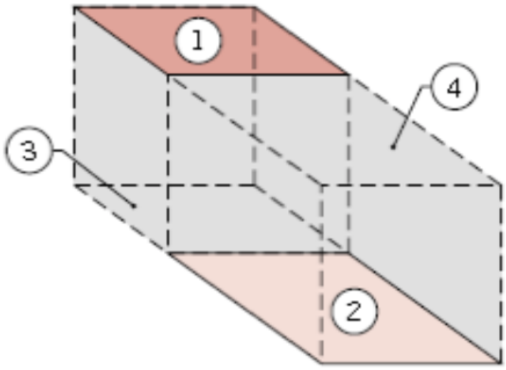
Exercise 13.1

Consider the parallel rectangles shown schematically. Show that the view factor F_{12} can be expressed as

$$F_{12} = \frac{1}{2A_1}\left[A_{1.4}F_{(1.4)(2.3)} - A_1F_{13} - A_4F_{42}\right]$$

Where all view factors on the right-hand side of the equation can be evaluated from the known relations for aligned parallel rectangles.



Solution

Analysis: Using the additive rule where the parenthesis denote a composite surface,

$$A_{(1,4)}F^*_{(1,4)(2,3)} = A_1F^*_{13} + A_1F_{12} + A_4F_{43} + A_4F^*_{42} \tag{0.1}$$

where the asterisk ($*$) denotes that the F_{ij} can be evaluated using the known relations . Now, find suitable relation for F_{43} . By symmetry,

$$F_{43} = F_{21} \tag{0.2}$$

and from reciprocity between A_1 and A_2

$$F_{21} = \frac{A_1}{A_2}F_{12} \tag{0.3}$$

Multiply Eq. (2) by A_4 and substitute Eq. (3), with $A_4 = A_2$

$$A_4F_{43} = A_4F_{21} = A_4\frac{A_1}{A_2}F_{12} = A_1F_{12} \tag{0.4}$$

Substituting for A_4F_{43} from Eq. (4) into Eq. (1), and rearranging,

$$F_{12} = \frac{1}{2A_1}\left[A_{(1,4)}F^*_{1,4)(2,3)} - A_1F^*_{13} - A_4F^*_{42}\right]$$

Exercise 13.2

A thermocouple whose surface is diffuse and gray with an emissivity of 0.6 indicates a temperature of $180°C$ when used to measure the temperature of a gas flowing through a large duct whose walls have an emissivity of 0.85 and a uniform temperature of $450°C$.

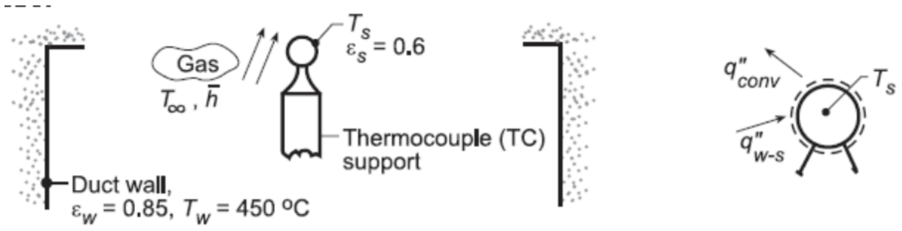
If the convection heat transfer coefficient between the thermocouple and the gas stream is $h = 125W/m^2K$ and there are negligible losses from the thermocouple, determine the temperature of the gas.

Hint 1: observe that the thermocouple and the surrounding duct form a two-surface cavity. *Hint 2*: what are the view factors for the thermocouple and the duct walls?

Solution

- $T_\infty = 117C$

Solution



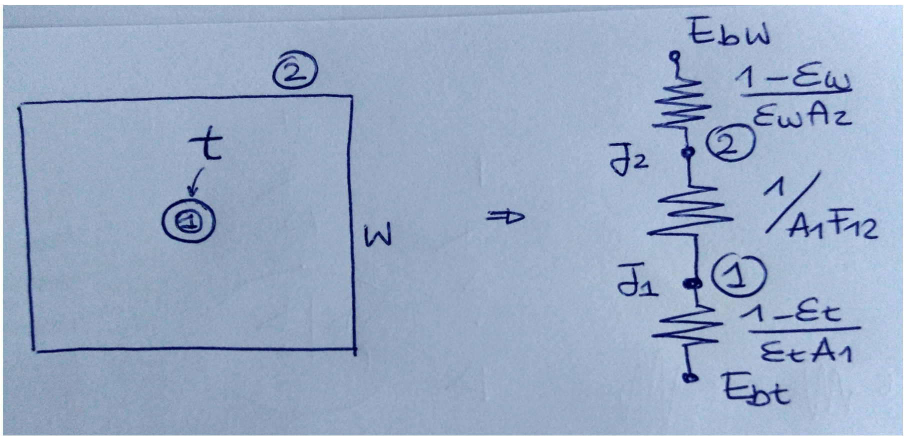
Assumptions: (1) steady-state conditions, (2) negligible heat loss from thermocouple (TC) sensing junction to support, (3) duct wall much larger than TC, (4) TC surface is diffuse-gray.

Analysis: Performing an energy balance on the thermocouple, we note that the steady-state temperature of the thermocouple must be determined from the balance of radiation exchange with the duct walls and the convection of the gas stream. Because the thermocouple surface is gray, the emissivity ϵ and absorptivity α coincide. It follows that:

$$q''_{w-s} - q''_{conv} = 0$$

We now need to determine the radiative component of the heat exchange. The net radiative heat transfer will be due to the balance of the radiation emitted and absorbed by the thermocouple. We observe that the thermocouple surface and the duct walls form an enclosure similar to slide 24 of W13L2 -1h. Therefore we know that the view factors between the thermocouple (named 1) and the duct wall (named 2) will be:

$$F_{11} = 0$$
$$F_{12} = 1$$
$$F_{21} = A_1/A_2$$
$$F_{22} = 1 - A_1/A_2$$



We can also draw the electrical circuit equivalent to this case and observe that, for $A_2 >> A_1$ then we obtain that the duct walls behave like a black body and therefore J_2 $E_{b,w}$.

On the other hand, the geometry resistance and the surface radiation resistance will be:

$$R_{geom} = 1/A_1F_{12}$$
$$R_{surf,rad} = (1-\epsilon_s)/(\epsilon_s A_1)$$

Based on the electrical circuit we draw we can calculate:

$$Q = \frac{E_{b,w} - E_{b,s}}{1/A_1 F_{12} + (1-\epsilon_s)/(\epsilon_s A_1)}$$

Simplifying by the thermocouple area A_1 and remembering that the blackbody emissions will be $E_{b,w} = \sigma T_w^4$ and $E_{b,s} = \sigma T_s^4$ for the walls and the thermocouple, respectively:

$$q^{"}_{w-s} = \epsilon_s \sigma (T_w^4 - T_s^4)$$

Hence,

$$\varepsilon_s \sigma (T_w^4 - T_s^4) - \overline{h}(T_s - T_\infty) = 0$$

Solving for T_∞ with $T_s = 180°C$:

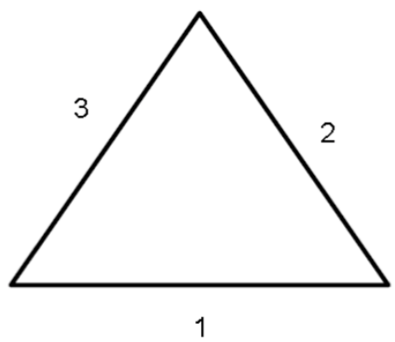
$$\begin{aligned} T_\infty &= T_s - \frac{\varepsilon_s \sigma}{\overline{h}}(T_w^4 - T_s^4) \\ &= (180+273)K - \frac{0.6(5.67 \times 10^{-8} W/m^2K^4)}{125 W/m^2K}\left([450+273]^4 - [180+273]^4\right)K^4 = 390K = 117°C \end{aligned}$$

Exercise 13.3 [DIFFICULT] FOR REVISION

A cavity composed of three, infinitely long, gray and opaque surfaces have the following conditions at the steady state:

$$\text{Surface 1}: \mathrm{T_1} = 300°\mathrm{C}, \mathrm{L_1} = 0.5\mathrm{m}, \varepsilon_1 = 0.7$$
$$\text{Surface 2}: \mathrm{T_2} = 200°\mathrm{C}, \mathrm{L_2} = 0.5\mathrm{m}, \varepsilon_2 = 1$$
$$\text{Surface 3}: \mathrm{T_3} = 100°\mathrm{C}, \mathrm{L_3} = 0.5\mathrm{m}, \varepsilon_3 = 1$$

Determine the net heat rate for each surface and verify that the sum of the heat rates is equal to 0.



Hint 1: what are the view factors for the various surfaces?

Hint 2: write the energy balance for surface 1

Solution

- $q_1 = 1.453e3W/m$
- $q_2 == 72.53W/mW/m$
- $q_3 = -1.379e3W/m$

Solution

Analysis: We start by calculating the View Factors: $F_{12} + F_{13} = 1, F_{12} = F_{13} = 0.5$. We do an energy balance for surface 1 and obtain:

$$q_1 = L_1 \varepsilon_1 \sigma \left(F_{12}\left(T_1^4 - T_2^4\right) + F_{13}\left(T_1^4 - T_3^4\right)\right) = 1.452\text{e}3W/m$$

We then proceed with calculating the radiosity and the irradiation on the surface 1

$$J_1 = \varepsilon_1 \sigma T_1^4 + (1 - \varepsilon_2) G_1 = 4.874\text{e}3\text{W/m}^2$$

$$\text{G}_1 = \text{F}_{12}\text{J}_2 + \text{F}_{13}\text{J}_3 = 1.97\text{e}3\text{W/m}^2$$

Finally we repeat the energy balances this time for the surfaces 2 and 3 obtaining respectively:

$$q_2 = L_2\left(F_{21}\left(E_{b2} - J_1\right) + F_{32}\left(E_{b2} - E_{b3}\right)\right) = -72.53\text{W/m}$$

$$q_3 = L_3\left(F_{31}\left(E_{b3} - J_1\right) + F_{32}\left(E_{b3} - E_{b2}\right)\right) = -1.379\text{e}3\text{W/m}$$

Note that $q_1 + q_2 + q_3 = 0$ is valid.

A second method for calculating q_1 would be based on its surface resistance as follows

$$Rs_1 = (1 - \varepsilon_1) / (L_1 \varepsilon_1) = 0.8571\ [1/m]$$

$$q_1 = \left(\sigma T_1^4 - J_1\right) / Rs_1 = 1.452\text{e}3\text{W/m}$$