Exercise 9.1

Saturated steam at $0.1bar$ condenses on the outside of a vertical stack of 10 brass tubes having inner and outer diameters of 16.5 and 19 mm respectively. The surface temperature of the tubes is kept $7.3K$ lower than the saturation temperature of the vapor by flowing water with a mean temperature of $30°C$ inside the tubes. Calculate:

a) the average external convection coefficient due to condensation on the bank of tubes

b) Assuming that the convection coefficient for water flowing inside the tube is $5200W/m^2K$, calculate the overall heat transfer coefficient for a single tube.[**Brass properties**: thermal conductivity - $k_B = 110W/mK$].

c) Under the previous assumptions, calculate the steam condensation rate per unit length of one tube

d) In an effort to increase condensation rate, an engineer proposes to apply $t = 100\mu m$ thick Teflon coating to the exterior surface of the brass tube to promote drop-wise condensation. Using the following correlation for the average external convection coefficient due to dropwise condensation:

$$\overline{h}_{dc} = 51104 + 2044T_{sat}[°C]$$

which is valid for $22°C < T_{sat} < 100°C$, estimate the new condensation convection coefficient and the steam condensation rate per unit length of the tube after the application of the coating. Comment on the proposed scheme's effect on the condensation rate. [**Teflon properties** thermal conductivity - $k_T = 0.35W/mK$.]

Solutions:

a) $\overline{h}_D \approx 6800W/m^2K$

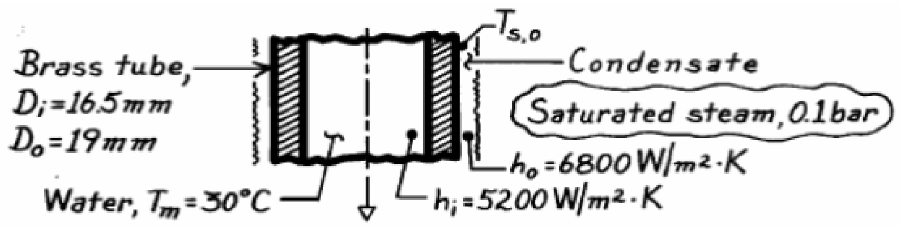
b) $U_{out} \approx 2627W/m^2K$

c) $\dot{m}' \approx 1.11x10^{-3}kg/sm$

d) $\dot{m}' \approx 7.56x10^{-4}kg/sm$

Solution

a)-c) Brass Tube



Assumptions: steady-state conditions.

Properties: *Table A-6*, Water, vapor ($0.1bar$): $T_{sat} \approx 320K$, $h_{fg} = 2390 \times 10^3 J/kg$. Given the small $\Delta T = T_{sat} - T_s = 7K$, the physical properties for the vapor/liquid can be estimated at the T_{sat} directly. You can also interpolate all of the properties at the average value $T_f = (T_{sat} + T_s)/2$ to get more accurate values.

Analysis: a) We are dealing with condensation on a bank of tubes with $N = 10$. Also $\Delta T = T_{sat} - T_s = 7K$. We therefore have that the average convection coefficient due to condensation outside of each tube is:

$$\overline{h}_D = 0.729[\frac{g\rho_l(\rho_l - \rho_v)k_l^3 h'_{fg}}{N\mu_l \Delta T D}]^{1/4}$$

Substituting all of the physical properties at T_{sat} from Table $A.6$ we get: $\overline{h}_D \approx 6800 W/m^2K$

Note that in the bank of tubes every single tube has a different external convection coefficient. However we cannot calculate the "local" convection coefficient on each single tube. Instead, we have determined the **average** external convection coefficient in the entire bank of tubes and we assume this single value for each and every one of the tubes. We cannot use the correlation for a single tube because the presence of the other tubes influences the external convection around each of them.

b) and c) The condensation rate per unit length is written as:

$$\dot{m}' = q'/h'_{fg}$$

where the heat rate is determined using the overall heat transfer coefficient with respect to the outside area:

$$q' = U_{out}\pi D_o(T_{sat} - T_m)$$

There are no fins in the system so the thermal resistances that we have to account for are external convection due to condensation, conduction and internal convection.

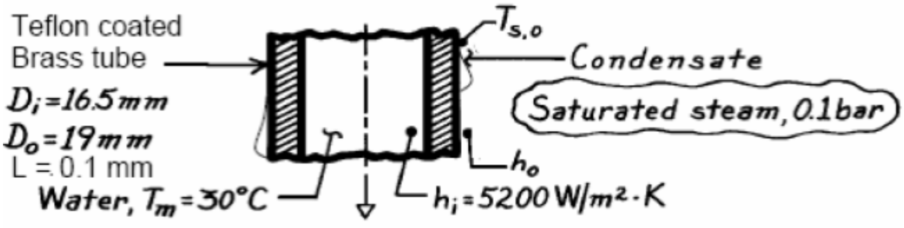
$$\begin{aligned}
U_{out} &= \left[\frac{1}{h_o} + \frac{D_o/2}{k}\ln\frac{D_o}{D_i} + \frac{D_o}{D_i}\frac{1}{h_i}\right]^{-1} \\
&= \left[\frac{1}{6800W/m^2K} + \frac{0.0095m}{110W/mK}\ln\frac{19}{16.5} + \frac{19}{16.5}\frac{1}{5200W/m^2K}\right]^{-1} \\
&= \left[147.1 \times 10^{-6} + 12.18 \times 10^{-6} + 192.3 \times 10^{-6}\right]^{-1} W/m^2K = 2627 W/m^2K
\end{aligned}$$

Combining the equations and substituting numerical values:

$$\begin{aligned}
\dot{m}' &= U_{out}\pi D_o(T_{sat} - T_m)/h'_{fg} \\
&= 2627 W/m^2K\pi(0.019m)(320 - 303)K/2410 \times 10^3 J/kg = 1.11 \times 10^{-3} kg/s
\end{aligned}$$

Comments: From the evaluation of the overall heat transfer coefficient, we note that the thermal resistance of the brass tube is not negligible.

d) Teflon coated tube



Assumptions: (1) Dropwise condensation, (2) correlations for a copper surface can be applied to Teflon, (3) negligible effect of noncondensable vapors.

Analysis: The condensation rate per unit length is written as:

$$\dot{m}' = q'/h'_{fg}$$

where the heat rate per unit length can be determined using the overall heat transfer coefficient:

$$q' = UP(T_{sat} - T_m)$$

where P is the perimeter.

We have:

$$UP = \left[\frac{1}{h_o\pi(D_o + 2L)} + \frac{\ln[(D_o + 2L)/D_o]}{2\pi k_t} + \frac{\ln(D_o/D_i)}{2\pi k_b} + \frac{1}{h_i\pi D_i}\right]^{-1}$$

In this case we do not know the outer convection coefficient due to condensation. Therefore we need to find a correlation to estimate it. For drop-wise condensation on a tube, the heat transfer coefficient, $h_o = \overline{h}_{dc}$, can be calculated as follows:

$$\overline{h}_{dc} = 51'104 + 2044T_{sat}(^\circ C) = 51'104 + 2044(319.9 - 273) = 144'900W/m^2K$$

Thus,

$$UP = \left[\frac{1}{144'900W/m^2K \times \pi(19.2 \times 10^{-3}m)} + \frac{\ln(19.2/19)}{2\pi \times 0.35W/mK} + \frac{\ln(19/16.5)}{2\pi \times 110W/mK} + \frac{1}{5200W/m^2K \times \pi(16.5 \times 10^{-3}m)}\right]^{-1}$$

$$UP \approx 113.77W/mK$$

Therefore we can calculate:

$$\dot{m}' = UP(T_{sat} - T_m)/h^{'}_{fg} = 114W/mK(318.9 - 303)K/2393 \cdot 10^3 J/kgK = 7.56 \cdot 10^{-4} kg/s$$

We thus observe that the Teflon coating induces a 21-fold increase in the condensation convection coefficient. However, the condensation rate decreases by 25%. This is because of the significant conduction resistance [posed by the thin teflon layer.

Comments:

a) Since the outer convection resistance is small relative to the sum of the remaining resistances $T_{s,o}$ T_{sat} and therefore $h^{'}_{fg} \approx h_{fg}$

b) In addition to the conduction resistance, a contact resistance would exist at the teflon-brass interface as well as constriction resistances at the droplet-teflon interfaces, further reducing the condensation rate.

Exercise 9.2

Consider a container exposed to saturated vapor T_{sat}, having a cold bottom surface $T_s < T_{sat}$ and with insulated sidewalls (see figure). Assuming a linear temperature distribution for the liquid, perform a surface energy balance on the liquid/vapor interface to obtain the following expression for the growth rate of the liquid layer:

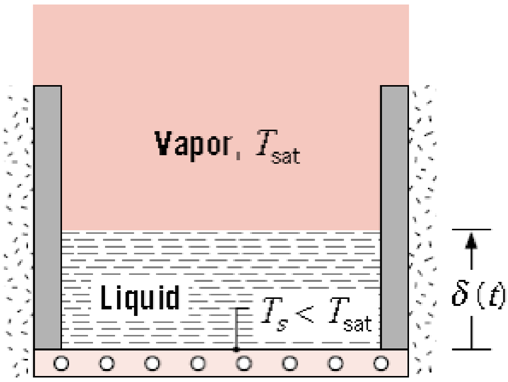
$$\delta(t) = \left[\frac{2k_l(T_{sat} - T_s)}{\rho_l h_{fg}} t\right]^{1/2}$$

Based on the above equation:

a) Calculate the thickness of the liquid layer ($\delta(1hr)$) and the total condensate mass (M) formed in $1h$ for a $200mm^2$ bottom surface maintained at $80°C$ and exposed to saturated steam at $1atm$.

b) Compare this result with the condensate formed by a *vertical* plate of the same dimensions for the same period of time. Can you explain why there is a difference between the two situations?

Hint: assume that the mass of condensate is uniformly distributed across the entire surface at all times and use this information to determine the change in the condensate layer thickness with time.

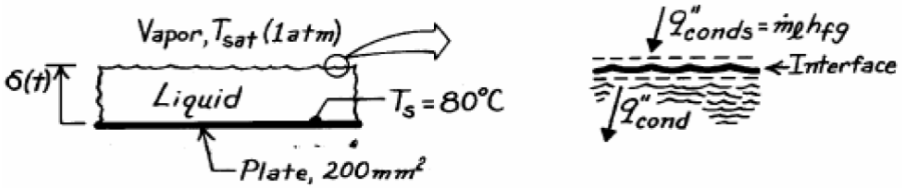


Solutions:

a) $\delta(1hr) = 6.69mm$, $M = 1.29 \cdot 10^{-3}kg$

b) $M_{vp} = 9.7 \cdot 10^{-2}kg$

Solution



Assumptions: (1) Side wall effects are negligible, (2) Vapor-liquid interface is at T_{sat}, (3) Temperature distribution in liquid in linear, (4) Constant properties.

Properties: *Table A-6,* Saturated vapor (p=1.0133 bar): $T_{sat} = 100°C, \rho_V = 0.596 kg/m^3, h_{fg} = 2257 kJ/kg$ *Table A-6,* Saturated liquid ($T_f = 90°C = 363K$): $\rho_\ell = 965 kg/m^3, \mu_\ell = 313 \cdot 10^{-6} N \cdot s/m^2, k_\ell = 0.676 W/m \cdot K, c_{p,\ell} = 4207 J/kg \cdot K, v_\ell = \mu_\ell/\rho_\ell = 3.24 \cdot 10^{-7} m^2/s$

Analysis: Perform a surface energy balance on the interface (see above) recognizing that $\dot{m}_\ell/A = \rho_\ell d\delta/dt$ from an overall mass rate balance on the liquid to obtain

$$\dot{E}''_{in} - \dot{E}''_{out} = q''_{conds} - q''_{cond} = \frac{\dot{m}}{A} h_{fg} - k_\ell \frac{T_{sat} - T_s}{\delta} = \rho_\ell \frac{d\delta}{dt} h_{fg} - k_\ell \frac{T_{sat} - T_s}{\delta} = 0 \quad (0.1)$$

where q''_{conds} is the condensation heat flux and the q''_{cond} is the conduction heat flux into the liquid layer of the thickness δ with linear temperature distribution Eq. (1) can be rewritten as

$$\rho_\ell h_{fg} \frac{d\delta}{dt} = k_\ell \frac{T_{Sat} - T_S}{\delta}$$

Separate variables and integrate with limits shown to obtain the liquid layer growth rate,

$$\int_0^\delta \delta d\delta = \int_0^t \frac{k_\ell (T_{sat} - T_s)}{\rho_\ell h_{fg}} dt \text{ or } \quad \delta = \left[\frac{2k_\ell (T_{sat} - T_s)}{\rho_\ell h_{fg}} t\right]^{1/2} \quad (0.2)$$

For the prescribed conditions, the liquid layer thickness and condensate formed in one hour are

$$\delta(1hr) = \left[2 \times 0.676 \frac{W}{m \cdot K} (100 - 80)°C \times 3600s / 965 \frac{kg}{m^3} \times 2257 \cdot 10^3 \frac{J}{kg}\right]^{1/2} = 6.69mm$$

$$M(1hr) = \rho_\ell A \delta = 965 kg/m^3 \times 200 \cdot 10^{-6} m^2 \times 6.69 \cdot 10^{-3} m = 1.29 \cdot 10^{-3} kg$$

The condensate formed on a vertical plane with the same conditions follows is:

$$M_{vp} = \dot{m} \cdot t = \frac{Q}{h'_{fg}} t = \bar{h}_L A (T_{sat} - T_s) \cdot t / h'_{fg}$$

where h'_{fg} follows from:

$$h'_{fg} = h_{fg} (1 + 0.68 c_{p,\ell} (T_{sat} - T_s)/h_{fg})$$

$$h'_{fg} = 2257 \cdot 10^3 J/kg \left(1 + 0.68 \times 4207 \frac{J}{kg \cdot K} (100 - 80)°C / 2257 \cdot 10^3 J/kg\right) = 2314 kJ/kg$$

To determine $\overline{h}_L$, instead, we first need to calculate Re_δ with the three possible correlations for laminar, laminare-wavy and turbulent flow. Let's try the correlation for the laminar case :

$$
\begin{aligned}
Re_\delta &= 3.78\left[\frac{k_\ell L\left(T_{sat}-T_s\right)}{\mu_\ell h'_{fg}\left(v_\ell^2/g\right)^{1/3}}\right]^{3/4} \\
&= 3.78\left[\frac{0.676W/mK\times 0.0141m(100-80)^\circ C}{313\times 10^{-6}N\cdot s/m^2\times 2314\cdot 10^3 J/kg\times\left[(3.24\cdot 10^{-7}m^2/s)^2/9.8m/s^2\right]^{1/3}}\right]^{3/4} = 24.3.
\end{aligned}
$$

Since $Re_\delta < 30$,this correlation is for the correct Re_δ range. Then we can substitute it into the expression for $\overline{h}_L$:

$$
\overline{h}_L = \frac{Re_\delta\,\mu_\ell h'_{fg}}{4L\left(T_{sat}-T_s\right)} = \frac{24.3\times 313\cdot 10^{-6}N\cdot s/m^2\times 2314\cdot 10^3 J/kg}{4\times 0.0141m\times(100-80)^\circ C} = 15,580W/m^2\cdot K
$$

Hence,

$$
M_{vp} = 15,580W/m^2\cdot K\times 200\cdot 10^{-6}m^2(100-80)^\circ C\times 3600s/2314\cdot 10^3 J/kg
$$

$$
M_{vp} = 9.7\cdot 10^{-2}kg
$$

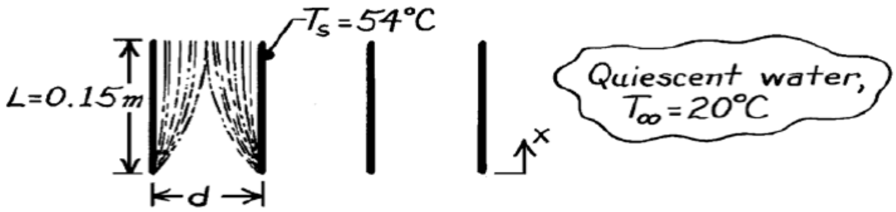
Comments: Note that the condensate formed by the vertical plate is almost two orders of magnitude larger. For the vertical plate the rate of the condensate formation is constant. For the container bottom surface, the rate decreases with increasing time since the conduction resistance increases as the liquid layer thickness increases.

Exercise 9.3

A number of thin plates are to be cooled by vertically suspending them in a water bath at a temperature of $20°C$. If the plates are initially at $54°C$ and are $0.15m$ long, what minimum spacing would prevent interference between their free convection boundary layers?

Solution $d = 12.6mm$

Solution



Assumptions: (a) water in bath is quiescent, (b) plates are at uniform temperature.

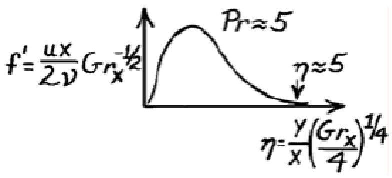
Properties: Water ($T_f = (T_s + T_\infty)/2 = (54 + 20)°C/2 = 310K$): $\rho = 1/v_f = 993.05 kg/m^3$, $\mu = 695 \times 10^{-6} Ns/m^2$, $\nu = \mu/\rho = 6.998 \times 10^{-7} m^2/s$, $\text{Pr} = 4.62$, $\beta = 361.9 \times 10^{-6} K^{-1}$.

Analysis: The minimum separation distance will be twice the thickness of the boundary layer at the trailing edge where $x = 0.15m$. Assuming laminar, free convection boundary layer conditions, the similarity parameter, η is given by:

$$\eta = \frac{y}{x}(\text{Gr}_x/4)^{1/4}$$

where y is measured normal to the plate.

According to the figure on the right, the boundary layer thickness occurs at a value of $\eta \approx 5$.
It follows then that:



$$y_{bl} = \eta x(\text{Gr}_x/4)^{-1/4}$$

where $\text{Gr}_x = \frac{g\beta(T_s - T_\infty)x^3}{\nu^2} = \frac{9.8m/s^2 \times 361.9 \times 10^{-6}K^{-1}(54 - 20)K \times (0.15m)^3}{(6.998 \times 10^{-7}m^2/s)^2} = 8.310 \times 10^8$.

Hence,

$$y_{bl} = 5 \times 0.15m(8.310 \times 10^8/4)^{-1/4} = 6.247 \times 10^{-3}m = 6.3mm$$

.

and the minimum separation is:

$$d = 2y_{bl} = 2 \times 6.3mm = 12.6mm$$

Comments: The critical Grashof number for the onset of turbulent conditions in the boundary layer is $\text{Gr}_{x,c}\text{Pr} \approx 10^9$. For the conditions above, $\text{Gr}_x\text{Pr} = 8.31 \times 10^8 \times 4.62 = 3.8 \times 10^9$. We conclude that the boundary layer is indeed turbulent at $x = 0.15m$ and our calculation is only an estimate which is likely to be low. Therefore, the plate separation should be greater than $12.6mm$.

Exercise 9.4 [DIFFICULT] FOR REVISION

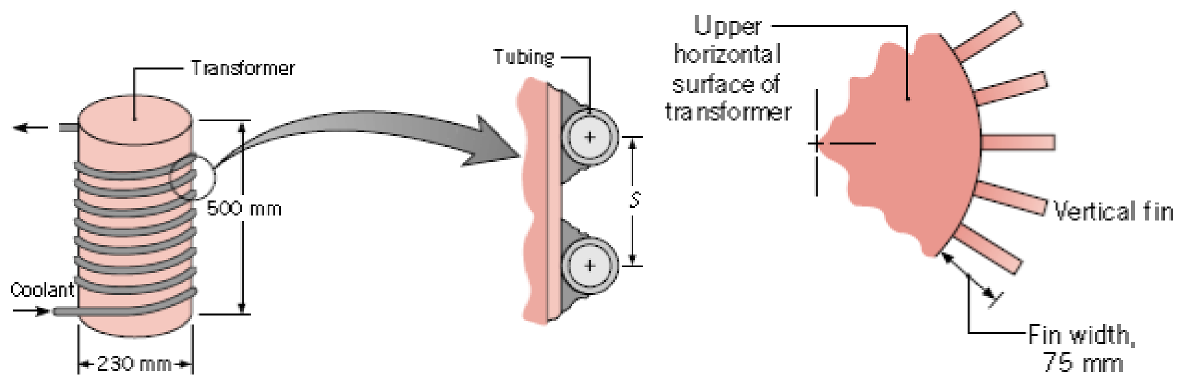
An electrical power transformer of diameter $230mm$ and height $500mm$ dissipates $1000W$. It is desired to maintain its surface temperature at $47°C$ and different solutions are considered:

a) Ethylene glycol at $24°C$ is supplied through a thin walled tubing of $20mm$ diameter welded to the lateral surface of the transformer (see figure below). All the heat dissipated by the transformer is assumed to be transferred to the ethylene glycol. Assuming the maximum allowable temperature rise of the coolant to be $6°C$, determine the required coolant flow rate, the total length of the tubing and the coil pitch S between turns of the tubing.

b) On the other hand, cooling of the transformer by free convection and radiation is explored. Assuming that the surface has an emissivity $\epsilon = 0.8$, determine how much power could be removed by free convection and radiation from the lateral and upper horizontal surfaces when the ambient temperature and the surroundings are at $27°C$.*Hint*: solve the free convection on each face choosing the appropriate correlations as if it were an infinite plate.

c) Finally, to improve free convection, vertical fins, $5mm$ thick, $75mm$ wide and $500mm$ long are welded to the lateral surface. Assuming that the fins have the same temperature of the transformer along their entire length (ideal fins), what is the heat removal rate by free convection if 30 such fins are attached?

d) Describe in words what you should do to solve the problem if the fins were not ideal and you wanted to use forced convection for the cooling. What advantages could you have?



Solutions

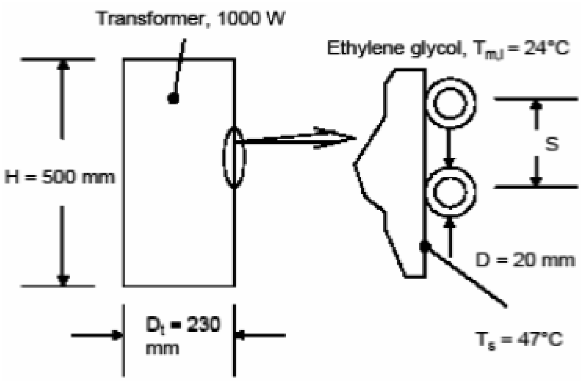
a) $\dot{m} = 6.9 \cdot 10^{-2} kg/s$, $L = 17.25m$, $S = 22.7mm$

b) $Q = 72.7W$

c) $Q = 483W$

Solution

Part a)



Assumptions: (1) constant properties, (2) incompressible liquid and negligible viscous dissipation, (3) steady-state conditions, (4) negligible tube wall thermal resistance, (5) fully-developed flow, (6) all heat dissipated by transformer is transferred to ethylene glycol.

Properties: Ethylene glycol ($\overline{T}_m = 300K$, assumed): $k = 0.252W/mK$, $c_p = 2415J/kgK$, $\mu_f = 1.57 \times 10^{-2}Ns/m^2$, $\mathrm{Pr} = 1151$.

Analysis: From an overall energy balance, the required flow rate is:

$$q = \dot{m}c_p(T_{m.o} - T_{m,i}) \quad \text{or} \quad \dot{m} = \frac{q}{c_p(T_{m.o} - T_{m,i})}$$

$$\dot{m} = \frac{1000W}{2415J/kg \times 6K} = 6.90 \times 10^{-2}kg/s$$

To determine the length of the tubing we observe that the heat is transferred to the coolant by forced internal convection in a circular tube. Furthermore, because the temperature of the transformer has to remain at $47C$ we observe that the heat transfer occurs under a constant surface temperature condition. Hence, the length of the tubing is determined using the following relation:

$$\frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \exp\left(\frac{-PL\overline{h}}{\dot{m}c_p}\right)$$

where $P = \pi D$.

If the average convection coefficient is known then it is immediate to determine the tubing length.

Because we are in forced convection we determine the Reynolds number as:

$$\mathrm{Re}_D = \frac{4\dot{m}}{\pi D\mu} = \frac{4 \times 6.90 \times 10^{-2kg/s}}{\pi \times 0.020m \times 1.57 \times 10^{-2}Ns/m^2} = 279.8 << 2300$$

hence we are in laminar flow in circular tubes. As we have constant surface temperature conditions, the Nusselt number is simply:

$$Nu_D = 3.66$$

Thus, we have:

$$\overline{h} = h = \mathrm{Nu}_D\frac{k}{D} = 3.66 \times 252 \times 10^{-3}W/mK/20 \times 10^{-3}m = 46.12W/m^2K$$

The length is then found using:

$$\frac{(47-30)^{\circ}C}{(47-24)^{\circ}C} = \exp\left[-\frac{\pi \times 0.02m \times 46.12W/m^2K \times L}{6.90 \times 10^{-2}kg/s \times 2415J/kgK}\right]$$

Solving the equation gives:

$$L = 17.25m$$

The number of turns of the tubing is $N = L/\pi D = 17.25m/\pi(0.25m) = 21.9$ and hence the spacing S is:

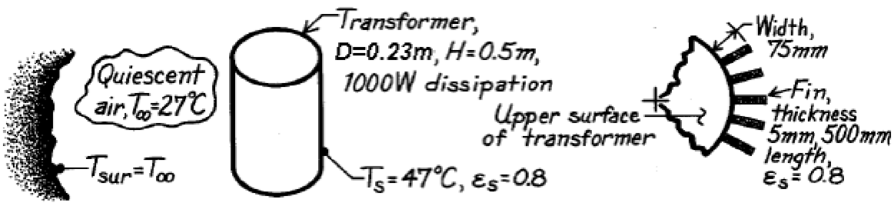
$$S = H/N = 500mm/22 = 22.7mm$$

Comments:

a) Coiling the tube results in a convective heat transfer coefficient that is $10.99/3.66 = 3$ times larger than the fully-developed value for a straight tube.

b) For a straight tube, the thermal entrance length is $x_{fd,t} = 0.05\mathrm{Re}_D\mathrm{Pr}D = 0.05 \times 279.8 \times 1151 \times 0.02m = 322m$. The flow will not be fully-developed, and care must be taken when using the predictions.

Part b)



Assumptions: (1) fins are isothermal at lateral surface temperature, T_s, (2) vertical fins and lateral surface behave as vertical plate, (3) transformer has isothermal surfaces and losses heat only on top and side.

Properties: Air($T_f = (27+47)^{\circ}C/2 = 310K$, $1atm$): $\nu = 16.90 \times 10^{-6}m^2/s$, $k = 27.0 \times 10^{-3}W/mK$, $\alpha = 23.98 \times 10^{-6}m^2/s$, $\mathrm{Pr} = 0.706$, $\beta = 1/T_f$.

Analysis: For the vertical lateral (lat) and top horizontal (top) surfaces, the heat loss by radiation and convection is:

$$Q = q_{lat} + q_{top} = (\overline{h}_{lat} + h_r)\pi DL(T_s - T_\infty) + (\overline{h}_{top} + h_r)(\pi D^2/4)(T_s - T_\infty)$$

where the linearized radiation coefficient is:

$$h_r = \epsilon\sigma(T_s + T_\infty)(T_s^2 + T_\infty^2) = 0.8 \times 5.67 \times 10^{-8}W/m^2K^4(320+300)K(320^2+300^2)K^2 = 5.41W/m^2K$$

To determine the free convection coefficient for the lateral and top surface we need to compute first the value of Ra adequate for that surface and then use the correct correlation for $\overline{Nu}$ in order to account for the orientation of the plate.

So we have:

- Lateral-vertical plate: the characteristic dimension is the height of the plate

$$\mathrm{Ra}_L = \frac{g\beta(T_s - T_\infty)H^3}{\nu\alpha} = \frac{9.8m/s^2(1/310K)(47-27)K(0.5m)^3}{16.90\times10^{-6}m^2/s \times 23.98\times10^{-6}m^2/s} = 1.950\times10^8$$

$$\overline{\mathrm{Nu}}_L = \left[0.825 + \frac{0.387\mathrm{Ra}_L^{1/6}}{\left(1+(0.492/\mathrm{Pr})^{9/16}\right)^{8/27}}\right]^2 = \left[0.825 + \frac{0.387(1.950\times10^8)^{1/6}}{\left(1+(0.492/0.706)^{9/16}\right)^{8/27}}\right]^2 = 74.5$$

$$\bar{h}_{lat} = \overline{\mathrm{Nu}}_L k/H = 74.5\times0.027W/mK/0.5m = 4.02W/m^2K$$

- Top-horizontal plate: the characteristic dimension is:

$$L_c = A_s/P = \frac{\pi D^2/4}{\pi D} = D/4 = 0.0575m$$

and we use the upper surface of a hot-plate correlation with:

$$\mathrm{Ra}_L = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu\alpha} = \frac{9.8m/s^2(1/310K)(47-27)K(0.0575m)^3}{16.90\times10^{-6}m^2/s \times 23.98\times10^{-6}m^2/s} = 2.97\times10^5$$

$$\overline{\mathrm{Nu}}_L = 0.54\mathrm{Ra}_L^{1/4} = 0.54(2.97\times10^5)^{1/4} = 12.6$$

$$\bar{h}_{top} = \overline{\mathrm{Nu}}_L k/L_c = 12.6\times0.027W/mK/0.0575m = 5.92W/m^2K$$

Hence, the heat loss by convection and radiation is:

$$Q = (4.02+5.41)W/m^2K(\pi\times0.23m\times0.50m)(47-27)K + (5.92+5.41)W/m^2K(\pi\times0.23^2m^2/4)(47-27)K$$
$$= (68.2+4.50)W = 72.7W$$

Part c)

Becasue we assume the fins to be isothermal with the base, the effect of adding the vertical fins in this case is to increase the area of the lateral surface to:

$$\begin{aligned} A_{wf} &= [\pi DH - 30(tH)] + 30\times2(wH) \\ &= \left[\pi 0.23m\times0.50m - 30(0.005\times0.500)m^2\right] + 30\times2(0.075\times0.500)m^2 \\ &= 2.536m^2 \end{aligned}$$

where t and w are the thickness and width of the fins, respectively. Hence, the heat loss is now:

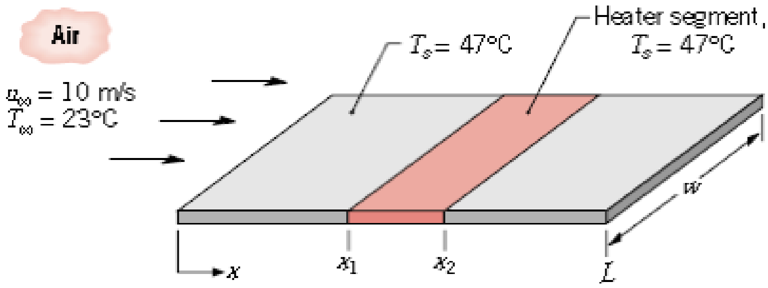
$$\begin{aligned} q &= q_{lat} + q_{top} \\ &= (\bar{h}_{lat} + h_r)A_{wf}(T_s - T_\infty) + q_{top} \\ &= (4.02+5.41)W/m^2\times2.536m^2\times20K + 4.50W = 483W \end{aligned}$$

Adding the fins to the lateral surface increases the heat loss by a factor of more than 6.

Comments: Since the fins are not likely to have 100% efficiency, our estimate is optimistic. Further, since thee fins see one another, as well as the lateral surface, the radiative heat loss is over preedicted.

Exercise 9.5 FOR REVISION

A highly polished aluminum plate of length $0.5m$ and width $0.2m$ is subjected to an air stream at a temperature of $23°C$ and a velocity of $10m/s$. Because of upstream conditions, the flow is turbulent over the entire length of the plate. A series of segmented, independently controlled heaters is attached to the lower side of the plate to maintain approximately isothermal conditions over the entire plate. The electrical heater covering the section between the positions $x_1 = 0.2m$ and $x_2 = 0.3m$ is shown in the schematic. Assuming that the bottom surface is thermally isolated and considering that the emissivity of highly polished aluminum is 0.03:

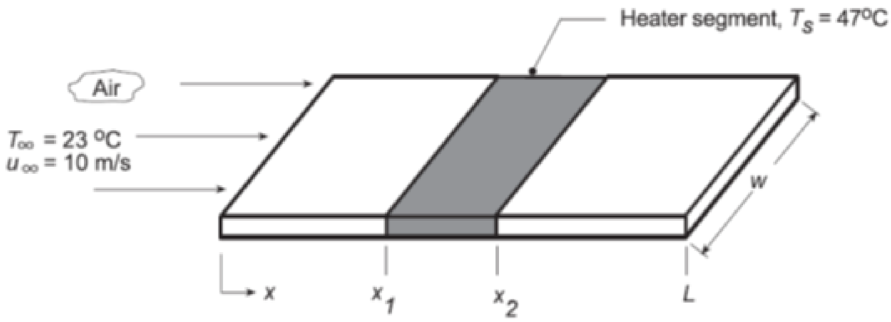


a) Estimate the electrical power that must be supplied to the designated heater segment to maintain the surface temperature at $T_s = 47°C$.

   Note: assume that the convection coefficient over the heater is the average value between the convection coefficient at x_1 and x_2.

b) If the blower that maintains the air stream velocity over the plate malfunctions, but the power to the heaters remains constant, estimate the surface temperature of the designated segment. Assume that the ambient air is extensive, quiescent and at $23°C$.

Solution



Assumptions: (1) Steady-state conditions, (2) Backside of plate is perfectly insulated, (3) Flow is turbulent over the entire length of the plate, part (a), (4) Ambient air is extensive, quiescent at 23°C for part (b).

Properties: *Table A-4*, Air $(T_f = (T_s + T_\infty)/2 = 308K) : \nu = 16.69 \cdot 10^6 \mathrm{m}^2/s,\ k = 0.02689 \mathrm{W/m} \cdot \mathrm{K},\ \alpha = 23.68 \cdot 10^6 \mathrm{m}^2/\mathrm{s},\ \mathrm{Pr} = 0.7059,\ \beta = 1/\mathrm{T_f}$; *Table A-12*, Aluminum, highly polished $\varepsilon = 0.03$.

Analysis: (a) The power required to maintain the segmented heater at the prescribed temperature T_s is: $(\mathrm{x_1} - \mathrm{x_2})$ is

$$P_\mathrm{e} = \overline{h}_{\mathrm{x1-x2}} (\mathrm{x_2} - \mathrm{x_1})\, \mathrm{w}\, (T_\mathrm{s} - T_\infty) \tag{0.3}$$

where $\overline{h}_{\mathrm{x1-x2}}$ is the average coefficient for the section between x_1 and x_2, and can be approximated as the average of the local values at $x_1 and x_2$,

$$\overline{\mathrm{h}}_{\mathrm{x1-x2}} = (\mathrm{h}(\mathrm{x_1}) + \mathrm{h}(\mathrm{x_2}))/2 \tag{0.4}$$

We observe that we are dealing with forced convection over an horizontal plate. The problem tells us that the flow is turbulent over the entire plate, therefore we use the correlation:

$$NU_x = 0.0296 Re_x^{4/5} Pr^{1/3}$$

where

$$\mathrm{Re_x} = \mathrm{u_\infty x}/\nu$$

$$\mathrm{Nu_{x1}} = 0.0296 \left(\frac{10\mathrm{m/s} \times 0.2\mathrm{m}}{16.69 \cdot 10^{-6} \mathrm{m}^2/\mathrm{s}}\right)^{4/5} (0.7059)^{1/3} = 304.6$$

$$\mathrm{h_{X1}} = \mathrm{Nu_{x1} k/x_1} = 304.6 \times 0.02689 \mathrm{W/m \cdot K/0.2m} = 40.9 \mathrm{W/m^2 \cdot K}$$
$$\mathrm{Nu_{x2}} = 421.3 \quad \mathrm{h_{x2}} = 37.8 \mathrm{W/m^2 \cdot K}$$

Hence, we obtain :

$$\overline{\mathrm{h}}_{\mathrm{X1-x2}} = (40.9 + 37.8) \mathrm{W/m^2 \cdot K/2} = 39.4 \mathrm{W/m^2 \cdot K}$$
$$\mathrm{P_e} = 39.4 \mathrm{W/m^2 \cdot K} (0.3 - 0.2)\mathrm{m} \times 0.2\mathrm{m} (47 - 23)°\mathrm{C} = 18.9\mathrm{W}$$

(b) If the blower malfunctions and there is no more airstream flow, the heater segment experiences free convection and radiation exchange with the surroundings

$$P_\mathrm{e} = \left[\overline{\mathrm{h}}_{\mathrm{cv}} (\mathrm{T_s} - \mathrm{T_\infty}) + \varepsilon\sigma \left(\mathrm{T_s^4} - \mathrm{T_{sur}^4}\right)\right] (\mathrm{x_2} - \mathrm{x_1})\, \mathrm{w} \tag{0.5}$$

We will assume that the free convection coefficient, $\bar{h}_{cv}$ for the segment is the same as that for the entire plate. When dealing with free convection on the upper surface of a hot-plate, we need to first determine the value of *Ra* to be able to choose the correct correlation. Therefore:

$$\mathrm{Ra_L} = \frac{g\beta\Delta T L_c^3}{\nu\alpha} \quad \mathrm{L_c} = \frac{A_s}{P} = \frac{0.2 \times 0.5\mathrm{m}^2}{2(0.2+0.5)\mathrm{m}} = 0.0714\mathrm{m}$$

and evaluating properties at $T_f = 308K$,

$$\mathrm{Ra_L} = \frac{9.8\mathrm{m/s^2}(1/308\mathrm{K})(47-23)(0.0714\mathrm{m})^3}{16.69 \cdot 10^{-6}\mathrm{m/s^2} \times 23.68 \cdot 10^{-6}\mathrm{m^2/s}} = 7.033 \cdot 10^5$$

We thus use as correlation:

$$\overline{\mathrm{Nu}}_L = 0.54\mathrm{Ra}_L^{1/4} = 0.54\left(7.033 \cdot 10^5\right)^{1/4} = 15.64$$

$$\bar{h}_{cv} = \overline{\mathrm{Nu}}_L k/L_c = 15.64 \times 0.02689\mathrm{W/m \cdot K}/0.0714\mathrm{m} = 5.89\mathrm{W/m^2 \cdot K}$$

Substituting numerical values into the expression for the electric power we get:

$$18.9\mathrm{W} = \left[5.89\mathrm{W/m^2 \cdot K}\left(T_s - 296\right) + 0.03 \times 5.67 \cdot 10^{-8}\mathrm{W/m^2 \cdot K^4}\left(T_s^4 - 296^4\right)\right](0.3-0.2)\mathrm{m} \times 0.2\mathrm{m}$$

$$T_s = 447\mathrm{K} = 174°\mathrm{C}$$

Comments: Recognise that in part (b), the assumed value for $T_f = 308K$ is a poor approximation. To obtain a more accurate result, we should now use the obtained surface temperature to determine the film temperature, re-evaluate all of the fluid properties and calculate a new surface temperature. Such iteration must be repeated until the value of the surface used for the film temperature estimation and the one obtained are in good agreement.