Exercise 2.1

The wall of a house is a composite structure consisting, from outside to inside, of a layer of wood ($L_w = 20mm$, $k_w = 0.12W/mK$), a fiberglass insulation ($L_f = 100mm$, $k_f = 0.038W/mK$, $\rho_f = 28kg/m^3$) and a plaster board($L_p = 10mm$, $k_p = 0.17W/mK$). On a winter day the air inside is at $T_{in} = 20°C$ while the air outside is at $T_{out} = -15°C$. Furthermore, due to the wind and internal ventilation the convection coefficients are $h_{out} = 60W/m^2K$ and $h_{in} = 30W/m^2K$. The wall has a total surface of $350m^2$.

a) Draw a sketch of the system indicating all the dimensions and material parameters of the problem (known and unknown). List all the assumptions.

b) Draw the equivalent electrical circuit and determine a symbolic expression for the total thermal resistance of the wall, including inside and outside convection effects.

c) Determine the total heat loss through the wall.

d) If the wind outside increases, raising h_{out} to $300W/m^2K$, determine the percentage increase in the heat loss

e) What is the controlling resistance that determines the amount of heat flux through the wall?

f) OPTIONAL QUESTION (Difficult) While the conditions inside remain unchanged, consider a varying external temperature of the form:

$$T_{\infty,o}(K) = 273 + 5sin(\frac{2\pi}{24}t)$$

for $0 \leq t \leq 12h$

$$T_{\infty,o}(K) = 273 + 11sin(\frac{2\pi}{24}t)$$

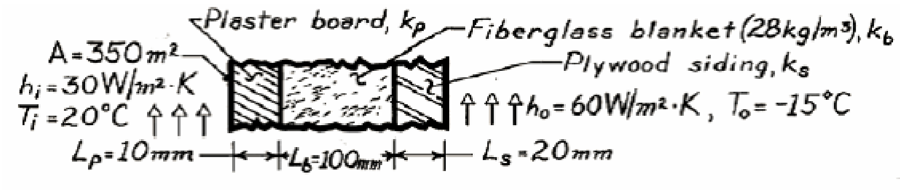
for $12 \leq t \leq 24h$.

The convection coefficient outside is always $h_{out} = 60W/m^2K$. Assume a quasi-steady condition for which changes in energy storage within the wall may be neglected. Can you estimate the daily heat-loss through the wall?

g) If the fuel to heat the room costs $0.1chf/kWh$, how much would be the daily bill to heat the room?

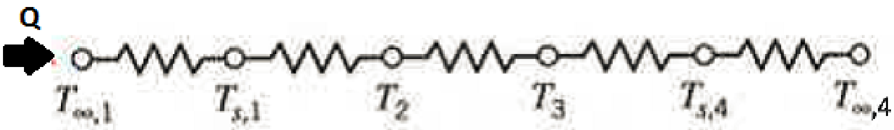
Solution

a)



Assumptions: i) 1D conduction, ii) steady-state, iii) negligible contact resistance

b)



Therefore the total thermal resistance of the composite wall is (from inside to outside):

$$R_{tot} = 1/(h_i A) + L_p/(k_p A) + L_f/(k_f A) + L_w/(k_w A) + 1/(h_o A)$$

c) The heat transfer rate along the wall must be constant so we can write:

$$Q_x = \frac{T_i - T_o}{R_{tot}}$$

Substituting all the numerical values we get:

$$R_{tot} = 831 \cdot 10^{-5} K/W$$

and therefore we obtain

$$Q_x = 4.21 kW$$

d) Recalculating R_{tot} with the new h_{out} we obtain:

$$R_{tot} = 826 \cdot 10^{-5} K/W$$

corresponding to a 0.6% decrease in the thermal resistance. This will lead to a 0.6% increase in heat transfer rate.

e) Looking at each term of the total thermal resistance individually we see that the fiberglass layer thermal resistance is $R_f = L_f/(k_f A) = 752 \cdot 10^{-5} K/W$ so for the conditions of part c) it constitutes $\approx 90\%$ of the total thermal resistance.

f) Based on the assumptions, we can obtain the total heat transfer exchanged in a day by integrating:

$$Q = \int_0^{24h} \frac{T_{\infty,i} - T_{\infty,o}}{R_{tot}} dt$$

Considered that the day and night functions are different we write:

$$Q = \frac{1}{R_{tot}} \left(\int_0^{12h} 293 - [273 + 5sin(\frac{2\pi}{24}t)]dt + \int_{12h}^{24h} 293 - [273 + 11sin(\frac{2\pi}{24}t)]dt \right)$$

After integration we get:

$$Q = \frac{1}{R_{tot}} \left(\left[20t + 5\frac{24}{2\pi} cos(\frac{2\pi}{24}t) \right]_0^{12} + \left[20t + 11\frac{24}{2\pi} cos(\frac{2\pi}{24}t) \right]_{12}^{24} \right)$$

and substituting all the numbers we find:

$$Q = 63.28kWh$$

g) Finally we can calculate:

$$Cost = Q \cdot Price_{kWh} = 6.3CHF/day$$

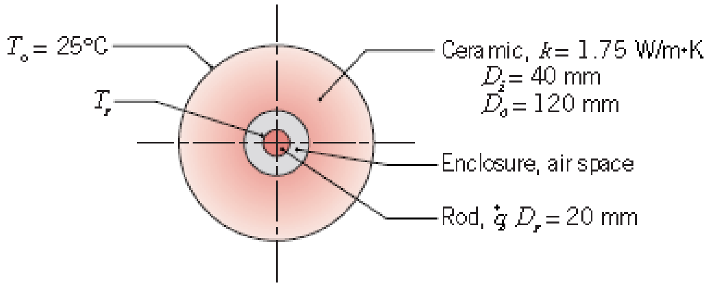
Exercise 2.2

An electric current flows through a long rod ($D_{rod} = 20mm$) generating thermal energy at a uniform volumetric rate $\dot{q} = 2 \cdot 10^6 W/m^3$. The rod is concentric with a hollow ceramic cylinder ($D_{cer,inner} = 40mm$, $D_{cer,outer} = 120mm$, $k_{cer} = 1.75W/mK$), thus creating an enclosure that is filled with air. The temperature of the outside surface of the ceramic cylinder is $T_o = 25°C$. The thermal resistance per unit length due to radiation between the enclosure surfaces is $R'_{rad} = 0.3mK/W$ and the natural convection coefficient in the enclosure is $h = 20W/m2K$.

a) Draw a schematic of the system including all dimensions and material parameters. List all of the assumptions.

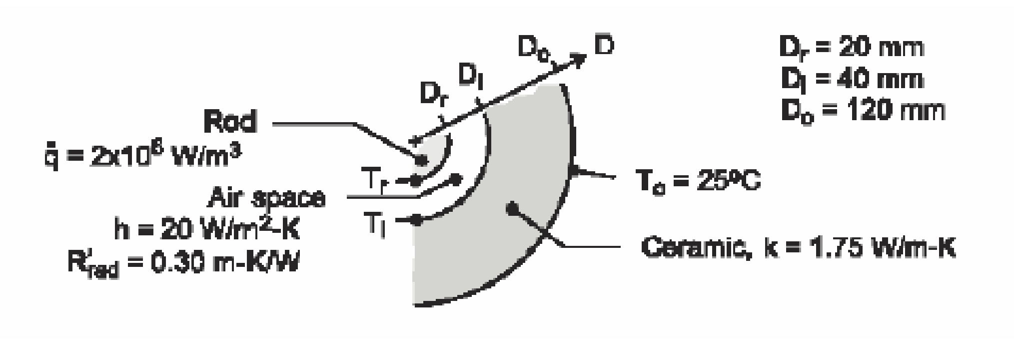
b) Construct a thermal circuit that can be used to calculate the surface temperature of the rod, T_r. Label all temperatures, heat rates and thermal resistances and evaluate each thermal resistance.

c) Calculate the surface temperature of the rod for the prescribed conditions.



Solution

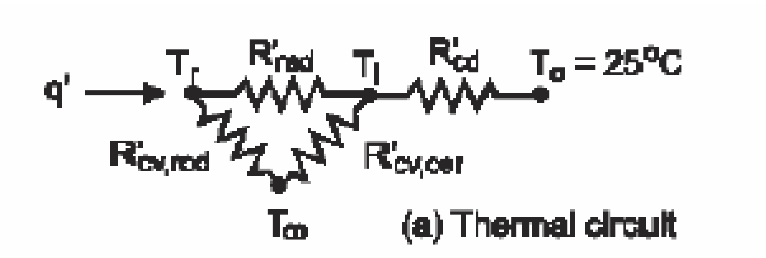
a)



Assumptions: i) steady state, ii) 1D radial conduction, iii) free convection and radiation in the enclosure

b)

We observe that in the enclosure between the rod and the hollow ceramic cylinder we have two heat transfer mechanisms: i) radiation between the two surface ii) convection at each surface. Therefore the radiation thermal resistance is in parallel to the two convection resistances.



We now write down all of the thermal resistances:

Radiation exchange in the enclosure

$$R'_{rad} = 0.3W/mK$$

Convection in the enclosure

$$R'_{cv,rod} = \frac{1}{h\pi D_{rod}} = 0.8mK/W$$

$$R'_{cv,cer} = \frac{1}{h\pi D_{cer,inner}} = 0.4mK/W$$

Conduction in the ceramic cylinder

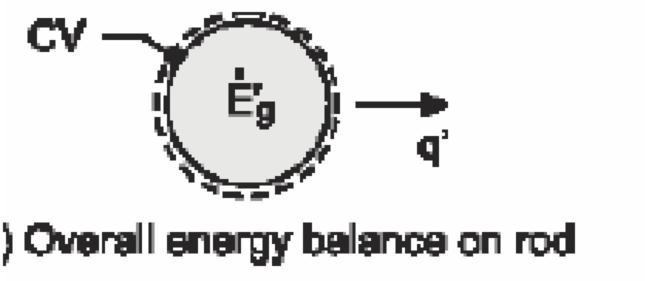
$$R'_{cd} = \frac{ln(D_{cer,outer}/D_{cer,inner})}{2\pi k_{cer}} = 0.1mK/W$$

We can now solve the thermal circuit and determine the overall thermal resistance:

$$\frac{1}{R'_{enclosure}} = \frac{1}{R'_{rad}} + \frac{1}{R'_{cv,rod} + R'_{cv,cer}}$$

$$R'_{tot} = R'_{enclosure} + R'_{cd} = 0.24 + 0.1 = 0.34mK/W$$

c)



To solve for the temperature of the rod surface we need to write the energy balance of the rod.

$$-q' + \dot{E}'_{gen} = 0$$

$$-q' + \dot{q}V'_{rod} = 0$$

Thus:

$$q' = \dot{q}\frac{\pi D_{rod}^2}{4}$$

Looking at the thermal cicuit of part b) we can also write:

$$q' = \frac{T_r - T_o}{R'_{tot}}$$

and combining the last two equations we get:

$$T_r = 239C$$

Exercise 2.3

Consider a plane composite wall that is composed of three materials (A,B and C from left to right) of thermal conductivities $k_A = 0.24 W/mK$, $k_B = 0.13 W/mK$, $k_C = 0.5 W/mK$. The three layers have thicknesses of $L_A = 20mm$, $L_B = 13mm$ and $L_C = 20mm$. A contact resistance of $R_{cont} = 0.01 m2K/W$ exists at the interface between materials A and B as well as at the interface of materials B and C. The left face of the composite wall is insulated while the right face is exposed to convection with $h = 10W/m^2K$ and $T_{air} = 20°C$. For Case 1, thermal energy is generated within material A at a rate ($q_A = 5000 W/m^3$). For Case 2 thermal energy is generated within material C at the rate ($q_C = 5000 W/m^3$).

a) Draw a schematic of the wall and indicate all dimensions and material parameters.

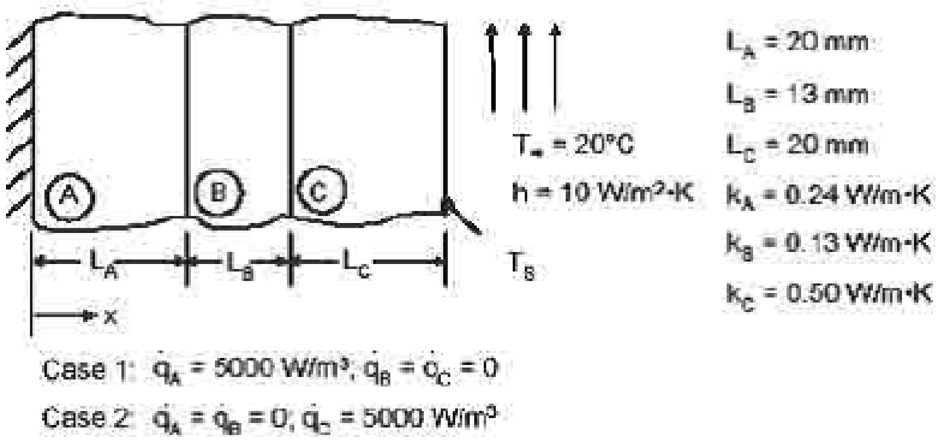
b) Determine the maximum temperature within the composite wall under steady state conditions for Case 1.

c) Sketch the steady state temperature distribution on T-x coordinates for Case 1.

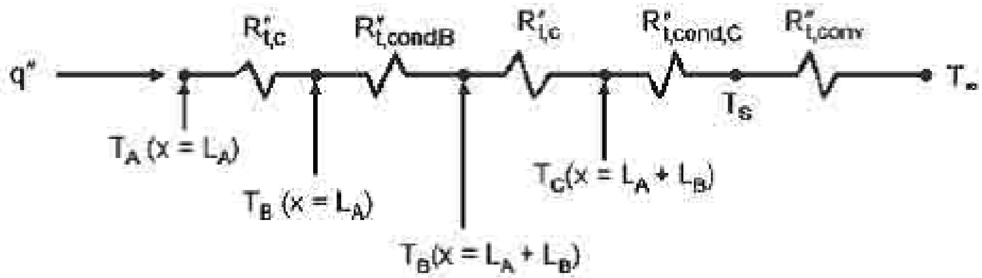
d) Sketch the steady state temperature distribution for Case 2 on the same T-x coordinates used for Case 1.

Solution

a)



b) We first sketch the thermal circuit to help identify the important temperatures and heat transfer mechanisms (rememeber that the thermal circuit analogy is not valid in layers with heat soruces, so in this case the layer A cannot be represented in the circuit).



It is clear that T_A will be the maximum value for this part of the circuit. However, we know that heat is generated within the layer A and we thus expect a parabolic profile within such layer. Overall, the maximum temperature wil thus occur at the adiabatic surface at $x = 0$.

Now we calculate each thermal resistance and the temperatures within the wall. As we are in planar geometry we calculate the quantities per-unit-area.

For Case 1, the heat flux along the layer B,C must be equal to the total heat generated in layer A. Therefore:

$$q^{"} = \dot{q_A} L_A = 100 W/m^2$$

$$R^{"}_{cont} = 0.01 m^2 K/W$$

$$R^{"}_{conv} = \frac{1}{h} = 0.1 m^2 K/W$$

$$R^{"}_{condB} = \frac{L_B}{k_B} = 0.1 m^2 K/W$$

$$R^{"}_{condC} = \frac{L_C}{k_C} = 0.04 m^2 K/W$$

so that :

$$R^{"}_{tot} = 2R^{"}_{cont} + R^{"}_{condB} + R^{"}_{condC} + R^{"}_{conv} = 0.26 m^2 K/W$$

and we can then calculate:

$$q^{"} = \frac{T_A - T_\infty}{R^{"}_{tot}}$$

and therefore:

$$T_A = T_\infty + q^{"} R^{"}_{tot} = 46°C$$

.

Now we can apply the quadratic temperature profile of a layer with heat sources and determine the temperature value at the adiabatic wall $x = 0$:

$$T_A(x=0) = T_A(x=L) + \frac{\dot{q_A} L_A^2}{2k_A} = 50.2°C = T_{max}$$

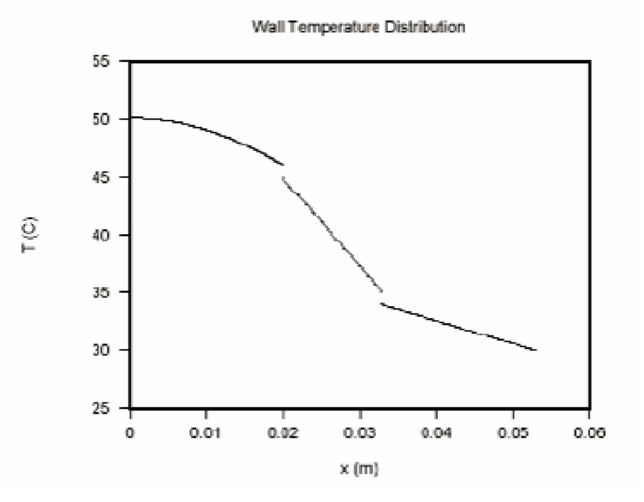
c) To sketch the temperature distribution we need to determine the intermediate temperatures.

Remembering that in general $q^{"} = (T_1 - T_2)/R^{"}_{1-2}$ we calculate:

$$T_s = T_\infty + q^{"} R^{"}_{conv} = 30°C$$
$$T_C(x = L_A + L_B) = T_s + q^{"} R^{"}_{condC} = 34°C$$
$$T_B(x = L_A + L_B) = T_C(x = L_A + L_B) + q^{"} R^{"}_{cont} = 35°C$$
$$T_B(x = L_A) = T_B(x = L_A + L_B) + q^{"} R^{"}_{condB} = 45°C$$

where we have accounted for the contact resistances that cause the two materials to have different temperatures at the same x-position.

So we can plot:

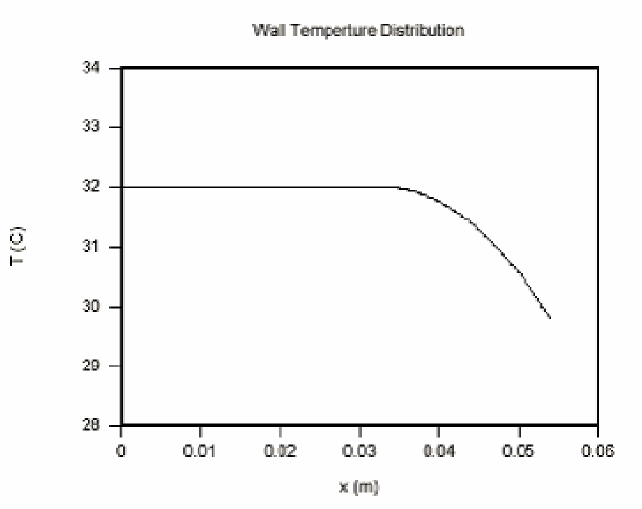


d) In Case 2, as the source is in layer C and the wall is adiabatic to the left, the heat flux in layer A, B will be zero and the maximum temperature will be reached at the layer C in the position $x = L_A + L_B$.

We apply again the parabolic temperature profile, this time at layer C and we obtain:

$$T_{max} = T_C(x = L_A + L_B) = T_s + \frac{\dot{q_C} L_C^2}{2k_C} = 32°C$$

and the temperature plot will look like:



Exercise 2.4 FOR REVISION

Superheated steam at $575°C$ has to be conveyed from the boiler to the turbine of an electric power plant. To do so, steel tubes are used ($k_{steel} = 35W/mK$, $D_1 = 300mm$ inner diameter and $30mm$ wall thickness). To reduce heat losses to the surroundings a layer of calcium silicate insulation ($k_{ins} = 0.1W/mK$) is applied to the tubes. Furthermore, a thin sheet of aluminum, having an emissivity of $\epsilon = 0.2$, is used to wrap the tubes. The air and wall temperatures of the power plant are $T_{sur} = 27°C$. Assume that the temperature of the inner surface of a steel tube corresponds to that of the steam and assume a convection coefficient outside the tube of $h_o = 6W/m^2K$.

a) Make a sketch of the system, indicating all the dimensions and material parameters. List the assumptions.

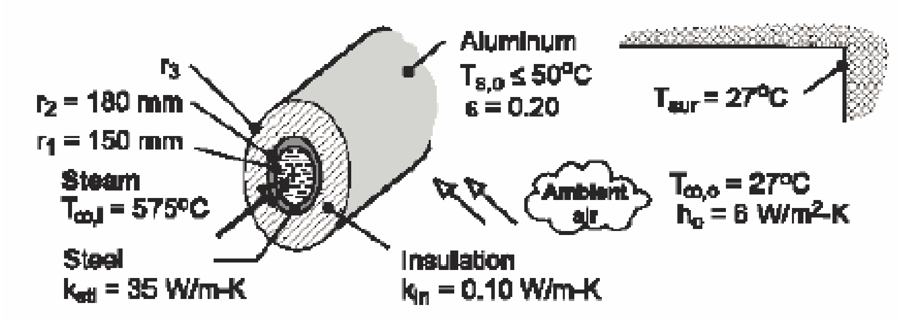
b) Draw the equivalent electrical circuit. HARD QUESTION What is the minimum insulation thickness needed to ensure that the temperature of the aluminum sheet does not exceed $T_{s,omax} = 50°C$? (Note: you will get a complex expression that can give you a numerical value only through iteration)

c) What is the corresponding heat loss per meter of tube length?

d) (Hard question) How would you expect the temperature of the aluminum sheet to change as a function of the thickness of the insulating layer?

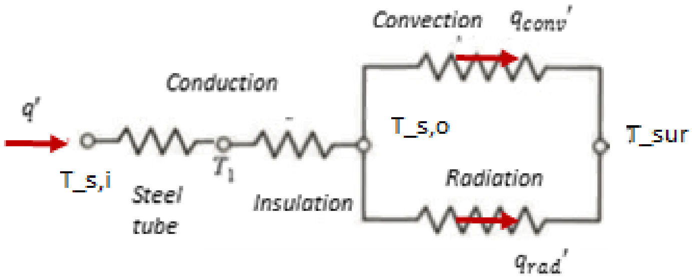
Solution

a)



Assumptions: i) steady state, ii) 1D radial conduction, iii) negligible contact resistance at all interfaces, iv) negligible steam side convection resistance, v) negligible conduction resistance for aluminum sheet, vi) constant properties vii) large surroundings.

b) We are in RADIAL geometry and we thus write all quantities per-unit-length of the tube.



The thickness of the insulating layer must guarantee that, for an external surface temperature $T_{s,o} = 50C$, convection and radiation enable the dissipation of the necessary the heat transfer rate q'.

$$q' = q'_{conv} + q'_{rad}$$

We can calculate the individual terms as:

$$q' = \frac{T_{s,i} - T_{s,o}}{R'_{cond,steel} + R'_{cond,ins}}$$

where $R'_{cond,steel} = \frac{ln(r_2/r_1)}{2\pi k_{steel}}$ and $R'_{cond,ins} = \frac{ln(r_3/r_2)}{2\pi k_{ins}}$.

$$q'_{conv} = h_o \pi D_3 (T_{s,o} - T_{sur})$$

$$q'_{rad} = \pi D_3 \epsilon \sigma (T^4_{s,o} - T^4_{sur})$$

So we have:

$$\frac{T_{s,i} - T_{s,o}}{\frac{ln(r_2/r_1)}{2\pi k_{steel}} + \frac{ln(r_3/r_2)}{2\pi k_{ins}}} = h_o \pi D_3 (T_{s,o} - T_{sur}) + \pi D_3 \epsilon \sigma (T^4_{s,o} - T^4_{sur})$$

where all the values are known except for r_3.

Through iteration (note that $r_3 > r_2$) we would find:

$$r_3 = 0.394m$$

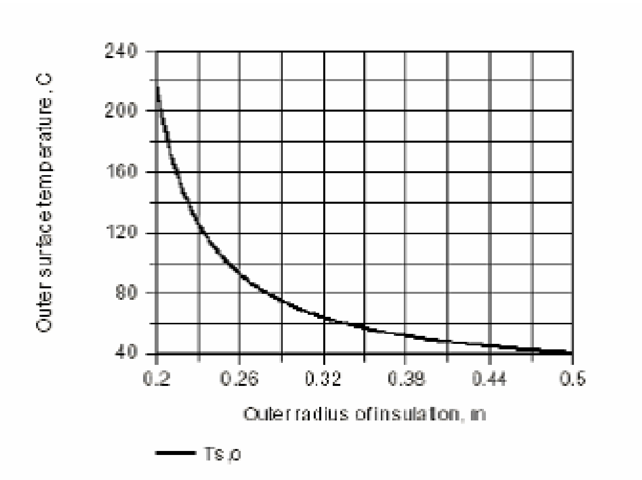
and therefore

$$T_{ins} = r_3 - r_2 = 214mm$$

c) It is straightforward to apply the previous equations and obtain:

$$q^{'} = \frac{T_{s,i} - T_{s,o}}{\frac{ln(r_2/r_1)}{2\pi k_{steel}} + \frac{ln(r_3/r_2)}{2\pi k_{ins}}} = 420W/m$$

d) For an increasing value of r_3 we can use the previous equations to calculate the corresponding $T_{s,o}(r_3)$:

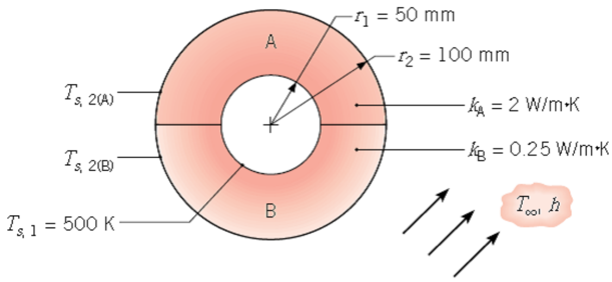


Exercise 2.5 FOR REVISION

Steam flowing through a long, thin-walled pipe maintains the pipe wall at a uniform temperature of $T_w = 500K$. The pipe is covered with an inhomogeneous insulation blanket comprised of two different materials, A ($k_A = 2W/mK$) and B ($k_B = 0.25W/mK$). The interface between the two materials may be assumed to have an infinite contact resistance and the entire outer surface is exposed to air at $T_{air} = 300K$ such that the convection coefficient is $h = 25W/m^2K$.

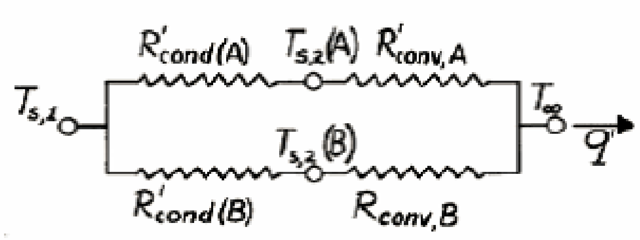
a) Sketch the thermal circuit of the system. Label all pertinent nodes and resistances.

b) For the given conditions, what is the total heat loss from the pipe? What are the outer surfaces T_A and T_B?



Solution

a)



Because there is an infinite thermal resistance between the two insulating materials (i.e. no heat is exchanged between them), we can treat each insulating material as an independent heat transfer path. hence the two material are in parallel and the only two common values are the temperature of the inner wall $T_{s,1}$ and the temperature of the surrounding air T_∞.

b) To determine the total heat loss we need to calculate the equivalent total thermal resistance. As each material covers half of the tube, we have:

$$R'_{conv,A} = \frac{1}{h\pi r_2} = 0.1273 \; [\frac{mK}{W}]$$

$$R'_{conv,B} = \frac{1}{h\pi r_2} = 0.1273 \; [\frac{mK}{W}]$$

$$R'_{cond,A} = \frac{ln(r_2/r_1)}{\pi k_A} = 0.1103 \; [\frac{mK}{W}]$$

$$R'_{cond,B} = \frac{ln(r_2/r_1)}{\pi k_B} = 0.8825 \; [\frac{mK}{W}]$$

We then calculate:

$$\frac{1}{R'_{tot}} = \frac{1}{R'_{conv,A} + R'_{cond,A}} + \frac{1}{R'_{conv,B} + R'_{cond,B}}$$

We can then obtain:

$$q' = \frac{T_{s,1} - T_\infty}{R_{tot}} = 1040 \; [\frac{W}{m}]$$

$$q'_A = \frac{T_{s,1} - T_\infty}{R'_{conv,A} + R'_{cond,A}} = 842 \; [\frac{W}{m}]$$

$$q'_B = \frac{T_{s,1} - T_\infty}{R'_{conv,B} + R'_{cond,B}} = 198 \; [\frac{W}{m}]$$

And from this calculate:

$$T_{s,2-A} = T_{s,1} - q'_A R'_{cond,A} = 407K$$

$$T_{s,2-B} = T_{s,1} - q'_B R'_{cond,B} = 325K$$