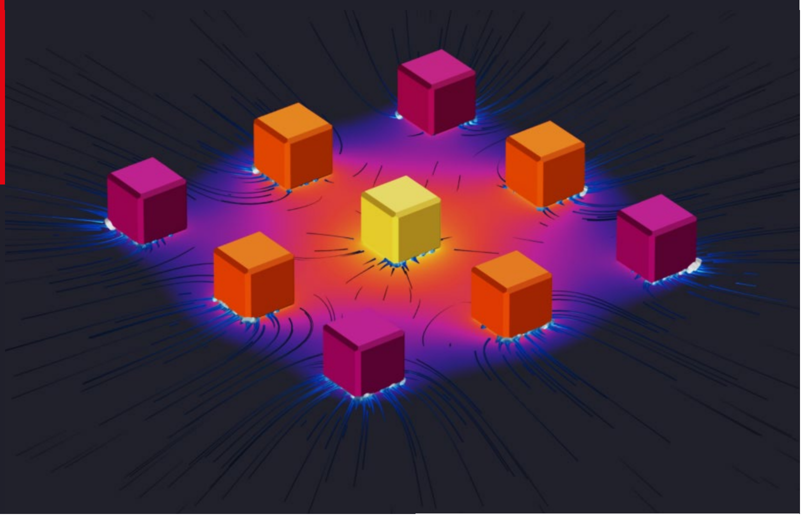
EPFL

Heat and Mass Transfer
ME-341

Instructor: Giulia Tagliabue



École polytechnique fédérale de Lausanne

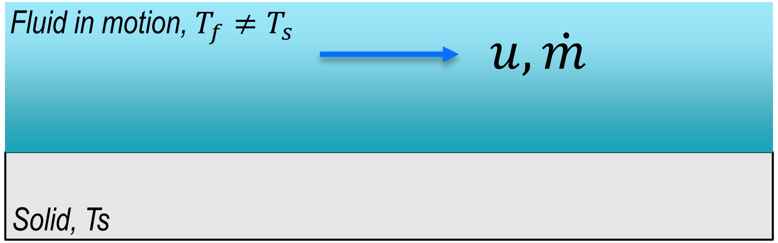
Spring Semester

RECAP of Forced and Free Convection

- ❑ Definition of Convection
- ❑ Boundary layer concept & Governing Equations
- ❑ Forced External/Internal Convection & Free Convection
- ❑ Non-dimensional numbers & Flow Conditions
- ❑ General procedure to determine the convection coefficient

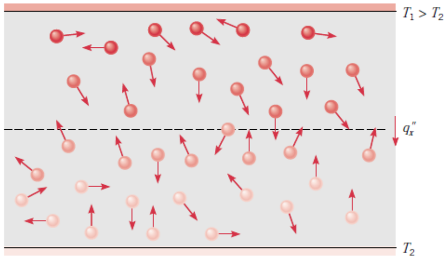
Definition of Convection

Convection refers to the heat transfer between a **solid** and a **fluid in motion** when they are at **different temperatures**.

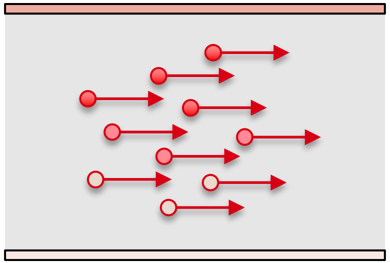


During convection heat is transferred through both **diffusion** (**random** molecular motion) and **advection** (**net** macroscopic mass transport)

diffusion



advection



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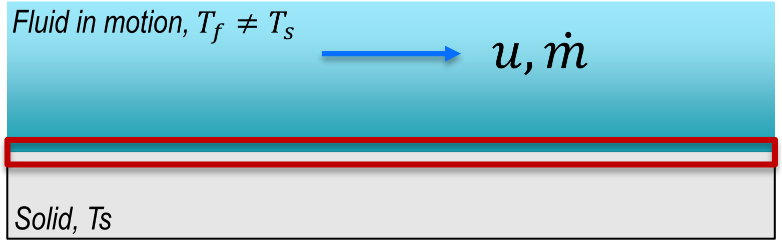
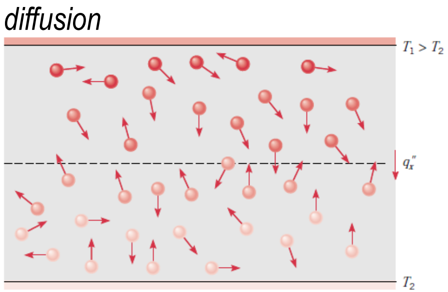
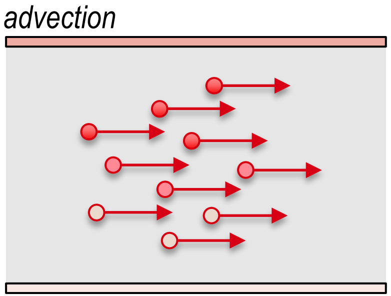


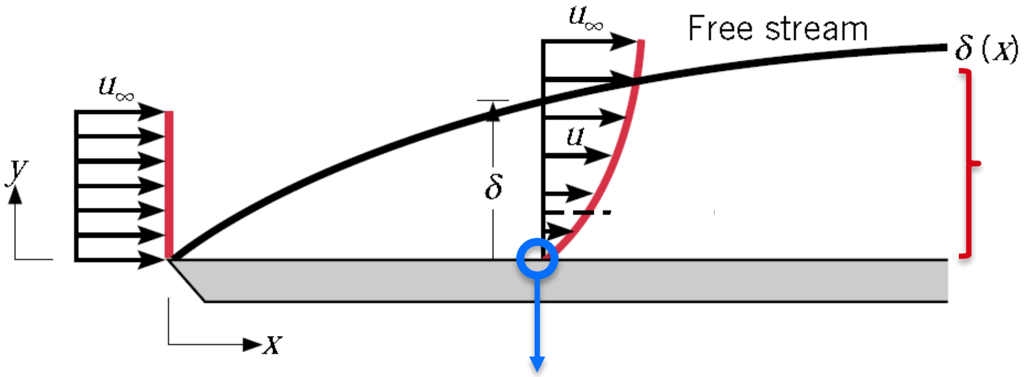
Energy transfer at the solid-fluid interface !! (Boundary layer)

During convection heat is transferred through both **diffusion** (**random** molecular motion) and **advection** (**net** macroscopic mass transport)





Velocity boundary layer



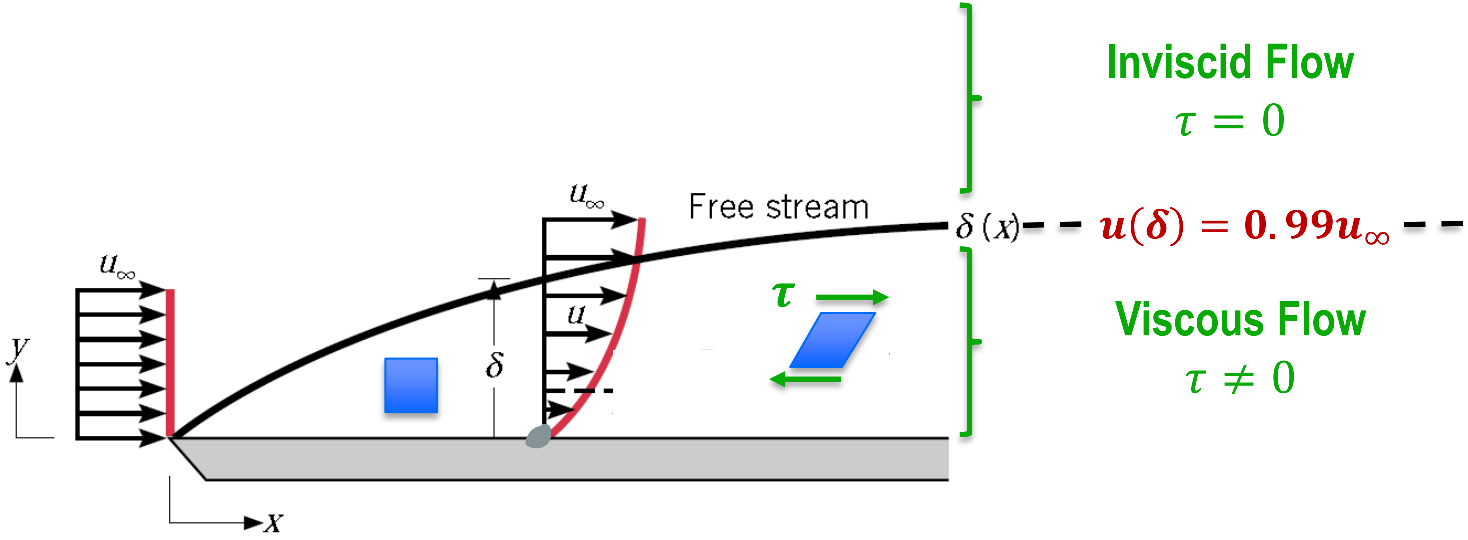
$$u(\delta) = 0.99u_\infty$$

Velocity Boundary Layer

$$u(0) = 0$$

No-slip condition

Velocity boundary layer



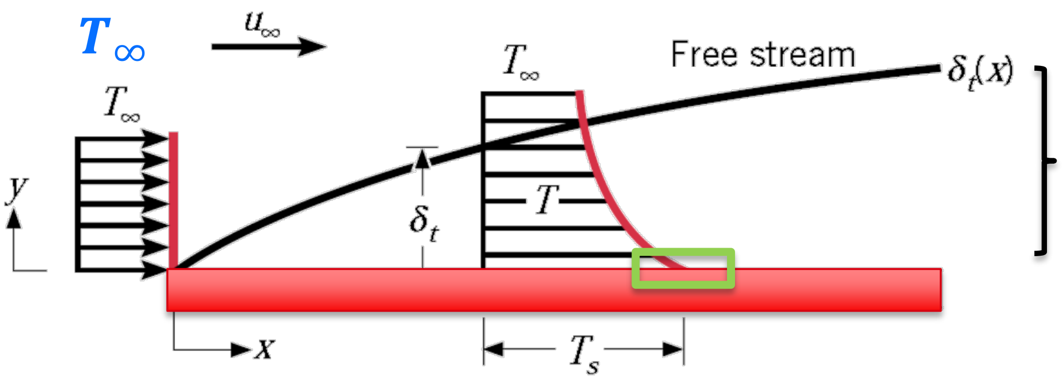
Shear stress τ = friction force per unit area

Newtonian fluids: $\tau(\bar{y}) = \mu \frac{\partial u}{\partial y}\Big|_{y=\bar{y}} \; [N/m^2]$ where $\mu \left[\frac{Ns}{m^2}\right] = dynamic\ viscosity = \rho[\frac{kg}{m^3}] \cdot \nu[\frac{m^2}{s}]$

At the wall ($y = 0$): $\tau(0) = \tau_w = C_f \frac{\rho u_\infty^2}{2}$ where $C_f = friction\ coefficient$

Thermal boundary layer

$$T_s > T_\infty$$



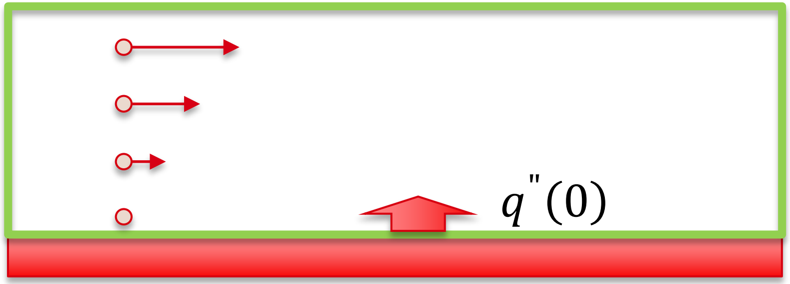
$$\frac{T_s - T(\delta_t)}{T_s - T_\infty} = 0.99$$

Thermal Boundary Layer

$$q''_{conv} = \mathrm{h}(T_s - T_\infty)$$

The total amount of heat removed by convection depends on what happens at the solid/fluid interface

Thermal boundary layer



The heat that is transferred to the fluid must satisfy **Newton's law**:

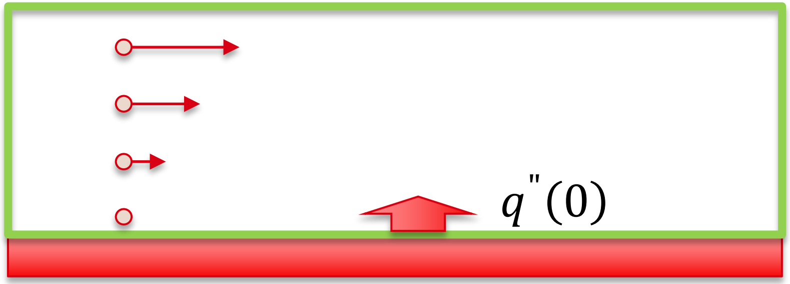
$$q''_{conv} = \mathrm{h}(T_s - T_\infty)$$

At the wall the **velocity is zero** (no advection, only diffusion).
Heat transfer by diffusion is described by **Fourier's law**:

$$q''_{conv} = q''(0) = -\boldsymbol{k_f} \left.\frac{\partial T}{\partial y}\right|_{y=0}$$

$$\Rightarrow \quad h = \frac{-k_f \left.\frac{\partial T}{\partial y}\right|_{y=0}}{(T_s - T_\infty)}$$

Thermal boundary layer

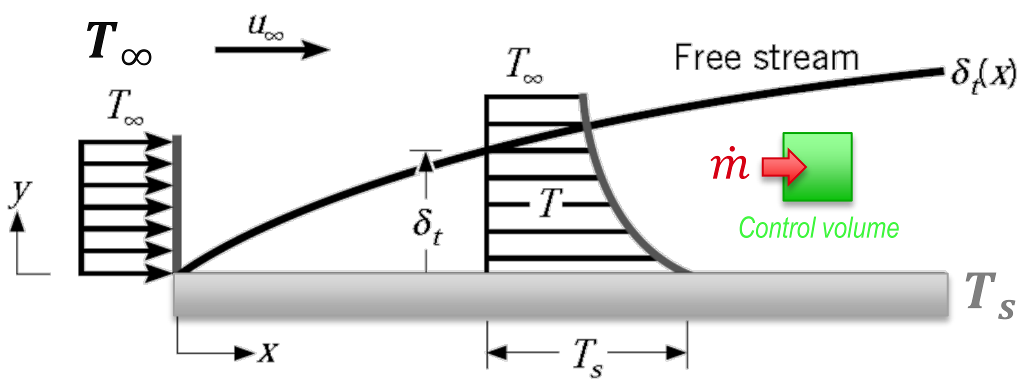


$$h = \frac{-k_f \left.\frac{\partial T}{\partial y}\right|_{y=0}}{(T_s - T_\infty)}$$

- To find the convection coefficient we need to know the temperature profile $T(x, y)$
- To find $T(x, y)$ we need to write the energy balance (1st law) for a control volume in the boundary layer.

Energy Conservation Equation (OPEN SYSTEM)

$$T_s > T_\infty$$

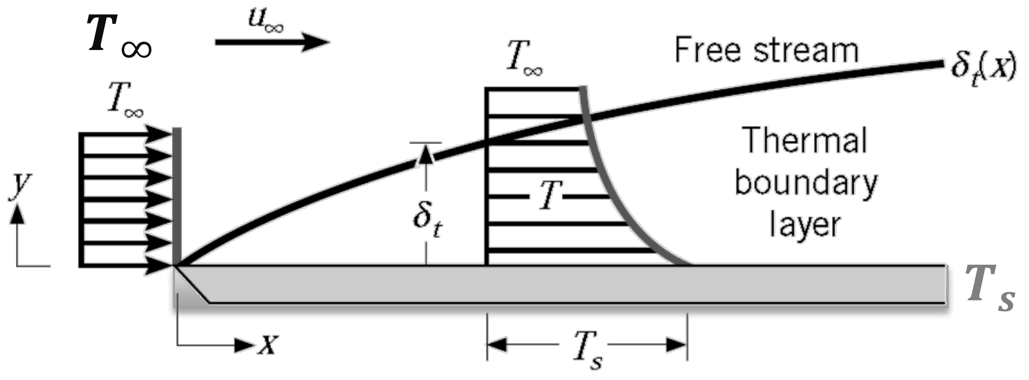


$$\rho c_p\left(\frac{\partial T}{\partial t} + \vec{u}\cdot\nabla T\right) = k\nabla^2 T + \dot{q}$$

Energy storage — *Enthalpy advection* — *Heat diffusion* — *Energy generation*

- To find $T(x,y)$ we need to know the velocity profile $\vec{u}(x,y)$
- To find $\vec{u}(x,y)$ we need to write the:
  - Mass conservation equation
  - Momentum conservation equations (Navier-Stokes)

Governing Equations



- Conservation of mass
- Conservation of momentum
- Conservation of energy

Conservation of mass and Conservation of momentum → *Velocity profile*

Conservation of energy → ***Temperature profile*** → $h = \dfrac{-k_f \left.\dfrac{\partial T}{\partial y}\right|_{y=0}}{(T_s - T_\infty)}$

Boundary Conditions (Temperature/Heat Flux)

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Forced Convection

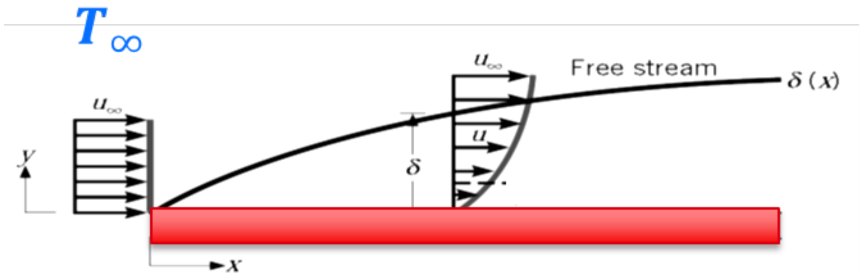
- Conservation of mass $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$
- Conservation of momentum $$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2}$$
- Conservation of energy $$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}$$

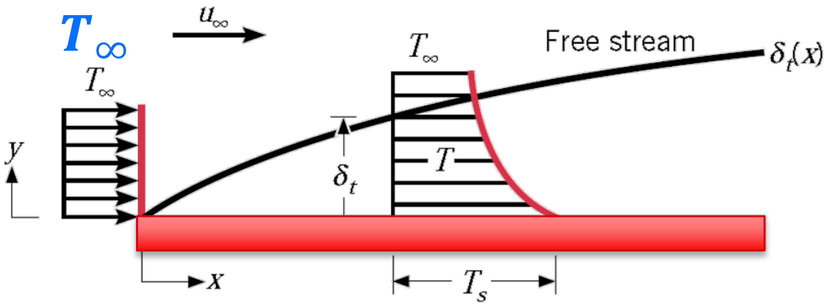
Velocity profile (mass and momentum)

Temperature profile (energy)

We can solve these equations to obtain u(x,y) and T(x,y), and hence h, only for very simple cases (laminar flow, planar geometry). For all other cases we use correlations!!

Forced External Convection – Laminar FLow





- Thermal boundary layer thickness: $\frac{\delta_t}{x} = \frac{4.92}{\sqrt{Re_x}} = \frac{\delta}{x}$

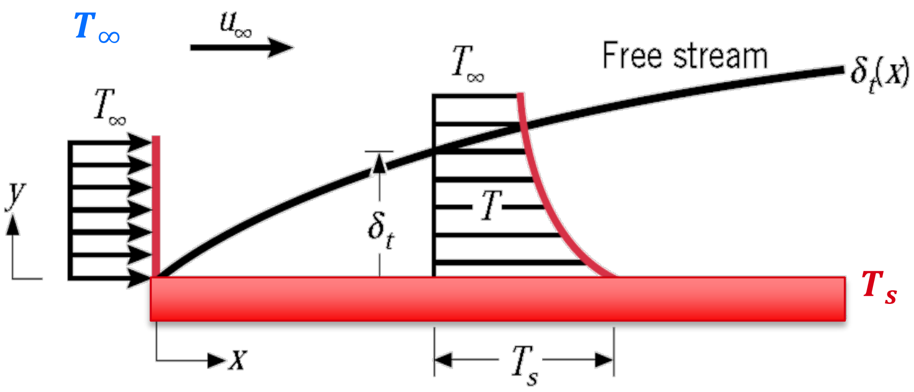
$$\tau_w = 0.332\,\frac{\mu u_\infty}{x}\sqrt{Re_x} \qquad C_f = \frac{0.664}{\sqrt{Re_x}}$$

$$\frac{hx}{k_f} = Nu_x = 0.332 {Re_x}^{1/2} Pr^{1/3}$$

$$\tau_w = 0.664\,\frac{\mu u_\infty}{x}\sqrt{Re_x} \qquad \overline{C_f} = \frac{1.328}{\sqrt{Re_x}}$$

$$\frac{\bar{h}x}{k_f} = \overline{Nu_x} = 0.664 {Re_x}^{1/2} Pr^{1/3}$$

Forced External Convection – Film Properties



$Pr > 0.6$

$$\frac{\delta}{\delta_t} \sim Pr^{1/3}$$

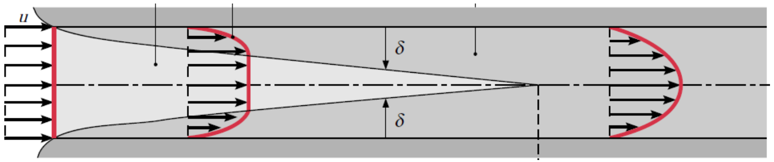
$$\frac{hx}{k_f} = Nu_x = C{Re_x}^a Pr^b$$

BL temperature: $T_f = \dfrac{(T_s + T_\infty)}{2}$

Fluid physical properties	
Density $\rho(T_f)$	
Viscosity $\nu(T_f)$	
Thermal diffusivity $\alpha_f(T_f)$	Specific heat $c_{p,f}(T_f)$ Thermal conductivity $k_f(T_f)$

Forced Internal Convection – Laminar Flow

Velocity Profile



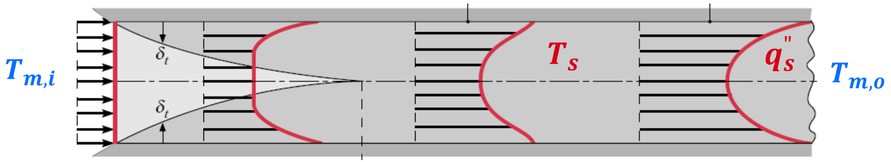
In the fully developed region $\partial u/\partial x = 0$

$$\frac{u(r)}{u_m} = 2\left[1 - \left(\frac{r}{r_o}\right)^2\right]$$

$$u_m = -\frac{r_o^2}{8\mu}\frac{dp}{dx}$$

$\vec{u}(r)$

Temperature Profile



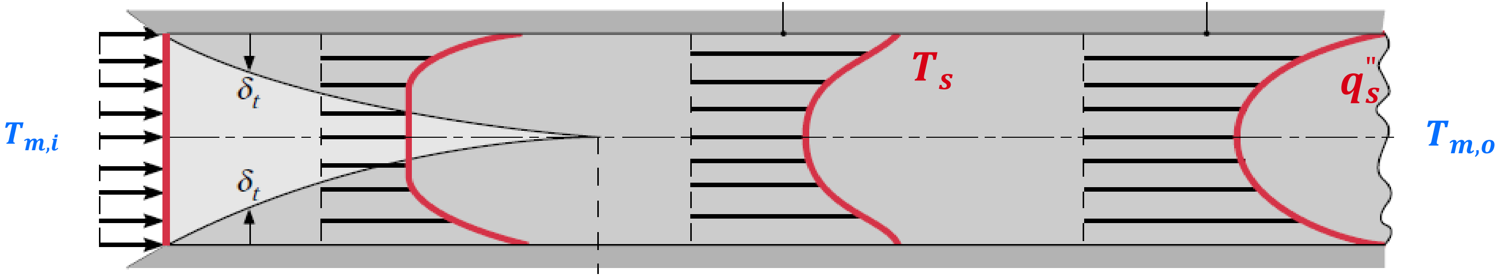
$$\theta = \frac{T_s(x) - T(r,x)}{T_s(x) - T_m(x)}$$

In the fully developed region $\partial \theta/\partial x = 0$

- Constant surface heat flux $\quad T_m(x) = \frac{q_s'' P}{\dot{m} c_p} x + T_{m,i} \quad Q_{conv} = q_s'' P L$
- Constant surface temperature $\quad \frac{T_s - T_{m,o}}{T_s - T_{m,i}} = exp\left(-\frac{\bar{h}A}{\dot{m}c_p}\right) \quad Q_{conv} = \bar{h}A\Delta T_{lm}$

- Constant surface heat flux $\quad \boldsymbol{Nu_D = hD/k_f = 4.36}$
- Constant surface temperature $\quad \boldsymbol{Nu_D = hD/k_f = 3.66}$

Forced Internal Convection – Film Properties

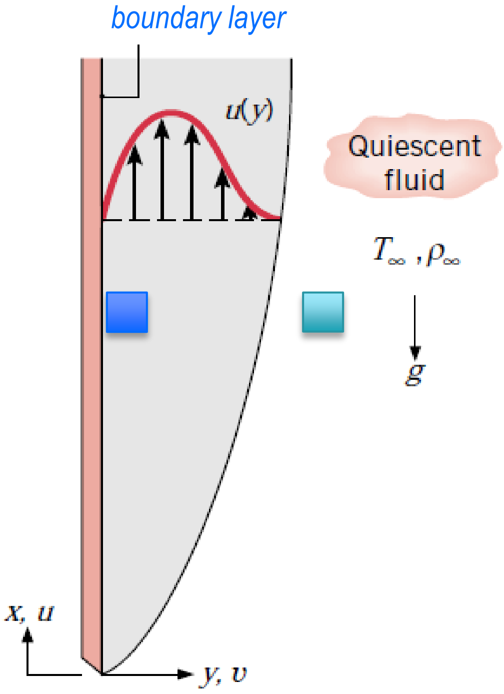


$$\frac{hD}{k_f} = Nu_D = C$$

BL temperature: $T_f = T_m = \frac{(T_{m,i} + T_{m,o})}{2}$

Fluid physical properties	
Density ρ(T_f)	
Viscosity ν(T_f)	
Thermal diffusivity $\alpha_f(T_f)$	Specific heat $c_{p,f}(T_f)$ Thermal conductivity $k_f(T_f)$

Free Convection



$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

(Mass conservation)

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = g\beta(T - T_\infty) + \nu\frac{\partial^2 u}{\partial y^2}$$

(Momentum conservation)

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}$$

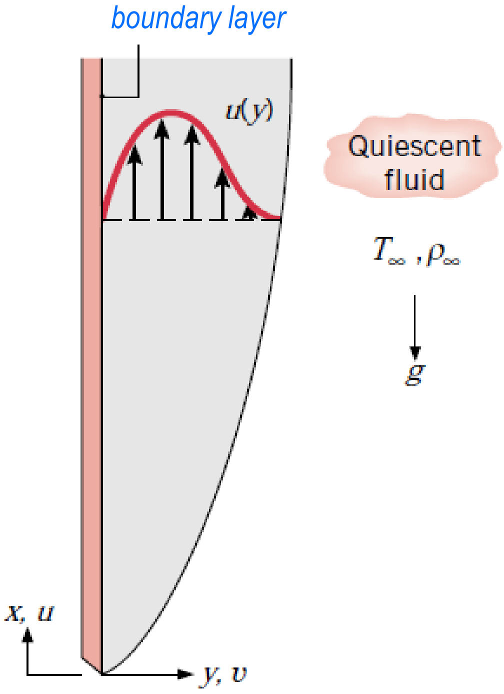
(Energy conservation)

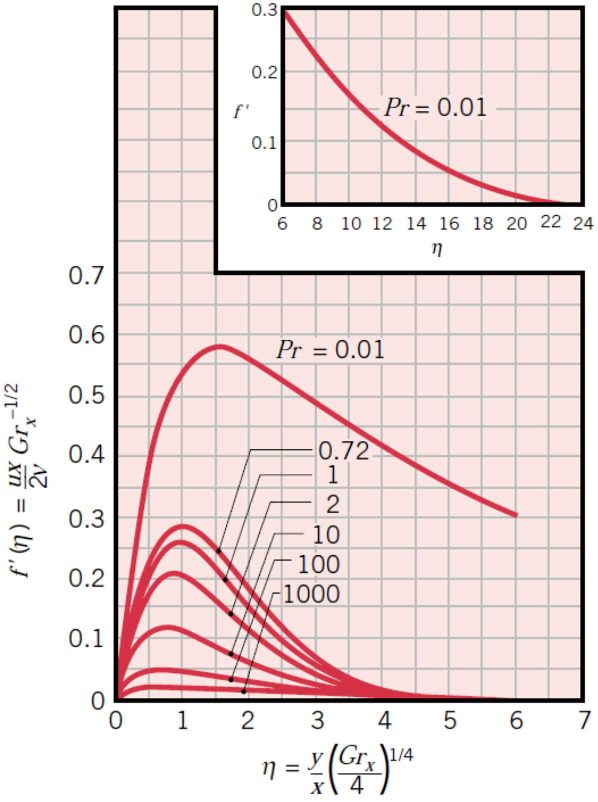
Temperature-driven fluid-motion!

Heat transfer includes advection!

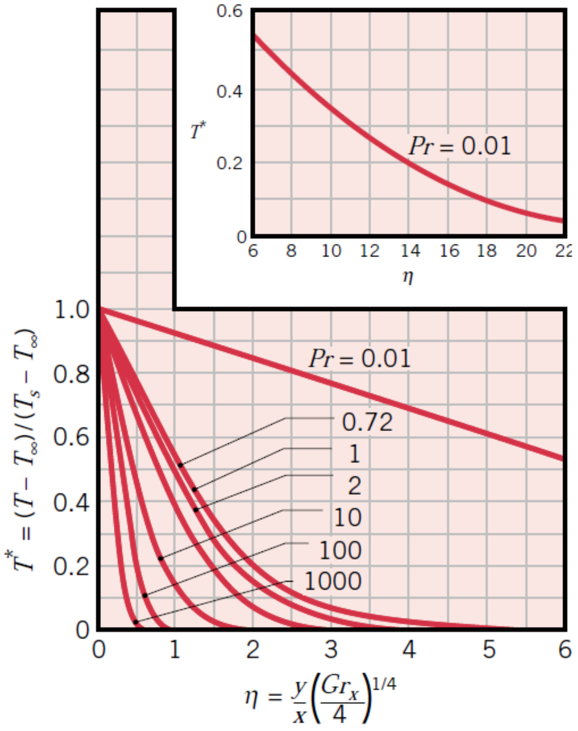
$$T_f = \frac{(T_s + T_\infty)}{2}$$

Free Convection on a Vertical Plate – Laminar Flow





FLUID DYNAMICS



HEAT TRANSFER

Today: RECAP of Forced and Free Convection

- [x] Definition of Convection
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- [ ] Non-dimensional numbers & Flow Conditions
- [ ] General procedure to determine the convection coefficient

Non-dimensional Numbers

Heat Transfer:

$$Pr = \frac{momentum\ diffusivity}{heat\ diffusivity} = \frac{\nu_f}{\alpha_f} = \frac{\mu_f c_{p,f}}{k_f} = \frac{\delta}{\delta_t}$$

Fluid dynamics:

(forced convection)

$$Re_x = \frac{inertia\ forces}{viscous\ forces} = \frac{\rho \boldsymbol{ux}}{\mu}$$

$\boldsymbol{u} = \boldsymbol{u}_\infty$ for external forced conv.

$\boldsymbol{u} = \boldsymbol{u}_m$ for internal forced conv.

(free convection)

$$Gr_x \equiv \frac{g\beta(T_s - T_\infty)x^3}{\nu^2}$$

$$Ra_x \equiv Gr_x Pr = \frac{g\beta(T_s - T_\infty)x^3}{\nu\alpha}$$

Convection coefficient:

$$\boldsymbol{Nu_x} = \frac{\boldsymbol{hx}}{\boldsymbol{k_f}}$$

$Nu_x = F(Re, Pr)$ *(forced convection)*

$Nu_x = F(Ra, Pr)$ *(free convection)*

Flow Conditions

Forced External Convection	$Re_x < 5 \cdot 10^5$	Laminar
	$Re_x > 5 \cdot 10^5$	Turbulent
Forced Internal Convection	$Re_D < 2300$	Laminar
	$Re_D > 2300$	Turbulent
Free Convection	$Ra_x < 10^9$	Laminar
	$Ra_x > 10^9$	Turbulent

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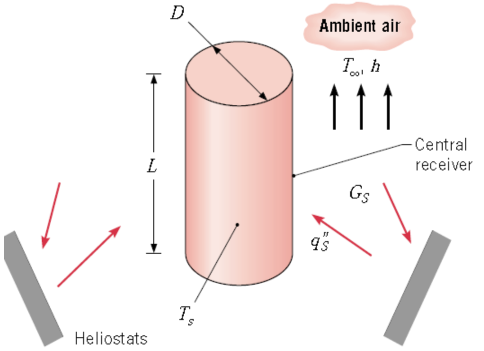
General methodology for calculating the convection coefficient

0. Identify the type of convection (Forced/External, Forced/Internal, Natural, Boiling/Condensation) **[Conv]**
1. Recognize the flow geometry (plate, cylinder, inner/outer flow etc.) **[GEOM]**
2. Specify the appropriate reference temperature and evaluate the pertinent fluid properties at that temperature $\boldsymbol{T_f}$
3. Calculate the Reynolds/Rayleigh number (be careful to use the right characteristic dimension - x, L, D – and velocity - u_m, u_∞) and determine the flow conditions (laminar/turbulent) **[FLOW]**
4. Decide whether a local or surface average coefficient is required **[Loc/Ave]**
5. Calculate Pr or get it from the table **[Pr]**
6. Select the appropriate correlation, determine Nu and the convection coefficient **[Nu, h]**

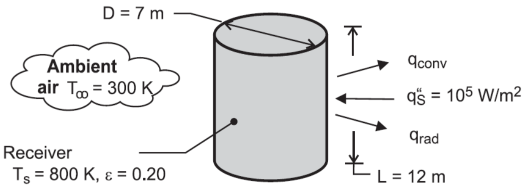
4. Free conv

In the *central receiver* concept of a solar power plant, many heliostats at ground level are used to direct a concentrated solar flux q''_S to the receiver, which is positioned at the top of a tower. However, even with absorption of all the solar flux by the outer surface of the receiver, losses due to free convection and radiation reduce the collection efficiency below the maximum possible value of 100%. Consider a cylindrical receiver of diameter $D = 7$ m, length $L = 12$ m, and emissivity $\varepsilon = 0.20$.



(a) If all of the solar flux is absorbed by the receiver and a surface temperature of $T_s = 800$ K is maintained, what is the rate of heat loss from the receiver? The ambient air is quiescent at a temperature of $T_\infty = 300$ K, and irradiation from the surroundings may be neglected. If the corresponding value of the solar flux is $q''_S = 10^5$ W/m², what is the collector efficiency?

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) Ambient air is quiescent, (3) Incident solar flux is uniformly distributed over receiver surface, (4) All of the incident solar flux is absorbed by the receiver, (5) Negligible irradiation from the surroundings, (6) Uniform receiver surface temperature, (7) Curvature of cylinder has a negligible effect on boundary layer development, (8) Constant properties.

PROPERTIES: *Table A-4*, air (T_f = 550 K): k = 0.0439 W/m·K, $\nu = 45.6 \times 10^{-6}$ m²/s, $\alpha = 66.7 \times 10^{-6}$ m²/s, Pr = 0.683, $\beta = 1.82 \times 10^{-3}$ K⁻¹.

ANALYSIS: (a) The total heat loss is

$$q = q_{rad} + q_{conv} = A_s \varepsilon \sigma T_s^4 + \bar{h} A_s (T_s - T_\infty)$$

With $Ra_L = g\beta(T_s - T_\infty)L^3/\nu\alpha = 9.8\ \text{m/s}^2\ (1.82 \times 10^{-3}\ \text{K}^{-1})\ 500\text{K}\ (12\text{m})^3/(45.6 \times 66.7 \times 10^{-12}\ \text{m}^4/\text{s}^2) = 5.07 \times 10^{12}$, Eq. 9.26 yields

$$\bar{h} = \frac{k}{L}\left\{0.825 + \frac{0.387\,Ra_L^{1/6}}{\left[1+(0.492/Pr)^{9/16}\right]^{8/27}}\right\}^2 = \frac{0.0439\ \text{W/m}\cdot\text{K}}{12\text{m}}\{0.825 + 42.4\}^2 = 6.83\ \text{W/m}^2\cdot\text{K}$$

Hence, with $A_s = \pi DL = 264\ \text{m}^2$

$$q = 264\,\text{m}^2 \times 0.2 \times 5.67 \times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4 (800\,\text{K})^4 + 264\,\text{m}^2 \times 6.83\ \text{W/m}^2\cdot\text{K}(500\,\text{K})$$

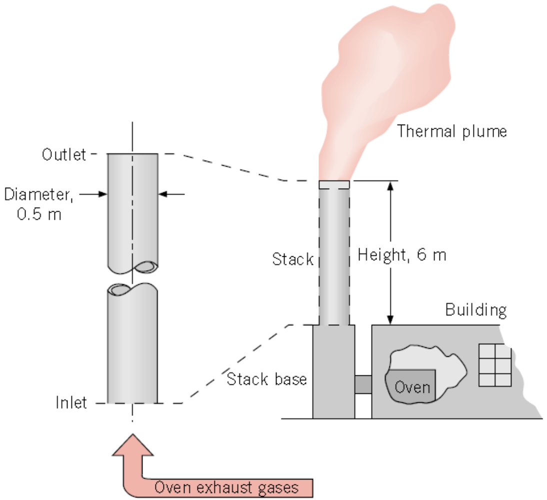
$$q = q_{rad} + q_{conv} = 1.23 \times 10^6\,\text{W} + 9.01 \times 10^5\,\text{W} = 2.13 \times 10^6\,\text{W}$$ <

With $A_s q''_S = 2.64 \times 10^7$ W, the collector efficiency is

$$\eta = \left(\frac{A_s q''_S - q}{A_s q''_S}\right)100 = \frac{\left(2.64 \times 10^7 - 2.13 \times 10^6\right)\text{W}}{2.64 \times 10^7\ \text{W}}(100) = 91.9\%$$ <

The problem considers only losses from the side surface area. However, one can also compute radiation and convection from the top and bottom surfaces (what correlations would you use for these?). Are they important or negligible compared to losses from the side surface?

8.52 Exhaust gases from a wire processing oven are discharged into a tall stack, and the gas and stack surface temperatures at the outlet of the stack must be estimated. Knowledge of the outlet gas temperature $T_{m,o}$ is useful for predicting the dispersion of effluents in the thermal plume, while knowledge of the outlet stack surface temperature $T_{s,o}$ indicates whether condensation of the gas products will occur. The thin-walled, cylindrical stack is 0.5 m in diameter and 6.0 m high. The exhaust gas flow rate is 0.5 kg/s, and the inlet temperature is 600°C.



(a) Consider conditions for which the ambient air temperature and wind velocity are 4°C and 5 m/s, respectively. Approximating the thermophysical properties of the gas as those of atmospheric air, estimate the outlet gas and stack surface temperatures for the given conditions.

Assume Tm,o =773K
Assume Ts = 523 K
Ignore the conduction resistance of the stack

ASSUMPTIONS: (1) Steady-state conditions, (2) Wall thermal resistance negligible, (3) Exhaust gas properties approximated as those of atmospheric air, (4) Radiative exchange with surroundings negligible, (5) Ideal gas with negligible viscous dissipation and pressure variation, (6) Fully developed flow, (7) Constant properties.

PROPERTIES: *Table A.4*, air (assume $T_{m,o}$ = 773 K, $\overline{T}_m$ = 823 K, 1 atm): c_p = 1104 J/kg·K, $\mu = 376.4 \times 10^{-7}$ N·s/m², k = 0.0584 W/m·K, Pr = 0.712; *Table A.4*, air (assume T_s = 523 K, T_∞ = 4°C = 277 K, T_f = 400 K, 1 atm): $\nu = 26.41 \times 10^{-6}$ m²/s, k = 0.0338 W/m·K, Pr = 0.690.

TABLE A.4 Thermophysical Properties of Gases at Atmospheric Pressure[a]

T (K)	ρ (kg/m³)	c_p (kJ/kg · K)	$\mu \cdot 10^7$ (N · s/m²)	$\nu \cdot 10^6$ (m²/s)	$k \cdot 10^3$ (W/m · K)	$\alpha \cdot 10^6$ (m²/s)	Pr
Air							
100	3.5562	1.032	71.1	2.00	9.34	2.54	0.786
150	2.3364	1.012	103.4	4.426	13.8	5.84	0.758
200	1.7458	1.007	132.5	7.590	18.1	10.3	0.737
250	1.3947	1.006	159.6	11.44	22.3	15.9	0.720
300	1.1614	1.007	184.6	15.89	26.3	22.5	0.707
350	0.9950	1.009	208.2	20.92	30.0	29.9	0.700
400	0.8711	1.014	230.1	26.41	33.8	38.3	0.690
450	0.7740	1.021	250.7	32.39	37.3	47.2	0.686
500	0.6964	1.030	270.1	38.79	40.7	56.7	0.684
550	0.6329	1.040	288.4	45.57	43.9	66.7	0.683
600	0.5804	1.051	305.8	52.69	46.9	76.9	0.685
650	0.5356	1.063	322.5	60.21	49.7	87.3	0.690
700	0.4975	1.075	338.8	68.10	52.4	98.0	0.695
750	0.4643	1.087	354.6	76.37	54.9	109	0.702
800	0.4354	1.099	369.8	84.93	57.3	120	0.709
850	0.4097	1.110	384.3	93.80	59.6	131	0.716
900	0.3868	1.121	398.1	102.9	62.0	143	0.720
950	0.3666	1.131	411.3	112.2	64.3	155	0.723
1000	0.3482	1.141	424.4	121.9	66.7	168	0.726
1100	0.3166	1.159	449.0	141.8	71.5	195	0.728

ASSUMPTIONS: (1) Steady-state conditions, (2) Wall thermal resistance negligible, (3) Exhaust gas properties approximated as those of atmospheric air, (4) Radiative exchange with surroundings negligible, (5) Ideal gas with negligible viscous dissipation and pressure variation, (6) Fully developed flow, (7) Constant properties.

PROPERTIES: *Table A.4*, air (assume $T_{m,o}$ = 773 K, $\overline{T}_m$ = 823 K, 1 atm): c_p = 1104 J/kg·K, μ = 376.4 × 10^{-7} N·s/m^2, k = 0.0584 W/m·K, Pr = 0.712; *Table A.4*, air (assume T_s = 523 K, T_∞ = 4°C = 277 K, T_f = 400 K, 1 atm): ν = 26.41 × 10^{-6} m^2/s, k = 0.0338 W/m·K, Pr = 0.690.

ANALYSIS: (a) From Eq. 8.45a,

$$T_{m,o} = T_\infty - \left(T_\infty - T_{m,i}\right)\exp\left[-\frac{PL}{\dot{m}c_p}\overline{U}\right] \qquad U = 1\Big/\left(\frac{1}{h_i}+\frac{1}{h_o}\right) \tag{1,2}$$

where h_i and h_o are average coefficients for internal and external flow, respectively.

Internal flow: With a Reynolds number of

$$Re_{D_i} = \frac{4\dot{m}}{\pi D\mu} = \frac{4\times 0.5\,\text{kg/s}}{\pi\times 0.5\,\text{m}\times 376.4\times10^{-7}\,\text{N}\cdot\text{s/m}^2} = 33{,}827 \tag{3}$$

and the flow is turbulent. Considering the flow to be fully developed throughout the stack (L/D = 12) and with $T_s < T_m$, the Dittus-Boelter correlation has the form

$$Nu_D = \frac{h_i D}{k} = 0.023\,Re_{D_i}^{4/5}\,Pr^{0.3} \tag{4}$$

$$h_i = \frac{58.4\times10^{-3}\,\text{W/m}\cdot\text{K}}{0.5\,\text{m}}\times 0.023\,(33{,}827)^{4/5}(0.712)^{0.3} = 10.2\,\text{W/m}^2\cdot\text{K}.$$

External flow: Working with the Churchill/Bernstein correlation, the Reynolds and Nusselt numbers are

$$Re_{D_o} = \frac{VD}{\nu} = \frac{5\,\text{m/s}\times 0.5\,\text{m}}{26.41\times10^{-6}\,\text{m}^2/\text{s}} = 94{,}660 \tag{5}$$

$$\overline{Nu}_D = 0.3 + \frac{0.62\,Re_D^{1/2}\,Pr^{1/3}}{\left[1+(0.4/Pr)^{2/3}\right]^{1/4}}\left[1+\left(\frac{Re_D}{282{,}000}\right)^{5/8}\right]^{4/5} = 205$$

Hence,

$$h_o = (0.0338\,\text{W/m}\cdot\text{K}/0.5\,\text{m})\times 205 = 13.9\,\text{W/m}^2\cdot\text{K} \tag{6}$$

The outlet gas temperature is then

$$T_{m,o} = 4^\circ\text{C} - (4-600)^\circ\text{C}\exp\left[-\frac{\pi\times0.5\,\text{m}\times6\,\text{m}}{0.5\,\text{kg/s}\times1104\,\text{J/kg}\cdot\text{K}}\left(\frac{1}{1/10.2+1/13.9}\,\text{W/m}^2\cdot\text{K}\right)\right] = 543^\circ\text{C} \quad <$$

The outlet stack surface temperature can be determined from a local surface energy balance of the form, $h_i(T_{m,o} - T_{s,o}) = h_o(T_{s,o} - T_\infty)$, which yields

$$T_{s,o} = \frac{h_i T_{m,o} + h_o T_\infty}{h_i + h_o} = \frac{(10.2\times543+13.9\times4)\,\text{W/m}^2}{(10.2+13.9)\,\text{W/m}^2\cdot\text{K}} = 232^\circ\text{C} \quad <$$