Exercise 6.1

A bank of tubes uses an aligned arrangement of $10mm$ diameter tubed with $S_T = S_L = 20mm$. There are 10 rows of tubes with 50 tubes in each row. Consider an application for which cold water flows through the tubes, maintaining the outer surface temperature at $27°C$, while flue gases at $427°C$ and a velocity of $5m/s$ are in cross flow over the tubes. The properties of the flue gas may be approximated as those of atmospheric air with temperature of $427°C$. What is the total rate of heat transfer per unit length of the tubes in the bank?

Note: For air use $\nu = 68.1 \cdot 10^{-6} m^2/s$ $k = 0.0524 W/mK$, $\rho = 0.498 kg/m^3$, $Pr = 0.695$ $c_p = 1075 J/kgK$.

Solution

Assumptions: (1) Steady-state conditions, (2) Negligible radiation effects, (3) Gas properties are approximately those of air.

Properties: *Table A-4*, Air (300 K, 1 atm): $Pr = 0.707$, *Table A-4*, Air (700 K, 1 atm): $\nu = 68.1 \cdot 10^{-6}, k = 0.0542 W/mK, Pr = 0.695, \rho = 0.498 kg/m^3, c_p = 1075 J/kgK$

Analysis: The rate of the heat transfer per unit length of tubes is

$$\mathrm{q}' = \overline{\mathrm{h}}\mathrm{N}\pi\mathrm{D}\Delta\mathrm{T}_{\mathrm{lm}} = \overline{\mathrm{h}}\mathrm{N}\pi\mathrm{D}\frac{(\mathrm{T_s}-\mathrm{T_i})-(\mathrm{T_s}-\mathrm{T_o})}{\ln\left[(\mathrm{T_s}-\mathrm{T_i})\,/\,(\mathrm{T_s}-\mathrm{T_o})\right]}.$$

With $\mathrm{V_{max}} = \frac{\mathrm{S_T}}{\mathrm{S_T-D}}\mathrm{V} = \frac{20}{10}5\mathrm{m/s} = 10\mathrm{m/s}$, $\mathrm{Re_{D,max}} = \frac{V_{\max}\mathrm{D}}{\nu} = \frac{10\mathrm{m/s}\times 0.01\mathrm{m}}{68.1\times 10^{-6}\mathrm{m^2/s}} = 1468$. Tables 7.7 and 7.8 give $C = 0.27, m = 0.63$ and $C_2 = 0.97$. Hence from the Zukauskas correlation, including the correction factor C_2 for a tube bank with less than 20 rows of tubes ($N_L < 20$) is:

$$\overline{\mathrm{Nu}}_{\mathrm{D}} = \mathrm{C}\mathrm{C_2}\mathrm{Re}_{\mathrm{D,max}}^{\mathrm{m}}\,\mathrm{Pr}^{0.36}\left(\mathrm{Pr}/\mathrm{Pr_S}\right)^{1/4} = 0.26(1468)^{0.63}(0.695)^{0.36}(0.695/0.707)^{1/4}$$

where $Pr_S = 0.707$ is the Prandtl number calculated for the fluid at the temperature of the tube surface (in this case $T_S = 27C$).

$$\overline{\mathrm{Nu}}_{\mathrm{D}} = 22.4 \quad \overline{\mathrm{h}} = \frac{\mathrm{k}}{\mathrm{D}}\overline{\mathrm{Nu}}_{\mathrm{D}} = 0.0524\mathrm{W/m\cdot K}\times 22.4/0.01\mathrm{m} = 117\mathrm{W/m^2\cdot K}.$$

Hence,

$$(T_s - T_o) = (T_s - T_i)\exp\left(-\frac{\pi D N \overline{h}}{\rho V N_T S_T c_p}\right) = -400\mathrm{K}\exp\left(-\frac{\pi\times 0.01\mathrm{m}\times 500\times 117\mathrm{W/m^2\cdot K}}{0.498\mathrm{kg/m^3}(5\mathrm{m/s})50(0.02\mathrm{m})1075\mathrm{J/kg\cdot K}}\right)$$

$$\mathrm{T_s} - \mathrm{T_0} = -201.3\mathrm{K}$$

and the heat rate is

$$\mathrm{q}' = \left(117\mathrm{W/m^2\cdot K}\right)500\pi(0.01\mathrm{m})\frac{(-400+201.3)\mathrm{K}}{\ell\mathrm{n}[(-400)/(-201.3)]} = -532\mathrm{kW/m}$$

Exercise 6.2

Ethylene glycol flows at $0.01 kg/s$ through a $3mm$ diameter, thin-walled tube. The tube is submerged in a well-stirred water bath maintained at $25°C$. If the fluid enters the tube at $85°C$ what heat transfer rate and tube length are required for the fluid to leave at $35°C$?

Use the following properties of ethylene glycol at $T_m = (85 + 35)/2 = 60°C = 333K$:

- $c_p = 2562 J/kgK$
- $\mu = 0.522 \cdot 10^{-2} Ns/m^2$
- $k = 0.260 W/mK$
- $Pr = 51.3$

Solution

Assumptions: (1) steady-state conditions; (2) tube wall thermal resistance negligible; (3) convection coefficient on water side infinite, cooling process approximates constant wall surface temperature distribution; (4) incompressible liquid with negligible viscous dissipation; (5) constant properties; (6) negligible heat transfer enhancement associated with the coiling.

Analysis: From an overall energy balance on the tube:

$$Q_{conv} = \dot{m}c_p(T_{m,o} - T_{m,i}) = 0.01kg/s \times 2562J/kg \times (35 - 85)^{\circ}C = -1281W$$

We thus see that convection removes heat from the fluid (as expected considered that the fluid is cooling).

Because the bath is well stirred and maintained at $25C$ we can use the constant surface temperature condition to write:

$$Q_{conv} = \overline{h}A_s\Delta T_{lm}$$

where

$$\Delta T_{\ell m} = \frac{\Delta T_o - \Delta T_i}{\ln\frac{\Delta T_o}{\Delta T_i}} = \frac{(25-35)^{\circ}C - (25-85)^{\circ}C}{\ln\frac{35-25}{85-25}} = -27.9^{\circ}C$$

We then observe that:

$$A_s = \frac{Q_{conv}}{\overline{h}\Delta T_{\ell m}} = \pi DL$$

Thus we need to find $\overline{h}$ to calculate A_s and to obtain L.

We know we are in forced internal convection. We now need to determine the flow condition to choose the right correlation. Therefore we calculate:

$$\mathrm{Re}_D = \frac{\rho u D}{\mu} = \frac{4\dot{m}}{\pi D\mu} = \frac{4 \times 0.01kg/s}{\pi \times 0.003m \times 0.522 \times 10^{-2}Ns/m^2} = 813 < 2300$$

Hence, the flow is laminar and, assuming the flow is fully developed, the appropriate correlation is:

$$\overline{\mathrm{Nu}}_D = \frac{\overline{h}D}{k} = 3.66, \quad \overline{h} = \mathrm{Nu}\frac{k}{D} = 3.66 \times \frac{0.260W/mK}{0.003m} = 317W/m^2K$$

Finally, the required area A_s and the tube length are:

$$A_s = \frac{1281W}{317W/m^2K \times 27.9^{\circ}C} = 0.1448m^2$$

$$L = \frac{A_s}{\pi D} = \frac{0.1448m^2}{\pi \times 0.003m} = 15.4m$$

Comments:

The hydrodynamics entry length is $x_{fd,h} = 0.05\mathrm{Re}_D D = 0.12m$, so it is reasonable to assume the flow is fully developed. However, with $x_{fd,t} = x_{fd,h}\mathrm{Pr} = 6.3m$, the temperature is developing over a significant portion of length. To be more accurate we should thus use the Hausen correlation is appropriate. Assuming $L = 15.4m$, this yields $\overline{\mathrm{Nu}}_D = 4.13$. $\overline{h} = 358W/m^2K$, $L = 13.6m$. Further iterations converge to $L = 13.4m$

Exercise 6.3

The evaporator section of a heat pump is installed in a large tank of water, which is used as a heat source during the winter. As energy is extracted from the water, it begins to freeze, creating an ice/water bath at $0°C$, which may be used for air conditioning during the summer. Consider summer cooling conditions for which air is passed through an array of copper tubes, each of inside diameter $D = 50mm$, submerged in the bath.

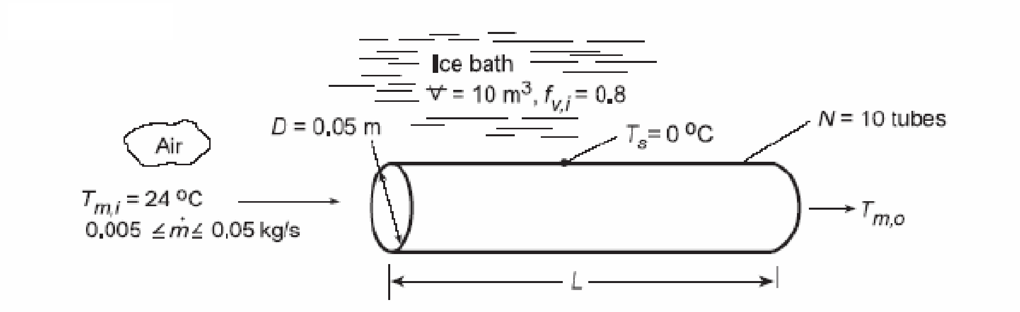
a) If air enters each tube at a mean temperature of $T_{m,i} = 24°C$ and a flow rate of $\dot{m} = 0.01kg/s$ what tube length L is needed to provide an exit temperature of $T_{m,o} = 14°C$? Assume fully developed flow inside the tube.

b) With 10 tubes passing through a tank of total volume $V = 10m^3$, which initially contains 80% ice by volume, how long would it take to completely melt the ice?

The density and latent heat of fusion of ice are $\rho_{ice} = 920kg/m^3$ and $f_{l,ice} = 3.34 \cdot 10^5 J/kg$, respectively.

Determine the properties of air from Table $A4$ (at the end of this document) at the appropriate temperature.

Solution



Assumptions: (1) Steady-state, (2) Ideal gas with negligible viscous dissipation and pressure variation, (3) Constant properties, (4) Fully developed flow throughout each tube, (5) Negligible tube wall thermal resistance.

Properties: *Table A.4*, air (assume $\overline{\mathrm{T}}_{\mathrm{m}} = 292\mathrm{K}$): $c_p = 1007$ J/kg $K, \mu = 180.6 \cdot 10^{-7}\mathrm{N \cdot s/m^2}$, $k = 0.0257\mathrm{W/m \cdot K}, \mathrm{Pr} = 0.709$; Ice: $\rho = 920\mathrm{kg/m^3}, \mathrm{h_{sf}} = 3.34 \cdot 10^5\mathrm{J/kg}$

Analysis:

(a)

Considered that the we have a water/ice bath whose temperature will stay constant at $T_s = 0C$, we can use the constant tube surface boundary condition. The overall energy balance over the tube gives:

$$\frac{T_s - T_{m,o}}{T_s - T_{m,i}} = exp(-\frac{PL}{\dot{m}c_p}\overline{h})$$

where $P = \pi D$

Thus to find the length of the tube we only have to determine the average convection coefficient. We note that we are in forced internal convection in circular tubes and we calculate:

$$\mathrm{Re_D = 4\dot{m}/\pi D\mu = 4(0.01kg/s)/\pi(0.05m)180.6 \times 10^{-7}N.N \cdot s/m^2 = 14,100 > 2300}$$

Thus the flow is turbulent and we use:

$$\mathrm{\overline{Nu}_D = Nu_D = 0.023Re_D^{0.8}Pr^{0.3} = 0.023(14,100)^{0.8}(0.709)^{0.3} = 43.3}$$

$$\mathrm{\overline{h} = \overline{Nu}_D(k/D) = 43.3(0.0257W/m \cdot K/0.05m) = 22.2W/m^2 \cdot K}$$

Substituting we obtain:

$$\mathrm{L = 1.56m}$$

(b) The time required to completely melt the ice may be obtained from an energy balance where the total heat needed to melt the volume of ice is removed through convection within the 10 tubes:

$$Q_{conv} = N\dot{m}c_p(T_{m,i} - T_{m,o}) = 10(0.01kg/s)1007J/kg \cdot K(10K) = 1007W$$
$$Q_{ice,melt}[J] = f_{l,ice}V_{ice}\rho_{ice} = Q_{conv}[W]t[s]$$

Hence,

$$t = \frac{0.8\left(10m^3\right)\left(920\mathrm{kg/m^3}\right)3.34 \cdot 10^5\mathrm{J/kg}}{1007\mathrm{W}} = 2.44 \times 10^6\mathrm{s} = 28.3\mathrm{days}$$

Exercise 6.4

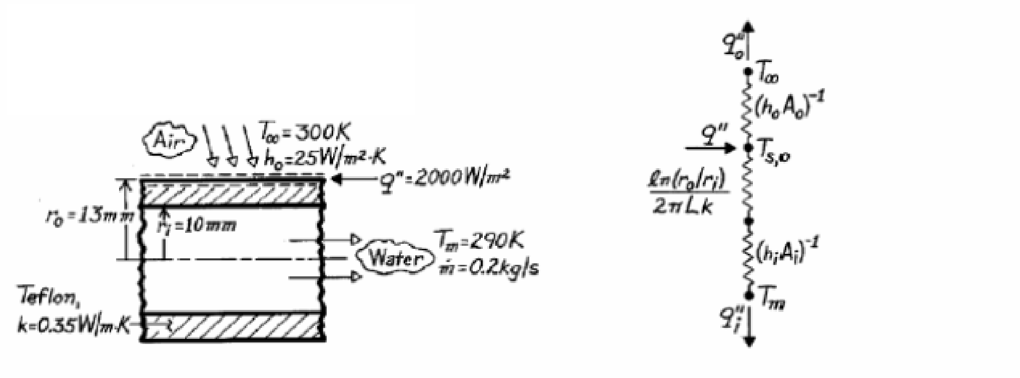
Water at $290K$ and $0.2kg/s$ flows through a Teflon tube ($k = 0.35W/mK$) of inner and outer radii equal to $10mm$ and $13mm$ respectively. A thin electrical heating tape wrapped around the outer surface of the tube delivers a uniform surface heat flux of $2000W/m^2$, while a convection coefficient of $25W/m^2K$ is maintained on the outer surface of the tape by ambient air at $300K$. What is the outer surface temperature of the Teflon tube? What is the fraction of the power dissipated by the tape, which is transferred to the water?

Determine the properties of water from Table $A6$ at the appropriate temperature.

Hint 1: start by drawing the equivalent thermal circuit for this problem. The heater can be assumed to have negligible thickness (no associated thermal resistance). It only contributes with the input heat flux.

Hint 2: you can use the inlet temperature of water to estimate its physical properties

Solution



Assumptions: (1) Steady-state conditions, (2) Fully developed flow, (3) One-dimensional conduction, (4) Negligible tape contact and conduction resistances.

Properties: *Table A-6* Water($T_m = 290K$): $\mu = 1080 \cdot 10^{-6}\mathrm{kg/s \cdot m}$, $\mathrm{k} = 0.598\mathrm{W/m \cdot K}$, $\mathrm{Pr} = 7.56$.

Analysis:

We can look at the thermal circuit and write the sum of the heat transfer rates at the node corresponding to the tube surface as:

$$(2\pi r_0 L)\, q'' = \frac{T_{s,o} - T_\infty}{(h_0 2\pi r_0 L)^{-1}} + \frac{\mathrm{T_{s,o} - T_m}}{(\ln(\mathrm{r_o/r_i})/2\pi\mathrm{Lk}) + (1/2\pi \mathrm{r_i L h_i})}$$

$$\mathrm{q''} = \mathrm{h_o}(\mathrm{T_{s,o}} - \mathrm{T_\infty}) + \frac{\mathrm{T_{s,o} - T_m}}{(\mathrm{r_o/k})\ln(\mathrm{r_o/r_i}) + (\mathrm{r_o/r_i})/\mathrm{h_i}}.$$

All the terms are known except for the h_i and therefore we have to use the appropriate internal forced convection correlation to find it. First we calculate:

$$\mathrm{Re_D} = 4\dot{\mathrm{m}}/(\pi \mathrm{D}\mu) = 4(0.2\mathrm{kg/s})/\left[\pi(0.02\mathrm{m})1080 \cdot 10^{-6}\mathrm{kg/s \cdot m}\right] = 11,789 > 2300$$

and therefore the flow is turbulent. We thus use:

$$\overline{Nu_D} = 0.023 Re_D^{4/5} Pr^{0.4}$$

from which we obtain:

$$\mathrm{h_i} = (\mathrm{k/D_i})\, 0.023\mathrm{Re_D^{4/5}Pr^{0.4}} = (0.598/0.02)(0.023)(11,789)^{4/5}(7.56)^{0.4} = 2792\mathrm{W/m^2 \cdot K}.$$

Hence

$$2000\mathrm{W/m^2} = 25\mathrm{W/m^2 \cdot K}\,(\mathrm{T_{s,o}} - 300\mathrm{K}) + \frac{\mathrm{T_{s,o}} - 290\mathrm{K}}{(0.013\mathrm{m}/0.35\mathrm{W/m \cdot K})\ln(1.3) + (1.3)/(2792\mathrm{W/m^2 \cdot K})}$$

Solving for $\mathrm{T_{s,o}}$,

$$\mathrm{T_{s,o}} = 308.3\mathrm{K}$$

We can now go back to the thermal circuit and calculate the heat flux to the air:

$$\mathrm{q''_o = h_o\,(T_{s,o} - T_\infty) = 25W/m^2 \cdot K(308.3 - 300)K = 207.5W/m^2.}$$

Hence, the fraction of heat that is transferred to the fluid inside the tubes is:

$$\mathrm{q}''_i/\mathrm{q}'' = \frac{q^{"} - q_o^{"}}{q^{"}} = (2000 - 207.5)\mathrm{W/m^2/2000W/m^2} = 0.90.$$

Comments: The resistance to heat transfer by convection to the air substantially exceeds that due to conduction in the teflon and convection in the water. Hence, most of the heat is transferred to the water.

Table A.4 Thermophysical Properties of Gases at Atmospheric Pressure[a]

T (K)	ρ (kg/m^3)	c_p (kJ/kg · K)	$\mu \cdot 10^7$ (N · s/m^2)	$\nu \cdot 10^6$ (m^2/s)	$k \cdot 10^3$ (W/m · K)	$\alpha \cdot 10^6$ (m^2/s)	Pr
Air							
100	3.5562	1.032	71.1	2.00	9.34	2.54	0.786
150	2.3364	1.012	103.4	4.426	13.8	5.84	0.758
200	1.7458	1.007	132.5	7.590	18.1	10.3	0.737
250	1.3947	1.006	159.6	11.44	22.3	15.9	0.720
300	1.1614	1.007	184.6	15.89	26.3	22.5	0.707
350	0.9950	1.009	208.2	20.92	30.0	29.9	0.700
400	0.8711	1.014	230.1	26.41	33.8	38.3	0.690
450	0.7740	1.021	250.7	32.39	37.3	47.2	0.686
500	0.6964	1.030	270.1	38.79	40.7	56.7	0.684
550	0.6329	1.040	288.4	45.57	43.9	66.7	0.683
600	0.5804	1.051	305.8	52.69	46.9	76.9	0.685
650	0.5356	1.063	322.5	60.21	49.7	87.3	0.690
700	0.4975	1.075	338.8	68.10	52.4	98.0	0.695
750	0.4643	1.087	354.6	76.37	54.9	109	0.702
800	0.4354	1.099	369.8	84.93	57.3	120	0.709
850	0.4097	1.110	384.3	93.80	59.6	131	0.716
900	0.3868	1.121	398.1	102.9	62.0	143	0.720
950	0.3666	1.131	411.3	112.2	64.3	155	0.723
1000	0.3482	1.141	424.4	121.9	66.7	168	0.726
1100	0.3166	1.159	449.0	141.8	71.5	195	0.728
1200	0.2902	1.175	473.0	162.9	76.3	224	0.728
1300	0.2679	1.189	496.0	185.1	82	238	0.719
1400	0.2488	1.207	530	213	91	303	0.703
1500	0.2322	1.230	557	240	100	350	0.685
1600	0.2177	1.248	584	268	106	390	0.688
1700	0.2049	1.267	611	298	113	435	0.685
1800	0.1935	1.286	637	329	120	482	0.683
1900	0.1833	1.307	663	362	128	534	0.677
2000	0.1741	1.337	689	396	137	589	0.672
2100	0.1658	1.372	715	431	147	646	0.667
2200	0.1582	1.417	740	468	160	714	0.655
2300	0.1513	1.478	766	506	175	783	0.647
2400	0.1448	1.558	792	547	196	869	0.630
2500	0.1389	1.665	818	589	222	960	0.613
3000	0.1135	2.726	955	841	486	1570	0.536

TABLE A.6 Thermophysical Properties of Saturated Water[a]

Temperature, T (K)	Pressure, p (bars)[b]	Specific Volume (m³/kg) $v_f \cdot 10^3$	v_g	Heat of Vaporization, h_{fg} (kJ/kg)	Specific Heat (kJ/kg · K) $c_{p,f}$	$c_{p,g}$	Viscosity (N · s/m²) $\mu_f \cdot 10^6$	$\mu_g \cdot 10^6$	Thermal Conductivity (W/m · K) $k_f \cdot 10^3$	$k_g \cdot 10^3$	Prandtl Number Pr_f	Pr_g	Surface Tension, $\sigma_f \cdot 10^3$ (N/m)	Expansion Coefficient, $\beta_f \cdot 10^6$ (K⁻¹)	Temperature, T (K)
273.15	0.00611	1.000	206.3	2502	4.217	1.854	1750	8.02	569	18.2	12.99	0.815	75.5	−68.05	273.15
275	0.00697	1.000	181.7	2497	4.211	1.855	1652	8.09	574	18.3	12.22	0.817	75.3	−32.74	275
280	0.00990	1.000	130.4	2485	4.198	1.858	1422	8.29	582	18.6	10.26	0.825	74.8	46.04	280
285	0.01387	1.000	99.4	2473	4.189	1.861	1225	8.49	590	18.9	8.81	0.833	74.3	114.1	285
290	0.01917	1.001	69.7	2461	4.184	1.864	1080	8.69	598	19.3	7.56	0.841	73.7	174.0	290
295	0.02617	1.002	51.94	2449	4.181	1.868	959	8.89	606	19.5	6.62	0.849	72.7	227.5	295
300	0.03531	1.003	39.13	2438	4.179	1.872	855	9.09	613	19.6	5.83	0.857	71.7	276.1	300
305	0.04712	1.005	29.74	2426	4.178	1.877	769	9.29	620	20.1	5.20	0.865	70.9	320.6	305
310	0.06221	1.007	22.93	2414	4.178	1.882	695	9.49	628	20.4	4.62	0.873	70.0	361.9	310
315	0.08132	1.009	17.82	2402	4.179	1.888	631	9.69	634	20.7	4.16	0.883	69.2	400.4	315
320	0.1053	1.011	13.98	2390	4.180	1.895	577	9.89	640	21.0	3.77	0.894	68.3	436.7	320
325	0.1351	1.013	11.06	2378	4.182	1.903	528	10.09	645	21.3	3.42	0.901	67.5	471.2	325
330	0.1719	1.016	8.82	2366	4.184	1.911	489	10.29	650	21.7	3.15	0.908	66.6	504.0	330
335	0.2167	1.018	7.09	2354	4.186	1.920	453	10.49	656	22.0	2.88	0.916	65.8	535.5	335
340	0.2713	1.021	5.74	2342	4.188	1.930	420	10.69	660	22.3	2.66	0.925	64.9	566.0	340
345	0.3372	1.024	4.683	2329	4.191	1.941	389	10.89	668	22.6	2.45	0.933	64.1	595.4	345
350	0.4163	1.027	3.846	2317	4.195	1.954	365	11.09	668	23.0	2.29	0.942	63.2	624.2	350
355	0.5100	1.030	3.180	2304	4.199	1.968	343	11.29	671	23.3	2.14	0.951	62.3	652.3	355
360	0.6209	1.034	2.645	2291	4.203	1.983	324	11.49	674	23.7	2.02	0.960	61.4	697.9	360
365	0.7514	1.038	2.212	2278	4.209	1.999	306	11.69	677	24.1	1.91	0.969	60.5	707.1	365
370	0.9040	1.041	1.861	2265	4.214	2.017	289	11.89	679	24.5	1.80	0.978	59.5	728.7	370
373.15	1.0133	1.044	1.679	2257	4.217	2.029	279	12.02	680	24.8	1.76	0.984	58.9	750.1	373.15
375	1.0815	1.045	1.574	2252	4.220	2.036	274	12.09	681	24.9	1.70	0.987	58.6	761	375
380	1.2869	1.049	1.337	2239	4.226	2.057	260	12.29	683	25.4	1.61	0.999	57.6	788	380
385	1.5233	1.053	1.142	2225	4.232	2.080	248	12.49	685	25.8	1.53	1.004	56.6	814	385
390	1.794	1.058	0.980	2212	4.239	2.104	237	12.69	686	26.3	1.47	1.013	55.6	841	390
400	2.455	1.067	0.731	2183	4.256	2.158	217	13.05	688	27.2	1.34	1.033	53.6	896	400
410	3.302	1.077	0.553	2153	4.278	2.221	200	13.42	688	28.2	1.24	1.054	51.5	952	410
420	4.370	1.088	0.425	2123	4.302	2.291	185	13.79	688	29.8	1.16	1.075	49.4	1010	420
430	5.699	1.099	0.331	2091	4.331	2.369	173	14.14	685	30.4	1.09	1.10	47.2		430

TABLE A.6 *Continued*

Temperature, T (K)	Pressure, p (bars)[b]	Specific Volume (m³/kg) $v_f \cdot 10^3$	v_g	Heat of Vaporization, h_{fg} (kJ/kg)	Specific Heat (kJ/kg · K) $c_{p,f}$	$c_{p,g}$	Viscosity (N · s/m²) $\mu_f \cdot 10^6$	$\mu_g \cdot 10^6$	Thermal Conductivity (W/m · K) $k_f \cdot 10^3$	$k_g \cdot 10^3$	Prandtl Number Pr_f	Pr_g	Surface Tension, $\sigma_f \cdot 10^3$ (N/m)	Expansion Coefficient, $\beta_f \cdot 10^6$ (K⁻¹)	Temperature, T (K)
440	7.333	1.110	0.261	2059	4.36	2.46	162	14.50	682	31.7	1.04	1.12	45.1		440
450	9.319	1.123	0.208	2024	4.40	2.56	152	14.85	678	33.1	0.99	1.14	42.9		450
460	11.71	1.137	0.167	1989	4.44	2.68	143	15.19	673	34.6	0.95	1.17	40.7		460
470	14.55	1.152	0.136	1951	4.48	2.79	136	15.54	667	36.3	0.92	1.20	38.5		470
480	17.90	1.167	0.111	1912	4.53	2.94	129	15.88	660	38.1	0.89	1.23	36.2		480
490	21.83	1.184	0.0922	1870	4.59	3.10	124	16.23	651	40.1	0.87	1.25	33.9	—	490
500	26.40	1.203	0.0766	1825	4.66	3.27	118	16.59	642	42.3	0.86	1.28	31.6	—	500
510	31.66	1.222	0.0631	1779	4.74	3.47	113	16.95	631	44.7	0.85	1.31	29.3	—	510
520	37.70	1.244	0.0525	1730	4.84	3.70	108	17.33	621	47.5	0.84	1.35	26.9	—	520
530	44.58	1.268	0.0445	1679	4.95	3.96	104	17.72	608	50.6	0.85	1.39	24.5	—	530
540	52.38	1.294	0.0375	1622	5.08	4.27	101	18.1	594	54.0	0.86	1.43	22.1	—	540
550	61.19	1.323	0.0317	1564	5.24	4.64	97	18.6	580	58.3	0.87	1.47	19.7	—	550
560	71.08	1.355	0.0269	1499	5.43	5.09	94	19.1	563	63.7	0.90	1.52	17.3	—	560
570	82.16	1.392	0.0228	1429	5.68	5.67	91	19.7	548	76.7	0.94	1.59	15.0	—	570
580	94.51	1.433	0.0193	1353	6.00	6.40	88	20.4	528	76.7	0.99	1.68	12.8	—	580
590	108.3	1.482	0.0163	1274	6.41	7.35	84	21.5	513	84.1	1.05	1.84	10.5	—	590
600	123.5	1.541	0.0137	1176	7.00	8.75	81	22.7	497	92.9	1.14	2.15	8.4	—	600
610	137.3	1.612	0.0115	1068	7.85	11.1	77	24.1	467	103	1.30	2.60	6.3	—	610
620	159.1	1.705	0.0094	941	9.35	15.4	72	25.9	444	114	1.52	3.46	4.5	—	620
625	169.1	1.778	0.0085	858	10.6	18.3	70	27.0	430	121	1.65	4.20	3.5	—	625
630	179.7	1.856	0.0075	781	12.6	22.1	67	28.0	412	130	2.0	4.8	2.6	—	630
635	190.9	1.935	0.0066	683	16.4	27.6	64	30.0	392	141	2.7	6.0	1.5	—	635
640	202.7	2.075	0.0057	560	26	42	59	32.0	367	155	4.2	9.6	0.8	—	640
645	215.2	2.351	0.0045	361	90	—	54	37.0	331	178	12	26	0.1	—	645
647.3[c]	221.2	3.170	0.0032	0	∞	∞	45	45.0	238	238	∞	∞	0.0	—	647.3[c]

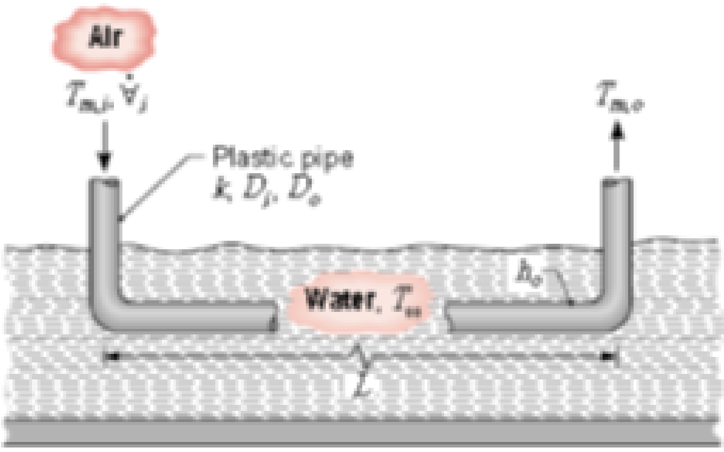
[a] Adapted from Reference 22.
[b] 1 bar = 10^5 N/m².
[c] Critical temperature.

Exercise 6.5 FOR REVISION

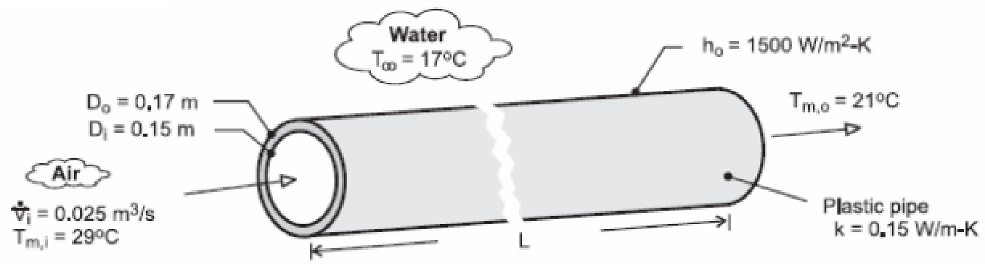
To cool a summer home without using a vapor-compression refrigeration cycle, air is routed through a plastic pipe ($k = 0.15 W/mK$, $D_i = 0.15m$, $D_o = 0.17m$) that is submerged in an adjoining body of water. The water temperature is nominally at $T_\infty = 17C$ and a convection coefficient of $h_o = 1500 W/m^2K$ is maintained at the outer surface of the pipe. If air from the home enters the pipe at a temperature of $T_{m,i} = 29C$ and a volumetric flow rate of $V_i = 0.025 m^3/s$, What pipe length L is needed to provide a discharge temperature of $T_{m,o} = 21°C$?

Additional Question: Now that you know the flow conditions and the length of the pipe, can you calculate the fan power required to move the air through this length of pipe if its inner surface is smooth? From your previous courses you should remember how the pumping power depends on the pressure drop and the volumetric flow rate as well as how the pressure drop is related to the friction factor.

Determine the properties of air from Table $A4$ at the appropriate temperature.



Solution



Assumptions: (1) Steady-state, (2) Ideal gas with negligible viscous dissipation and pressure variation, (3) Constant properties, (4) Smooth interior surface, (5) Negligible heat transfer from air in vertical legs of pipe.

Properties: *Table A.4*, air$(T_{m,i} = 29°C)$: $\rho_i = 1.155\mathrm{kg/m^3}$. Air $\left(\overline{\mathrm{T}}_\mathrm{m} = 25°\mathrm{C}\right) : \mathrm{c_p} = 1007 J/kgK$, $\mu = 183.6 \cdot 10^{-7}\mathrm{N \cdot s/m^2}, \mathrm{k_a} = 0.0261\mathrm{W/m \cdot K}, \mathrm{Pr} = 0.707$.

Analysis:

The surface of the pipe is maintained at a constant value through convection by the external water flow. Therefore, from an energy balance over the tube we would get:

$$\frac{\mathrm{T}_\infty - \mathrm{T}_{\mathrm{m,o}}}{\mathrm{T}_\infty - \mathrm{T}_{\mathrm{m,i}}} = \exp\left(-\frac{\overline{\mathrm{U}}\mathrm{A_s}}{\dot{\mathrm{m}}\mathrm{c_p}}\right)$$

where

$$\left(\overline{U}A_s\right)^{-1} = R_{tot} = \frac{1}{\overline{h}_i \pi D_i L} + \frac{\ln\left(D_0/D_i\right)}{2\pi L k} + \frac{1}{h_0 \pi D_0 L}$$

as we have to include not only the internal convection but also the conduction through the wall of the pipe and the external convection.

The internal convection coefficient $\overline{h_i}$ is the only missing information. We know that we are in internal forced convection and we have to determine the flow condition. So we calculate:

$$\mathrm{Re_D} = 4\dot{\mathrm{m}}/\pi \mathrm{D_i}\mu = 4\rho\dot{V}/\pi \mathrm{D_i}\mu = 13,350 > 2300$$

and so the flow in the pipe is turbulent.

Assuming fully developed flow throughout the pipe we use:

$$\overline{Nu_D} = 0.023 Re_D^{4/5} Pr^{0.3}$$

Therefore:

$$\overline{\mathrm{h}}_\mathrm{i} = \frac{\mathrm{k_a}}{\mathrm{D_i}} 0.023 \mathrm{Re_D^{4/5}} \mathrm{Pr}^{0.3} = \frac{0.0261\mathrm{W/m \cdot K} \times 0.023}{0.15\mathrm{m}} (13,350)^{4/5} (0.707)^{0.3} = 7.20\mathrm{W/m^2 \cdot K}$$

$$(\overline{U}A_s)^{-1} = \frac{1}{L}\left(\frac{1}{7.21W/m^2 \cdot K \times \pi \times 0.15m} + \frac{\ln(0.17/0.15)}{2\pi \times 0.15\mathrm{W/m \cdot K}} + \frac{1}{1500\mathrm{W/m^2 \cdot K} \times \pi \times 0.17\mathrm{m}}\right)$$

$$\mathrm{\overline{U}A_s} = \frac{\mathrm{L}}{(0.294 + 0.133 + 0.001)} = 2.335\mathrm{L\ W/K}$$

$$\frac{\mathrm{T_\infty - T_{m,o}}}{\mathrm{T_\infty - T_{m,i}}} = \frac{17-21}{17-29} = 0.333 = \exp\left(-\frac{2.335\mathrm{L}}{0.0289\mathrm{kg/s} \times 1007\mathrm{J/kg \cdot K}}\right) = \exp(-0.0802\mathrm{L})$$

$$L = -\frac{\ln(0.333)}{0.0802} = 13.7\mathrm{m}$$

Additional Question:

We know that for turbulent flow the friction coefficient in a smooth circular pipe is equal to:

$$\mathrm{f} = (0.790 ln \mathrm{Re_D} - 1.64)^{-2} = 0.029$$

Furthermore, we know that :

$$f = -\frac{\Delta p/\Delta x D_i}{\rho u_m^2/2} = 0.029$$

where

$$u_m = \dot{V}/A = \dot{V}/(\pi D_i^2/4) = 1.415 m/s$$

.

If we consider $\Delta x = L$ The pumping power will be given by:

$$P = -\Delta p \dot{V} = f\frac{\rho u_m^2}{2D_i} L\dot{V} = 0.029 \frac{1.155\mathrm{kg/m^3}(1.415\mathrm{m/s})^2}{2(0.15\mathrm{m})} 13.7\mathrm{m} \times 0.025\mathrm{m^3/s} = 0.078\mathrm{W}$$

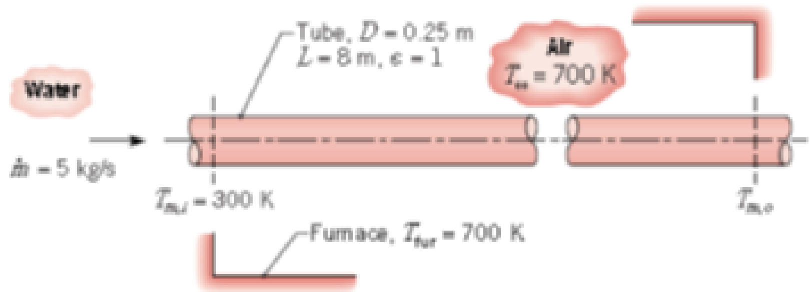
Comments: (1) with $L/D_i = 91$, the assumption of fully developed flow throughout the pipe is justified. (2) The fan power requirement is small and the process is economical. (3) The resistance to heat transfer associated with convection at the outer surface is negligible.

Exercise 6.6 FOR REVISION

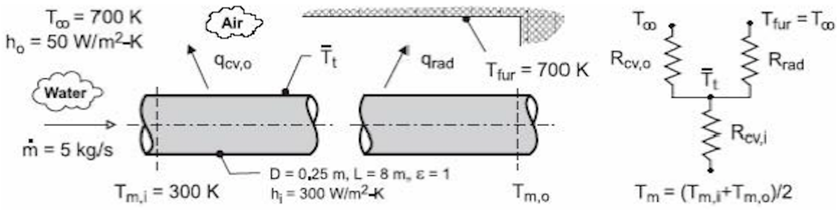
Water at $300K$ and a flow rate of $5kg/s$ enters a black, thin-walled tube, which passes through a large furnace whose walls and air are at a temperature of $700K$. The diameter and length of the tube are $0.25m$ and $8m$, respectively. Convection coefficients associated with water flow through the tube and air flow over the tube are $300W/m^2K$ and $50W/m^2K$, respectively.

a) Write an expression for the linearized radiation coefficient corresponding to radiation exchange between the outer surface of the pipe and the furnace walls. Explain how to calculate this coefficient if the surface temperature of the tube is represented by the arithmetic mean of its inlet and outlet values.

Determine the properties of water from Table $A6$ (at the end of this document) using $T_m = (T_{m,i} + T_{m,o})/2 = 331K$.



Solution



Assumptions: (1) steady-state conditions; (2) tube is small object with large, isothermal surroundings; (3) furnace air and walls are the same temperature; (4) tube is thin-walled with black surface ($\varepsilon = 1$); (5) incompressible liquid with negligible viscous dissipation.

Properties: Table $A6$, Water($T_m = (T_{m,i} + T_{m,o})/2 = 331K$): $c_p = 4192J/kgK$.

Analysis:

a) The linearized radiation coefficient follows from:

$$\overline{h}_{rad} = \sigma(\overline{T}_t + \overline{T}_{fur})(\overline{T}_t^2 + \overline{T}_{fur}^2)$$

where $\overline{T}_t$ represents the average tube wall surface temperature, which can be evaluated from an energy balance on the tube as represented by the thermal circuit above.

$$T_m = \frac{T_{m,i} + T_{m,o}}{2}$$

$$R_{tot} = R_{cv,i} + \frac{1}{\frac{1}{R_{cv,o}} + \frac{1}{R_{rad}}}$$

$$\frac{T_m - \overline{T}_t}{R_{cv,i}} = \frac{\overline{T}_t - T_{fur}}{\frac{1}{R_{cv,o}} + \frac{1}{R_{rad}}}$$

The thermal resistances, with $A_s = PL = \pi DL$, are:

$$R_{cv,i} = \frac{1}{h_i A_s}, \quad R_{cv,o} = \frac{1}{h_o A_s}, \quad R_{rad} = \frac{1}{\overline{h}_{rad}}$$

b) The outlet temperature can be calculated using the energy balance relation with $T_{fur} = T_\infty$:

$$\frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = \exp\left(-\frac{1}{mc_p R_{tot}}\right) \tag{0.1}$$

where c_p is evaluated at T_m.

Using IHT, the following results were obtained:

$$R_{cv,i} = 6.631 \times 10^{-5} K/W, \quad R_{cv,o} = 3.978 \times 10^{-4} K/W, \quad R_{rad} = 4.724 \times 10^{-4} K/W$$

$$T_m = 331K, \quad \overline{T}_t = 418K, \quad T_{m,o} = 362K$$

Comments: Since $T_\infty = T_{fur}$, it was possible to use R_{tot} in expression 0.1. How would you write the energy balance relation if $T_\infty \neq T_{fur}$?