Exercise 8.1

The surface of a horizontal $20mm$ diameter cylinder is maintained at an excess temperature of $5°C$ in saturated water at $1atm$. Estimate the heat flux using the appropriate free convection correlation and compare your result with the boiling curve of Figure 1. For nucleate boiling, estimate the maximum value of the heat transfer coefficient from the boiling curve.

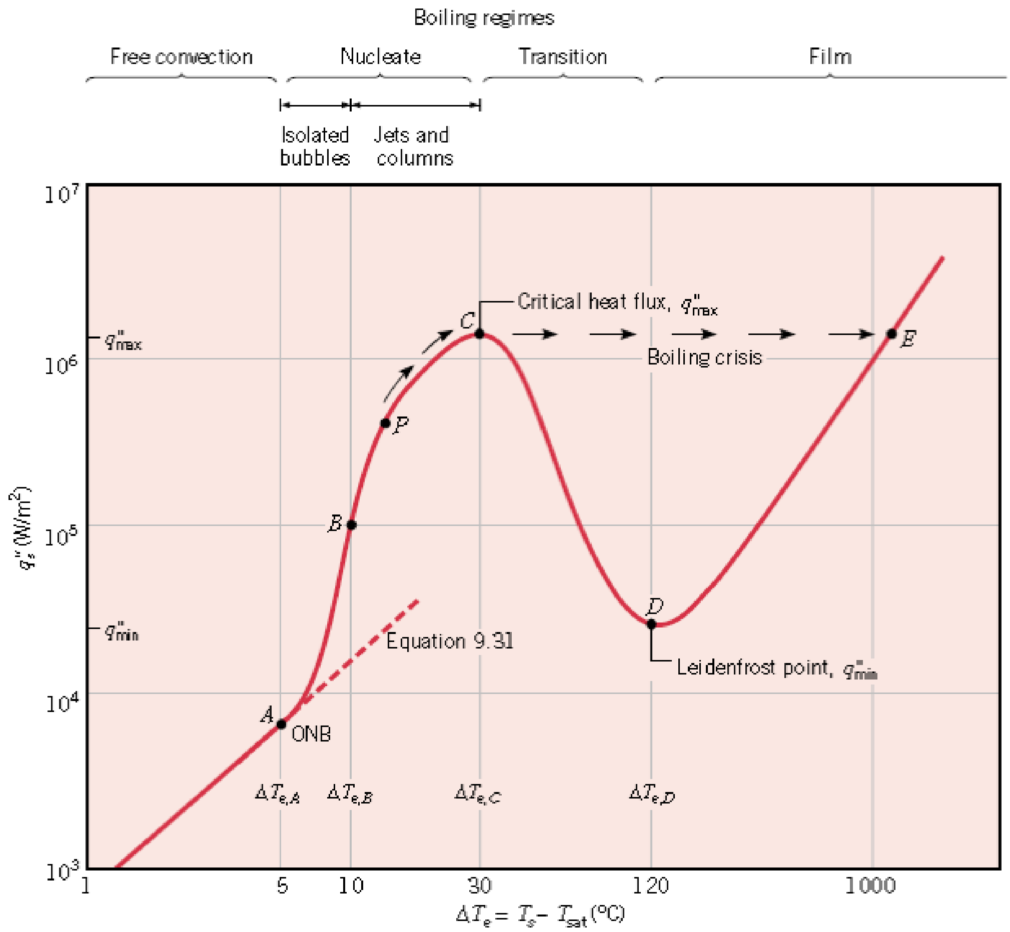
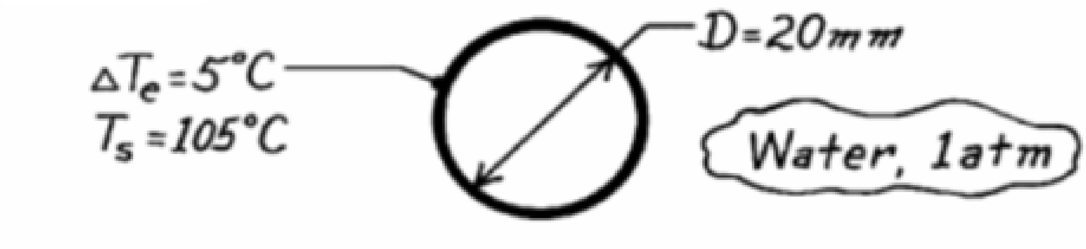


Figure 1: Typical boiling curve for water at $1atm$: surface heat flux q''_s as a function of excess temperature, $\Delta T_e \equiv T_s - T_{sat}$.

Solution



Assumptions: (1) Horizontal cylinder, (2) Free convection, no bubble nucleation.

Properties: *Table A-6*, Water (Saturated liquid, $T_f = (T_{sat} + T_s)/2 = 102.5°C \approx 375K$) : $\rho_\ell = 956.9 kg/m^3$, $c_{p,\ell} = 4220 J/kg \cdot K$, $\mu_\ell = 274 \cdot 10^{-6} N \cdot s/m^2$, $k_\ell = 0.681 W/m$, $Pr = 1.70$, $\beta = 761 \cdot 10^{-6} K^{-1}$

Analysis: To estimate the free convection heat transfer coefficient, use the Churchill-Chu correlation for an infinite cylinder:

$$\overline{Nu}_D = \frac{\bar{h}D}{k} = \left\{0.60 + \frac{0.387 Ra_D^{1/6}}{\left[1 + (0.559/Pr)^{9/16}\right]^{8/27}}\right\}^2$$

To determine a numerical value we must first calculate the *Ra* value and, using $\Delta T = \Delta T_e = 5°C$, we find:

$$Ra_D = \frac{g\beta\Delta T D^3}{\nu\alpha} = \frac{9.8 m/s^2 \times 761 \cdot 10^{-6} K^{-1} \times 5°C (0.020m)^3}{[274 \cdot 10^{-6} N \cdot s/m^2 / 956.9 kg/m^3] \times 1.686 \cdot 10^{-7} m^2/s} = 6.178 \cdot 10^6$$

where $\alpha = k/\rho c_p = \left(0.681 W/m \cdot K / 956.9 kg/m^3 \times 4220 Jkg \cdot K\right) = 1.686 \cdot 10^{-7} m^2/s$. Note that Ra_D is within the prescribed limits of the correlation. Hence,

$$\overline{Nu}_D = \left\{0.60 + \frac{0.387\left(6.178 \cdot 10^6\right)^{1/6}}{\left[1 + (0.559/1.70)^{9/16}\right]^{-8/27}}\right\}^2 = 27.22$$
$$\bar{h}_{fc} = Nu_D \frac{k}{D} = \frac{27.22 \times 0.681 W/m \cdot K}{0.020 m} = 928 W/m^2 \cdot K$$

Hence, $q_s'' = h_{fc} \Delta T_e = 4640 W/m^2$

From the typical boiling curve for water at 1 atm, Fig. 1, find at $\Delta T_e = 5°C$ that

$$q_s'' \approx 8.5 \cdot 10^3 W/m^2$$

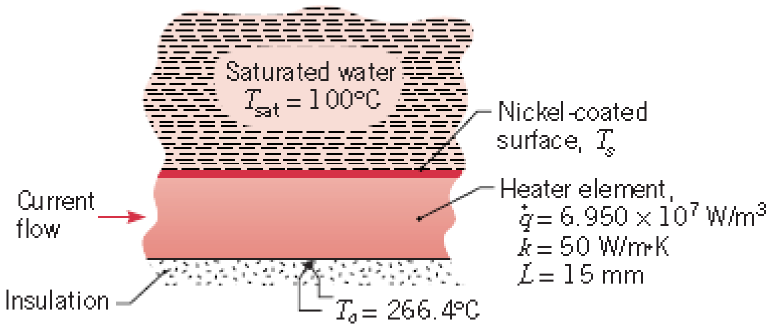
The free convection correlation underpredicts (by 1.8 times) the boiling curve. The maximum value of h_{bc} can be estimated as

$$h_{max} \approx q_{max}'' / \Delta T_e = 1.2 \cdot 10^6 MW/m^2 / 30°C = 40,000 W/m^2 \cdot K.$$

Comments: (1) Note the large increase in h with a slight change in ΔT_e. (2) The maximum value of h occurs at point P on the boiling curve.

Exercise 8.2

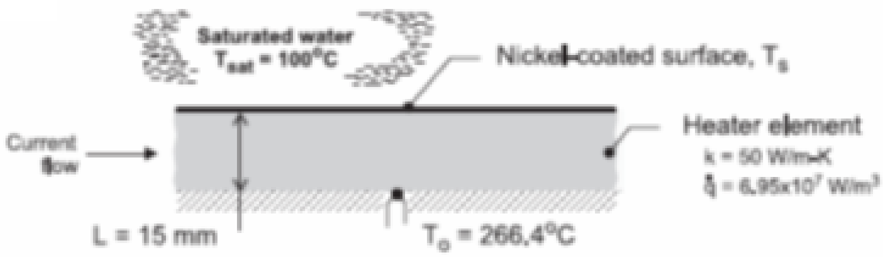
A nickel-coated heater element with a thickness of $15mm$ and a thermal conductivity of $50W/mK$ is exposed to saturated water at atmospheric pressure. A thermocouple is attached to the back surface, which is well insulated. Measurements at a particular operating condition yield an electrical power dissipation in the heater element of $6.95 \times 10^7 W/m^3$ and a temperature of $T_0 = 266.4°C$.



a) From the foregoing data, calculate the surface temperature T_s and the heat flux at the exposed surface. *Hint*: the heater has a uniform volumetric heat source. What is the temperature profile in the heater?

b) Using the surface heat flux determined in part (a), determine the boiling mode and estimate the surface temperature by applying the appropriate boiling correlation.

Solution



Assumptions: (1) Steady-state conditions, (2) Water exposed to standard atmospheric pressure and uniform temperature, T_{sat}, (3) Uniform volumetric generation in element, and (4) Backside of heater is perfectly insulated.

Properties: *Table A-6*, Saturated water, liquid (100 °C): $\rho_\ell = 1/\mathrm{v_f} = 957.9\mathrm{kg/m^3}, c_{\mathrm{p},\ell} = \mathrm{c_{p,f}} = 4.217\mathrm{kJ/kg \cdot K}, \mu_\ell = \mu_\mathrm{f} = 279 \cdot 10^{-6}\mathrm{N \cdot s/m^2}, Pr_\ell = Pr_\mathrm{f} = 1.76,\ \mathrm{h_{fg}} = 2257\mathrm{kJ/kg}\sigma = 58.9 \cdot 10^{-3}\mathrm{N/m}$; Saturated water, vapor (100°C) : $\rho_\mathrm{v} = 1/\mathrm{v_g} = 0.5955\mathrm{kg/m^3}$.

Analysis:

(a) The heater presents a uniform heat source. Therefore the temperature profile in this layer will be parabolic instead of linear. Considered that the bottom surface is adiabatic (insulated), the temperature distribution is maximum at the bottom surface and the temperature of the upper surface, T_s is described by:

$$\mathrm{T_s} = \mathrm{T_o} - \frac{\dot{\mathrm{q}}\mathrm{L}^2}{2\mathrm{k}} = 266.4°\mathrm{C} - \frac{6.95 \cdot 10^7 \mathrm{W/m^3} (0.015\mathrm{m})^2}{2 \times 50 \mathrm{W/m \cdot K}}$$
$$\mathrm{T_s} = 110.0°\mathrm{C}$$

The heat flux at the exposed surface must equal the total heat generated within the heater:

$$\mathrm{q_s''} = \dot{\mathrm{q}}\mathrm{L} = 6.95 \cdot 10^7 \mathrm{W/m^3} \times 0.015\mathrm{m} = 1.043 \cdot 10^6 \mathrm{W/m^2}$$

(b) Since $\Delta\mathrm{T_e} = \mathrm{T_s} - \mathrm{T_{sat}} = (110 - 100)°\mathrm{C} = 10°\mathrm{C}$, nucleate pool boiling occurs and the Rohsenow correlation with $\mathrm{q_s''}$ from part (a) can be used to estimate the surface temperature, $T_{\mathrm{s,c}}$,

$$\mathrm{q_s''} = \mu_\ell \mathrm{h_{fg}} \left[\frac{\mathrm{g}\left(\rho_\ell - \rho_\mathrm{v}\right)}{\sigma}\right]^{1/2} \left(\frac{\mathrm{c}_{\mathrm{p},\ell}\Delta\mathrm{T_{e,c}}}{\mathrm{C_{s,f}h_{fg}Pr}_\ell^\mathrm{n}}\right)^3$$

From Table 10.1, for the water-nickel surface-fluid combination, $C_{\mathrm{s,f}} = 0.006$ and $n = 1.0$.

Substituting numerical values, find $\Delta\mathrm{T_{e,c}}$ and $\mathrm{T_{s,c}}$.

$$1.043 \cdot 10^6 \mathrm{W/m^2} = 279 \cdot 10^{-6}\mathrm{N \cdot s/m^2} \times 2257 \cdot 10^3 \mathrm{J/kg}$$
$$\times \left[\frac{9.8\mathrm{m/s^2}(957.9 - 0.5955)\mathrm{kg/m^3}}{58.9 \cdot 10^{-3}\mathrm{N/m}}\right]^{1/2}$$
$$\times \left(\frac{4.217 \cdot 10^3 \mathrm{J/kg \cdot K} \times \Delta\mathrm{T_{e,c}}}{0.006 \times 2257 \cdot 10^3 \mathrm{J/kg} \times 1.76}\right)^3$$
$$\Delta\mathrm{T_{e,c}} = \mathrm{T_{s,c}} - \mathrm{T_{sat}} = 9.1°\mathrm{C} \quad \mathrm{T_{s,c}} = 109.1°\mathrm{C}$$

Comments: From the experimental data, part (a), the surface temperature is determined from the conduction analysis as $\mathrm{T_s} = 110.0°\mathrm{C}$. Using the traditional nucleate boiling correlation with the experimental value for the flux, the surface temperature is estimated as $\mathrm{T_{s,c}} = 109.1°\mathrm{C}$. The two approaches provide excess temperatures that are 10.0 vs 9.1 °C, which amounts to nearly a 10% difference.

Exercise 8.3

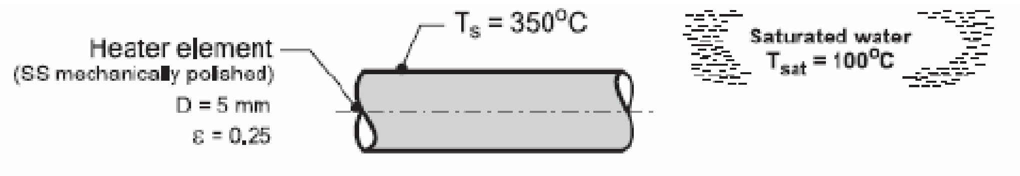
A heater element of $5mm$ diameter is maintained at a surface temperature of $350°C$ when immersed horizontally in water under atmospheric pressure. The element sheath is stainless steel with a mechanically polished finish having an emissivity of 0.25.

a) Calculate the electrical power dissipation

b) If the heater were operated at the same power dissipation rate in the nucleate boiling regime, what temperature would the surface achieve? Calculate the rate of vapor production per unit length for this operating condition.

c) Sketch the boiling curve and represent the two operating conditions of parts (a) and (b). Compare the results of your analysis. If the heater element is operated in the power controlled mode explain how you would achieve these two operating conditions beginning with a cold element.

Solution



Assumptions: (1) steady-state conditions, (2) water exposed to standard atmospheric pressure and uniform T_{sat}.

Properties: *Table A6*, Saturated water, liquid (100°C): $\rho_l = 957.9 kg/m^3$, $c_{p,l} = 4217 J/kgK$, $\mu_l = 279 \times 10^{-6} Ns/m^2$, $\text{Pr}_l = 1.76$, $h_{fg} = 2257 kJ/kg$, $h'_{fg} = h_{fg} + 0.8 c_{p,v}(T_s - T_{sat}) = 2654 kJ/kg$, $\sigma = 58.9 \times 10^{-3} N/m$.

Saturated water, vapor (100°C): $\rho_v = 0.5955 kg/m^3$.

Table A4, water vapor ($T_f \approx 500K$): $\rho_v = 0.4405 kg/m^3$, $c_{p,v} = 1985 J/kgK$, $k_v = 0.0339 W/mK$, $\nu_v = 38.68 \times 10^{-6} m^2/s$.

Analysis:

a) Since $\Delta T_e > 120°C$, the element is operating in the *film-boiling* (FB) regime. The electrical power dissipation per unit length is:

$$q'_s = \overline{h}(\pi D)(T_s - T_{sat})$$

where the total heat transfer coefficient is:

$$\overline{h} = \overline{h}_{conv} + \frac{3}{4}\overline{h}_{rad} \quad (\text{if } h_{conv} > h_{rad})$$

The convection coefficient is given by the the expression for film pool boiling (with $C = 0.62$):

$$\frac{\overline{h}D}{k_v} = C\left[\frac{g(\rho_l - \rho_v)h'_{fg}D^3}{\nu_v k_v (T_s - T_{sat})}\right]^{1/4}$$

$$\begin{aligned}\overline{h}_{conv} &= \frac{0.0339 W/mK}{0.005 m}(0.62)\left[\frac{9.8 m/s^2(957.9 - 0.5955) kg/m^3 \times 2.654 \times 10^6 J/kgK(0.005 m)^3}{38.68 \times 10^{-6} m^2/s \times 0.0339 W/mK(350 - 100)K}\right]^{1/4} \\ &= 233 W/m^2K\end{aligned}$$

The radiation coefficient, with $\sigma = 5.67 \times 10^{-8} W/m^2K^4$ is:

$$\begin{aligned}\overline{h}_{rad} &= \frac{\epsilon\sigma(T_s^4 - T_{sat}^4)}{T_s - T_{sat}} \\ &= \frac{0.25\sigma(623^4 - 373^4)K^4}{(350 - 100)K} = 7.4 W/m^2K\end{aligned}$$

Substituting numerical values into the total heat transfer and electrical power dissipation equations:

$$\overline{h} = 239 W/m^2K$$

$$q'_s = 231 W/m^2K(\pi \times 0.005 m)(350 - 100)K = 939 W/m$$

$$q''_s = \frac{q'_s}{\pi D} = 59.75 kW/m^2$$

b) For the same heat flux, $q_s'' = 57.8kW/m^2$, using the Rohsenow correlation for tthe *nucleate boiling* (NB) regime, find ΔT_e and hence T_s.

$$q_s'' = \mu_l h_{fg}\left[\frac{g(\rho_l-\rho_v)}{\sigma}\right]^{1/2}\left(\frac{c_{p,l}\Delta T_e}{C_{s,f}h_{fg}\mathrm{Pr}_l^n}\right)^3$$

where from Table 10.1, for stainless steel mechanically polished finish with water, $C_{s,f} = 0.0132$ and $n = 1.0$:

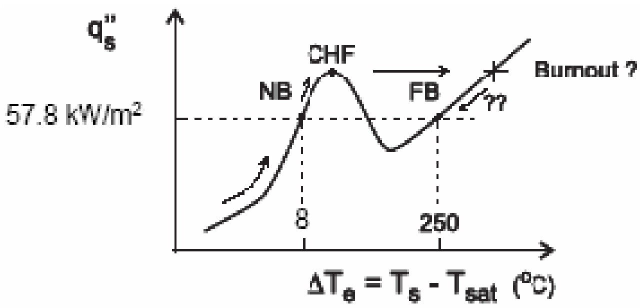
$$57.8\times10^3 W/m^2 = 270\times10^{-6}Ns/m^2\times2.257\times10^6 J/kg \times\left[\frac{9.8m/s^2(957.9-0.5955)kg/m^3}{58.9\times10^{-3}N/m}\right]^{1/2} \times\left(\frac{4217J/kgK\times\Delta T_e}{0.0132\times2.257\times10^6 J/kg\times1.76}\right)^3$$

$$\Rightarrow \Delta T_e = T_s - T_{sat} = 7.6K \qquad T_s = 107.6^\circ C$$

The evaporation rate per unit length is:

$$\dot{m}_b = q_s''(\pi D)/h_{fg} = 1.4kg/hm$$

c) The two operating conditions are shown on the boiling curve below. For FB, the surface temperature is $T_s = 350^\circ C$ ($\Delta T_e = 250^\circ C$). The element can be operated at NB with the same heat flux, $q_s'' = 57.8kW/m^2$, with a surface temperature of $T_s = 108^\circ C$ ($\Delta T_e = 8^\circ C$). Since the heat fluxes are the same for both conditions, the evaporation rates are the same.



If the element is cold, and operated in a power-controlled mode, the element would be brought to the NB condition following the arrow shown next to the boiling curve near $\Delta T_e = 0$. If the power is increased beyond that for the NB point, the element will approach the critical heat flux (CHF) condition. If q_s'' is increased beyond q''_{max}, the temperature of the element will increase abruptly, and the burnout condition will likely occur. If burnout does not occur, reducing the heat flux would allow the element to reach the FB point.

Exercise 8.4 FOR REVISION

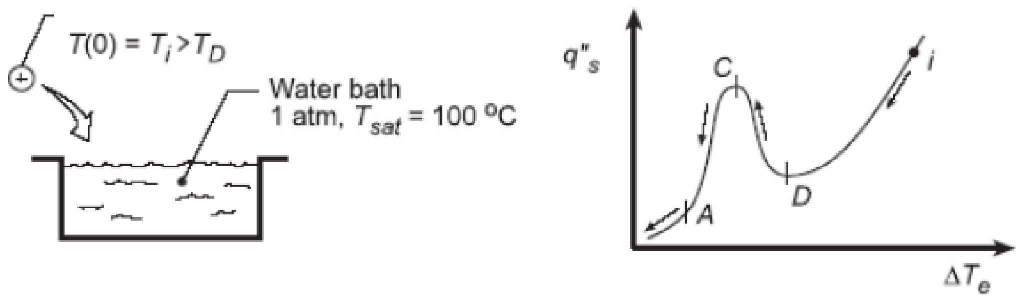
A small steel bar, initially at a uniform, elevated temperature $T(0) = T_i$ is suddenly immersed in a large fluid bath maintained at T_{sat}. The initial temperature of the steel bar exceeds the Leidenfrost point corresponding to the temperature T_D of the boiling curve.

a) Sketch the variation of the average sphere temperature $T(t)$, with time during the quenching process. Indicate on this sketch the temperatures T_i, T_D and T_{sat} as well as the regimes of film, transition and nucleate boiling and the regime of single-phase convection. Identify the key features of the temperature history

b) At what time(s) in the cooling process do you expect the surface temperature of the steel bar to deviate most from its center temperature? Explain your answer.

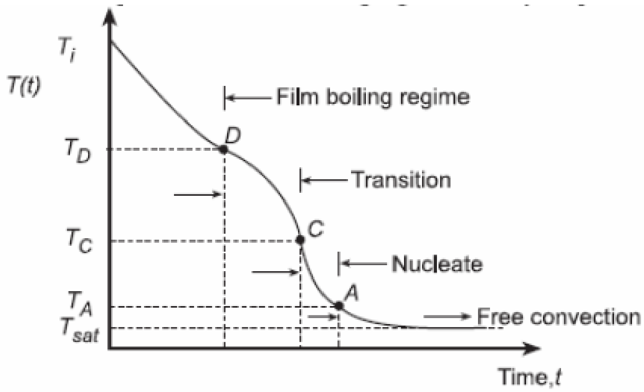
c) The steel bar has $20mm$ in diameter, is $200mm$ long and has an emissivity of 0.9. It is indeed removed from a furnace at $455°C$ and suddenly submerged horizontally in a water bath under atmospheric pressure. What is the initial convection mechanism/mode? Estimate the initial heat transfer rate from the bar using the appropriate correlation.

Solution



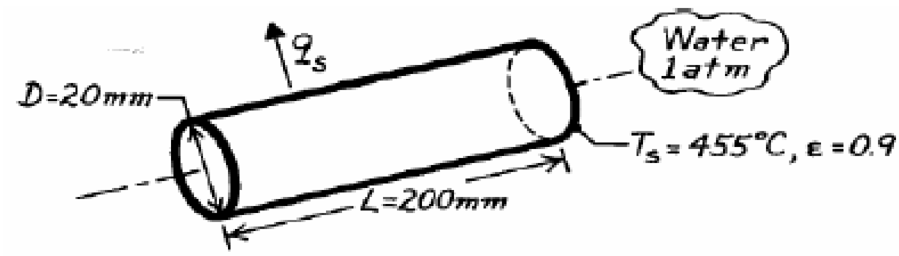
Analysis:

a) In the right-hand side schematic above, the quench process is shown on the 'boiling curve'. Beginning at an initial temperature, $T_i > T_D$, the process proceeds as indicated by the arrows: film regime from i to D, transition regime from D to C, nucleate regime from C to A and single-phase (free convection) from A to the condition when $\Delta T_e = T_s - T_{sat} = 0$. The quench process is shown on the temperature-time plot below and the boiling regimes and key temperatures are labeled. The highest temperature-time change should occur in the nucleate pool boiling regime,



especially near the critical flux condition, T_c. The lowest temperature-time change will occur in the single-phase, free convection regime.

b) The difference between the center and surface temperatures will be greatest when the Biot number is largest, which occurs in regimes with the highest convection coefficients. The convection coefficient is maximum between C and A on the above plots.



c) **Assumptions:** (1) uniform bar surface temperature, (2) film pool boiling conditions.

Properties: *Table A6*, Water, liquid ($1atm$, $T_{sat} = 100°C$): $\rho_l = 957.9kg/m^3$, $h_{fg} = 2257kJ/kg$.

Table A4: Water, vapor ($T_f = (T_s + T_{sat})/2 = 550K$): $\rho_v = 0.4005kg/m^3$, $c_{p,v} = 1997J/kgK$, $\nu_v = 47.04 \times 10^{-6}m^2/s$, $k_v = 0.0379W/mK$.

Analysis: The total heat transfer rate from the bar at the instant of time it is removed from the furnace and immersed in the water is:

$$q_s = \overline{h} A_s (T_s - T_{sat}) = \overline{h} A_s \Delta T_e$$

where $\Delta T_e = 455 - 100 = 355K$.

According to the boiling curve, with such a high ΔT_e film pool boiling will occur. Furthermore, because the temperature of the surface is above $300C$ we have to include also the radiative effects into the heat transfer coefficient. We have:

$$\overline{h} = \overline{h}_{conv} + \frac{3}{4}\overline{h}_{rad} \quad (\text{if } h_{conv} > h_{rad})$$

To estimate the convection coefficient, we use:

$$\overline{\mathrm{Nu}}_D = \frac{\overline{h}_{conv} D}{k_v} = C\left[\frac{g(\rho_l - \rho_v)h'_{fg}D^3}{\nu_v k_v \Delta T_e}\right]^{1/4}$$

where $C = 0.62$ for the horizontal cylinder and $h'_{fg} = h_{fg} + 0.8c_{p,v}(T_s - T_{sat})$. Thus:

$$\overline{h}_{conv} = \frac{0.0379W/mK}{0.020m}0.62\left[\frac{9.8m/s^2(957.9 - 0.4005)kg/m^3(2257 \times 10^3 + 0.8 \times 1997 \times 355)J/kg(0.020m)^3}{(47.04 \times 10^{-6})m^2/s \times 0.0379W/mK \times 355K}\right]^{1/4}$$
$$= 159W/m^2K$$

To estimate the radiation coefficient, we use:

$$\overline{h}_{rad} = \frac{\epsilon\sigma(T_s^4 - T_{sat}^4)}{T_s - T_{sat}} = \frac{0.9 \times 5.67 \times 10^{-8}w/m^2K^4(728^4 - 373^4)K^4}{355K} = 37.6W/m^2K$$

Substituting numerical values into the equation for $\overline{h}$:

$$\overline{h} = (159 + (3/4)37.6)W/m^2K = 187W/m^2K$$

Using the first equation, the heat rate, with $A_s = \pi DL$, is:

$$q_s = 187W/m^2K(\pi \times 0.020m \times 0.200m) \times 355K = 835W$$

Comments: For these conditions, the combined radiation and convection heat transfer coefficient is 18% larger than the convection coefficient alone.