# Exercise 4.1

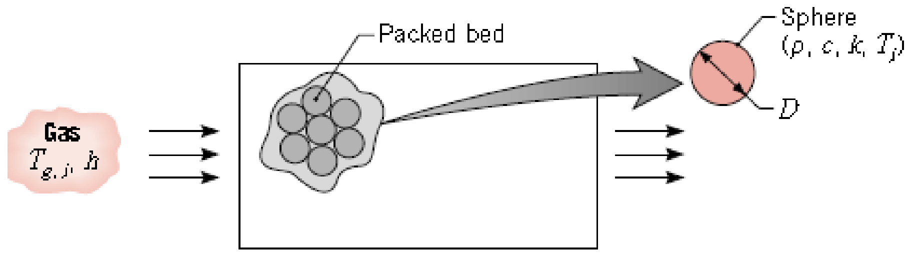
Thermal energy storage systems commonly involve packed bed of solid spheres, through which a hot gas flows if the system is being charged, or a cold gas if it is being discharged. In a charging process, heat transfer from the hot gas increases thermal energy stored within the colder spheres; during discharge, the stored energy decreases as heat is transferred from the warmer spheres to the cooler gas. Consider a packed bed of 75 mm diameter aluminum spheres ($\rho = 2700$ kg/m$^3$, $c = 950$J/kgK, $k = 150$ W/mK) and a charging process for which gas enters the storage unit at a temperature of $T_{g,i} = 300$°C. If the initial temperature of the spheres is $T_i = 25$°C and the convection coefficient is $h = 75$ W/m$^2$K, calculate:

a) How long does it take a sphere near the inlet of the system to accumulate 90% of the maximum possible thermal energy?

b) What is the corresponding temperature at the center of the sphere?

c) Is there any disadvantage to using copper instead of aluminum $\rho_{Cu} = 8900$kg/m$^3$ and $c_{Cu} = 400$ J/kgK) ?

d) Calculate the time needed to accumulate 90% of the maximum possible thermal energy and the corresponding temperature at the center of the sphere if the spheres are made of Pyrex, with $\rho = 2225$kg/m$^3$ ,$c = 835$ J/kg K, $k = 1.4$W/mK.

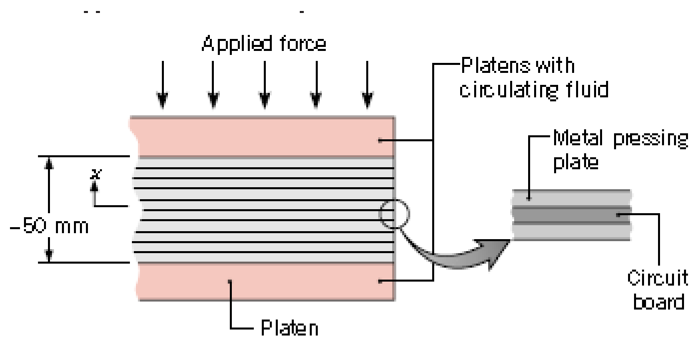


## Exercise 4.2

Circuit boards are treated by heating a stack of them under high pressure. The platens at the top and bottom of the stack are maintained at a uniform temperature by a circulating fluid. To achieve a curing condition, the epoxy has to be maintained at or above $170°C$ for at least 5 min. The effective thermo-physical properties of the stack are $k = 0.613 W/mK$ and $\rho c_p = 2.73 \cdot 10^6 J/m^3K$.

a) If the stack is initially at $15°C$ and, following application of pressure, the platens are suddenly brought to a uniform temperature of $190°C$, calculate the elapsed time $t_e$ required for the mid-plane of the stack to reach the cure temperature of $170°C$.

b) If at this time $t = t_e$, the platen temperature were reduced suddenly to $15°C$, how much energy would have to be removed from the stack by the coolant circulating in the platen in order to return the stack to its initial uniform temperature?



## Exercise 4.3

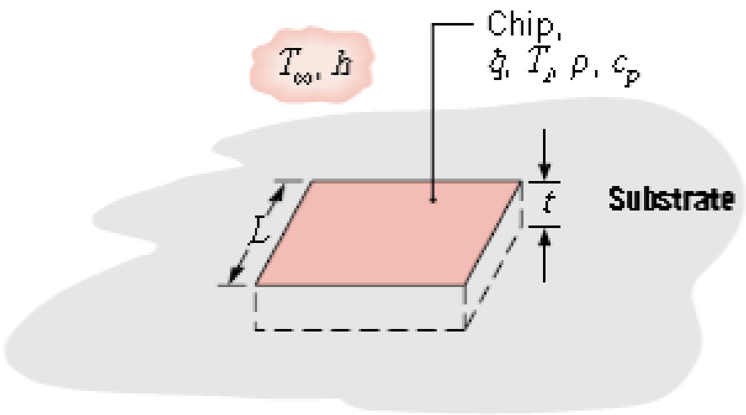
A procedure for determining the thermal conductivity of a solid material involves embedding a thermocouple in a thick slab of the solid and measuring the response to a prescribed change in temperature at one surface. Consider an arrangement for which the thermocouple is embedded 10mm from a surface that is suddenly brought to a temperature of $100°C$ by exposure to boiling water. If the initial temperature of the slab was $30°C$ and the thermocouple measures a temperature of $65°C$ 2min after the surface is brought to $100°C$, what is the thermal conductivity? The density and specific heat of the solid are known to be $\rho = 2200 kg/m^3$ and $c_p = 700 J/kgK$.

## Exercise 4.4 FOR REVISION

A chip that is of length $L = 5$mm on a side and thickness $t = 1$mm is encased in a ceramic substrate, and its exposed surface is convectively cooled by a dielectric liquid for which $h = 150\text{W/m}^2\text{K}$ and $T_\infty = 20°\text{C}$. In the off-mode the chip is in thermal equilibrium with the coolant ($T_i = T_\infty$). When the chip is energized, however, its temperature increases until a new steady-state is established. For purposes of analysis, the energized chip is characterized by uniform volumetric heating with $\dot{q} = 9 \times 10^6$ $\text{W/m}^3$.

a) Assuming an infinite contact resistance between the chip and substrate and negligible conduction resistance within the chip, determine the steady-state chip temperature $T_f$. Following activation of the chip, how long does it take to come within 1°C of this temperature? The chip density and specific heat are $\rho = 2000$ kg/m$^3$ and $c = 700$J/kgK, respectively.

b) For a more realistic analysis the indirect transfer from the chip to the substrate and then from the substrate to the coolant needs to be accounted for. The total thermal resistance associated with this indirect route includes contributions due to the chip-substrate interface (a contact resistance), multidimensional conduction in the substrate and convection from the surface of the substrate to the coolant. If this total thermal resistance is $R_t = 200$W/K, what is the steady state chip temperature $T_f$ ? Following the activation of the chip, how long does it take to come within 1°C of this temperature?




**Hint:** The general equation for heat transfer accounting for ALL of the mechanisms is:

$$q_s'' A_{s,h} + \dot{E}_g - [h(T - T_\infty) + \varepsilon\sigma(T^4 - T_{sur}^4)]A_{s(c,r)} = \rho V c \frac{dT}{dt}$$

An exact solution to Equation 5.15 may also be obtained if radiation may be neglected and $h$ is independent of time. Introducing a temperature difference $\theta \equiv T - T_\infty$, where $d\theta/dt = dT/dt$, Equation 5.15 reduces to a linear, first-order, nonhomogeneous differential equation of the form

$$\frac{d\theta}{dt} + a\theta - b = 0 \tag{5.20}$$

where $a \equiv (hA_{s,c}/\rho Vc)$ and $b \equiv [(q''_s A_{s,h} + \dot{E}_g)/\rho Vc]$. Although Equation 5.20 may be solved by summing its homogeneous and particular solutions, an alternative approach is to eliminate the nonhomogeneity by introducing the transformation

$$\theta' \equiv \theta - \frac{b}{a} \tag{5.21}$$

Recognizing that $d\theta'/dt = d\theta/dt$, Equation 5.21 may be substituted into (5.20) to yield

$$\frac{d\theta'}{dt} + a\theta' = 0 \tag{5.22}$$

Separating variables and integrating from 0 to $t$ ($\theta'_i$ to $\theta'$), it follows that

$$\frac{\theta'}{\theta'_i} = \exp(-at) \tag{5.23}$$

or substituting for $\theta'$ and $\theta$,

$$\frac{T - T_\infty - (b/a)}{T_i - T_\infty - (b/a)} = \exp(-at) \tag{5.24}$$

Hence

$$\frac{T - T_\infty}{T_i - T_\infty} = \exp(-at) + \frac{b/a}{T_i - T_\infty}[1 - \exp(-at)] \tag{5.25}$$

# Exercise 4.5 FOR REVISION

A plastic coating is applied to wood panels by first depositing molten polymer on a panel and then cooling the surface of the polymer by subjecting it to air flow at $25°C$. As first approximation, the heat of reaction associated with solidification of the polymer may be neglected and the polymer/wood interface may be assumed to be adiabatic. If the thickness of the coating is $L = 2mm$ and it has an initial uniform temperature of $T_i = 200°C$, how long will it take for the surface to achieve a safe-to-touch temperature of $42°C$ if the convection coefficient is $h = 200W/m^2K$ ?

What is the corresponding value of the interface temperature?

The thermal conductivity and diffusivity of the plastic are $k = 0.25W/mK$ and $\alpha = 1.2 \cdot 10^{-7} m^2/s$.