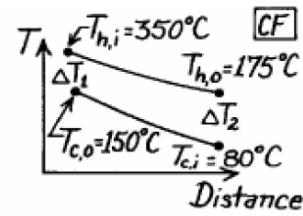
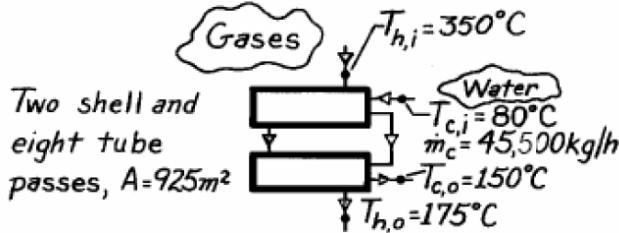


## Exercise 11.1

Water at a rate of  $45000\text{kg}/\text{h}$  is heated from  $80^\circ\text{C}$  to  $150^\circ\text{C}$  in a heat exchanger having two shell passes and eight tube passes with a total surface area of  $925\text{m}^2$ . Hot exhaust gases having approximately the same thermophysical properties as air enter at  $350^\circ\text{C}$  and exit at  $175^\circ\text{C}$ . Determine the overall heat transfer coefficient.

**Solutions**  $NTU = 1.27$ ,  $U = 29.5\text{W}/\text{m}^2 \cdot \text{K}$

## Solution



**Assumptions:** (1) Negligible losses to surroundings, (2) Constant properties, (3) Exhaust gas properties are approximated as those of atmospheric air.

**Properties:** Table A-6, Water ( $\bar{T}_C = (80 + 150)^\circ\text{C}/2 = 388\text{K}$ ) :  $c_c = c_{p,f} = 4236\text{J/kg} \cdot \text{K}$

**Analysis:** Since this is a shell-and-tube heat exchanger, we will use the  $\varepsilon - NTU$  method, for which

$$C_c = \dot{m}_c c_c = \frac{45,500\text{kg/h}}{3600\text{s/h}} \times 4236\text{J/kg} \cdot \text{K} = 5.35 \cdot 10^4\text{W/K}$$

$$Q = C_c (T_{c,o} - T_{c,i}) = 5.35 \cdot 10^4\text{W/K} (150 - 80)^\circ\text{C} = 3.75 \cdot 10^6\text{W}$$

Then we can find  $C_h$  from an energy balance on the hot stream,

$$C_h = Q / (T_{h,i} - T_{h,o}) = 3.75 \cdot 10^6\text{W} / (350 - 175)^\circ\text{C} = 2.14 \cdot 10^4\text{W/K}$$

Thus

$$C_r = C_{\min} / C_{\max} = 0.40$$

$$\varepsilon = Q / C_{\min} (T_{h,i} - T_{c,i}) = 3.75 \cdot 10^6\text{W} / 2.14 \cdot 10^4\text{W/K} (350 - 80)^\circ\text{C} = 0.648$$

From Table 11.4, we use the equation for a shell-tube heat exchanger with multiple shell passes ( $n = 2$ ) - Eqs. 11.31b and c - and we follow the indicated procedure to calculate the NTU for the overall heat exchanger.

First, knowing the overall heat exchanger efficiency  $\varepsilon$  we calculate the factor  $F$  and from this the single-shell heat exchanger efficiency  $\varepsilon_1$  :

$$F = \left( \frac{\varepsilon C_r - 1}{\varepsilon - 1} \right)^{1/n} = 1.45, \quad \varepsilon_1 = \frac{F - 1}{F - C_r} = 0.429$$

We then use the equation for a single shell heat exchanger (Eqs. 11.30c and 11.30b) to determine the factor  $E$  and from this the number of heat transfer units of a single-shell pass ( $NTU_1$ ):

$$E = \frac{2/\varepsilon_1 - (1 + C_r)}{(1 + C_r^2)^{1/2}} = 3.0$$

$$(NTU)_1 = - (1 + C_r^2)^{-1/2} \ln \left[ \frac{E-1}{E+1} \right] = 0.637$$

Finally, we use Eq. 11.31d to calculate the multi-shell heat exchanger  $NTU$  value from the single-shell pass value:

$$NTU = n(NTU)_1 = 1.27$$

Now, knowing  $NTU$ ,  $C_{min}$  and  $A$  of the heat exchanger we can determine the overall heat transfer coefficient:

$$U = NTU \times C_{min}/A = 1.27 \times 2.14 \cdot 10^4 \text{W/K} / (925 \text{m}^2) = 29.5 \text{W/m}^2 \cdot \text{K}$$

## Exercise 11.2

Consider a concentric tube heat exchanger characterized by a uniform overall heat transfer coefficient and operating under the following conditions:

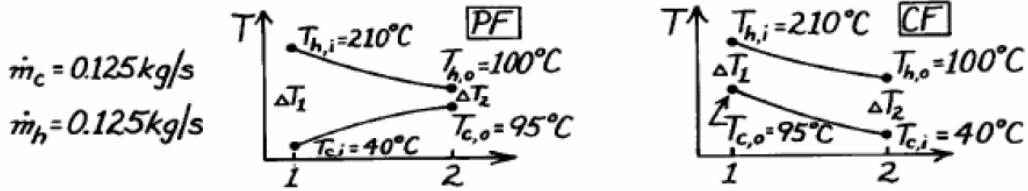
	$\dot{m}$ (kg/s)	$c_p$ (J/kg · K)	$T_i$ (°C)	$T_o$ (°C)
Cold fluid	0.125	4200	40	95
Hot fluid	0.125	2100	210	—

Determine:

- the maximum possible heat transfer rate
- the heat exchanger effectiveness
- what is the ratio of the required areas to operate the heat exchanger in counter-flow ( $A_{CF}$ ) or in parallel-flow ( $A_{PF}$ ) conditions? If we want to minimize the area which of the two configurations is more advantageous?

### Solutions

- $Q_{max} = 44625W$
- $\epsilon = 0.65$
- $\frac{A_{CF}}{A_{PF}} = 0.55$ , counter-flow

**Solution**

**Assumptions:** (1) Negligible heat loss to surroundings, (2) Constant properties, (3) Overall heat transfer coefficient remains unchanged for PF or CF conditions.

**Properties:** Hot fluid (given):  $c = 2100 \text{ J/kg} \cdot \text{K}$ ; Cold fluid (given):  $c = 4200 \text{ J/kg} \cdot \text{K}$

**Analysis:** (a) The maximum possible heat transfer rate is given by Eq. 11.18

$$Q_{\max} = C_{\min} (T_{h,i} - T_{c,i})$$

The minimum capacity fluid is the hot fluid with  $C_{\min} = \dot{m}_h c_h$ , giving

$$Q_{\max} = \dot{m}_h c_h (T_{h,i} - T_{c,o}) = 0.125 \frac{\text{kg}}{\text{s}} \times 2100 \frac{\text{J}}{\text{kg} \cdot \text{K}} (210 - 40) \text{K} = 44,625 \text{ W}$$

(b) The effectiveness is defined as the ratio of the actual heat transfer rate  $Q$  and the maximum heat transfer rate  $Q_{\max}$ . As we know both inlet and outlet temperatures of the cold fluid,  $Q$  can be determined from an energy balance on the cold fluid.

$$\begin{aligned} \varepsilon &= Q/Q_{\max} = \dot{m}_c c_c (T_{c,o} - T_{c,i}) / Q_{\max} \\ &= 0.125 \text{ kg/s} \times 4200 \text{ J/kg} \cdot \text{K} (95 - 40) \text{ K} / 44,625 \text{ W} = 0.65 \end{aligned}$$

(c) The studied heat exchanger will have to transfer  $Q$  from the hot to the cold fluid. We consider the specific geometry of the heat exchanger fixed (i.e. its overall heat transfer coefficient is fixed and equal to  $U$ ), and we consider only the overall heat exchange area as a variable parameter. Considering that this is a concentric heat exchanger, we can use the LMTD method to estimate the ratio of the areas between the parallel and counter flow configurations. We can indeed write:

$$Q = U A_{PF} \Delta T_{lm,PF}$$

$$Q = U A_{CF} \Delta T_{lm,CF}$$

Hence we can write:

$$\frac{A_{CF}}{A_{PF}} = \frac{Q/U \Delta T_{lm,CF}}{Q/U \Delta T_{lm,PF}} = \frac{\Delta T_{lm,PF}}{\Delta T_{lm,CF}}$$

To calculate the LMTD, we first find  $T_{h,o}$  from the overall energy balances on the two fluids.

$$T_{h,o} = T_{h,i} - \frac{\dot{m}_c c_c}{\dot{m}_h c_h} (T_{c,o} - T_{c,i}) = 210^\circ\text{C} - \frac{0.125 \times 4200}{0.125 \times 2100} (95 - 40)^\circ\text{C} = 100^\circ\text{C}$$

In all cases  $\Delta T_1$  and  $\Delta T_2$  as shown below, find  $\Delta T_{\ell m} = (\Delta T_1 - \Delta T_2) / \ln(\Delta T_1 / \Delta T_2)$ .

Yet, as illustrated in the schematic at the beginning of the exercise, the  $\Delta T_{1,2}$  will be different for the parallel and counterflow cases:

$$\frac{A_{CF}}{A_{PF}} = \frac{[(210 - 40) - (100 - 95)]/\ell n(170/5)}{[(210 - 95) - (100 - 40)]\ell n(115/60)} = \frac{46.8^{\circ}\text{C}}{84.5^{\circ}\text{C}} = 0.55$$

**Comments:** Alternatively in solving part (c), it is also possible to use Figs. 11.11 and 11.12 to evaluate NTU values for corresponding  $\varepsilon$  and  $C_{min}/C_{max}$  values. With knowledge of NTU it is then possible to find  $A_{CF}/A_{PF}$ .

**Exercise 11.3**

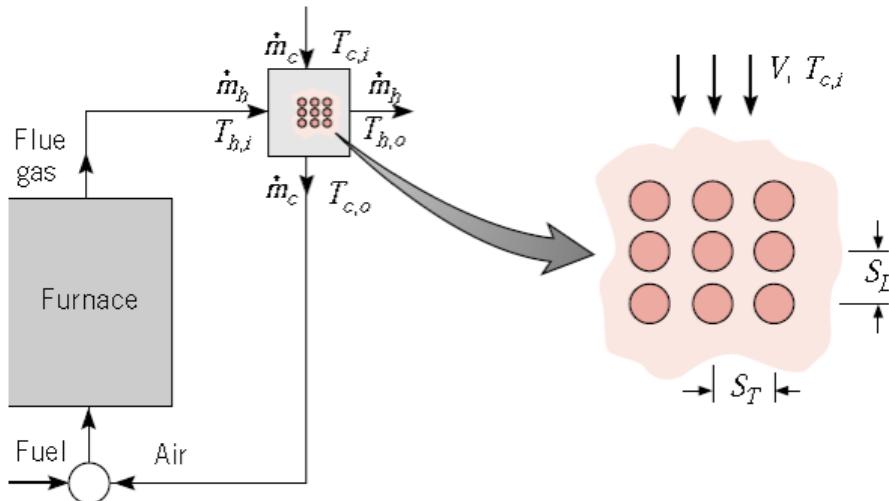
A recuperator is a heat exchanger that heats the air used in a combustion process by extracting energy from the products of combustion (the flue gas). Consider using a single-pass cross-flow heat exchanger as a recuperator.

Eighty silicon carbide ceramic tubes ( $k = 20W/mK$ ) of inner and outer diameters equal to  $55mm$  and  $80mm$  respectively, and of length  $L = 1.4m$  are arranged as an aligned tube bank of longitudinal and transverse pitches  $S_L = 100mm$  and  $S_T = 120mm$ , respectively. Cold air is in cross flow over the tube bank with upstream conditions of  $V = 1m/s$  and  $T_{c,i} = 300K$ , while hot-flue gases of inlet temperature  $T_{h,i} = 1400K$  pass through the tubes. The tube outer surface is clean while the inner surface is characterized by a fouling factor of  $R_f^i = 0.0002m^2K/W$ . The air and flue gas flow rates are  $\dot{m}_c = 1kg/s$  and  $\dot{m}_h = 1.05kg/s$ , respectively.

Use the following assumptions:

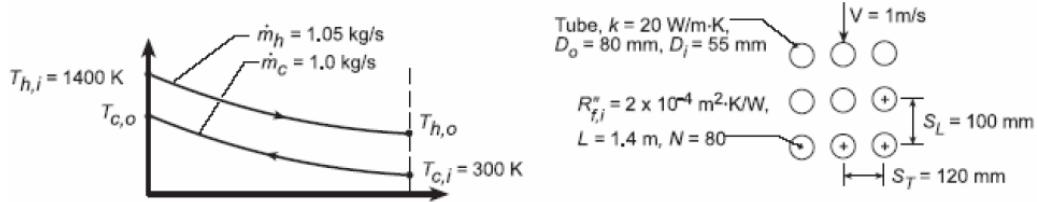
- evaluate all required air properties at  $1atm$  and  $300K$
- assume the flue gas to have the properties of air at  $1atm$  and  $1400K$
- assume the tube wall temperature to be at  $800K$  for the purpose of treating the effect of variable properties on convection heat transfer.

If there is a 1% fuel savings associated with each  $10^\circ C$  increase in temperature of the combustion air  $T_{c,o}$  above  $300K$ , what is the percentage fuel savings for the prescribed conditions? (See the exercise in series 10 for your reference).



**Solutions**  $T_{c,o} = 594K$ ,  $\Delta T_c = 294K$ , fuel savings = 29.4%

## Solution



**Assumptions:** (1) Negligible heat loss to surroundings, (2) Air properties are those of atmospheric air at 300 K, (3) Gas properties are those of atmospheric air at 1400 K, (4) Tube wall temperature may be approximated as 800 K from treating variable property effects.

**Properties:** Table A-4, Air (1 atm, T = 300K) :  $\nu = 15.89 \cdot 10^{-6} \text{ m/s}$ ,  $c_p = 1007 \text{ J/kg·K}$ ,  $k = 0.0263 \text{ W/m·K}$ ,  $\text{Pr} = 0.707$ ; (T = 1400K) :  $\mu = 530 \cdot 10^{-7} \text{ kg/s·m}$ ,  $c_p = 1207 \text{ J/kg}$ ,  $k = 0.091 \text{ W/m·K}$ ,  $\text{Pr} = 0.703$ ; (T = 800K) :  $\text{Pr} = 0.709$

### Analysis:

To solve this problem we need to find the outlet temperature of the cold gas flow in the heat exchanger. Because this is a cross-flow heat exchanger, we have to use the  $\varepsilon - NTU$  method to determine the overall heat transfer rate of the heat exchanger and from this the  $T_{c,o}$ .

We thus want to determine the NTU value of the exchanger and from this we will estimate the effectiveness and the actual heat transfer rate. Hence, we must first evaluate the overall heat transfer coefficient and  $C_{min}$  of our system.

With capacity rates of  $C_c = \dot{m}_c c_{p,c} = 1 \text{ kg/s} \times 1007 \text{ J/kg·K} = 1007 \text{ W/K} = C_{min}$  and  $C_h = \dot{m}_h c_{p,h} = 1.05 \text{ kg/s} \times 1207 \text{ J/kg·K} = 1267 \text{ W/K} = C_{max}$ ,  $C_{min}/C_{max} = 0.795$ . The overall coefficient is

$$\frac{1}{UA} = \frac{1}{h_i A_i} + \frac{R_{f,i}''}{A_i} + \frac{\ln(D_o/D_i)}{(2\pi k L)N} + \frac{1}{h_o A_o}$$

For flow through a single tube (consider that the total mass flow rate is flowing around all the 80 tubes of the recuperator), we calculate  $Re_D$  as follows:

$$Re_D = \frac{4\dot{m}_h}{N\pi D_i \mu} = \frac{4 \times 1.05 \text{ kg/s}}{80\pi(0.055 \text{ m})530 \cdot 10^{-7} \text{ kg/s·m}} = 5733 > 2300$$

Therefore the flow is turbulent. To determine the convection coefficient for internal forced convection under fully developed and turbulent flow conditions we use the Gnielinski correlation,

$$Nu_D = \frac{(f/8)(Re_D - 1000)\Pr}{1 + 12.7(f/8)^{1/2}(\Pr^{2/3} - 1)} = 18.8$$

where  $f = (0.79 \ln Re_D - 1.64)^{-2} = 0.0370$

$$h_i = Nu_D k / D_i = 18.8(0.091 \text{ W/mK}) / 0.055 \text{ m} = 31.1 \text{ W/m}^2 \cdot \text{K}$$

For the external flow over the tube bank, we need to determine the maximum velocity of the flow and then  $Re_{D,max}$ . We note that we have to distinguish between the staggered and aligned tube arrangement. In this case we have aligned arrangement and we calculate:

$$V_{\max} = [S_T / (S_T - D_O)] V = [0.12m / (0.12 - 0.08)m] 1m/s = 3m/s$$

$$Re_{D,\max} = \frac{V_{\max} D_0}{\nu} = \frac{3m/s \cdot 0.08m}{15.89 \cdot 10^{-6} m^2/s} = 15,100$$

We then use the Zukauskas correlation for a tube bank, as the  $Re_{D,\max}$  is within its validity range:

$$\overline{Nu}_D = C(Re_{D,\max})^m (Pr)^{0.36} (Pr/Pr_s)^{1/4}$$

For  $Re_{D,\max} = 10^3 - 2 \cdot 10^5$  and aligned tube banks we have:  $C = 0.27$  and  $m = 0.63$ . Therefore:

$$\overline{Nu}_D = 0.27(15,100)^{0.63}(0.707)^{0.36}(0.707/0.709)^{1/4} = 102.3$$

$$\overline{h}_o = \overline{Nu}_D (k/D_o) = 102.3(0.0263W/mK)/0.08m = 33.6W/m^2 \cdot K$$

Hence, based on the inner surface, the *overall coefficient* is

$$\frac{1}{U_i} = \frac{1}{h_i} + R''_{f,i} + \frac{D_i \ln(D_O/D_i)}{2k} + \frac{D_i}{D_0 h_o}$$

$$\frac{1}{U_i} = \left( 0.0322 + 0.0002 + \frac{0.055 \ln(0.08/0.055)}{20} + \frac{0.055}{0.08 \times 33.6} \right) m^2 \cdot K/W$$

$$U_i = [0.0322 + 0.0002 + 0.001 + 0.0205m^2 \cdot K/W]^{-1} = 18.6W/m^2 \cdot K$$

Hence:

$$(UA)_i = U_i N \pi D_i L = 18.6W/m^2 \cdot K \times 80\pi(0.055m)1.4m = 360W/K$$

The number of transfer units is then

$$NTU = UA/C_{\min} = 360W/K / 1007W/K = 0.357,$$

We also calculate:

$$C_{\text{mixed}} / C_{\text{unmixed}} = C_c / C_h = C_{\min} / C_{\max} = 0.795,$$

We can now use Table 11.13 and Eq. 11.34a to calculate  $\varepsilon$ :

$$\varepsilon = 1 - \exp(-C_r^{-1} \{ 1 - \exp[-C_r \cdot NTU] \}) = 0.267$$

Finally we can determine:

$$Q_{\max} = C_{\min} (T_{h,i} - T_{c,i}) = 1007W/K(1100K) = 1.11 \cdot 10^6 W$$

$$Q = \varepsilon Q_{\max} = 0.267 \times 1.11 \cdot 10^6 W = 295,800W$$

$$T_{c,o} = T_{c,i} + Q/C_{\min} = 300K + (295,800W/1007W/K) = 594K$$

Hence,

$$\% \text{ fuel savings} \equiv FS = (\Delta T_c / 10K) \times 1\% = (294K / 10K) \times 1\% = 29.4\%$$

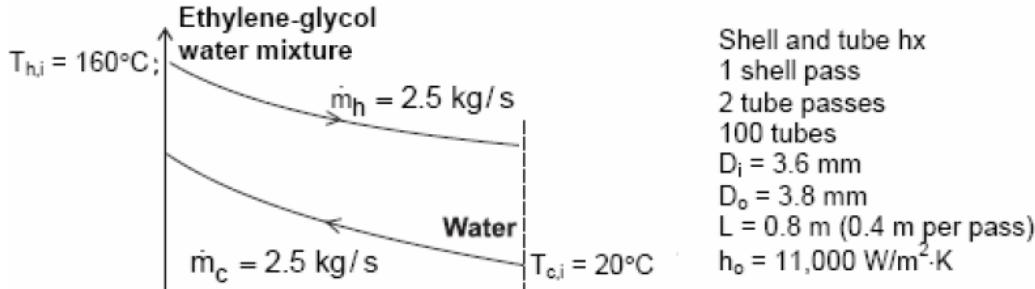
## Exercise 11.4 FOR REVISION

A shell-tube heat exchanger consisting of one shell pass and two tube passes is used to transfer heat from an ethylene glycol-water solution (shell side) supplied from a rooftop solar collector to pure water (tube side) used for household purposes. The tubes are of inner and outer diameters  $D_i = 3.6\text{mm}$  and  $D_o = 3.8\text{mm}$ , respectively. Each of the 100 tubes is  $0.8\text{m}$  long ( $0.4\text{m}$  per pass), and the heat transfer coefficient associated with the ethylene glycol-water mixture is  $h_o = 11000\text{W/m}^2\text{K}$ .

- For pure copper tubes, calculate the heat transfer rate from the ethylene glycol-water solution ( $\dot{m} = 2.5\text{kg/s}$ ,  $T_{h,i} = 80^\circ\text{C}$ ) to the pure water ( $\dot{m} = 2.5\text{kg/s}$ ,  $T_{c,i} = 20^\circ\text{C}$ ). Determine the outlet temperatures of both streams of fluid. The density and specific heat of the ethylene glycol-water mixture are  $1040\text{kg/m}^3$  and  $3660\text{J/kgK}$ , respectively.
- It is proposed to replace the copper tube bundle with a bundle composed of high-temperature nylon tubes of the same diameter and tube wall thickness. The nylon is characterized by a thermal conductivity of  $k_n = 0.31\text{W/mK}$ . Determine the tube length required to transfer the same amount of energy as in part (a).

### Solutions

- $UA = 5522\text{W/K}$ ,  $Q = 204120\text{W}$ ,  $T_{h,o} \approx 57^\circ\text{C}$ ,  $T_{c,o} \approx 43^\circ\text{C}$
- $L = 2.33\text{m}$

**Solution**

**Assumptions:** (1) Negligible heat loss to the surroundings, (2) Constant properties, (3) Fully developed water flow in tubes.

**Properties:** Table A-6, water ( $T \approx 300\text{K}$ ) :  $k = 0.613 \text{ W/m} \cdot \text{K}$ ,  $c_p = 4179 \text{ J/kg} \cdot \text{K}$ ,  $\mu = 855 \cdot 10^{-6} \text{ N} \cdot \text{s/m}^2$ ,  $\text{Pr} = 5.83$ . Ethylene-glycol water mixture (given):  $\rho = 1040 \text{ kg/m}^3$ ,  $c_p = 3660 \text{ J/kg} \cdot \text{K}$ . Copper ( $T \approx 300\text{K}$ ) :  $k_c = 401 \text{ W/m} \cdot \text{K}$ . Nylon (given):  $k_n = 0.31 \text{ W/m} \cdot \text{K}$

**Analysis:** (a) We need to determine the overall heat transfer in the heat exchanger for which all of the physical dimensions are given. Because it is a shell-tube HE we will use the  $\varepsilon$ -NTU method to determine  $Q$  once we have calculated  $Q_{max}$ . Hence we first need to determine NTU and  $C_r$  to then calculate the effectiveness. We remember that  $NTU = UA/C_{min}$  therefore we start by calculating all of these parameters.

We begin by finding the overall heat transfer coefficient and we notice that we will have to account for convection inside and outside the tubes as well as conduction along the tube wall:

$$\frac{1}{UA} = \left[ \frac{1}{h_i \pi D_i L N} + \frac{\ln(D_o/D_i)}{2 \pi k_c L N} + \frac{1}{h_o \pi D_o L N} \right]$$

where  $L$  is the length of each tube and  $N$  the total number of tubes.

We know all the physical properties except for the convection coefficient inside the tubes. We recognize we are dealing with internal forced convection and we use  $Re$  to determine the flow condition:

$$Re_D = \frac{4\dot{m}_1}{\pi D_i \mu_c} = \frac{4 \times 2.5 \text{ kg/s} / 100}{\pi \times 0.0036 \text{ m} \times 855 \cdot 10^{-6} \text{ N} \cdot \text{s/m}^2} = 1.03 \cdot 10^4 > 2300$$

Hence the flow is turbulent and we can use the Dittus-Boettler correlation,

$$h_c = (k/D_i) 0.023 Re_D^{4/5} Pr^{0.4} = (0.613 \text{ W/m} \cdot \text{K} / 0.0036 \text{ m}) 0.023 (1.03 \cdot 10^4)^{4/5} (5.83)^{0.4} = 1.29 \cdot 10^4 \text{ W/m}^2 \cdot \text{K}$$

We can now calculate:

$$UA = [6.85 \cdot 10^{-3} + 2.15 \cdot 10^{-5} + 7.62 \cdot 10^{-3}]^{-1} \text{ W/m} \cdot \text{K} \times 0.8 \text{ m} \times 100 = 5522 \text{ W/K}$$

We now calculate:

$$C_h = 2.5 \text{ kg/s} \times 3660 \text{ J/kg} \cdot \text{K} = 9150 \text{ W/K} = C_{min}$$

$$C_c = 2.5 \text{ kg/s} \times 4179 \text{ J/kg} \cdot \text{K} = 10,450 \text{ W/K} = C_{max}$$

$$C_r = \frac{C_{min}}{C_{max}} = 0.876$$

$$NTU = UA/C_{min} = 0.603$$

Then from table 11.13 and Eq. 11.30a we get:

$$\varepsilon = 2 \left\{ 1 + C_r + (1 + C_r^2)^{1/2} \times \frac{1 + \exp \left[ -NTU (1 + C_r^2)^{1/2} \right]}{1 - \exp \left[ -NTU (1 + C_r^2)^{1/2} \right]} \right\}^{-1} = 0.373$$

We can then calculate the overall heat transfer rate as:

$$Q = \varepsilon Q_{max} = \varepsilon C_{min} (T_{h,i} - T_{c,i}) = 0.373 \times 9150(80 - 20) = 204777W \approx 205000$$

and from this:

$$T_{h,o} = T_{h,i} - Q/C_h = 80^\circ\text{C} - 205,000\text{W}/9150\text{W/K} = 57.6^\circ\text{C}$$

$$T_{c,o} = T_{c,i} + Q/C_c = 20^\circ\text{C} + 205,000\text{W}/10,450\text{W/K} = 39.6^\circ\text{C}$$

Note: once you have calculated all the temperatures you can double check your numbers to verify that there are no obvious errors (e.g.  $T_{c,o} < T_{c,i}$  or similar relationships that must be true)

(b) In order to maintain the same heat rate, we must have the same effectiveness, which means that NTU and UA must be the same as in part (a). When the tubes are nylon, we can recalculate UA from,

$$\begin{aligned} UA &= \left[ \frac{1}{h_i \pi D_i} + \frac{\ln(D_o/D_i)}{2\pi k_n} + \frac{1}{h_o \pi D_o} \right]^{-1} LN \\ &= [6.85 \cdot 10^{-3} + 2.78 \cdot 10^{-2} + 7.62 \cdot 10^{-3}]^{-1} \text{W/m} \cdot \text{K} \times 100 \times L(\text{m}) = 5522\text{W/K} \end{aligned}$$

Solving for L,

$$L = 2.33\text{m}$$

**Comments:** (1) The nylon tube bundle is significantly larger due to nylon's low thermal conductivity relative to the copper. Based upon a nylon density of 1150 kg/m<sup>3</sup>, the masses of the two tube bundles are 0.83 kg and 0.39 kg for the copper and nylon, respectively. The cost difference between the two raw materials is negligible. However, the nylon heat exchanger may ultimately be less expensive when assembly costs are considered. Time-consuming and expensive brazing, joining and welding processes associated with construction of the copper heat exchanger are avoided with use of materials such as nylon. (2) With L/D ≈ 200, the fully developed assumption is excellent. (3) The properties of the cold stream should have calculated at the mean temperature of 304 K, very close to the assumed value.