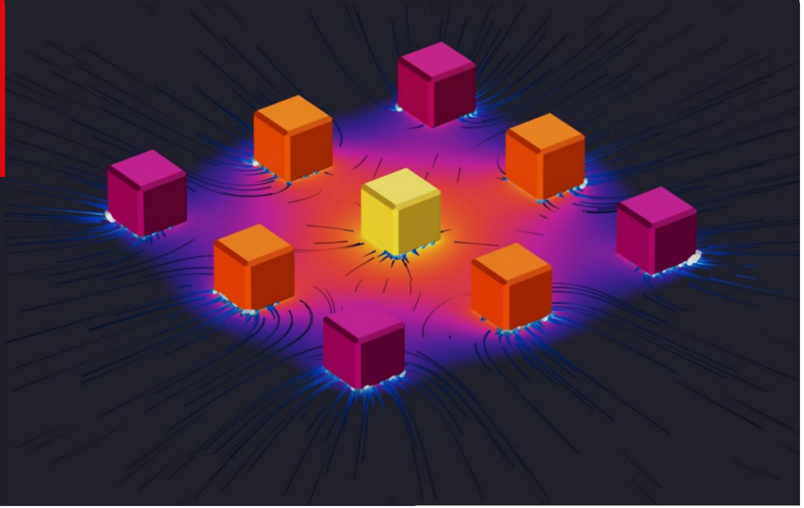
Heat and Mass Transfer
ME-341

Instructor: Giulia Tagliabue



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Spring Semester

Introduction to Heat Transfer

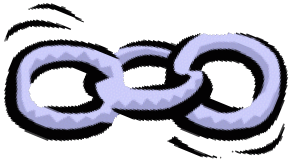
- ☑ Heat Transfer Mechanisms
- ☑ From Thermodynamics to Heat Transfer
- ☑ Heat Transfer Rate Equations

Learning Objectives:

- ☑ From real-world to model:
  - identify the system and its boundaries
  - Identify heat transfer mechanisms involved
- ☑ Solve basic heat transfer problems

From Thermodynamics to Heat Transfer

Thermodynamics



Heat Transfer

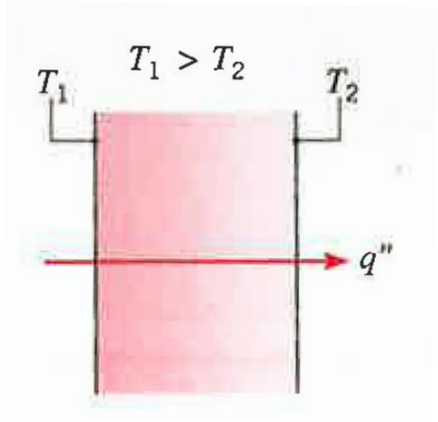
Although thermodynamics may be used to determine the amount of heat needed for a system to pass from one equilibrium state to another it does not acknowledge that heat transfer is inherently a non-equilibrium process. In fact, for heat transfer to occur there MUST be a TEMPERATURE GRADIENT. (Incropera, Ch. 1.3)

Nomenclature & Units

- $Q = heat\ transfer\ rate\ [W]$
- $q'' = heat\ flux\ [W/m^2]$
- $\dot{q} = volumetric\ heat\ source\ [\frac{W}{m^3}]$
- $q' = heat\ flux\ unit\ length\ [W/m]$

Transport Laws

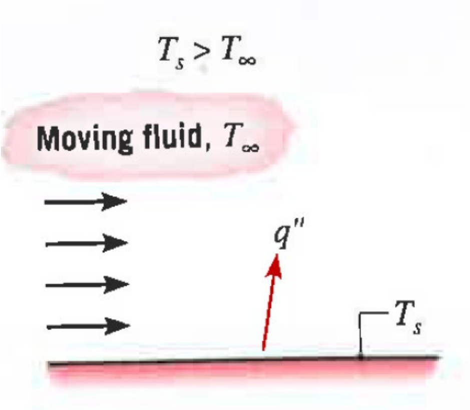
Conduction



Fourier's Law

$$q'' = -k\frac{dT}{dx}$$

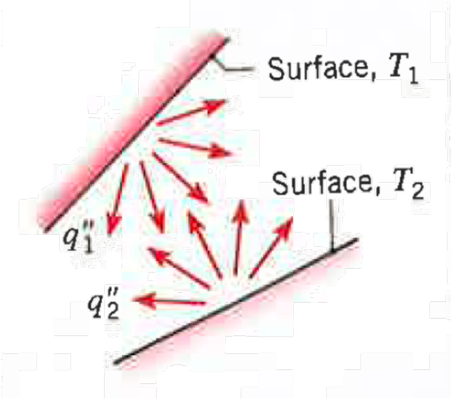
Convection



Newton's Law

$$q'' = \bar{h}\,(T_s - T_\infty)$$

Radiation



Stefan-Boltzmann Law

$$Q_{rad} = \varepsilon\sigma A_s(T^4 - T_{sur}^4)$$

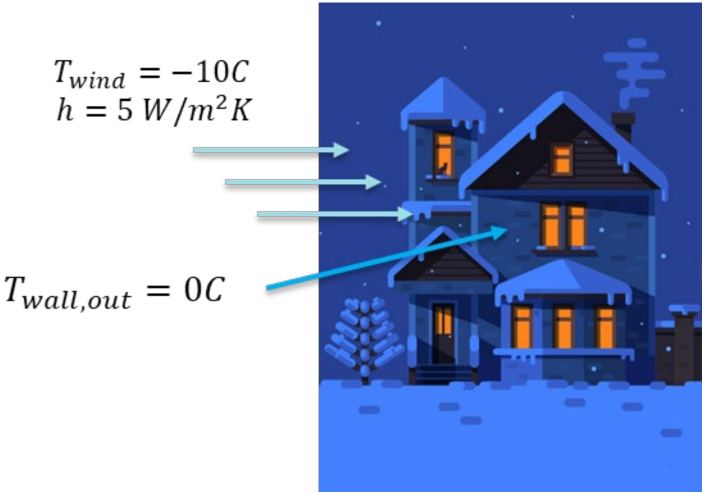
Transport Laws

$T_{wind} = -10C$
$h = 5\, W/m^2K$

$T_{wall,out} = 0C$

- Draw a simplified system representing the heat transfer.
- Calculate the amount of heat transferred per unit area (heat flux)

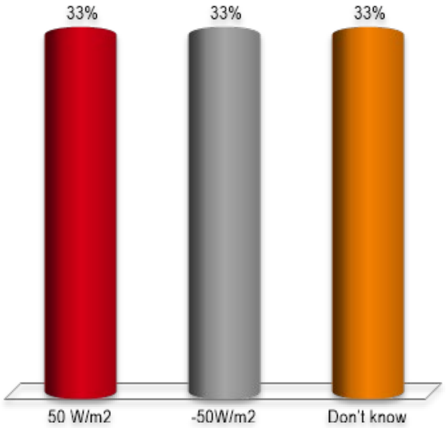
Heat Flux in Winter:



A. 50 W/m^2

B. -50W/m^2

C. Don't know



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Transport Laws

$T_{wall,out} = 30C$

$t_{wall} = 30\ cm$

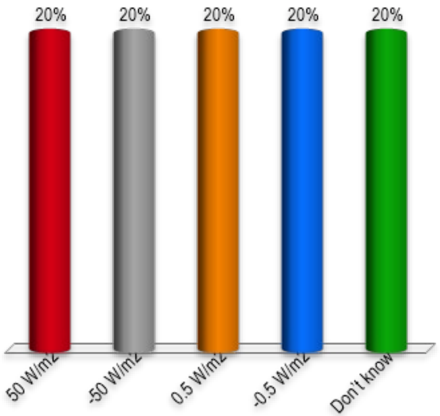
$k_{wall} = 1.5 \frac{W}{mK}$

$T_{indoor} = 20C$

- Draw a simplified system representing the heat transfer.
- Calculate the amount of heat transferred per unit area (heat flux)

Heat Flux in Summer

A. 50 W/m^2
B. -50 W/m^2
C. 0.5 W/m^2
D. -0.5 W/m^2
E. Don't know



$T_{wall,out} = 30C$

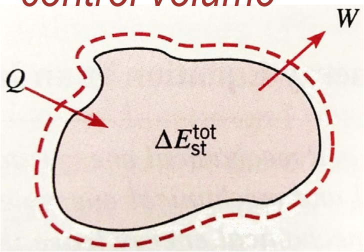
$t_{wall} = 30\ cm$

$k_{wall} = 1.5 \frac{W}{mK}$

$T_{indoor} = 20C$

Part I – Fourier's Law and Heat Conduction

control volume



$$\frac{dE_{st}}{dt} = \dot{U} = Q - W + \dot{E}_{gen}$$

$$q" = -k\frac{dT}{dx}$$

- Heat Diffusion Equation (HDE) 3D
- HDE Steady-state 1D Solutions with/without Heat Sources
- Thermal Resistances and Equivalent Electrical Circuits
- Fins and Arrays of Fins
- Transient HDE
    - Lumped Capacitance Model T(t)
    - 1D Spatial Effects T(X,t)
    - Semi-Infinite Solid
    - Periodic BC

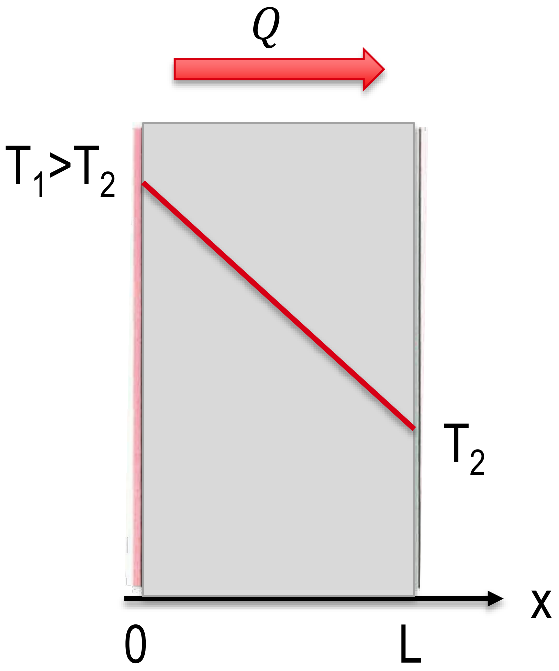
Conductive Heat Transfer - 1

- ❑ Heat diffusion equation
- ❑ Thermal conductivity and diffusivity

Learning Objectives:

- ❑ Understand material parameters and know their magnitude
- ❑ Derive the general heat diffusion equation

The Conduction Rate Equation – From 1D to 3D



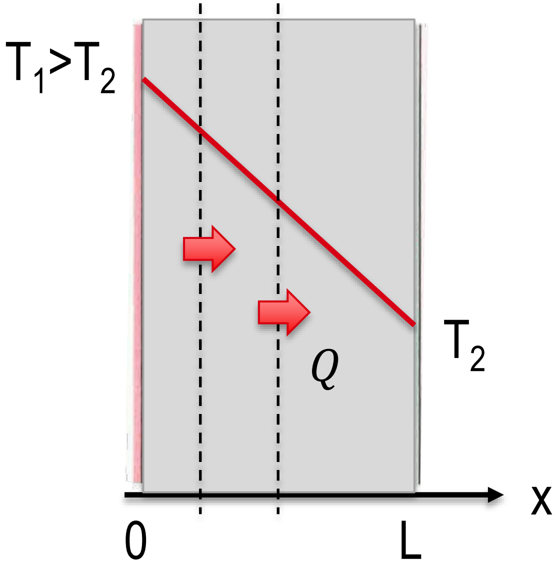
$$Q = -kA\frac{T_2 - T_1}{L} \Rightarrow \frac{Q}{A} = q'' = -k\frac{\Delta T}{L}$$

In the limit of an infinitesimal thickness *dx* :

$$\boldsymbol{q'' = -k\frac{dT}{dx}} \quad [\mathrm{W/m^2}]$$

***k* = thermal conductivity, [W/mK]**

The Conduction Rate Equation – From 1D to 3D



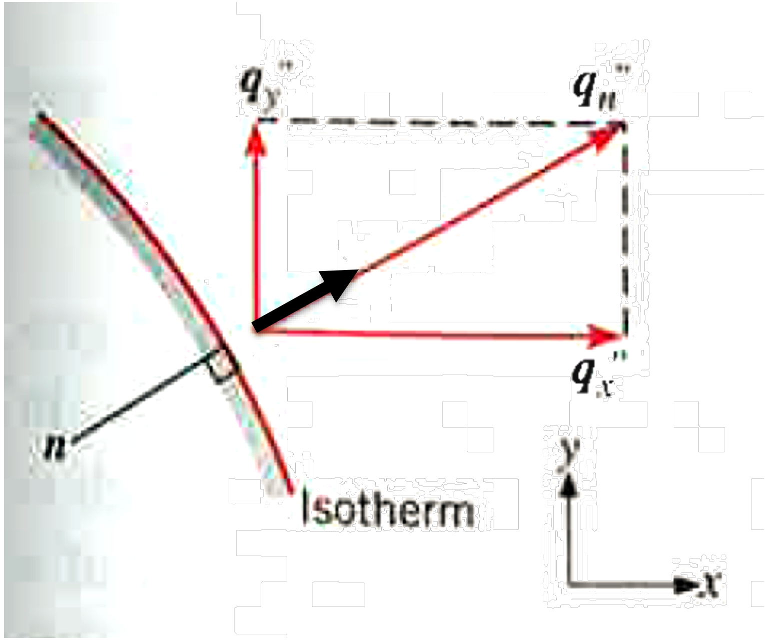
Heat flux is perpendicular to the isotherm surface,

i.e. it along the temperature gradient

$$q'' = -k\frac{dT}{dx} \quad [\,W/m^2]$$

k **= thermal conductivity, [W/mK]**

The Conduction Rate Equation – From 1D to 3D



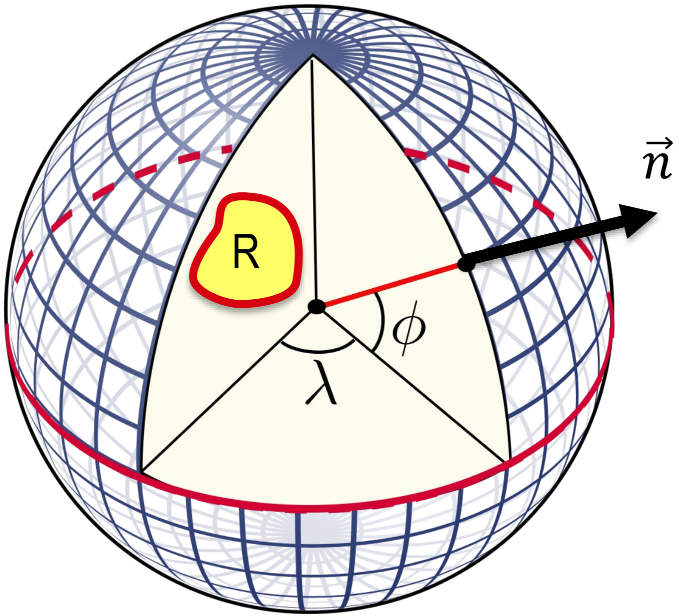
Heat flux is perpendicular to the isotherm surface,

i.e. it along the temperature gradient

$$\overrightarrow{q_n''} = -k\frac{\partial T}{\partial n}\vec{n}$$

$$\overrightarrow{q''} = \overrightarrow{q_x''} + \overrightarrow{q_y''} = -k\frac{\partial T}{\partial x}\vec{i} - k\frac{\partial T}{\partial y}\vec{j} = -k\,\nabla T$$

The Conduction Rate Equation – From 1D to 3D



3D isothermal surface

$$\overrightarrow{q''} = -k\,\nabla T = -k\left(\frac{\partial T}{\partial x}\vec{i} + \frac{\partial T}{\partial y}\vec{j} + \frac{\partial T}{\partial y}\vec{k}\right)$$

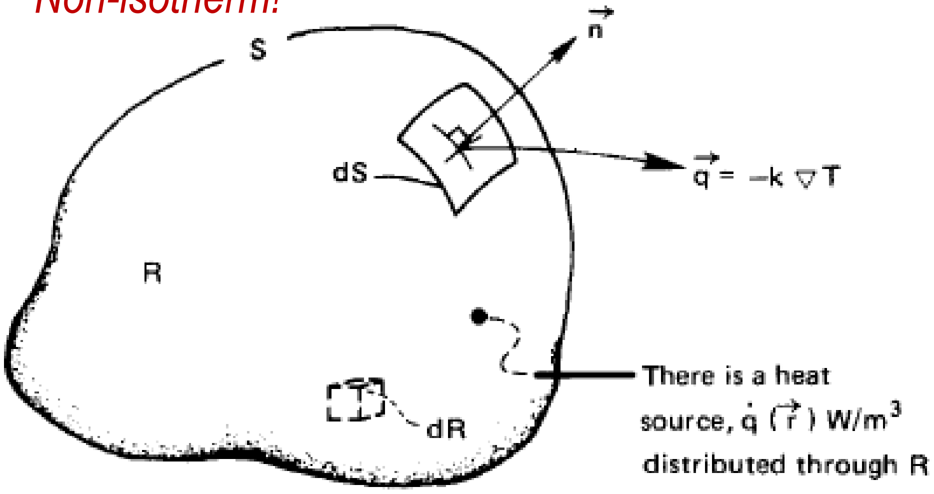
If the material is ANISOTROPIC:

$$\overrightarrow{q''} = -\bar{\bar{k}}\,\nabla T \qquad \bar{\bar{k}} = \begin{bmatrix} k_x & 0 & 0 \\ 0 & k_y & 0 \\ 0 & 0 & k_z \end{bmatrix}$$

$$\overrightarrow{q''} = -\left(k_x\frac{\partial T}{\partial x}\vec{i} + k_y\frac{\partial T}{\partial y}\vec{j} + k_z\frac{\partial T}{\partial y}\vec{k}\right)$$

Heat Diffusion Equation – 3D

Non-isotherm!



The energy conservation equation requires:

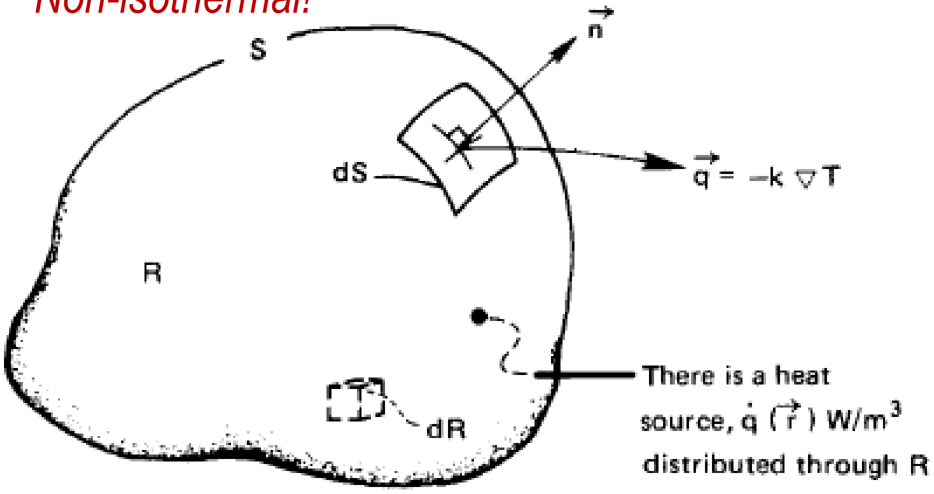
$$\dot{U} = mc\frac{dT}{dt} = Q - \cancelto{0}{W} + \dot{E}_{gen}$$

Remember that Q>0 when flows INTO the volume. Hence its sign is opposite to the normal vector $\vec{n}$

<u>Assumption 1</u>: incompressible medium
<u>Assumption 2</u>: medium at rest (no convection)
<u>Assumption 3</u>: isotropic material

Heat Diffusion Equation – 3D

Non-isothermal!



Assumption 1: incompressible medium
Assumption 2: medium at rest (no convection)
Assumption 3: isotropic material

Heat transfer across S only through conduction

Heat flux is along T gradient

$$\overrightarrow{q''} = -k\,\nabla T$$

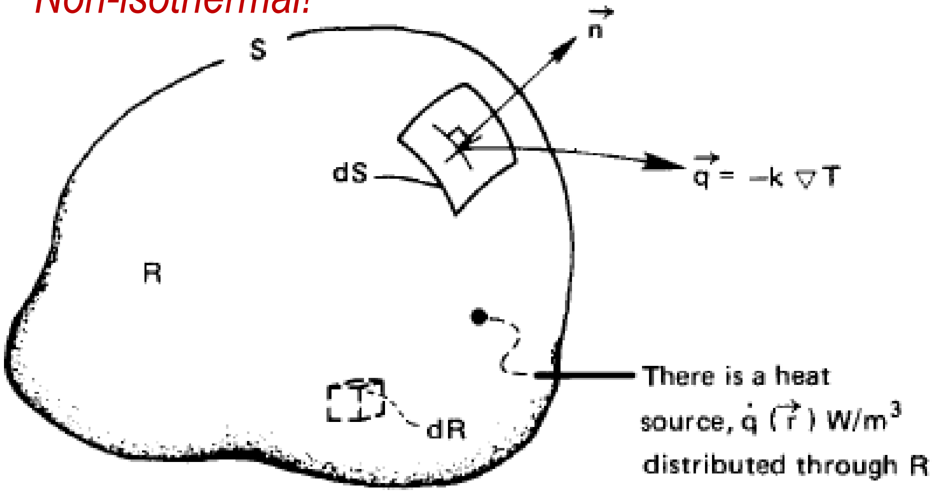
⇩

Heat transfer rate OUT of dS

$$\boldsymbol{\delta Q = (-k\,\nabla T)\cdot(\vec{n}\,dS)}$$

Heat Diffusion Equation – 3D

Non-isothermal!



Assumption 1: incompressible medium
Assumption 2: medium at rest (no convection)
Assumption 3: isotropic material

Total Heat Transfer Rate INTO non-isothermal S

$$Q = -\int_S (-k\,\nabla T)\cdot(\vec{n}\,dS) = \int_S (k\,\nabla T)\cdot(\vec{n}\,dS)$$

Total Generation Rate in Volume R

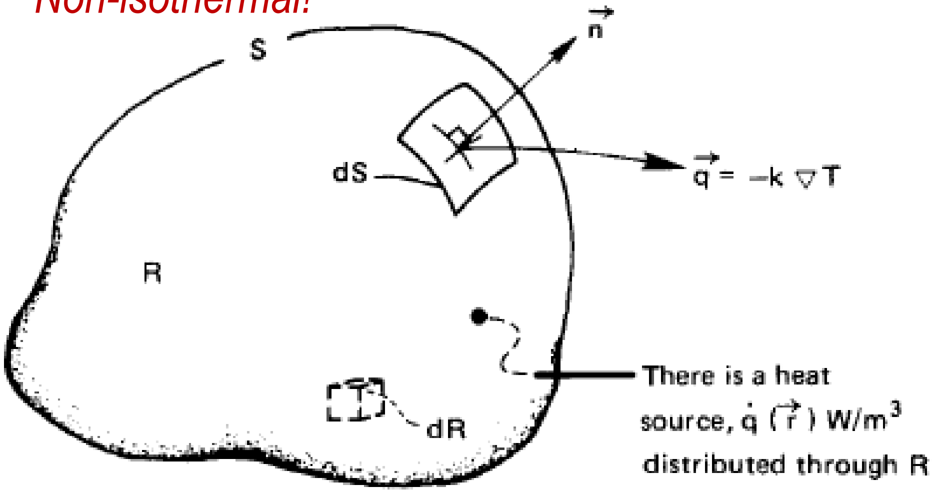
$$\dot{E}_{gen} = \int_V \dot{q}\,dR$$

Change in Internal Energy of the Volume

$$\frac{dU}{dt} = \int_V \rho c \frac{\partial T}{\partial t}\,dR$$

Heat Diffusion Equation – 3D

Non-isothermal!



Assumption 1: incompressible medium
Assumption 2: medium at rest (no convection)
Assumption 3: isotropic material

So we can write: $\dot{U} = Q + \dot{E}_{gen}$

$$\int_V \rho c \frac{\partial T}{\partial t} dR = \int_S (k\,\nabla T) \cdot (\vec{n} dS) + \int_V \dot{q} dR$$

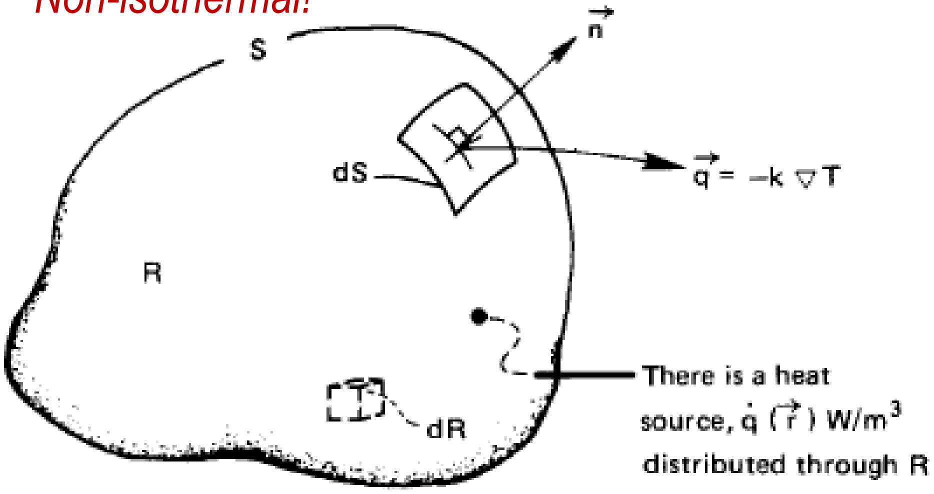
Gauss' law

$$\int_V \rho c \frac{\partial T}{\partial t} dR = \int_V (\nabla \cdot (k\,\nabla T) + \dot{q}) dR$$

$$\mathbf{0} = \int_V \left(\nabla \cdot (k\,\nabla T) + \dot{q} - \rho c \frac{\partial T}{\partial t} \right) dR$$

Heat Diffusion Equation – 3D

Non-isothermal!



Assumption 1: incompressible medium
Assumption 2: medium at rest (no convection)
Assumption 3: isotropic material

Heat Diffusion Equation:

$$\boldsymbol{\nabla \cdot (k\, \nabla T) + \dot{q} = \rho c \frac{\partial T}{\partial t}}$$

Assumption 4: *k* is independent of *T*

$$k\nabla^2 T + \dot{q} = \rho c \frac{\partial T}{\partial t}$$

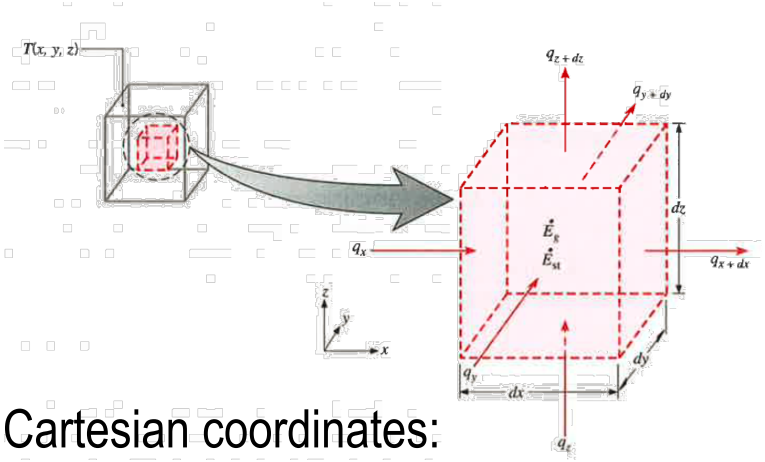
$$\boldsymbol{\nabla^2 T + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}}$$

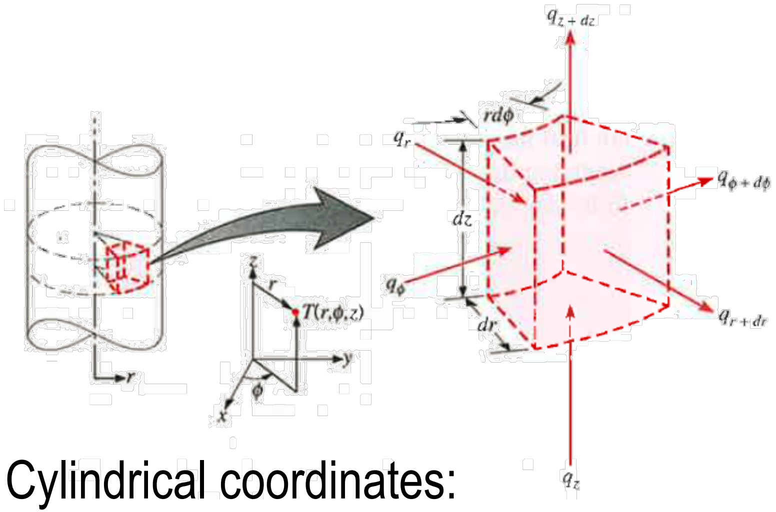
$$\alpha = \frac{k}{\rho c} = thermal\ diffusivity\ [\frac{m^2}{s}]$$

Heat Diffusion Equation – 3D

Assumption 1: incompressible medium
Assumption 2: medium at rest (no convection)
Assumption 3: isotropic material

$$\nabla \cdot (k\, \nabla T) + \dot{q} = \rho c \frac{\partial T}{\partial t}$$





Cartesian coordinates:

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

Cylindrical coordinates:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

Material Properties

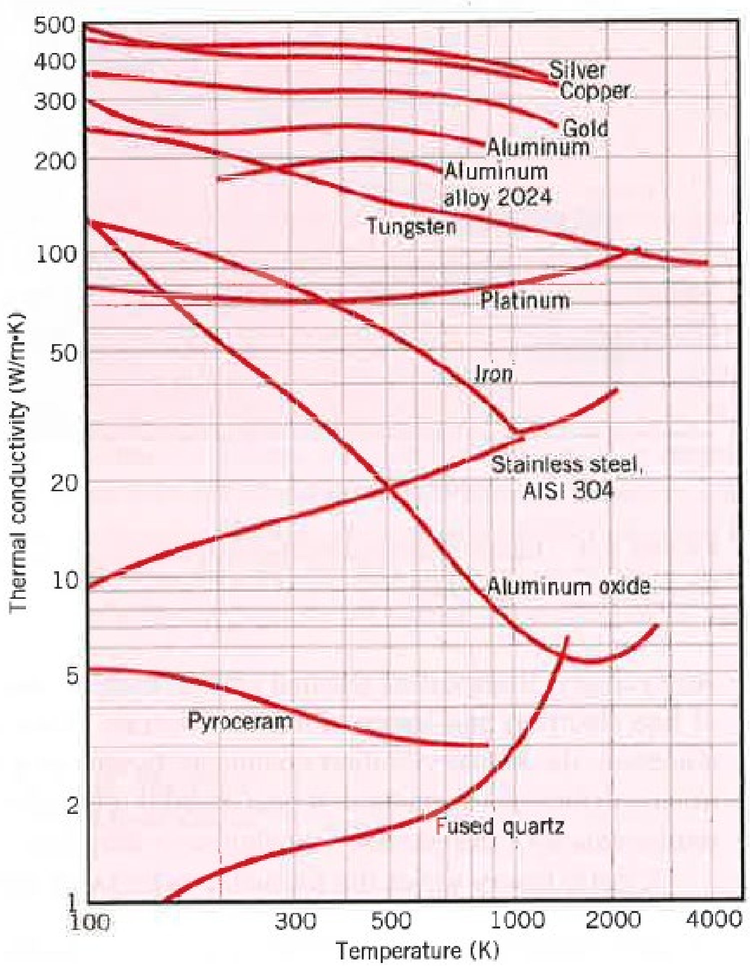
Thermal Conductivity k [W/mK]

For a general material we have:

$$\overrightarrow{q''} = -\bar{\bar{k}}\,\nabla T \quad \Rightarrow \quad q''_x = -k_x \frac{\partial T}{\partial x}$$

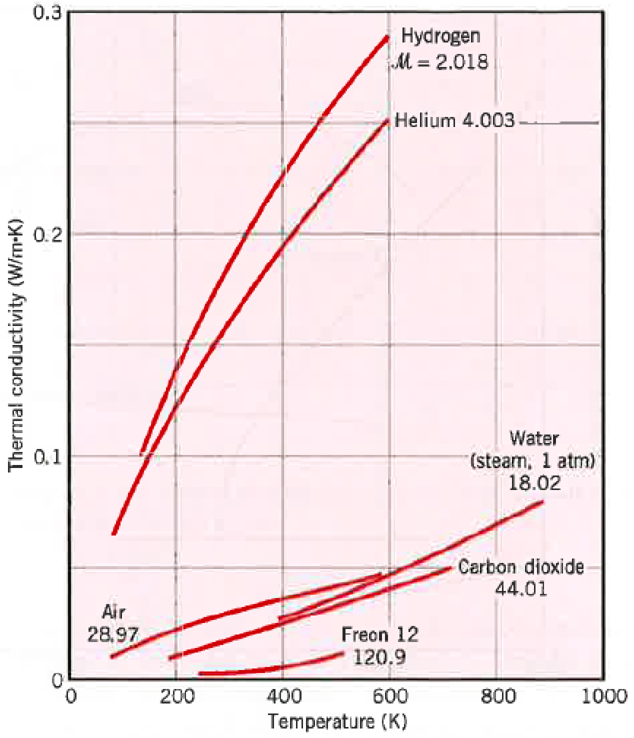
So we can define:

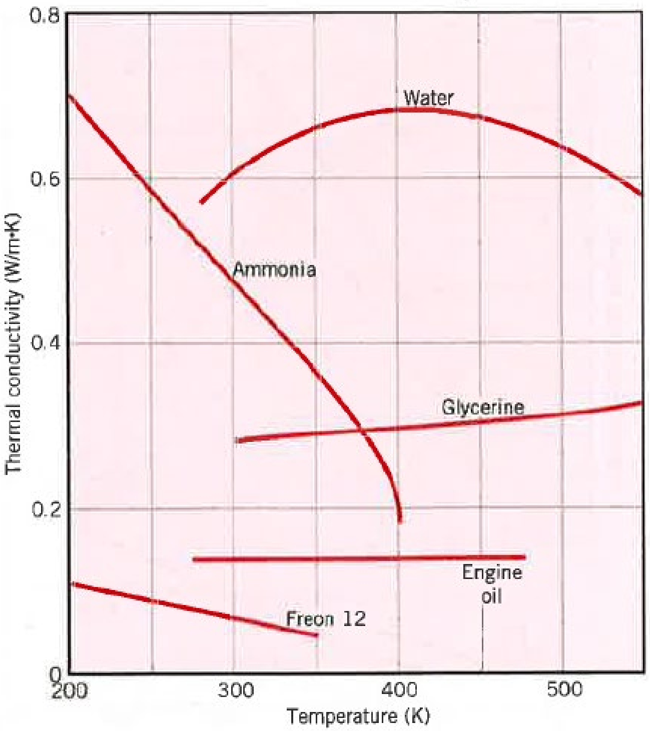
$$k_i \equiv \frac{q''_i}{\frac{\partial T}{\partial x}} \qquad i = x,y,z$$



Material Properties

Thermal Conductivity k [W/mK]





Material Properties

Thermal Diffusivity α [m²/s]

$$\boldsymbol{\alpha = \frac{k}{\rho c}}$$

Relates to the speed with which a body responds to changes in the thermal environment. Indeed:

- Conduction *k* removes heat
- Thermal capacity (ρc) cumulates energy

The higher the thermal conductivity and the lower the thermal capacity, the higher the material diffusivity, i.e. the material will respond quickly to thermal changes.

Material Properties

Thermal Diffusivity α [m²/s]

$$\alpha = \frac{k}{\rho c}$$

Table A.1, pure aluminum (300 K):

$$\left.\begin{aligned} \rho &= 2702\ \text{kg/m}^3 \\ c_p &= 903\ \text{J/kg}\cdot\text{K} \\ k &= 237\ \text{W/m}\cdot\text{K} \end{aligned}\right\} \alpha = \frac{k}{\rho c_p} = \frac{237\ \text{W/m}\cdot\text{K}}{2702\ \text{kg/m}^3 \times 903\ \text{J/kg}\cdot\text{K}}$$

$$= 97.1 \times 10^{-6}\ \text{m}^2/\text{s}$$

paraffin (300 K):

$$\left.\begin{aligned} \rho &= 900\ \text{kg/m}^3 \\ c_p &= 2890\ \text{J/kg}\cdot\text{K} \\ k &= 0.24\ \text{W/m}\cdot\text{K} \end{aligned}\right\} \alpha = \frac{k}{\rho c_p} = \frac{0.24\ \text{W/m}\cdot\text{K}}{900\ \text{kg/m}^3 \times 2890\ \text{J/kg}\cdot\text{K}}$$

$$= 9.2 \times 10^{-8}\ \text{m}^2/\text{s}$$

Transport Laws

$T_{wind} = -10C$
$h = 5\,W/m^2K$

$T_{wall,out} = 0C$

$T_{wall,out} = 30C$

$t_{wall} = 30\ cm$

$k_{wall} = 1.5\,\frac{W}{mK}$

$T_{indoor} = 20C$

- Draw a simplified system representing the heat transfer.
- Calculate the amount of heat transferred per unit area (heat flux) in the two situations.

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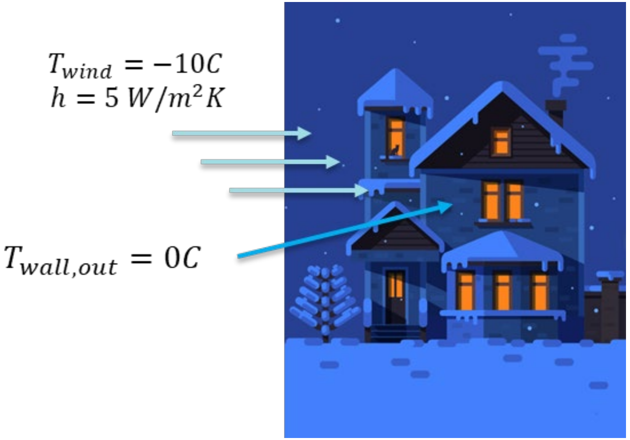
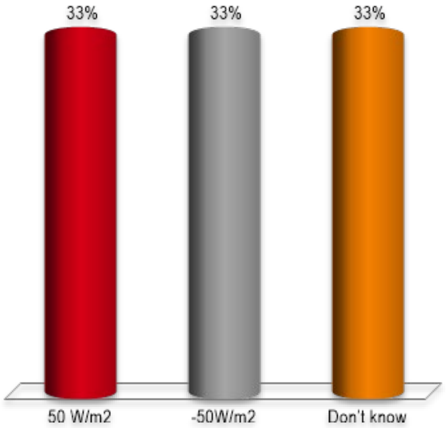
Heat Flux in Winter:



A. 50 W/m^2

B. -50W/m^2

C. Don’t know



Heat Flux in Summer

A. 50 W/m^2

B. -50 W/m^2

C. 0.5 W/m^2

D. -0.5 W/m^2

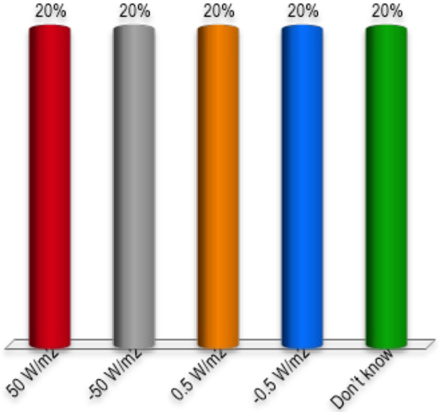
E. Don't know

$T_{wall,out} = 30C$

$t_{wall} = 30\ cm$

$k_{wall} = 1.5 \frac{W}{mK}$

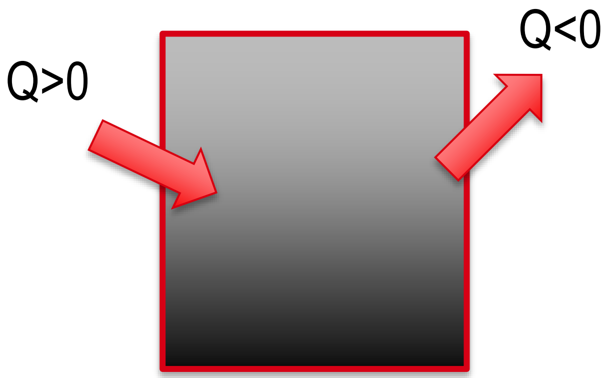
$T_{indoor} = 20C$



Transport Laws

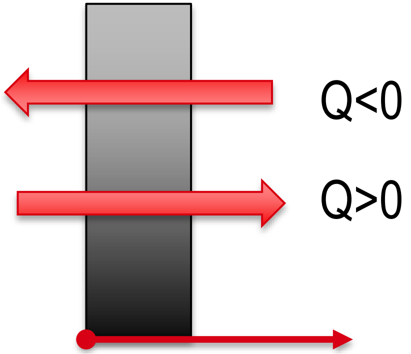
Be careful with the signs and their meaning!

1st Law of Thermodynamics



Control volume

Fourier's Law (1D)



X-axis

Transport Laws

$T_{wind} = -10C$
$h = 5\, W/m^2K$

$T_{wall,out} = 0C$

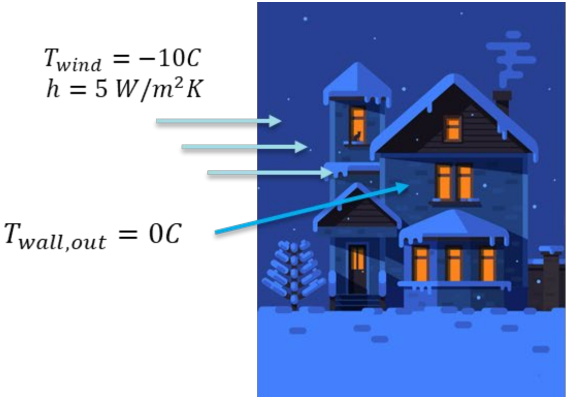
$T_{wall,out} = 30C$

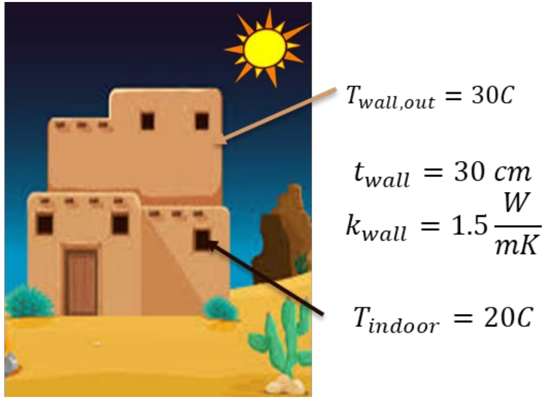
$t_{wall} = 30\ cm$
$k_{wall} = 1.5\, \frac{W}{mK}$

$T_{indoor} = 20C$

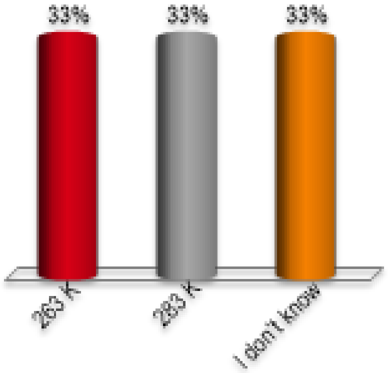
- If the wall is the same in both houses, what is the indoor wall temperature in the winter case?

What is the indoor wall temperature in the winter case?

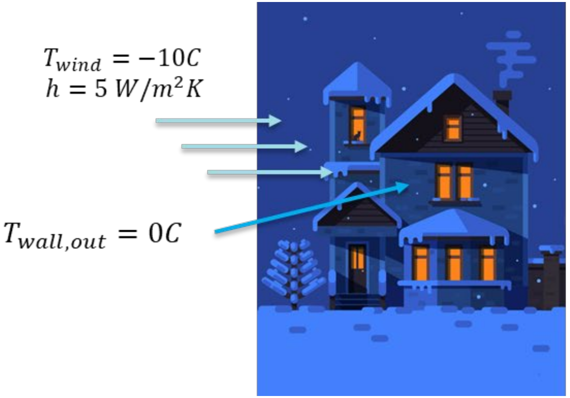


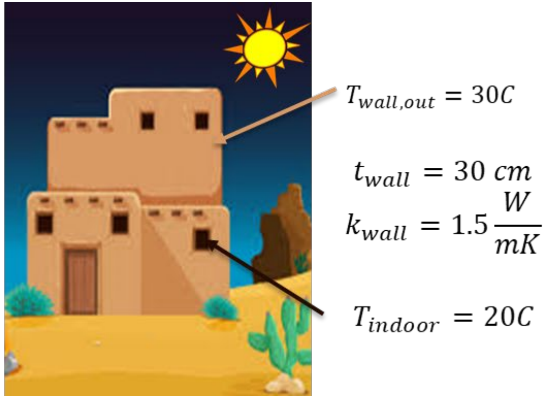


A. 263 K
B. 283 K
C. I don't know



Transport Laws





- If the wall is the same in both houses, what is the indoor wall temperature in the winter case?

Conductive Heat Transfer - 1

- [x] Heat diffusion equation
- [x] Thermal conductivity and diffusivity

Learning Objectives:

- [x] Understand material parameters and know their magnitude
- [x] Derive the general heat diffusion equation

Tomorrow:

- ❑ Types of boundary conditions
- ❑ Planar and cylindrical (1D) solutions
- ❑ Thermal resistance
- ❑ Bi Number
- ❑ Intro to Thermal Circuits

Learning Objectives:

- ❑ Identify the possible boundary conditions
- ❑ Express mathematically the various boundary conditions
- ❑ Calculate the temperature profile in a planar or cylindrical wall
- ❑ Calculate the thermal resistances
- ❑ Calculate the Bi number
- ❑ Solve 1D problems using thermal circuits