Exercise 1.1

Consider the common case of a room, with internal air temperature T, having a glass window through which solar radiation can enter. Windows can have a single pane structure or a double pane construction in which adjoining panes are separated by an air space.

a) For each case draw a sketch of the window system and identify the relevant heat transfer processes leading to heat transfer across the window.

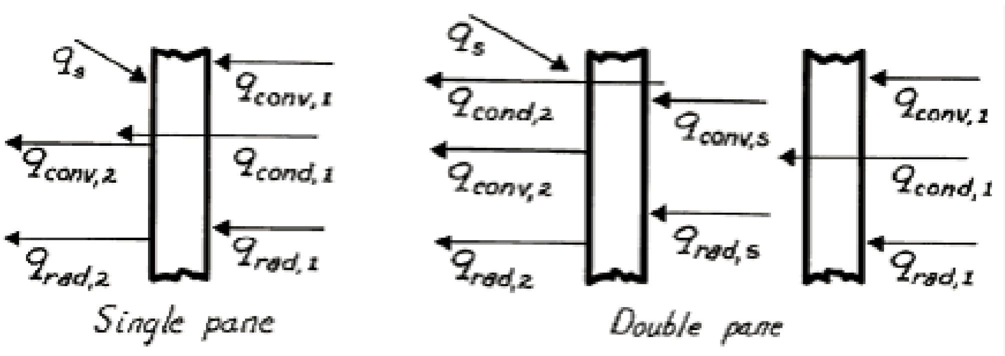
b) Consider a single pane glass window with $W = 1m$, $H = 2m$ and a thickness $L = 5mm$. The glass thermal conductivity is $k_g = 1.4W/mK$. In a winter day the inner and outer surface temperatures of the glass are $15°C$ and $-20°C$ respectively. What is the rate of heat loss through the glass?

c) Now consider a double pane glass window with the same W,H as in the previous part. In this case the two adjoining glass panes, each of thickness $t = 5mm$, are separated by an air space of thickness $t_{air} = 10mm$ ($k_{air} = 0.024K/mK$). Under the assumption that the air in the gap space is at rest and radiation effects are negligible, if the glass surfaces in contact with the air space have a temperature of $10°C$ and $-15°C$ respectively, what is the rate of heat loss?

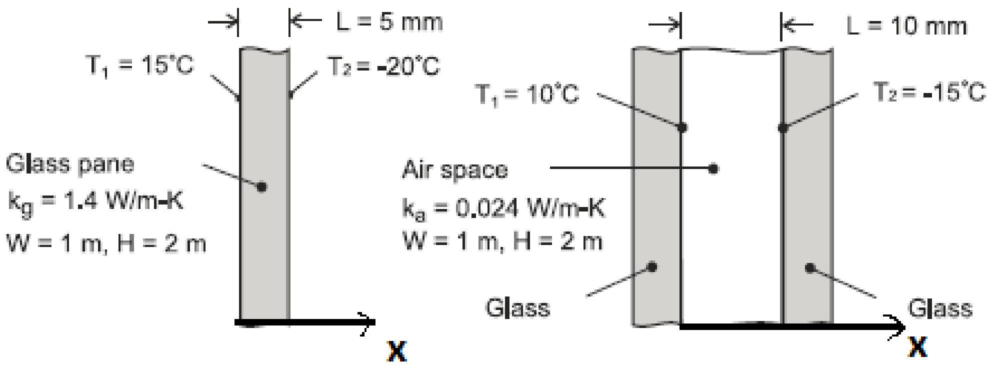
d) Could doubling the thickness of the single glass pane make the performance of the single pane window acceptable compared to a double pane window?

Solution

a)



b)



For the single pane window we express the conduction heat transfer rate as:

$$Q_{single} = -k_g A \frac{T2 - T1}{L} = 19.6kW$$

c) For the double pane window we express the conduction heat transfer rate in the air gap as:

$$Q_{double} = -k_{air} A \frac{T2 - T1}{t_{air}} = 120W$$

d) If we double the thickness of the single pane window $L^{'} = 2L$:

$$Q^{'}_{single} = -k_g A \frac{T2 - T1}{2L} = 9.8kW >> Q_{double}$$

so the performance would still be unacceptable.

Exercise 1.2

A hot-wire anemometer is a common instrument to measure the velocity of an air stream. It consists of a heated wire which is placed into the air flow with the axis oriented perpendicular to the flow direction. The electrical energy dissipated in the wire is transferred to the air by air convection and it is assumed that no other heat transfer mechanism plays a role. Thus, for a given electrical power, the temperature of the wire depends on the convection coefficient, which in turn depends on the velocity of the air flow. Let's thus consider a wire with $L = 20mm$ and a diameter $D = 0.5mm$. This has been calibrated to have:

$$v = 6.25 \cdot 10^{-5} h^2$$

where h is the convection coefficient.

a) Make a sketch of the system, identify the boundaries, the heat transfer mechanisms and the energy source terms. List the most important assumptions.

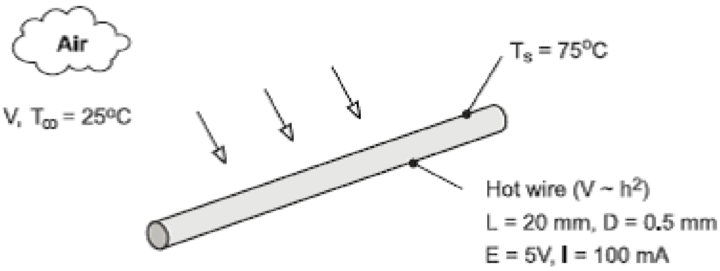
b) Write the energy balance for the system you identified in part a)

c) If the air temperature is $T_{air} = 25°C$ and the surface temperature of the anemometer is $T_s = 75°C$ when a voltage of $5V$ is applied to the wire and a current of $0.1A$ is flowing through it, what is the velocity of the air stream?

Note: remember that the power dissipated due to Joule heating is $P_{Joule} = VI$

Solution

a)



Assumptions: i) constant k; ii) steady state; iii) negligible conduction and radiation b) The energy balance for the anemometer is

$$0 = -Q_{out,conv} + E_{joule}$$

where the heat transfer rate by convection as a negative sign because it is leaving the wire.

c) The power dissipated by joule heating is: $E_{joule} = VI$. We compute the heat transfer rate by convection as:

$$Q_{out,conv} = hA(T_s – T_{air})$$

Combining these expressions:

$$h = \frac{VI}{A(T_s - T_{air})} = 318W/m^2K$$

And hence:

$$v = 6.25 \cdot 10^{-5} h^2 = 6.3m/s$$

Note: We see that $T_s > T_{air}$ and therefore with the expression we used $Q_{out,conv} > 0$ and therefore it is consistent with the addition of a negative sign in the energy balance.

Alternatively we could have written the energy balance as:

$$0 = Q'_{out,conv} + E_{joule}$$

where $Q'_{out,conv} = hA(T_{air} – T_s)$.

In either case make sure that the overall sign of the heat transfer rate is correct in the energy balance equation you write.

Exercise 1.3

A hair dryer can be modelled as a circular duct through which a small fan draws ambient air and within which the air is heated as it flows over a coiled electrical resistance.

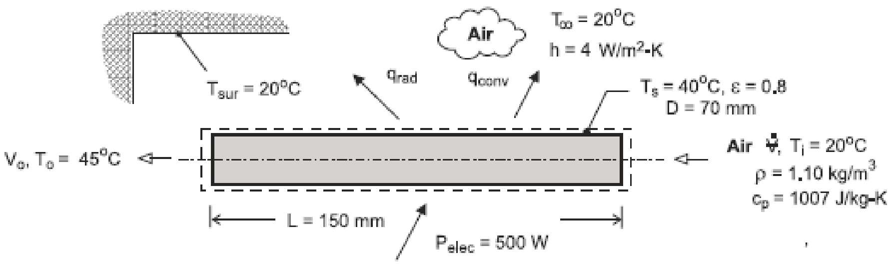
a) Make a sketch of the air drier, identify the control volume, the heat transfer mechanisms and heat sources.

b) An hair drier is designed to operate with an electric power consumption of $P_{el} = 500W$ when heating air from $T_i = 20°C$ to $T_o = 45°C$. Heat loss from the casing to the ambient air and the surroundings can be neglected. The duct diameter is $D = 70mm$, the density and specific heat of the air are $\rho = 1.1kg/m^3$ and $c_p = 1007J/kgK$. What is the volumetric flow rate $V[m^3/s]$ of air in the drier? What is the discharge velocity v_o?

c) Consider a dryer duct length $L = 150mm$ and a surface emissivity $\epsilon = 0.8$. If the heat transfer coefficient for natural convection from the casing to the ambient air is $h = 4W/m^2K$ and the temperature of the surrounding air is $T_{air} = 20°C$, confirm that the heat loss from the casing is negligible. Use $T_s = 40°C$ as the average casing temperature.

Solution

a)



Assumptions: i) steady state; ii) constant air properties; iii) negligible potential and kinetic energy changes of air flow; iv) negligible work done by fan; v) negligible heat transfer from casing of dryer to ambient air.

b) The volumetric flow rate is equal to $\dot{V} = \frac{\dot{m}}{\rho}$ therefore to determine it we need to determine the mass flow rate. We can do so through the energy balance of the hair drier duct.

We note that mass flows in and out of the hair drier duct hence we need to use the OPEN SYSTEM energy balance to solve this problem. We have:

$$\dot{m}(u + pv)_i - \dot{m}(u + pv)_o - Q_{rad} - Q_{conv} + P_{elec} = 0$$

We have assumed that the casing is well insulated so there is not heat exchange to the environment through convection or radiation, therefor we are left with:

$$\dot{m}(i_i - i_o) + P_{elec} = 0$$

where we have used $i = u+pv = enthalpy$ [we used the symbol i to avoid confusions with he convection coefficient].

For a gas we can write:

$$\dot{m}(i_i - i_o) = \dot{m}c_p(T_i - T_o)$$

Therefore:

$$\dot{m}c_p(T_i - T_o) = P_{elec}$$

from which we obtain:

$$\dot{m} = \frac{P_{elec}}{c_p(T_i - T_o)} = 0.0199kg/s$$

We can now obtain:

$$\dot{V} = \frac{\dot{m}}{\rho} = 0.0181m^3/s$$

Finally we know that:

$$\dot{V} = vA_c = v\frac{\pi D^2}{4}$$

and therefore the discharge velocity is

$$v_o = \frac{\dot{V}}{A_c} = 4.7m/s$$

c) We can now calculate the two heat transfer rate terms due to convection and radiation respectively. We have:

$$Q_{conv} = hA_s(T_s - T_\infty) = 2.64W$$

$$Q_{rad} = \epsilon A_s \sigma (T_s^4 - T_{sur}^4) = 3.33W$$

Therefore

$$Q_{tot} = Q_{conv} + Q_{rad} = 5.97W << P_{elec} = 500W$$

and our initial assumption of negligible heat losses due to convection and radiation was correct.

Note: in part b) we should account for the change in density of the air as it heats up inside the hair drier. Indeed $\dot{m}$ is constant but $\dot{V} = \frac{\dot{m}}{\rho(T)}$ will change as $\rho(T)$ changes. Yet, given the small temperature change within the air drier we have neglected the change in density.

Exercise 1.4 FOR REVISION

A square isothermal electronic chip has a width w=5mm and it is mounted on a substrate. Its side and back surfaces are well insulated while the front surface is exposed to the flow of a coolant at $T_{cool} = 15°C$. During operation, for reliability reasons, the chip temperature must not exceed $T_{max} = 85°C$.

a) Make a sketch of the system, identify the boundaries, heat transfer mechanisms and heat sources.

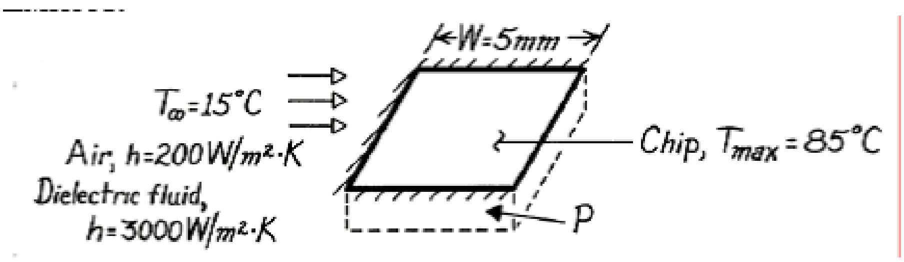
b) If the coolant is air and the convection coefficient is $h_1 = 200W/m^2K$, what is the maximum allowable chip power?

c) If the coolant is a dielectric liquid for which $h_2 = 3000W/m^2K$, what is the maximum allowable chip power?

d) Now consider case b) and include the heat transfer by radiation from the chip surface to the surroundings at $15°C$. If the chip has an emissivity of $\epsilon = 0.9$, what is the maximum allowable chip power?

Solution

a)



Assumptions: i) Steady-state; ii) isothermal chip; iii) negligible heat transfer through sides and bottom; iv) negligible radiative heat transfer.

b) The energy balance equation for the chip is:

$$0 = -Q_{out,conv} + P_{gen}$$

where

$$Q_{out,conv} = h_1 A(T_{max} - T_{cool})$$

Therefore:

$$P_{gen,max1} = h_1 A(T_{max} - T_{cool}) = 0.35W$$

c) In this case:

$$P_{gen,max2} = h_2 A(T_{max} - T_{cool}) = 5.25W$$

d) If radiation is NOT negligible we need to account for it in the energy balance. As the chip surface is hotter than the surrounding the heat transfer rate by radiation will have to have a negative sign. We write:

$$Q_{rad} = A\epsilon\sigma(T^4 - T_{sur}^4) = 0.0122W$$

and then the energy balance becomes:

$$0 = -Q_{out,conv} - Q_{rad} + P_{gen}$$

Therefore:

$$P_{gen,max3} = Q_{rad} + Q_{out,conv1} = 0.3622W$$

and we have:

$$\frac{\Delta P_{gen,max1,3}}{P_{gen,max1}} \cdot 100 = 3.5\%$$

Exercise 1.5 FOR REVISION

A pan is used to boil water by placing it on the stove, from which heat is transferred at a fixed rate Q_0. The boiling happens in two stages. In stage 1, the water is taken from its initial room temperature T_i to the boiling point, as heat is transferred from the pan by natural convection. During this stage, a constant value of the convection coefficient h may be assumed while the temperature of the bulk water increases with time as $T_\infty = T_\infty(t)$. In stage 2 the water temperature remains at a fixed value, T_B as heating continues. The pan bottom has a thickness L and a diameter D with a coordinate system corresponding to $x = 0$ and $x = L$ for the surfaces in contact with the stove and water respectively.

a) Make a sketch of the system and the heat transfer mechanisms.

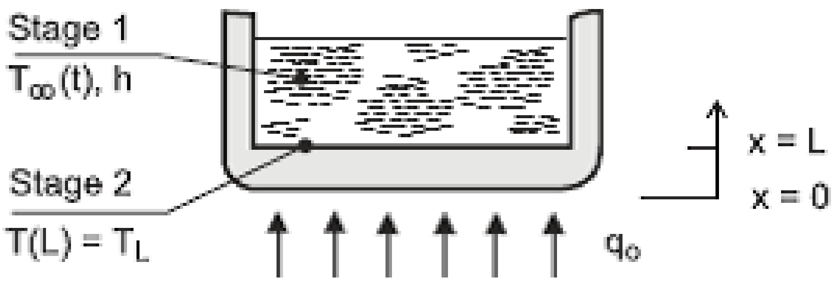
b) Write the form of the heat diffusion equation and the boundary/initial conditions that determine the variation of the temperature of the pan with position and time, $T(x, t)$ in the pan bottom during stage 1. Write all the equations in terms of the parameters Q_0, D, L, h and $T_\infty(t)$ as well as appropriate properties of the pan material.

c) During stage 2, the surface of the pan in contact with the water is at a fixed temperature $T(L, t) = T_L > T_B$. Write the form of the heat diffusion equation and boundary conditions that determine the temperature distribution $T(x)$ in the pan bottom. Express your results in terms of the parameters Q_0, D, L, T_L as well as appropriate properties of the pan material.

d) Consider a pan with $L = 5mm$, $D = 200mm$ made of aluminum ($k = 240W/mK$) or copper ($k = 390W/mK$). When used to boil water, the surface of the bottom exposed to the water is at $T_L = 110°C$. If the heat transferred from the stove to the pan is $Q_0 = 600W$, what is the temperature of the surface in contact with the stove for each of the two materials?

Solution

a)



Assumptions: i) 1D conduction in the pan bottom; ii) uniform heat transfer from the stove to the bottom of the pan; iii) constant properties.

b) *Stage* 1

As there are no heat sources in the pan and we have a one-dimensional problem, the heat diffusion equation reduces to:

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

The first boundary condition is that at the bottom of the pan ($x = 0$) we have a constant heat transfer rate and we express it as:

$$-kA_{pan}\left.\frac{\partial T}{\partial x}\right|_{x=0} = Q_0$$

where $A_{pan} = \pi D^2/4$.

The second boundary condition is that at the top surface of the pan we have convection with a constant convection coefficient h therefore:

$$-kA_{pan}\left.\frac{\partial T}{\partial x}\right|_{x=L} = hA_{pan}[T(L,t) - T_\infty(t)]$$

The initial condition is:

$$T(x, 0) = T_i$$

c) *Stage* 2

During stage 2 the temperature of the pan does not change with time, therefore the heat diffusion equation simplifies further to:

$$\frac{d^2T}{dx^2} = 0$$

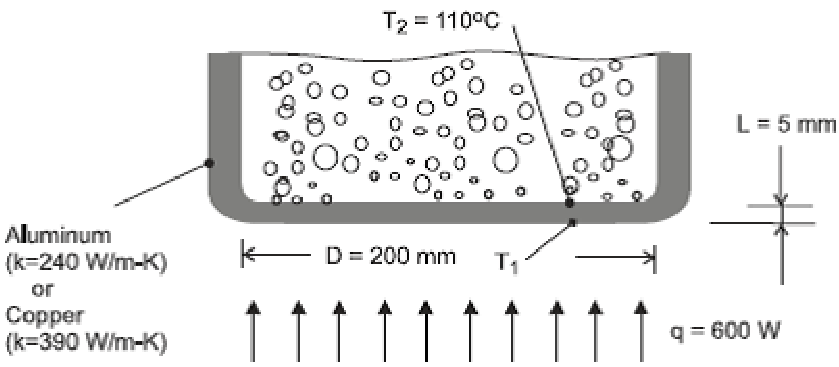
The boundary conditions are the heat transfer rate at the bottom surface ($x = 0$) and the constant temperature at the top surface ($x = L$):

$$-kA_{pan} \left.\frac{\mathrm{d}T}{\mathrm{d}x}\right|_{x=0} = Q_0$$
$$T(L) = T_L$$

We do not need the initial condition.

If we integrate this solution we will obtain the linear temperature profile expected from Fourier law of conduction.

d)



From the result of part c), using Fourier law we have that the heat transfer rate by conduction through the bottom of the pan is:

$$Q_0 = kA_{pan}\frac{T_1 - T_2}{L}$$

where $T_1 = T(x = 0)$ and $T_2 = T_L$.

Therefore we can find the temperature of the bottom surface as:

$$T_1 = T_L + \frac{Q_0 L}{kA_{pan}}$$

where $A_{pan} = \pi D^2/4 = 0.0314m^2$.

Substituting the different thermal conductivities, for the two materials we obtain:

- *Aluminium* $T_1 = 110.40°C$
- *Copper* $T_1 = 110.24°C$

We therefore see that, irrespective of the material $T_1 \approx T_L$ and therefore the bottom of the pan can be considered isothermal.