Exercise 3.1

Turbine blades mounted to a rotating disk in a gas turbine engine are exposed to a gas stream that is at $T_\infty = 1200°\text{C}$ and maintains a convection coefficient of $h = 250\ \text{W/m}^2\text{K}$ over the blade.

The blades are fabricated from Inconel with $k = 20\ \text{W/mK}$ and have a length of 50 mm. The blade profile has a constant cross-sectional area of $A_c = 0.0006\ \text{m}^2$ and a perimeter $P = 110$ mm. A proposed blade cooling scheme, which involves routing air through the supporting disc, is able to maintain the base of each blade at a temperature $T_b = 300°\text{C}$.

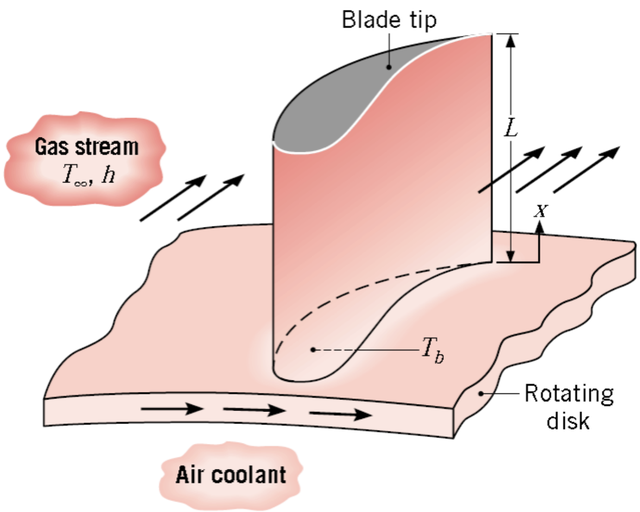
a) If the maximum allowable blade temperature is 1050°C, is the proposed cooling scheme satisfactory?

   Hint 1: treat the turbine blade as a fin of constant cross-section and assume that the tip of the blade is adiabatic.

   Hint2: carefully consider the physical problem and imagine where the temperature would be maximum in this fin!!

b) For the proposed cooling scheme, what is the rate at which heat is transferred from each blade to the coolant?

If necessary use the hyperbolic function table at the end of the document to determine the necessary values.



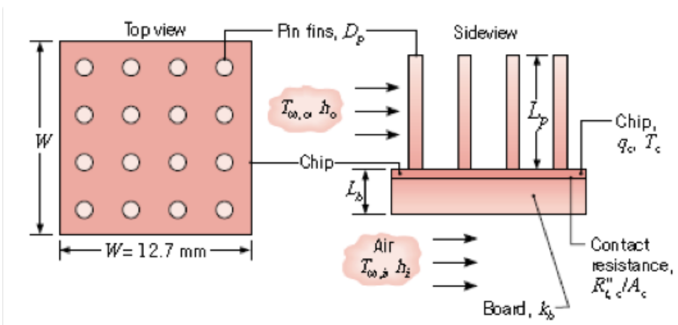
Exercise 3.2

The maximum operation temperature for an electronic chip is 75°C. To maximize the dissipation from a square chip with side length $W = 12.7mm$, it is proposed that a 4x4 array of copper fins ($k = 400$W/mK) be metallurgically joined to the outer surface of the chip.

a) Sketch the equivalent thermal circuit for the pin-chip-board assembly assuming 1D steady state conditions and negligible contact resistance between the pins and the chip. Hint: consider the heat removed from a unit cell of the periodic arrangement of fins. Consider the thermal resistance of one fin and do not forget to account for convection at the exposed surface of the chip which is not covered by the fins.

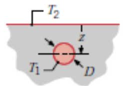
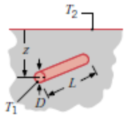
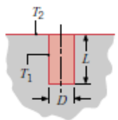
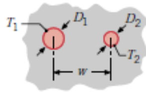
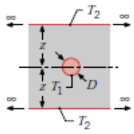
b) Write the expression for all the thermal resistances involved in the problem. Consider the convection heat transfer at the tip of each fin.

c) the pin diameter and length are $D_p = 1.5$mm and $L_p = 15$mm respectively and the contact resistance is $R''_t = 10^{-4}$ $[\frac{m^2 K}{W}]$, find what is the maximum heat dissipation rate Q_c when $T_c = 75$°C and $h = 1000$W/m2K



Exercise 3.3

Two parallel pipelines spaced 0.5m apart are buried in soil having a thermal conductivity of 0.5W/mK. The pipes have outer diameter of 100mm and 75mm with surface temperatures of 175°C and 5°C respectively. Estimate the heat transfer rate per unit length between the two pipelines.

System	Schematic	Restrictions	Shape Factor
Case 1 Isothermal sphere buried in a semi-infinite medium		$z > D/2$	$\frac{2\pi D}{1 - D/4z}$
Case 2 Horizontal isothermal cylinder of length L buried in a semi-infinite medium		$L \gg D$ $L \gg D$ $z > 3D/2$	$\frac{2\pi L}{\cosh^{-1}(2z/D)}$ $\frac{2\pi L}{\ln(4z/D)}$
Case 3 Vertical cylinder in a semi-infinite medium		$L \gg D$	$\frac{2\pi L}{\ln(4L/D)}$
Case 4 Conduction between two cylinders of length L in infinite medium		$L \gg D_1, D_2$ $L \gg w$	$\frac{2\pi L}{\cosh^{-1}\left(\frac{4w^2 - D_1^2 - D_2^2}{2D_1D_2}\right)}$
Case 5 Horizontal circular cylinder of length L midway between parallel planes of equal length and infinite width		$z \gg D/2$ $L \gg z$	$\frac{2\pi L}{\ln(8z/\pi D)}$

Exercise 3.4

Consider an alloyed aluminum ($k = 180\text{W/mK}$) rectangular fin of length $L = 10(mm)$, thickness $t = 1\text{mm}$ and width $w >> t$. The base temperature of the fin is $T_b = 100°\text{C}$ and the fin is exposed to a fluid of temperature $T_\infty = 25°\text{C}$. Assuming a uniform convection coefficient $h = 100\text{W/m}^2\text{K}$ over the entire fin surface, determine the fin heat transfer rate per unit width q'_f, efficiency η_f, effectiveness ϵ_f and thermal resistance per unit length R'_f and the tip temperature $T(L)$ for the case of:

a) Convective heat transfer at the tip

b) Adiabatic tip

c) How do these numbers compare to the values in the infinite fin approximation?

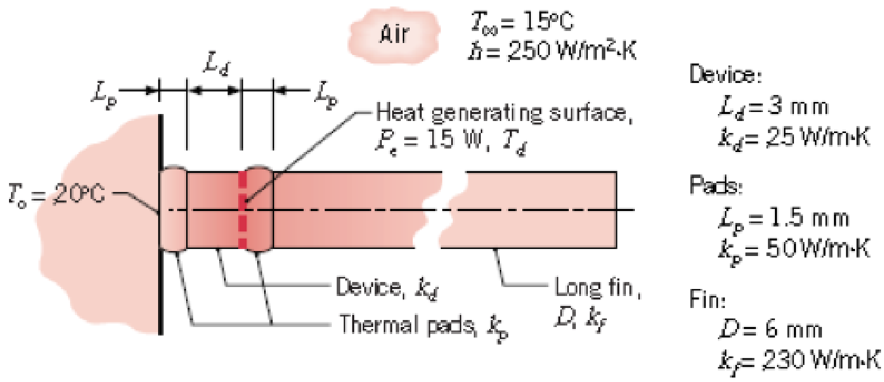
Exercise 3.5 FOR REVISION

A disk shaped electronic device of thickness L_d, diameter L and thermal conductivity k_d, dissipates electrical power at a steady rate of P_e along one of its surfaces. The device is bonded to a cooled base at T_0 using a thermal pad of thickness L_p and a thermal conductivity k_p. A long fin of diameter D and thermal conductivity k_f is bonded to the heat generating surface of the device using an identical thermal pad. The fin is cooled by an air stream which is at temperature T_∞ and provides a convection coefficient h.

a) Construct the thermal circuit of the system and write the expression of the thermal resistances involved.

b) Derive an expression for the temperature T_d of the heat-generating surface of the device in terms of the circuit thermal resistances , T_0 and T_∞

c) Calculate T_d for the prescribed conditions.



Note: considered the much shorter length of the thermal pads and electronic device compared to the fin, neglect the effect of convection along these parts.

Exercise 3.6 FOR REVISION

Copper tubing is joined to the absorber of a flat-plate solar collector as shown in this figure. The aluminum alloy absorber plate is 6mm thick with $k = 180$ $[\frac{W}{mK}]$ and well insulated on its bottom. The top surface of the plate is separated from a transparent cover plate by an evacuated space. The tubes are spaced at a distance $L = 0.2$m from each other. Water is circulated through the tubes to remove the collected energy and it may be assumed to have a constant temperature of $T_w = 60$°C. Under steady-state operating conditions the net radiation heat flux to the surface is $q''_{rad} = 800\text{W/m}^2$. (Note: this value accounts for both the radiation absorption by the collector plate and radiative heat exchange between the collector plate and the cover-plate).

a) Draw a schematic of the flat-plate solar collector and the water tubing with all the relevant physical parameters and known temperature/heat flux values. Treating the flat-plate collector as a fin, identify an infinitesimal section of it with length dx and write the energy balance for it. Then express it as a function of the temperature. Integrate the obtained differential equation to find the function that describes the temperature profile in the collector plate.

b) Assume that the temperature of the absorber plate directly above a tube is equal to that of the water. What is the maximum temperature on the collector plate?

c) What is the heat transfer rate per unit length of tube?

d) Let's now imagine that the cover plate is removed. In this case the surface of the collector plate is directly cooled by air flowing over it with temperature T_∞ and convection coefficient h. Derive the differential equation that governs the temperature distribution $T(x)$ in the plate and define the appropriate boundary conditions. What is the solution for this differential equation? (DIFFICULT)