

Exercise 2.1

The wall of a house is a composite structure consisting, from outside to inside, of a layer of wood ($L_w = 20mm$, $k_w = 0.12W/mK$), a fiberglass insulation ($L_f = 100mm$, $k_f = 0.038W/mK$, $\rho_f = 28kg/m^3$) and a plaster board ($L_p = 10mm$, $k_p = 0.17W/mK$). On a winter day the air inside is at $T_{in} = 20^\circ C$ while the air outside is at $T_{out} = -15^\circ C$. Furthermore, due to the wind and internal ventilation the convection coefficients are $h_{out} = 60W/m^2K$ and $h_{in} = 30W/m^2K$. The wall has a total surface of $350m^2$.

- Draw a sketch of the system indicating all the dimensions and material parameters of the problem (known and unknown). List all the assumptions.
- Draw the equivalent electrical circuit and determine a symbolic expression for the total thermal resistance of the wall, including inside and outside convection effects.
- Determine the total heat loss through the wall.
- If the wind outside increases, raising h_{out} to $300W/m^2K$, determine the percentage increase in the heat loss
- What is the controlling resistance that determines the amount of heat flux through the wall?
- OPTIONAL QUESTION (Difficult) While the conditions inside remain unchanged, consider a varying external temperature of the form:

$$T_{\infty,o}(K) = 273 + 5\sin\left(\frac{2\pi}{24}t\right)$$

for $0 \leq t \leq 12h$

$$T_{\infty,o}(K) = 273 + 11\sin\left(\frac{2\pi}{24}t\right)$$

for $12 \leq t \leq 24h$.

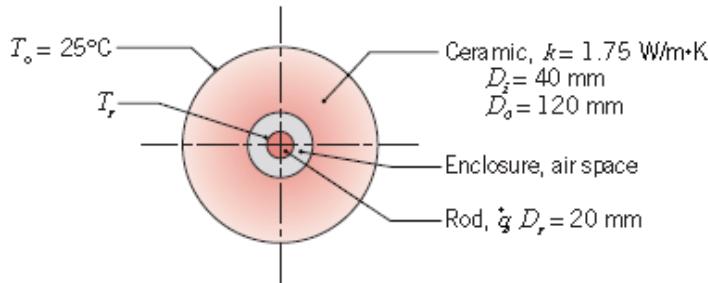
The convection coefficient outside is always $h_{out} = 60W/m^2K$. Assume a quasi-steady condition for which changes in energy storage within the wall may be neglected. Can you estimate the daily heat-loss through the wall?

- If the fuel to heat the room costs $0.1chf/kWh$, how much would be the daily bill to heat the room?

Exercise 2.2

An electric current flows through a long rod ($D_{rod} = 20\text{mm}$) generating thermal energy at a uniform volumetric rate $\dot{q} = 2 \cdot 10^6 \text{W/m}^3$. The rod is concentric with a hollow ceramic cylinder ($D_{cer,inner} = 40\text{mm}$, $D_{cer,outer} = 120\text{mm}$, $k_{cer} = 1.75 \text{W/m}\cdot\text{K}$), thus creating an enclosure that is filled with air. The temperature of the outside surface of the ceramic cylinder is $T_o = 25^\circ\text{C}$. The thermal resistance per unit length due to radiation between the enclosure surfaces is $R'_{rad} = 0.3 \text{mK/W}$ and the natural convection coefficient in the enclosure is $h = 20 \text{W/m}^2\text{K}$.

- Draw a schematic of the system including all dimensions and material parameters. List all of the assumptions.
- Construct a thermal circuit that can be used to calculate the surface temperature of the rod, T_r . Label all temperatures, heat rates and thermal resistances and evaluate each thermal resistance.
- Calculate the surface temperature of the rod for the prescribed conditions.



Exercise 2.3

Consider a plane composite wall that is composed of three materials (A,B and C from left to right) of thermal conductivities $k_A = 0.24W/mK$, $k_B = 0.13W/mK$, $k_C = 0.5W/mK$. The three layers have thicknesses of $L_A = 20mm$, $L_B = 13mm$ and $L_C = 20mm$. A contact resistance of $R_{cont} = 0.01m^2K/W$ exists at the interface between materials A and B as well as at the interface of materials B and C. The left face of the composite wall is insulated while the right face is exposed to convection with $h = 10W/m^2K$ and $T_{air} = 20^\circ C$. For Case 1, thermal energy is generated within material A at a rate ($q_A = 5000W/m^3$). For Case 2 thermal energy is generated within material C at the rate ($q_C = 5000W/m^3$).

- a) Draw a schematic of the wall and indicate all dimensions and material parameters.
- b) Determine the maximum temperature within the composite wall under steady state conditions for Case 1.
- c) Sketch the steady state temperature distribution on T-x coordinates for Case 1.
- d) Sketch the steady state temperature distribution for Case 2 on the same T-x coordinates used for Case 1.

Exercise 2.4 FOR REVISION

Superheated steam at $575^{\circ}C$ has to be conveyed from the boiler to the turbine of an electric power plant. To do so, steel tubes are used ($k_{steel} = 35W/mK$, $D_1 = 300mm$ inner diameter and $30mm$ wall thickness). To reduce heat losses to the surroundings a layer of calcium silicate insulation ($k_{ins} = 0.1W/mK$) is applied to the tubes. Furthermore, a thin sheet of aluminum, having an emissivity of $\epsilon = 0.2$, is used to wrap the tubes. The air and wall temperatures of the power plant are $T_{sur} = 27^{\circ}C$. Assume that the temperature of the inner surface of a steel tube corresponds to that of the steam and assume a convection coefficient outside the tube of $h_o = 6W/m^2K$.

- a) Make a sketch of the system, indicating all the dimensions and material parameters. List the assumptions.
- b) Draw the equivalent electrical circuit. HARD QUESTION What is the minimum insulation thickness needed to ensure that the temperature of the aluminum sheet does not exceed $T_{s,omax} = 50^{\circ}C$? (Note: you will get a complex expression that can give you a numerical value only through iteration)
- c) What is the corresponding heat loss per meter of tube length?
- d) (Hard question) How would you expect the temperature of the aluminum sheet to change as a function of the thickness of the insulating layer?

Exercise 2.5 FOR REVISION

Steam flowing through a long, thin-walled pipe maintains the pipe wall at a uniform temperature of $T_w = 500K$. The pipe is covered with an inhomogeneous insulation blanket comprised of two different materials, A ($k_A = 2W/mK$) and B ($k_B = 0.25W/mK$). The interface between the two materials may be assumed to have an infinite contact resistance and the entire outer surface is exposed to air at $T_{air} = 300K$ such that the convection coefficient is $h = 25W/m^2K$.

- Sketch the thermal circuit of the system. Label all pertinent nodes and resistances.
- For the given conditions, what is the total heat loss from the pipe? What are the outer surfaces T_A and T_B ?

