Exercise 4.1

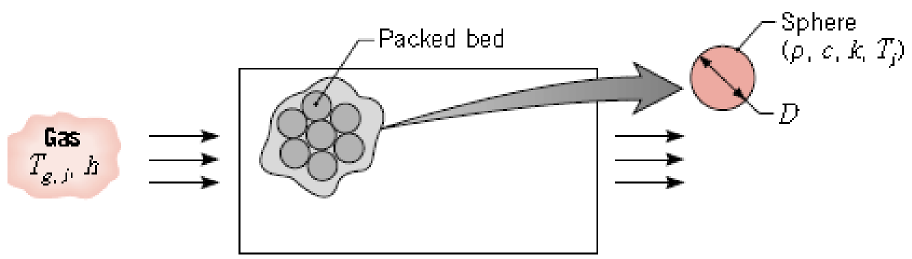
Thermal energy storage systems commonly involve packed bed of solid spheres, through which a hot gas flows if the system is being charged, or a cold gas if it is being discharged. In a charging process, heat transfer from the hot gas increases thermal energy stored within the colder spheres; during discharge, the stored energy decreases as heat is transferred from the warmer spheres to the cooler gas. Consider a packed bed of 75 mm diameter aluminum spheres ($\rho = 2700$ kg/m^3, $c = 950$J/kgK, $k = 150$ W/mK) and a charging process for which gas enters the storage unit at a temperature of $T_{g,i} = 300$°C. If the initial temperature of the spheres is $T_i = 25$°C and the convection coefficient is $h = 75$ W/m^{2}K, calculate:

a) How long does it take a sphere near the inlet of the system to accumulate 90% of the maximum possible thermal energy?

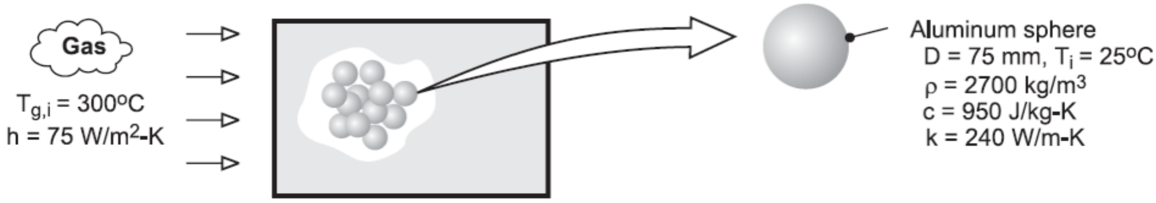
b) What is the corresponding temperature at the center of the sphere?

c) Is there any disadvantage to using copper instead of aluminum $\rho_{Cu} = 8900$kg/m^3 and $c_{Cu} = 400$ J/kgK) ?

d) Calculate the time needed to accumulate 90% of the maximum possible thermal energy and the corresponding temperature at the center of the sphere if the spheres are made of Pyrex, with $\rho = 2225$kg/m^3 ,$c = 835$ J/kg K, $k = 1.4$W/mK.



Solution



First of all, we compute the Biot number:

$$\text{Bi} \equiv \frac{h(r_0/3)}{k} = \frac{75\text{W/m}^2\text{K}(0.0125)\text{m}}{150\text{W/m}\cdot\text{ K}} = 0.00625 < 0.1$$

The total thermal energy that a particle can accumulate is equal to its thermal capacity times the maximum $\Delta T_{max} = T_{g,i} - T_i = \theta_i$. Therefore $Q_{removed} = 0.9 \cdot \rho c V \theta_i$. Therefore, remebering that the total heat removed added during a transient is expressed as $Q(t) = \rho c V \theta_i (1 - exp(-t/\tau_t))$, we get:

$$\frac{Q_removed}{\rho c V \theta_i} = 0.90 = 1 - \exp(-t/\tau_t)$$

$$\tau_t = \frac{\rho V c}{h A_s} = \frac{\rho D c}{6h} = \frac{2700\text{kg/m}^3 \times 0.075\text{m} \times 950\text{J/kgK}}{6 \times 75\text{W/m}^2\text{K}} = 427\text{s}$$

$$t = -\tau_t \ln(0.1) = 427\text{s} \times 2.30 = 984\text{s}$$

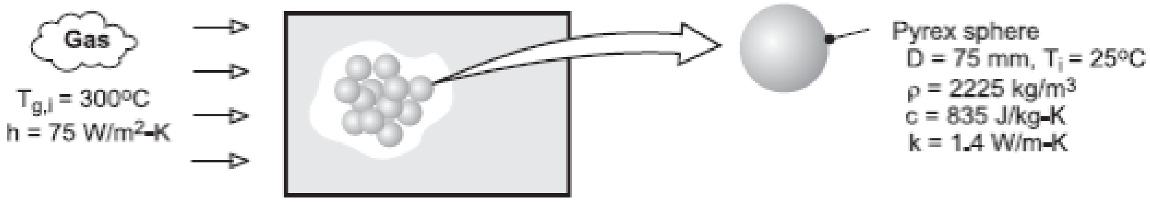
$$\begin{aligned} T(984\text{s}) &= T_{g,i} + (T_i - T_{g,i}) \exp\left(\frac{-6ht}{\rho D c}\right) \\ &= 300°\text{C} - 275°\text{C} \exp\left(\frac{-6 \times 75\text{W/m}^2\text{K} \times 984\text{s}}{2700\text{kg/m}^3 \times 0.075\text{m} \times 950\text{J/kgK}}\right) \\ &= 272.5°\text{C} \end{aligned}$$

For Cu, the heat capacity is:

$$\rho c = 8900\text{kg/m}^3 \times 400\text{J/kgK} = 3.56 \times 10^6\text{J/m}^3\text{K} > (\rho c)_{\text{Al}} = 2.57 \times 10^6\text{J/m}^3\text{K}$$

So for Cu spheres, we could store ∼38% more thermal energy.

Comments: Before the packed bed becomes fully charged, the temperature of the gas decreases as it passes through the bed. Hence, the time required for a sphere to reach a prescribed state of thermal energy storage increases with increasing distance from bed inlet.



We re-calculate the Biot number for the Pyrex spheres:

$$\text{Bi} \equiv \frac{h(r_0/3)}{k} = \frac{75\text{W/m}^2\text{K}(0.0125\text{m})}{1.4\text{W/m}\cdot\text{ K}} = 0.67$$

We see that the lumped capacitance model approximation is no longer valid. So, we need ti use the solution for transient heat transfer from a sphere under convective boundary conditions.

We have for a sphere:

$$\frac{Q}{Q_0} = 1 - \frac{3}{\lambda_1^3}\theta_0[\sin\lambda_1 - \lambda_1\cos\lambda_1] = 0.9$$

$$\theta_0 = A_1\exp\left(-\lambda_1^2\text{Fo}\right)$$

We have to remember that the Bi definition for the spatial effects equation is different from the lumped capacitance model definition. So, we re-calculate:

$$\text{Bi} = \frac{hr_0}{k} = 2.01$$

And from the interpolation in the table, we get: $\lambda_1 = 2.03$ and $A_1 = 1.48$.

So, we have:

$$\theta_0 = \frac{\lambda_1^3}{3[\sin\lambda_1 - \lambda_1\cos\lambda_1]}\left(1 - \frac{Q}{Q_0}\right) = \frac{8.37}{3[0.896 - 2.03\times(-0.443)]}\times 0.1 = 0.155$$

So, we can get the temperature at the center as:

$$\frac{T_0 - T_{gas,i}}{T_i - T_{gas,i}} = \theta_0 = 0.155$$

Therefore, $T_0 = 257.3$°C.

Finally, we use:

$$\theta_0 = A_1\exp\left(-\lambda_1^2\frac{\alpha t}{r_0^2}\right)$$

$$t = -\frac{r_0^2}{\alpha\lambda_1^2}\ln\left(\frac{\theta_0}{A_1}\right) = 1020\text{s}$$

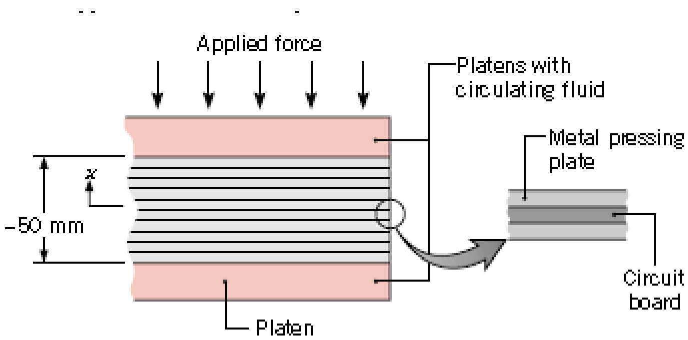
Bi	Plate			Cylinder			Sphere		
	$\hat{\lambda}_1$	A_1	D_1	$\hat{\lambda}_1$	A_1	D_1	$\hat{\lambda}_1$	A_1	D_1
0.01	0.09983	1.0017	1.0000	0.14124	1.0025	1.0000	0.17303	1.0030	1.0000
0.02	0.14095	1.0033	1.0000	0.19950	1.0050	1.0000	0.24446	1.0060	1.0000
0.05	0.22176	1.0082	0.9999	0.31426	1.0124	0.9999	0.38537	1.0150	1.0000
0.10	0.31105	1.0161	0.9998	0.44168	1.0246	0.9998	0.54228	1.0298	0.9998
0.15	0.37788	1.0237	0.9995	0.53761	1.0365	0.9995	0.66086	1.0445	0.9996
0.20	0.43284	1.0311	0.9992	0.61697	1.0483	0.9992	0.75931	1.0592	0.9993
0.30	0.52179	1.0450	0.9983	0.74646	1.0712	0.9983	0.92079	1.0880	0.9985
0.40	0.59324	1.0580	0.9971	0.85158	1.0931	0.9970	1.05279	1.1164	0.9974
0.50	0.65327	1.0701	0.9956	0.94077	1.1143	0.9954	1.16556	1.1441	0.9960
0.60	0.70507	1.0814	0.9940	1.01844	1.1345	0.9936	1.26440	1.1713	0.9944
0.70	0.75056	1.0918	0.9922	1.08725	1.1539	0.9916	1.35252	1.1978	0.9925
0.80	0.79103	1.1016	0.9903	1.14897	1.1724	0.9893	1.43203	1.2236	0.9904
0.90	0.82740	1.1107	0.9882	1.20484	1.1902	0.9869	1.50442	1.2488	0.9880
1.00	0.86033	1.1191	0.9861	1.25578	1.2071	0.9843	1.57080	1.2732	0.9855
1.10	0.89035	1.1270	0.9839	1.30251	1.2232	0.9815	1.63199	1.2970	0.9828
1.20	0.91785	1.1344	0.9817	1.34558	1.2387	0.9787	1.68868	1.3201	0.9800
1.30	0.94316	1.1412	0.9794	1.38543	1.2533	0.9757	1.74140	1.3424	0.9770
1.40	0.96655	1.1477	0.9771	1.42246	1.2673	0.9727	1.79058	1.3640	0.9739
1.50	0.98824	1.1537	0.9748	1.45695	1.2807	0.9696	1.83660	1.3850	0.9707
1.60	1.00842	1.1593	0.9726	1.48917	1.2934	0.9665	1.87976	1.4052	0.9674
1.80	1.04486	1.1695	0.9680	1.54769	1.3170	0.9601	1.95857	1.4436	0.9605
2.00	1.07687	1.1785	0.9635	1.59945	1.3384	0.9537	2.02876	1.4793	0.9534
2.20	1.10524	1.1864	0.9592	1.64557	1.3578	0.9472	2.09166	1.5125	0.9462
2.40	1.13056	1.1934	0.9549	1.68691	1.3754	0.9408	2.14834	1.5433	0.9389
3.00	1.19246	1.2102	0.9431	1.78866	1.4191	0.9224	2.28893	1.6227	0.9171
4.00	1.26459	1.2287	0.9264	1.90808	1.4698	0.8950	2.45564	1.7202	0.8830
5.00	1.31384	1.2402	0.9130	1.98981	1.5029	0.8721	2.57043	1.7870	0.8533
6.00	1.34955	1.2479	0.9021	2.04901	1.5253	0.8532	2.65366	1.8338	0.8281
8.00	1.39782	1.2570	0.8858	2.12864	1.5526	0.8244	2.76536	1.8920	0.7889
10.00	1.42887	1.2620	0.8743	2.17950	1.5677	0.8039	2.83630	1.9249	0.7607
20.00	1.49613	1.2699	0.8464	2.28805	1.5919	0.7542	2.98572	1.9781	0.6922
50.00	1.54001	1.2727	0.8260	2.35724	1.6002	0.7183	3.07884	1.9962	0.6434
100.00	1.55525	1.2731	0.8185	2.38090	1.6015	0.7052	3.11019	1.9990	0.6259
∞	1.57080	1.2732	0.8106	2.40483	1.6020	0.6917	3.14159	2.0000	0.6079

Exercise 4.2

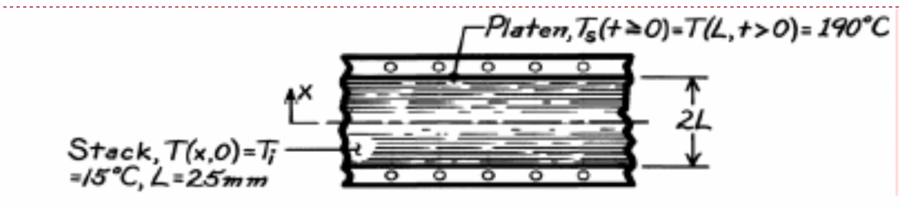
Circuit boards are treated by heating a stack of them under high pressure. The platens at the top and bottom of the stack are maintained at a uniform temperature by a circulating fluid. To achieve a curing condition, the epoxy has to be maintained at or above $170°C$ for at least 5 min. The effective thermo-physical properties of the stack are $k = 0.613W/mK$ and $\rho c_p = 2.73 \cdot 10^6 J/m^3K$.

a) If the stack is initially at $15°C$ and, following application of pressure, the platens are suddenly brought to a uniform temperature of $190°C$, calculate the elapsed time t_e required for the mid-plane of the stack to reach the cure temperature of $170°C$.

b) If at this time $t = t_e$, the platen temperature were reduced suddenly to $15°C$, how much energy would have to be removed from the stack by the coolant circulating in the platen in order to return the stack to its initial uniform temperature?



Solution



First of all we must determine the missing property : $\alpha = \frac{K}{\rho \cdot c_p} = 2.245 \cdot 10^{-7}$.

Having all the required properties, we can begin the analysis and solving of the problem.

a) Here we can recognize that sudden application of surface temperature is equivalent to having an infinite Biot number ($Bi \to \infty$), or to $h \to \infty$. With $T_s = T_\infty$.

$$\theta_0 = \frac{T(0,t) - T_s}{T_i - T_s} = \frac{170 - 190}{15 - 190} = 0.114$$

$$\theta_0 = A_1 \cdot \exp\{-\lambda_1^2 \cdot \text{Fo}\}$$

with $\lambda_1 = 1.5707 \quad A_1 = 1.2733$. The equation can be used to find the value of Fo using the same table as in exercise 4.1:

$$\text{Fo} = \frac{-1}{\lambda_1^2} \cdot \ln\left(\frac{\theta_0^*}{A_1}\right) = 0.977$$

Then using the definition of $\text{Fo} = \alpha t / L^2$, we find out the required t:

$$t = \frac{\text{Fo} \cdot L^2}{\alpha} = \frac{0.977 \cdot (25 \cdot 10^{-3})^2}{2.245 \cdot 10^{-7}} = 2720s = 45.3min$$

b) The energy removal is equivalent to the energy gained by the stack per unit area for the time interval going from 0 to t_e. With $Q_0^"$ corresponding to the maximum amount of energy the could be transferred.

$$Q_0^" = \rho \cdot c_p \cdot 2L \cdot (T_i - T_\infty) = 2.73 \cdot 10^6 \cdot 2 \cdot 25 \cdot 10^{-3} \cdot (15 - 190) = -2.389 \cdot 10^7 J/m^2$$

We can also determine $Q_0^"$ using the following equation:

$$\frac{Q^"}{Q_0^"} = 1 - \frac{\sin \lambda_1}{\lambda_1} \cdot \theta_0 = 0.927$$

Having the heat removed by unit of volume we can find out the heat to be removed per unit area. This heat must be removed to return to T_i.

$$Q^" = 0.927 \cdot Q_0^" = 0.927 \cdot 2.389 \cdot 10^7 = 2.21 \cdot 10^7 J/m^2$$

Note:

For question b if we compute θ_0 the value will be zero, as such the Fourier number will be infinite. Since $Fo = \frac{t\alpha}{L^2}$, then the time required for this heat transfer will be infinite as well and as such the heat to be transferred is equal to maximum amount of energy transfer that could occur if the process were continued to infinite time Q_0. Specifically if we compute it for case b

$$Q_0^{"} = \rho \cdot c_p \cdot 2L \cdot (T_i - T_\infty) = 2.73 \cdot 10^6 \cdot 2 \cdot 25 \cdot 10^{-3} \cdot (170 - 15) = 2.12 \cdot 10^7 J/m^2$$

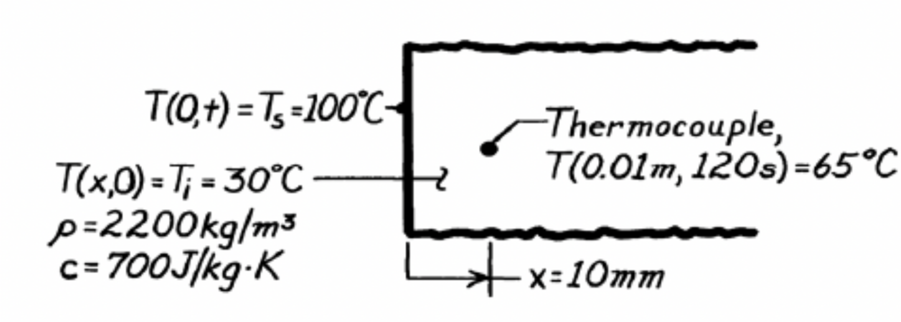
that is only 4% lower than the value obtained in the solution for b.

However, we can see the problem in b as such: We want to remove the heat transferred to the stack between 0 and t_e and this heat can be actually estimated with the approximated solution given in chapter 5.5.3 at page 275. In this formula Q_0 is the maximum amount of energy transfer that could occur if the process were continued to infinite time. However, this heat transfer takes place in a finite time in question a and as such the actual heat transferred would be a fraction of Q_0 and to estimate this rate we use a formula that is an approximated solution. Since both formulas are from approximated solutions their values are not exact and which one to prefer will depend on how much precision is required.

Exercise 4.3

A procedure for determining the thermal conductivity of a solid material involves embedding a thermocouple in a thick slab of the solid and measuring the response to a prescribed change in temperature at one surface. Consider an arrangement for which the thermocouple is embedded 10mm from a surface that is suddenly brought to a temperature of $100°C$ by exposure to boiling water. If the initial temperature of the slab was $30°C$ and the thermocouple measures a temperature of $65°C$ 2min after the surface is brought to $100°C$, what is the thermal conductivity? The density and specific heat of the solid are known to be $\rho = 2200 kg/m^3$ and $c_p = 700 J/kgK$.

Solution



Assumptions:

a) One-dimensional conduction in x

b) Slab semi-infinite medium

c) Constant properties

Analysis:

To solve this problem we use the semi-infinite medium model with constant surface temeprature boundary condition. Therefore we have:

$$\frac{T(x,t) - T_s}{T_i - T_s} = erf\Big(\frac{x}{2 \cdot (\alpha \cdot t)^{0.5}}\Big) \rightarrow \frac{65 - 100}{30 - 100} = erf\Big(\frac{0.01}{2 \cdot (\alpha \cdot 120)^{0.5}}\Big) = 0.5$$

The equation is non linear so the values for the erf function must be taken from a table and it is possible to find: $erf(x) = 0.5 \iff x = 0.477$, hence:

$$\alpha \cdot 120 = \Big(\frac{0.01}{0.477}\Big)^2 \rightarrow \alpha = 9.156 \cdot 10^{-7} m^2/s$$

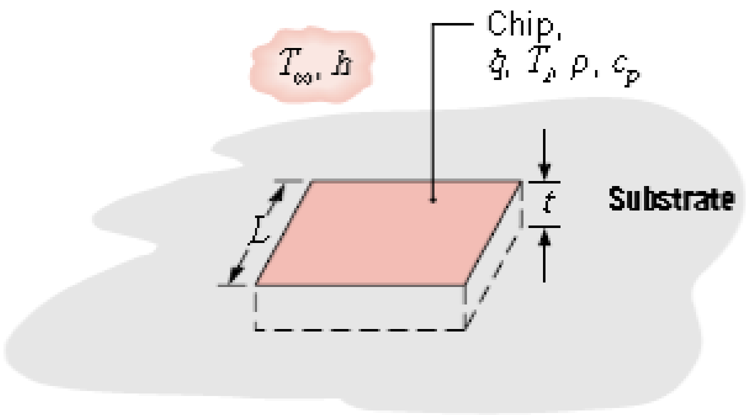
Since $\alpha = \dfrac{k}{\rho c_p} \rightarrow k = \alpha \cdot \rho \cdot c_p = 1.41 W/mK$

Exercise 4.4 FOR REVISION

A chip that is of length $L = 5$mm on a side and thickness $t = 1$mm is encased in a ceramic substrate, and its exposed surface is convectively cooled by a dielectric liquid for which $h = 150$W/m^{2}K and $T_\infty = 20$°C. In the off-mode the chip is in thermal equilibrium with the coolant ($T_i = T_\infty$). When the chip is energized, however, its temperature increases until a new steady-state is established. For purposes of analysis, the energized chip is characterized by uniform volumetric heating with $\dot{q} = 9 \times 10^6$ W/m^3.

a) Assuming an infinite contact resistance between the chip and substrate and negligible conduction resistance within the chip, determine the steady-state chip temperature T_f. Following activation of the chip, how long does it take to come within 1°C of this temperature? The chip density and specific heat are $\rho = 2000$ kg/m^3 and $c = 700$J/kgK, respectively.

b) For a more realistic analysis the indirect transfer from the chip to the substrate and then from the substrate to the coolant needs to be accounted for. The total thermal resistance associated with this indirect route includes contributions due to the chip-substrate interface (a contact resistance), multidimensional conduction in the substrate and convection from the surface of the substrate to the coolant. If this total thermal resistance is $R_t = 200$W/K, what is the steady state chip temperature T_f ? Following the activation of the chip, how long does it take to come within 1°C of this temperature?



Hint: The general equation for heat transfer accounting for ALL of the mechanisms is:

$$q''_s A_{s,h} + \dot{E}_g - [h(T - T_\infty) + \varepsilon\sigma(T^4 - T^4_{sur})]A_{s(c,r)} = \rho V c \frac{dT}{dt}$$

An exact solution to Equation 5.15 may also be obtained if radiation may be neglected and h is independent of time. Introducing a temperature difference $\theta \equiv T - T_\infty$, where $d\theta/dt = dT/dt$, Equation 5.15 reduces to a linear, first-order, nonhomogeneous differential equation of the form

$$\frac{d\theta}{dt} + a\theta - b = 0 \qquad (5.20)$$

where $a \equiv (hA_{s,c}/\rho Vc)$ and $b \equiv [(q_s'' A_{s,h} + \dot{E}_g)/\rho Vc]$. Although Equation 5.20 may be solved by summing its homogeneous and particular solutions, an alternative approach is to eliminate the nonhomogeneity by introducing the transformation

$$\theta' \equiv \theta - \frac{b}{a} \tag{5.21}$$

Recognizing that $d\theta'/dt = d\theta/dt$, Equation 5.21 may be substituted into (5.20) to yield

$$\frac{d\theta'}{dt} + a\theta' = 0 \tag{5.22}$$

Separating variables and integrating from 0 to t (θ_i' to θ'), it follows that

$$\frac{\theta'}{\theta_i'} = \exp(-at) \tag{5.23}$$

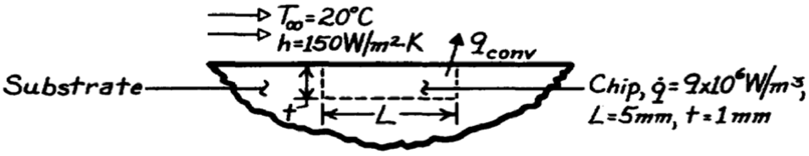
or substituting for θ' and θ,

$$\frac{T - T_\infty - (b/a)}{T_i - T_\infty - (b/a)} = \exp(-at) \tag{5.24}$$

Hence

$$\frac{T - T_\infty}{T_i - T_\infty} = \exp(-at) + \frac{b/a}{T_i - T_\infty}[1 - \exp(-at)] \tag{5.25}$$

Solution



Properties: chip material (given): $\rho = 2000 \text{kg/m}^3$, $c = 700$ J/kgK.

Analysis: At steady-state, conservation of energy yields:

$$-\dot{E}_{out} + \dot{E}_g = 0$$

$$-hL^2(T_f - T_\infty) + \dot{q}L^2 t = 0$$

$$T_f = T_\infty + \frac{\dot{q}t}{h}$$

$$T_f = 20°\text{C} + \frac{9 \times 10^6 \text{W/m}^3 \times 0.001\text{m}}{150\text{W/m}^2\text{K}} = 80°\text{C}$$

From the lumped capacitance analysis the general heat transfer equation reduces to:

$$\rho L^2 tc \frac{\text{d}T}{\text{d}t} = \dot{q}L^2 t - h(T - T_\infty)L^2$$

with

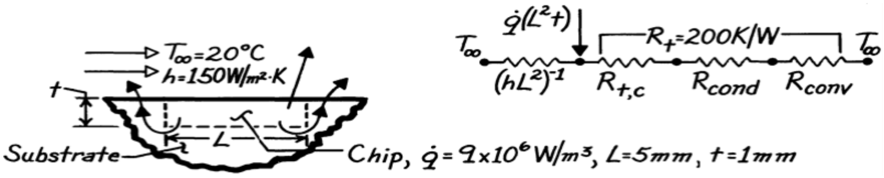
$$a \equiv \frac{h}{\rho tc} = \frac{150\text{W/m}^2\text{K}}{(2000\text{kg/m}^3)(0.001\text{m})(700\text{J/kgK})} = 0.107\text{s}^{-1}$$

$$b \equiv \frac{\dot{q}}{\rho c} = \frac{9 \times 10^6 \text{W/m}^3}{(2000\text{kg/m}^3)(700\text{J/kgK})} = 6.429\text{K/s}$$

From equation 5.25,

$$\exp(-at) = \frac{T - T_\infty - b/a}{T_i - T_\infty - b/a} = \frac{(79 - 20 - 60)\text{K}}{(20 - 20 - 60)\text{K}} = 0.01667$$

$$t = -\frac{\ln(0.01667)}{0.107\text{s}^{-1}} = 38.3\text{s}$$



Assumptions: Constant properties

Analysis: the direct and indirect path for heat transfer from the chip to the coolant are in parallel, and the equivalent resistance is:

$$R_{equiv} = \left[hL^2 + R_t^{-1}\right]^{-1} = \left[\left(3.75 \times 10^{-3} + 5 \times 10^{-3}\right)\mathrm{W/K}\right]^{-1} = 114.3\mathrm{K/W}$$

The corresponding overall heat transfer coefficient is:

$$U = \frac{(R_{equiv})^{-1}}{L^2} = \frac{0.00875\mathrm{W/K}}{(0.005\mathrm{m})^2} = 350\mathrm{W/m^2K}$$

To obtain the steady-state temperature, apply conservation of energy to a control surface about the chip:

$$-\dot{E}_{out} + \dot{E}_g = 0 \qquad -UL^2(T_f - T_\infty) + \dot{q}L^2t = 0$$

$$T_f = T_\infty + \frac{\dot{q}t}{U} = 20°\mathrm{C} + \frac{9 \times 10^6\mathrm{W/m^3} \times 0.001\mathrm{m}}{350\mathrm{W/m^2K}} = 45.7°\mathrm{C}$$

From the general lumped capacitance analysis, the general equation for heat transfer yields:

$$\rho L^2 tc\frac{\mathrm{d}T}{\mathrm{d}t} = \dot{q}L^2t - U(T - T_\infty)L^2$$

with

$$a \equiv \frac{U}{\rho tc} = \frac{350\mathrm{W/m^2K}}{(2000\mathrm{kg/m^3})(0.001\mathrm{m})(700\mathrm{J/kgK})} = 0.250\mathrm{s^{-1}}$$

$$b = \frac{\dot{q}}{\rho c} = \frac{9 \times 10^6\mathrm{W/m^3}}{(2000\mathrm{kg/m^3})(700\mathrm{J/kgK})} = 6.429\mathrm{K/s}$$

Equation 5.24 yields

$$\exp(-at) = \frac{T - T_\infty - b/a}{T_i - T_\infty - b/a} = \frac{(44.7 - 20 - 25.7)K}{(20 - 20 - 25.7)K} = 0.0389$$

$$t = -\ln(0.0389)/0.250\mathrm{s^{-1}} = 13.0\mathrm{s}$$

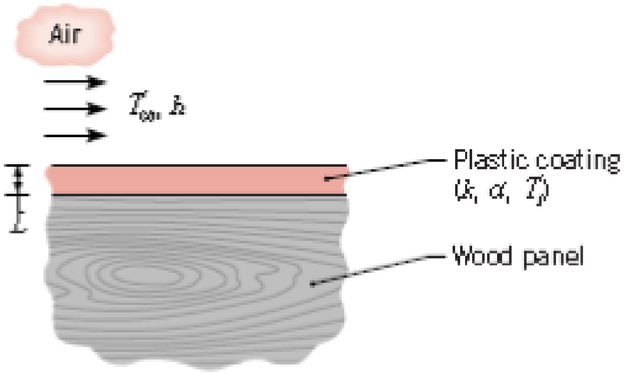
Comments: Heat transfer through the substrate is comparable to that associated with direct convection to the coolant.

Exercise 4.5 FOR REVISION

A plastic coating is applied to wood panels by first depositing molten polymer on a panel and then cooling the surface of the polymer by subjecting it to air flow at $25°C$. As first approximation, the heat of reaction associated with solidification of the polymer may be neglected and the polymer/wood interface may be assumed to be adiabatic. If the thickness of the coating is $L = 2mm$ and it has an initial uniform temperature of $T_i = 200°C$, how long will it take for the surface to achieve a safe-to-touch temperature of $42°C$ if the convection coefficient is $h = 200W/m^2K$?

What is the corresponding value of the interface temperature?

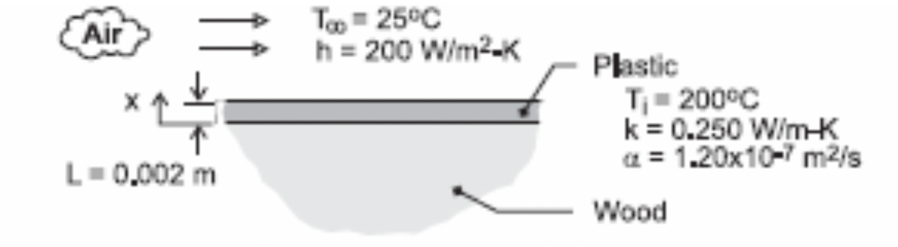
The thermal conductivity and diffusivity of the plastic are $k = 0.25W/mK$ and $\alpha = 1.2{\cdot}10^{-7}m^2/s$.



Solution

Assumptions:

a) One-dimensional conduction in coating
b) Negligible radiations
c) Constant properties
d) Negligible heat from reaction
e) Negligible heat transfer across plastic/wood interface



With a $Bi = hL/k = 200 \cdot 0.002/0.25 = 1.6 > 0.1$, we must avoid using the lumped capacity method. To use the approximate solution we first have to consider more closely the physics of this problem and the conditions under which the formulas have been derived in class. The expression reported in the formula sheet is obtained for a plate of thickness $2L_0$ that is convectively cooled on both sides. Clearly the solution is symmetric around the centerline of the wall and indeed we have observed that the characteristic dimension describing the temperature change in the wall and the Bi number is half the length of such a wall, L_0. In this case, however, the wall is convectively cooled on one side but insulated on the other side. We then remember that the adiabatic boundary condition is equivalent to a symmetry condition for the temperature profile. Therefore, the plate under study, of thickness L, will have a temperature profile that is equal to that of half a plate with thickness $2L_0$. We thus realize that the thickness L of the plate studied in this problem is equal to half the thickness of the convectively cooled plate we used to derive the expressions in the formula sheet. Therefore, the total thickness of the plate of this problem is the characteristic dimension we have to use to calculate the Bi anf Fo numbers for this problem. Thus with $Bi = hL/k = 1.6$ we get $A_1 = 1.1593$ and $\lambda_1 = 1.00842$:

$$\theta = \frac{T - T_\infty}{T_i - T_\infty} = \frac{42 - 25}{200 - 25} = 0.0971$$

$$\theta = A_1 \cdot \exp\left\{-\lambda_1^2 \cdot \text{Fo}\right\} \cdot \cos\left(\lambda_1 \cdot x/L\right) = 1.1593 \exp\left\{-1.00842^2 \cdot \text{Fo}\right\} \cdot cos(1.00842)$$

For the case where $x = L$

$$\text{Fo} = \frac{-\ln\left(\frac{0.0971}{1.1593 \cdot cos(1.00842)}\right)}{1.00842^2} = 1.82$$

$$t = \frac{\text{Fo} \cdot L^2}{\alpha} = \frac{1.82 \cdot 0.002^2}{1.20 \cdot 10^{-7}} = 60.7s$$

From the expression for the dimensionless temperature at the center of the sphere $\theta_0 = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 exp(-\lambda_1^2 \cdot Fo)$ we get:

$$T_0 = T_\infty + (T_i - T_\infty) \cdot A_1 \exp\left\{-\lambda_1^2 \cdot \text{Fo}\right\} = 25 + 175 \cdot 1.1593 \cdot \exp\left\{-1.00842^2 \cdot 1.82\right\} = 56.88°C$$