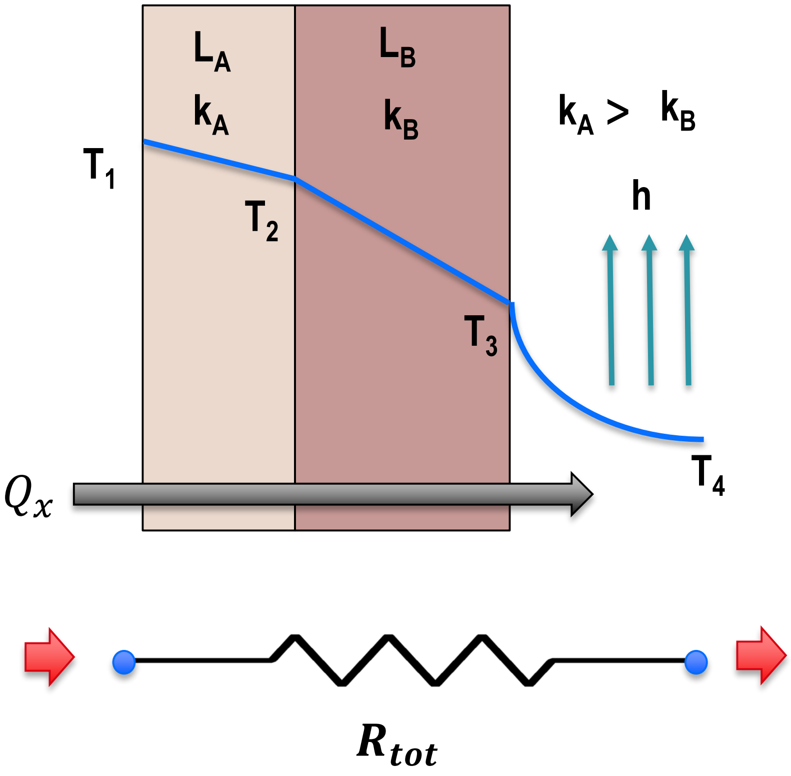
$L_A = 10\ cm = 0.1m$
$L_B = 50\ \mathrm{cm} = 0.5\mathrm{m}$

$k_A = 100\ {}^{W}/_{mK}$
$k_B = 0.1\ {}^{W}/_{mK}$

$h = 50\ {}^{W}/_{m^2K}$

$$Q_x = \frac{T_1 - T_2}{R_{th,cond,A}} = \frac{T_2 - T_3}{R_{th,cond,B}} = \frac{T_3 - T_4}{R_{th,conv}}$$

Thermal Resistance



$$A = 4m^2 \qquad T_1 = 900C = 1173K \qquad L_A = 10\ cm = 0.1m$$

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$$k_A = 100\ {}^{W}\!/_{mK} \qquad k_B = 0.1\ {}^{W}\!/_{mK} \qquad h = 50\ {}^{W}\!/_{m^2K}$$

$$Q_x = \frac{T_1 - T_2}{R_{th,cond,A}} = \frac{T_2 - T_3}{R_{th,cond,B}} = \frac{T_3 - T_4}{R_{th,conv}}$$

$$Q_x = \frac{\boldsymbol{T_1 - T_4}}{\boldsymbol{R_{tot,series}}} = \frac{T_1 - T_4}{R_{th,cond,A} + R_{th,cond,B} + R_{th,conv}}$$

Q = 708.6 W, T2 = 899.8 C, T3 = 13.54 C

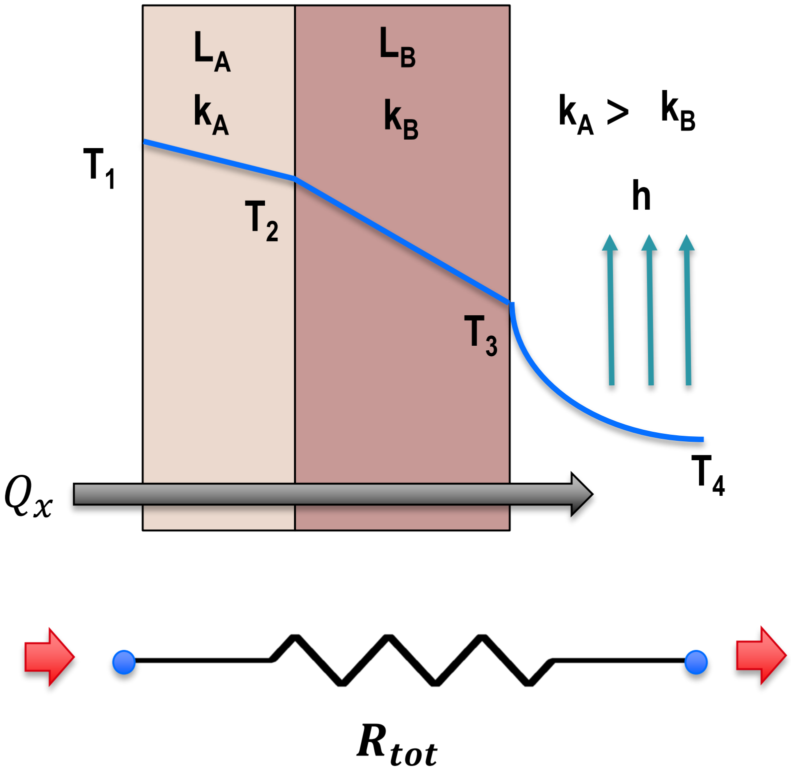
This Lecture

- Overall Heat Transfer Coefficient
- Bi number
- Thermal Circuits

Learning Objectives:

- Understand the Overall Heat Transfer Coefficient
- Solve 1D steady state heat transfer problems without heat sources using thermal circuits

Overall Heat Transfer Coefficient



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$k_A = 100\ {}^{W}/_{mK}$
$k_B = 0.1\ {}^{W}/_{mK}$

$h = 50\ {}^{W}/_{m^2K}$

$$\boldsymbol{Q_x} = \frac{T_1 - T_4}{R_{tot,series}} = \boldsymbol{UA(T_1 - T_4)}$$

***U* = Overall heat transfer coefficient**

$$U = \frac{1}{R_{tot,series}A} = \frac{1}{R''_{tot,series}}$$

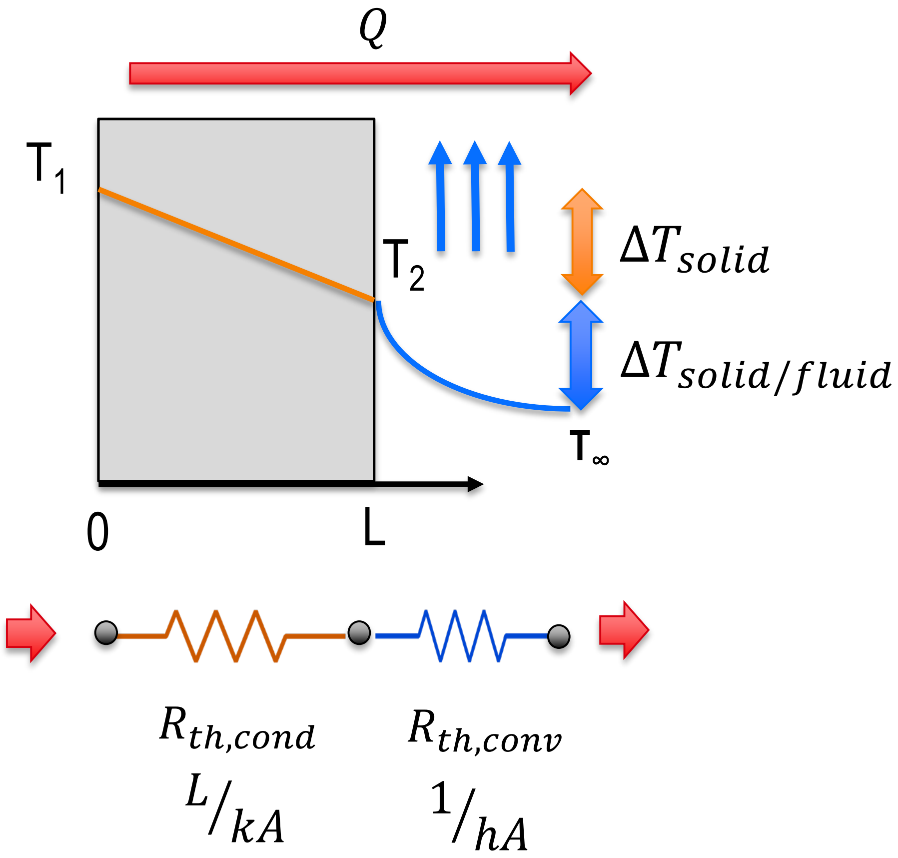
This Lecture

- ☑ Overall Heat Transfer Coefficient
- ❑ Bi number
- ❑ Thermal Circuits

Learning Objectives:

- ☑ Understand the Overall Heat Transfer Coefficient
- ❑ Solve 1D steady state heat transfer problems without heat sources using thermal circuits

Biot Number



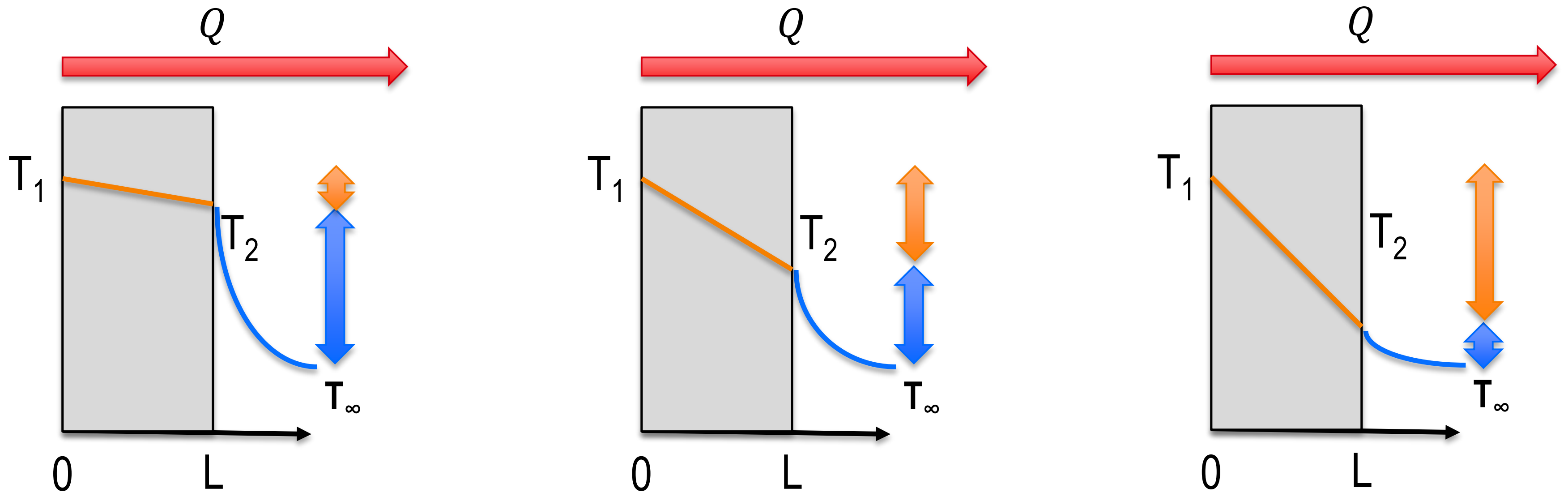
$$Q = \frac{(T_1 - T_2)}{R_{th,cond}} = \frac{(T_2 - T_\infty)}{R_{th,conv}}$$

$$\Rightarrow \frac{(T_1 - T_2)}{(T_2 - T_\infty)} = \frac{R_{th,cond}}{R_{th,conv}} \equiv Bi$$

$$\Rightarrow \boldsymbol{Bi \equiv \frac{hL}{k}}$$

Note: L can be generalized to be a characteristic dimension of a body (e.g. diameter of a sphere)

Biot Number



$\Delta T_{solid} \ll \Delta T_{solid/fluid}$

$\boldsymbol{Bi \equiv \frac{hL}{k} \ll 1}$

Solid has ~constant temperature

$\Delta T_{solid} \sim \Delta T_{solid/fluid}$

$\boldsymbol{Bi \equiv \frac{hL}{k} \sim 1}$

$\Delta T_{solid} \gg \Delta T_{solid/fluid}$

$\boldsymbol{Bi \equiv \frac{hL}{k} \gg 1}$

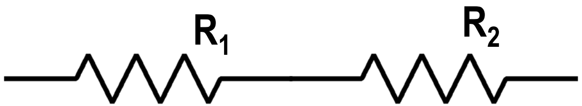
This Lecture

- [x] Overall Heat Transfer Coefficient
- [x] Bi number
- [ ] Thermal Circuits

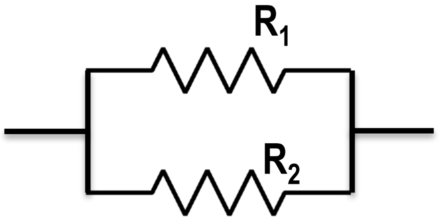
Learning Objectives:

- [x] Understand the Overall Heat Transfer Coefficient
- [ ] Solve 1D steady state heat transfer problems without heat sources using thermal circuits

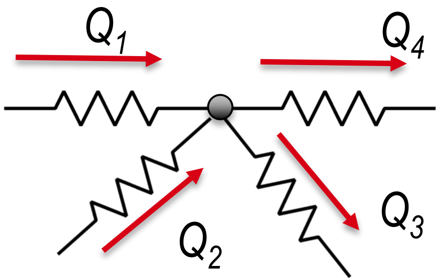
Thermal Circuits



$$R_{series} = R_1 + R_2 + .. = \sum R_i$$



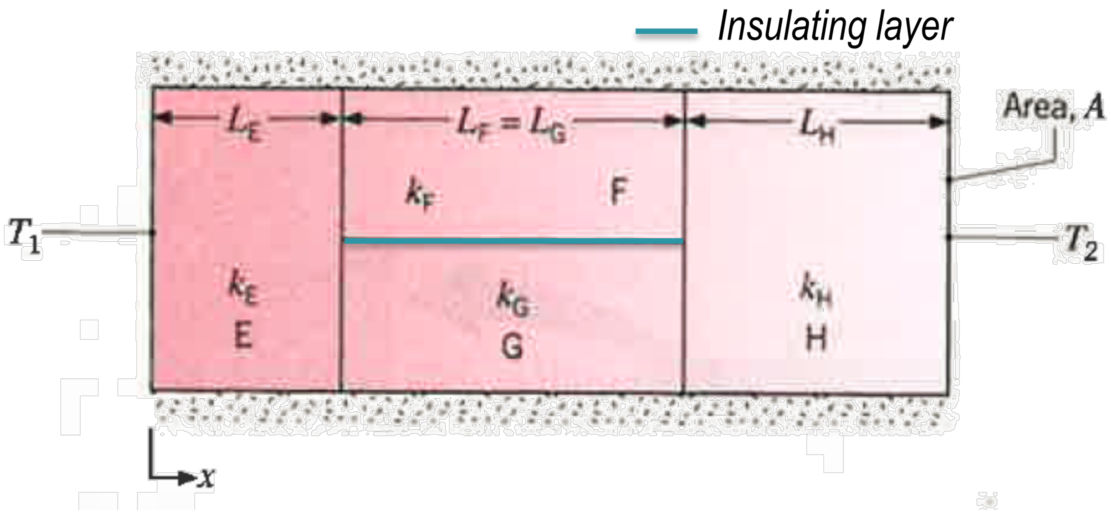
$$\frac{1}{R_{parallel}} = \frac{1}{R_1} + \frac{1}{R_2} + .. = \sum \frac{1}{R_i}$$



Kirchoff's 1st Law:

$$\sum Q_i = 0 \qquad \begin{matrix} Q_i > 0 \; enters \\ Q_i < 0 \; exits \end{matrix}$$

Thermal Circuits

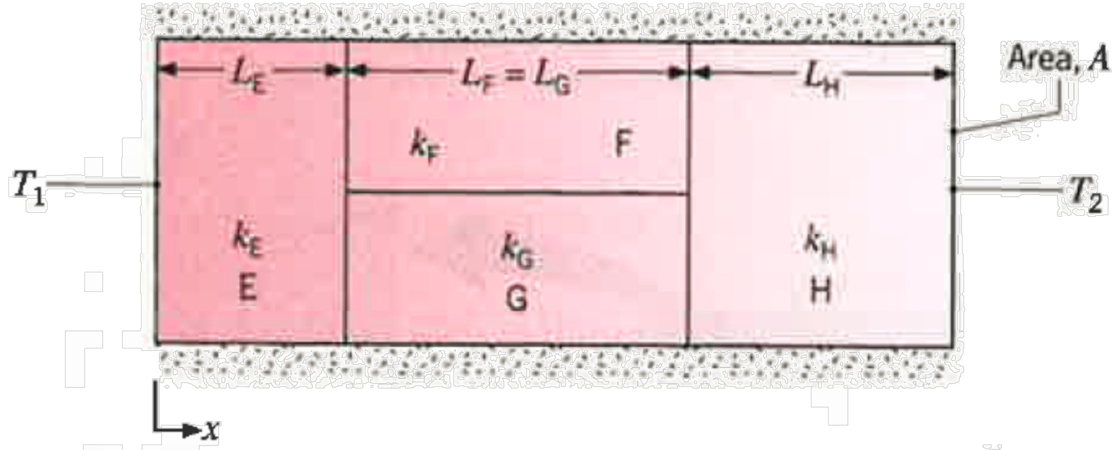


$$Q_x = \frac{(T_1 - T_2)}{R_{tot}}$$

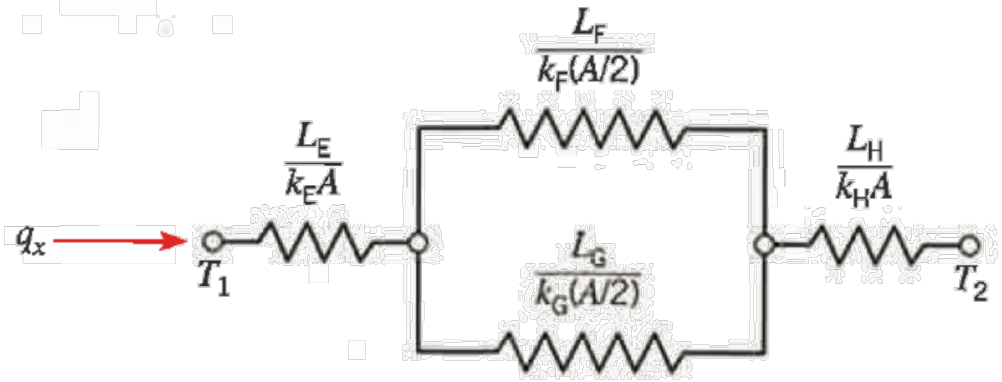
Draw the electrical circuit equivalent to this heat conduction problem.

Reduce it to a single resistor.

Thermal Circuits



$$Q_x = \frac{(T_1 - T_4)}{R_{tot}}$$

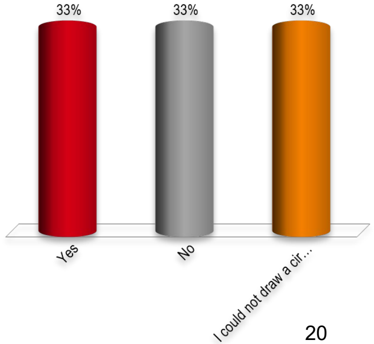


Did you draw this circuit?

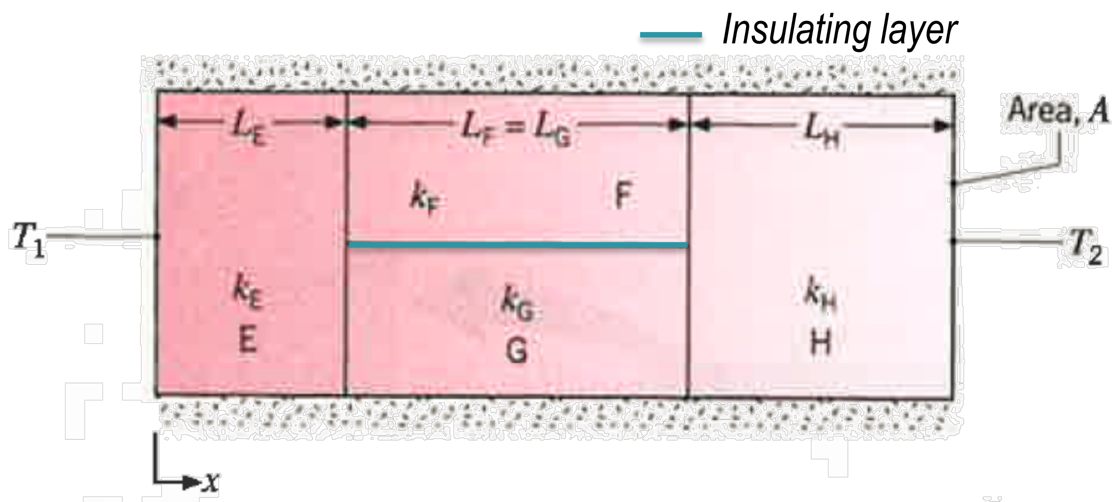
A. Yes

B. No

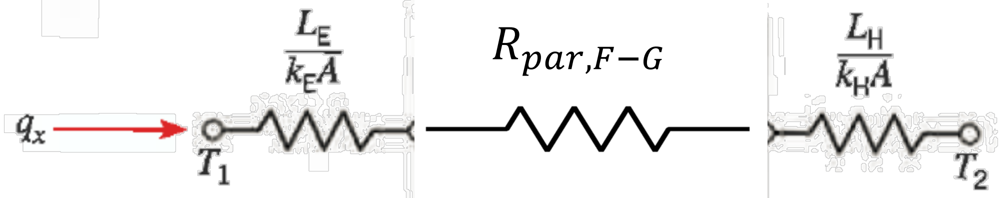
C. I could not draw a circuit



Thermal Circuits



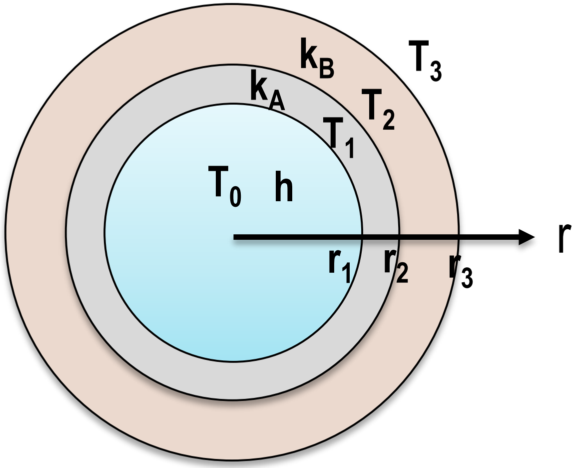
$$Q_x = \frac{(T_1 - T_4)}{R_{tot}}$$



$$R_{par,F-G} = \left(\frac{1}{R_F} + \frac{1}{R_G}\right)^{-1}$$

$$R_{tot} = R_E + R_{par,F-G} + R_H$$

Thermal Circuits



$\mathrm{L} = 10m$

$T_0 = 10C = 283K$
$T_3 = 35C = 208K$

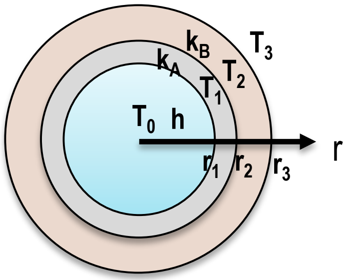
$h = 50\,{}^{W}\!/_{m^2K}$

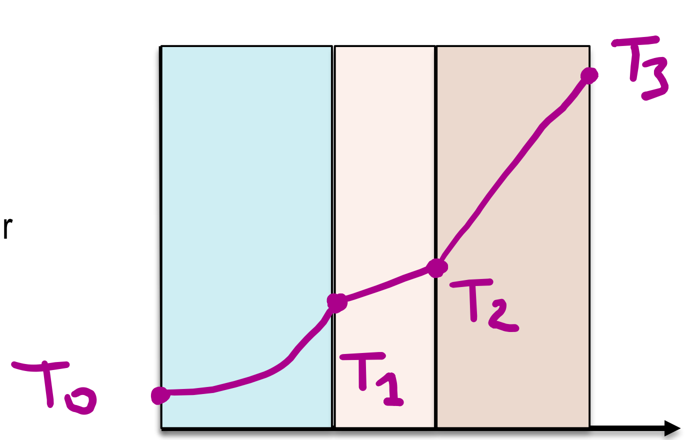
$r_1 = 10\ cm = 0.1m$
$r_2 = 15\ \mathrm{cm} = 0.15\mathrm{m}$
$r_3 = 30\ \mathrm{cm} = 0.3\mathrm{m}$

$k_A = 100\,{}^{W}\!/_{mK}$
$k_B = 0.1\,{}^{W}\!/_{mK}$

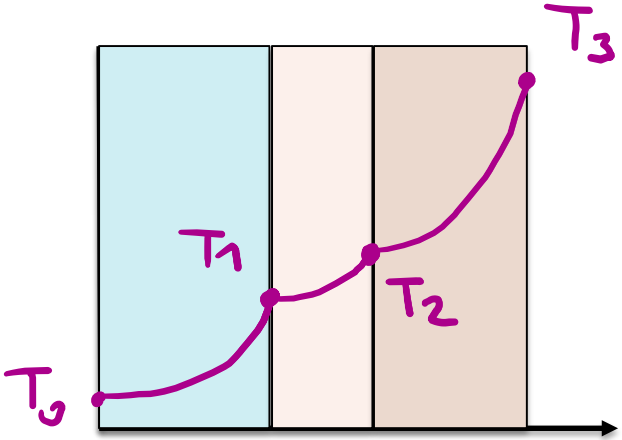
- **Draw the temperature profile**
- **Draw the equivalent circuit**

Which temperature profile did you draw?





A



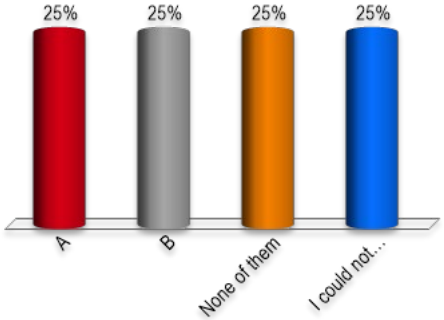
B

A. A

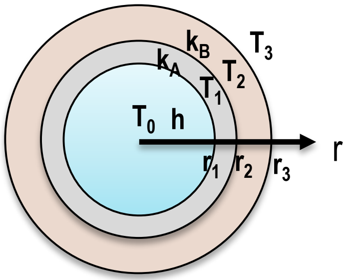
B. B

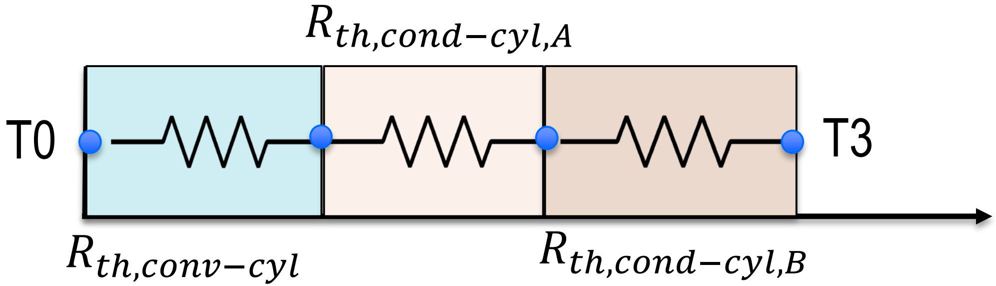
C. None of them

D. I could not draw any T profile

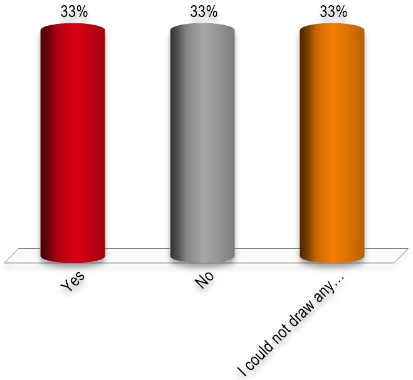


Did you draw this equivalent circuit?

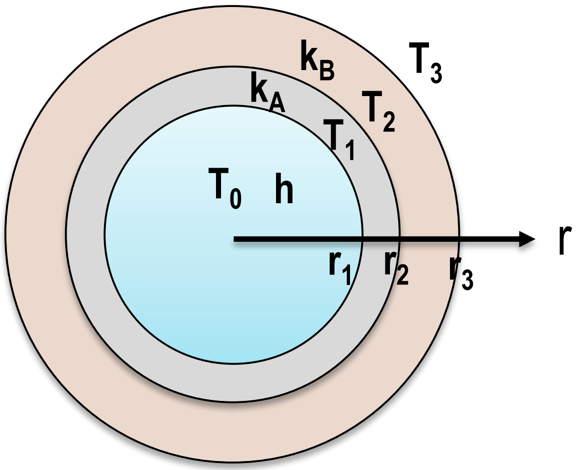




A. Yes
B. No
C. I could not draw any circuit



Thermal Circuits



$\mathrm{L} = 10m$

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$r_1 = 10\ cm = 0.1m$
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$r_3 = 30\ \mathrm{cm} = 0.3\mathrm{m}$

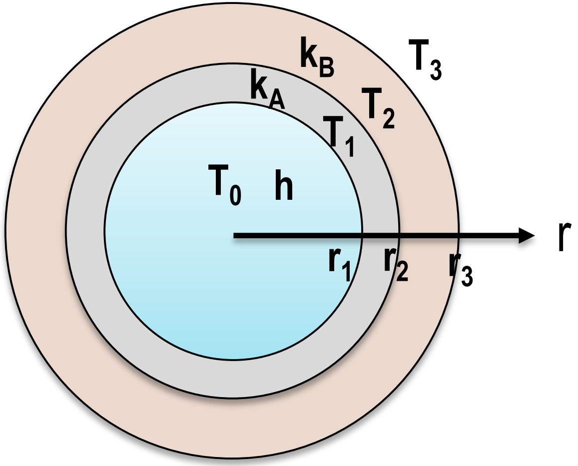
$h = 50\,{}^{W}/_{m^2K}$

$k_A = 100\,{}^{W}/_{mK}$
$k_B = 0.1\,{}^{W}/_{mK}$

- **Calculate Q and U**
- **Calculate T_1, T_2**

Hint: first reduce the equivalent electrical circuit to a single resistor.

Thermal Circuits



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$k_A = 100\, {}^{W}/_{mK}$
$k_B = 0.1\, {}^{W}/_{mK}$

$$\boldsymbol{Q_r} = \frac{\boldsymbol{T_0 - T_3}}{\boldsymbol{R_{tot,series}}} = UA(T_0 - T_3)$$

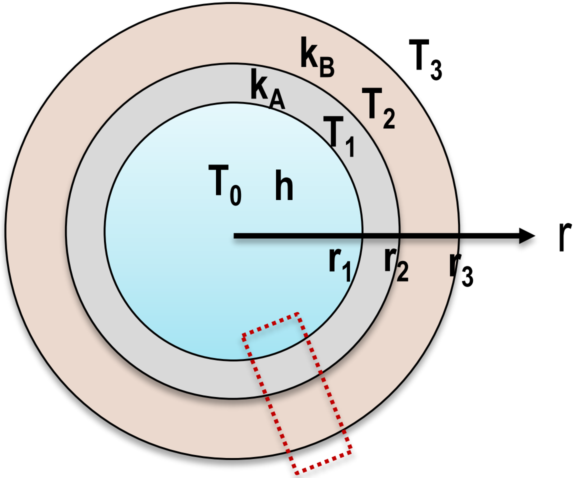
$R_{th,conv-cyl}$ $R_{th,cond-cyl,A}$ $R_{th,cond-cyl,B}$

$$Q_r = \frac{T_0 - T_1}{R_{th,conv-cyl}}$$

$$R_{th,conv} = \frac{1}{h2\pi rL} \qquad R_{th,cond-cyl} = \frac{ln(r_2/r_1)}{2\pi Lk}$$

$$Q_r = \frac{T_1 - T_2}{R_{th,cond-cyl,A}}$$

Thermal Circuits



$R_{th,conv-cyl}$ $R_{th,cond-cyl,A}$ $R_{th,cond-cyl,B}$

$$R_{th,conv} = \frac{1}{h2\pi rL} \qquad R_{th,cond-cyl} = \frac{ln(r_2/r_1)}{2\pi Lk}$$

$$R_{th,conv} = 0.0032K/W$$
$$R_{th,cond-cyl,A} = 0.000065K/W$$
$$R_{th,cond-cyl,B} = 0.11K/W$$

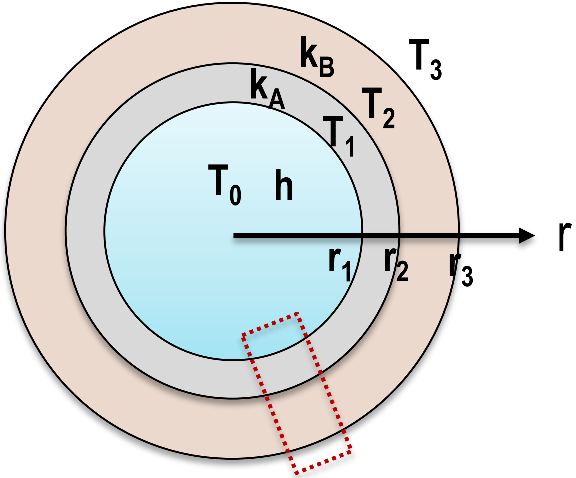
$$Q_r = \frac{T_0 - T_3}{R_{tot,series}} = -220.7W$$

$$UA = \frac{Q_r}{T_0 - T_3} = 8.8W/K$$

$$U(r) = \frac{UA}{2\pi rL} \quad \begin{cases} U_{r1} = \\ U_{r3} = \end{cases}$$

UA is a constant of the system.
However, if the area is not constant,
U depends on the position!!

Thermal Circuits



$L = 10m$

$T_0 = 10C = 283K$
$T_3 = 35C = 208K$

$h = 50 \,{}^{W}/_{m^2K}$

$r_1 = 10\ cm = 0.1m$
$r_2 = 15\ \text{cm} = 0.15\text{m}$
$r_3 = 30\ \text{cm} = 0.3\text{m}$

$k_A = 100 \,{}^{W}/_{mK}$
$k_B = 0.1 \,{}^{W}/_{mK}$

$R_{th,conv} = 0.0032K/W$
$R_{th,cond-cyl,A} = 0.000065K/W$
$R_{th,cond-cyl,B} = 0.11K/W$

$R_{th,conv-cyl}$ $R_{th,cond-cyl,A}$ $R_{th,cond-cyl,B}$

$$Q_r = \frac{T_0 - T_3}{R_{tot,series}} = -220.7W$$

$T_1 = 10.7C$

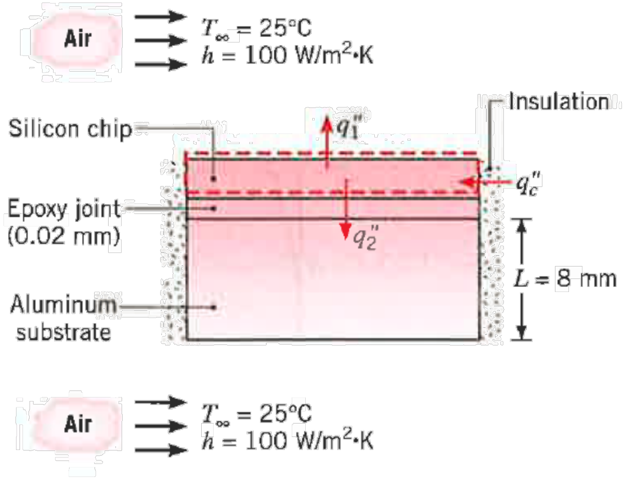
$T_2 = 10.714C$

$$R_{th,conv} = \frac{1}{h2\pi rL} \qquad R_{th,cond-cyl} = \frac{ln(r_2/r_1)}{2\pi Lk}$$

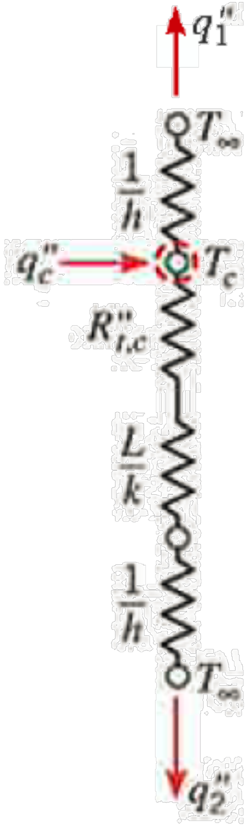
Thermal Circuits

A thin silicon chip of area 100 mm^2 at a uniform temperature Tc is bonded to a 8mm thick aluminum substrate by a thin epoxy layer whose thermal resistance per unit area is $R_e^{''} = 0.9\, m^2K/W$. The exposed surfaces of the chip and the aluminum substrate are cooled by air at 25C with a convection coefficient of 100W/m^2K. If the chip produces 10^4 W/m^2, is its temperature below the safety limit of 85C ?

- Draw the equivalent electrical circuit indicating the heat fluxes, temperatures and thermal resistances



Thermal Resistance of a Composite Wall (Example)



$$q_c'' = q_1'' + q_2''$$

$$q_c'' = \frac{T_c - T_\infty}{(1/h)} + \frac{T_c - T_\infty}{R_{t,c}'' + (L/k) + (1/h)}$$

$$T_c = T_\infty + q_c''\left[h + \frac{1}{R_{t,c}'' + (L/k) + (1/h)}\right]^{-1}$$

$$T_c = 25°\text{C} + 10^4\ \text{W/m}^2$$
$$\times\left[100 + \frac{1}{(0.9 + 0.33 + 100) \times 10^{-4}}\right]^{-1} \text{m}^2 \cdot \text{K/W}$$
$$T_c = 25°\text{C} + 50.3°\text{C} = 75.3°\text{C}$$

This Lecture

- ☑ Overall Heat Transfer Coefficient
- ☑ Bi number
- ☑ Thermal Circuits

Learning Objectives:

- ☑ Understand the Overall Heat Transfer Coefficient
- ☑ Solve 1D steady state heat transfer problems without heat sources using thermal circuits

Next Lecture

- ❑ 1D steady-state conduction with heat sources
- ❑ Introduction to Fins

Learning Objectives:

- ❑ Solve 1D steady state heat conduction problems in different geometries, with heat sources