Exercise 10.1

A steel tube ($k = 50W/mK$) of inner and outer diameters $D_i = 20mm$ and $D_o = 26mm$ respectively, is used to transfer heat from hot gases flowing over the tube ($h_h = 200W/m^2K$) to cold water flowing through the tube ($h_c = 8000W/m^2K$).

a) What is the cold-side overall heat transfer coefficient U_c?

b) To enhance heat transfer, 16 straight fins of rectangular profile are installed longitudinally along the outer surface of the tube. The fins are equally spaced around the circumference of the tube, each having a thickness of $2mm$ and a length of $15mm$. What is the corresponding overall cold-side heat transfer coefficient U_c? Assume adiabatic fin tip.

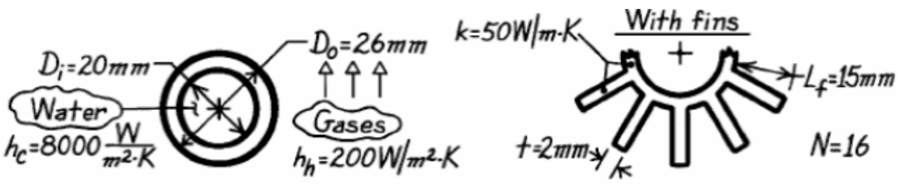
Reflect on the effect of adding the fins.

Solutions

a) $U_c = 249W/m^2K$

b) $U_c = 1138W/m^2K$

Solution



Assumptions: (1) Negligible fouling, (2) Negligible contact resistance between fins and tube wall, (3) h_h is not affected by fins, (4) one-dimensional conduction in fins, (5) adiabatic fin tip.

Analysis:

$$\frac{1}{U_c} = \frac{1}{(\eta_o h)_c} + \frac{D_i \ln(D_o/D_i)}{2k} + \frac{A_c}{(\eta_o h A)_h}$$

Without fins: $\eta_{o,c} = \eta_{o,h} = 1$

$$\frac{1}{U_c} = \frac{1}{8000W/m^2K} + \frac{(0.02m)\ln(26/20)}{100W/mK} + \frac{1}{200W/m^2K}\frac{20}{26}$$
$$= 4.02 \times 10^{-3} m^2K/W$$

$$U_c = 249W/m^2K$$

With fins: $\eta_{o,c} = 1$, $\eta_{o,h} = 1 - (A_f/A)(1 - \eta_f)$ per unit length along the tube axis

$$A_f = N(2L_f + t)L = 16(0.03 + 0.002)L = 0.0512L\ m^2$$
$$A_h = A_f + (piD_o - Nt)L = (0.512 + 0.0497)L = 0.5617L\ m^2$$

with $m = (2h/kt)^{1/2} = \left(400W/m^2K/50W/mK \times 0.002m\right)^{1/2} = 63.3m^{-1}$ and so:

$$mL_f = (63.3m^{-1})(0.015m) = 0.95$$

Thus,

$$\eta_f = \frac{tanh(mL_f)}{mL_f} = \frac{0.739}{0.95} = 0.778$$

The overall surface efficiency is then

$$\eta_o = 1 - (A_f/A_h)(1 - \eta_f) = 1 - (512/561.7)(1 - 0.778) = 0.798$$

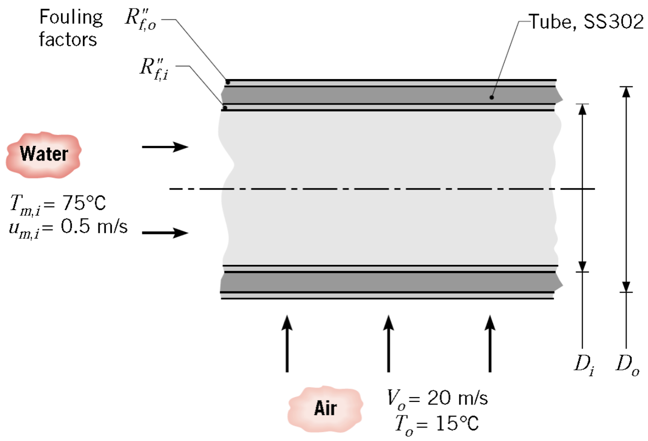
Hence,

$$\frac{1}{U_c} = \left(1.25 \times 10^{-4} + 5.25 \times 10^{-5} + \frac{\pi(20)}{0.798(200)561.7}\right) m^2K/W = 8.78 \times 10^{-4} m^2K/W$$

$$U_c = 1138W/m^2K$$

Exercise 10.2

A type-302 stainless steel tube of inner and outer diameters $D_i = 22mm$ and $D_o = 27mm$, respectively, is used in a cross-flow heat exchanger. The fouling factors, R'_f for the inner and outer surfaces are estimated to be 0.0004 and $0.0002m^2K/W$, respectively.



After depicting the thermal resistance circuit of the considered system:

a) Determine the overall heat transfer coefficient based on the outside area of the tube, U_o. Compare the thermal resistances due to convection, tube-wall conduction and fouling.

Hint: initially assume that the temperature of the outer surface of the tube is $T_{f,o} = 315K$. Verify at the end of the exercise whether this assumption was reasonably accurate. Also, assume that the flow inside the tube is fully developed.

b) Instead of air flowing over the tube, consider a situation for which the cross-flow fluid is water at $15°C$ with a velocity of $V_o = 1m/s$. For this case assume $T_{f,o} = 292K$ as it is expected that the convection effects will be stronger in water. Determine the overall heat transfer coefficient based on the outside area of the tube U_o. Compare the thermal resistances due to convection, tube-wall conduction and fouling.

Hint: determine first the conditions of flow on the inner and outer sides of the tube and select the appropriate correlations for the convection coefficient.

Solutions

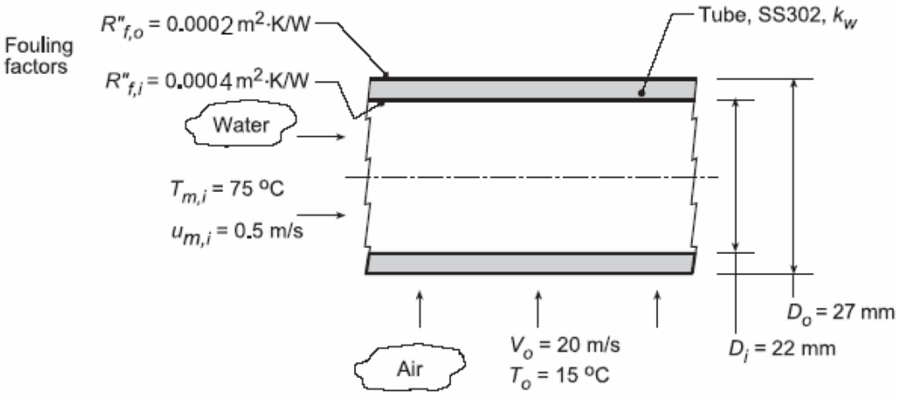
a)

$R_{cv,i}$	$R_{f,i}$	R_w	$R_{f,o}$	$R_{cv,o}$	U_o	R_{tot}
0.00436	0.00578	0.00216	0.00236	0.1134	92.1	0.128

b)

$R_{cv,i}$	$R_{f,i}$	R_w	$R_{f,o}$	$R_{cv,o}$	U_o	R_{tot}
0.00436	0.00579	0.00216	0.00236	0.00240	691	0.0171

Solution



Assumptions: (1) steady-state conditions, (2) fully developed internal flow.

Properties: *Table A.1*, Stainless steel, AISI 302 ($300K$): $k_w = 15.1W/mK$.

Table A.6, Water ($\overline{T}_{m,i}$): $\rho_i = 974.8kg/m^3$, $\mu_i = 3.746\times10^{-4}Ns/m^2$, $k_i = 0.668W/mK$, $\mathrm{Pr}_i = 2.354$.

Table A.4, Air (assume $\overline{T}_{f,o} = 315K$, $1atm$): $k_o = 0.02737W/mK$, $\nu_o = 17.35\times10^{-6}m^2/s$, $\mathrm{Pr}_o = 0.705$.

Analysis:

a) For the water-air condition, the overall coefficient based upon the outer area can be expressed as the sum of the thermal resistances due to convection (cv), tube wall conduction (w) and fouling (f):

$$\frac{1}{U_oA_o} = R_{tot} = R_{cv,i} + R_{f,i} + R_w + R_{f,o} + R_{cv,o}$$

with $R_{cv,i} = \dfrac{1}{\overline{h}_iA_i}$, $R_{cv,o} = \dfrac{1}{\overline{h}_oA_o}$, $R_{f,i} = \dfrac{R'_{f,i}}{A_i}$, $R_{f,o} = \dfrac{R'_{f,o}}{A_o}$ and $R_w = \dfrac{\ln(D_o/D_i)}{2\pi Lk_w}$.

The convection coefficients can be estimated from appropriate correlations.

Estimating $\overline{h}_i$*:* In this case we have internal forced convection in a smooth tube of circular cross section. Having assumed fully developed flow we still need to determine the flow conditions (laminar/turbulent etc.) hence we calculate the Re number. Evaluating the thermophysical properties at $T_{m,i}$:

$$\mathrm{Re}_{D,i} = \frac{u_{m,i}D_i}{\nu_i} = \frac{0.5m/s\times0.022m}{3.746\times10^{-4}Ns/m^2/974.8kg/m^3} = 28625 > 2300$$

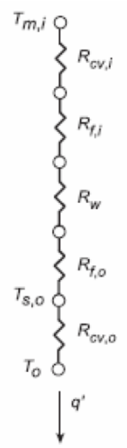
Therefore the flow is turbulent and we need to use the (Dittus-Boelter) correlation:

$$\mathrm{Nu}_{D,i} = 0.023\,\mathrm{Re}_{D,i}^{0.8}\,\mathrm{Pr}_i^{0.3} = 0.023(28625)^{0.8}(2.354)^{0.3} = 109.3$$

$$\overline{h}_i = \frac{\mathrm{Nu}_{D,i}k_i}{D_i} = \frac{109.3\times0.668W/mK}{0.022m} = 3313W/m^2K$$

Estimating $\overline{h}_o$: For external flow, we have forced convection around a cylindrical tube. We characterize the flow with:

$$\mathrm{Re}_{D,o} = \frac{V_o D_o}{\nu_o} = \frac{20m/s \times 0.027m}{17.35 \times 10^{-6} m^2/s} = 31124$$



We evaluate the thermophysical properties of the fluid at $T_{f,o} = 315K$.
Note: the more precise evaluation of the thermophysical properties should be done at the average fluid temperature, i.e. the mean value between the free flow temperature T_o and the wall surface (outside of the fouling layer), i.e. $T_{s,o}$ $T_{f,o} = (T_{s,o}+T_o)/2$. The surface temperature T_s, o is determined from the thermal circuit analysis result (see sketch on the left):

$$\frac{T_{m,i} - T_o}{R_{tot}} = \frac{T_{s,o} - T_o}{R_{cv,o}}$$

After solving the exercise you can verify whether the approximation of $T_{f,o} = T_{s,o}$ is reasonable or not: calculate the heat transfer from the inner to the outer fluid and then determine $T_{s,o}$.

Because we are interested in the average value of the convection coefficient, for cross-flow around a cylinder we use the Churchill-Bernstein correlation which is valid at any value of Re_D:

$$\overline{\mathrm{Nu}}_{D,o} = 0.3 + \frac{0.62\,\mathrm{Re}_{D,o}^{1/2}\,\mathrm{Pr}_o^{1/3}}{\left[1 + (0.4/\mathrm{Pr}_o)^{2/3}\right]^{1/4}} \left[1 + \left(\frac{\mathrm{Re}_{D,o}}{282000}\right)^{5/8}\right]^{4/5}$$

$$= 0.3 + \frac{0.62\,(31124)^{1/2}\,(0.705)^{1/3}}{\left[1 + (0.4/0.705)^{2/3}\right]^{1/4}} \left[1 + \left(\frac{31124}{282000}\right)^{5/8}\right]^{4/5} = 102.6$$

$$\overline{h}_o = \frac{\overline{\mathrm{Nu}}_{D,o} k_o}{D_o} = \frac{102.6 \times 0.02737W/mK}{0.027m} = 104.0W/mK$$

Using the above values for $\overline{h}_i$ and $\overline{h}_o$ and other prescribed values, the thermal resistances in $[K/W]$ and overall coefficient in $[W/m^2K]$ can be evaluated and are tabulated below:

$R_{cv,i}$	$R_{f,i}$	R_w	$R_{f,o}$	$R_{cv,o}$	U_o	R_{tot}
0.00436	0.00578	0.00216	0.00236	0.1134	92.1	0.128

The major thermal resistance is due to outside air convection, accounting for 89% of the total resistance. The other thermal resistances are of similar magnitude, nearly 50 times smaller than $R_{cv,o}$.

b) For the water-water condition, the method of analysis follows that of part (a). For the internal flow, the estimated convection coefficient is the same as part (a). For an assumed outer film coefficient, $\overline{T}_{f,o} = 292K$, the convection correlation for the outer water flow condition $V_o = 1m/s$ and $T_o = 15°C$, find:

$$\mathrm{Re}_{D,o} = 26260, \qquad \mathrm{Nu}_{D,o} = 220.6, \qquad \overline{h}_o = 4914W/m^2K$$

The thermal resistances in $[K/W]$ and overall coefficient in $[W/m^2K]$ are tabulated below:

$R_{cv,i}$	$R_{f,i}$	R_w	$R_{f,o}$	$R_{cv,o}$	U_o	R_{tot}
0.00436	0.00579	0.00216	0.00236	0.00240	691	0.0171

Note that the thermal resistances are of similar magnitude. In contrast with the results for the water-air condition of part (a), the thermal resistance of the outside convection process, $R_{cv,o}$, is nearly 50 times smaller. The overall coefficient for the water-water condition is 7.5 times greater than that for the water-air condition.

Exercise 10.3

A recuperator is a heat exchanger that heats the air used in a combustion process by extracting energy from the products of combustion (the flue gas). Consider using a single-pass cross-flow heat exchanger as a recuperator.

Eighty silicon carbide ceramic tubes ($k = 20W/mK$) of inner and outer diameters equal to $55mm$ and $80mm$ respectively, and of length $L = 1.4m$ are arranged as an aligned tube bank of longitudinal and transverse pitches $S_L = 100mm$ and $S_T = 120mm$, respectively. Cold air is in cross flow over the tube bank with upstream conditions of $V = 1m/s$ and $T_{c,i} = 300K$, while hot-flue gases of inlet temperature $T_{h,i} = 1400K$ pass through the tubes. The tube outer surface is clean while the inner surface is characterized by a fouling factor of $R^i_f = 0.0002m^2K/W$. The air and flue gas flow rates are $\dot{m}_c = 1kg/s$ and $\dot{m}_h = 1.05kg/s$, respectively.

Use the following assumptions:

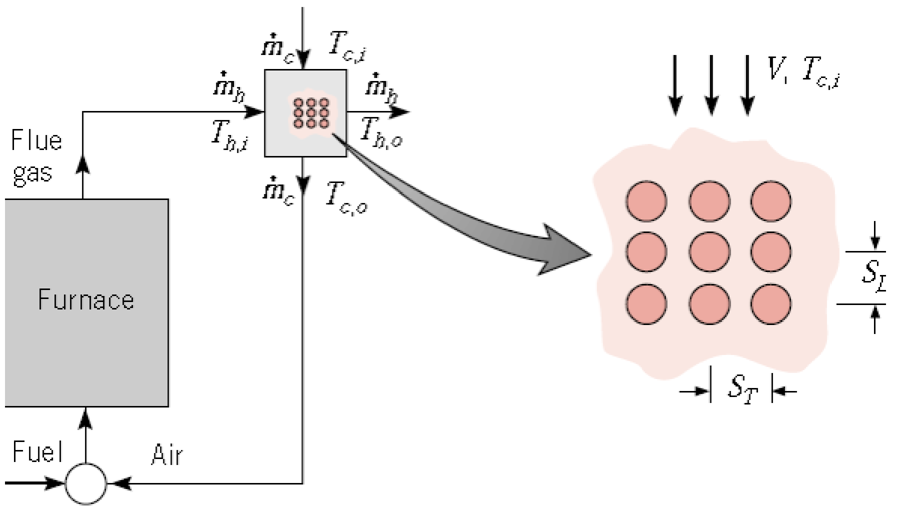
a) evaluate all required air properties at $1atm$ and $300K$

b) assume the flue gas to have the properties of air at $1atm$ and $1400K$

c) assume the tube wall temperature to be at $800K$ for the purpose of treating the effect of variable properties on convection heat transfer.

d) assume fully developed flow inside the tubes

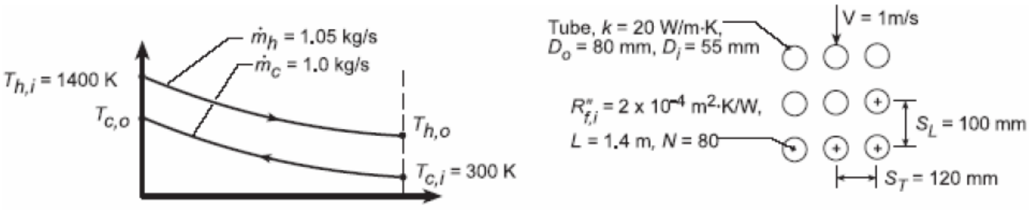
Determine the overall heat transfer coefficient referred to the internal surface.



Solutions

- $h_i = 31.1W/m^2K$, $h_o = 33.6W/m^2K$, $U_i = 18.6W/m^2K$

Solution



Assumptions: (1) Negligible heat loss to surroundings, (2) Air properties are those of atmospheric air at 300 K, (3) Gas properties are those of atmospheric air at 1400 K, (4) Tube wall temperature may be approximated as 800 K fro treating variable property effects.

Properties: *Table A-4,* Air (1 atm, T = 300K) : $\nu = 15.89 \cdot 10^{-6}\mathrm{m/s}, \mathrm{c_p} = 1007\mathrm{J/kg,K}, \mathrm{k} = 0.0263\mathrm{W/m \cdot K}, \mathrm{Pr} = 0.707$; $(\mathrm{T} = 1400\mathrm{K}) : \mu = 530 \cdot 10^{-7}\mathrm{kg/s \cdot m}, \mathrm{c_p} = 1207\mathrm{J/kg}, \mathrm{k} = 0.091\mathrm{W/m \cdot K}, \mathrm{Pr} = 0.703$; $(\mathrm{T} = 800\mathrm{K}) : \mathrm{Pr} = 0.709$

Analysis: (a) With capacity rates of $C_c = \dot{m}_c c_{p,c} = 1kg/s \times 1007\mathrm{J}/kg \cdot K = 1007W/K = C_{\min}$ and $C_h = \dot{m}_h c_{p,h} = 1.05\mathrm{kg/s} \times 1207\mathrm{J/kg \cdot K} = 1267\mathrm{W/K} = \mathrm{C_{max}}$, $\mathrm{C_{min}/C_{max}} = 0.795$. The overall coefficient is

$$\frac{1}{\mathrm{UA}} = \frac{1}{\mathrm{h_i A_i}} + \frac{\mathrm{R''_{f,i}}}{\mathrm{A_i}} + \frac{\ln\left(\mathrm{D_O/D_i}\right)}{(2\pi \mathrm{kL})\mathrm{N}} + \frac{1}{\mathrm{h_o A_o}}$$

For flow through a single tube (consider that the total mass flow rate is flowing around all the 80 tubes of the recuperator), we calculate Re_D as follows:

$$\mathrm{Re_D} = \frac{4\dot{\mathrm{m}}_\mathrm{h}}{\mathrm{N}\pi \mathrm{D_i}\mu} = \frac{4 \times 1.05\mathrm{kg/s}}{80\pi(0.055\mathrm{m})530 \cdot 10^{-7}\mathrm{kg/s \cdot m}} = 5733 > 2300$$

Therefore the flow is turbulent. To determine the convection coefficient for internal forced convection under fully developed and tubulent flow conditions we use the Gnielinski correlation,

$$\mathrm{Nu_D} = \frac{(\mathrm{f}/8)\left(\mathrm{Re_D} - 1000\right)\mathrm{Pr}}{1 + 12.7(\mathrm{f}/8)^{1/2}\left(\mathrm{Pr}^{2/3} - 1\right)} = 18.8$$

where $\mathrm{f} = \left(0.79 \ln \mathrm{Re_D} - 1.64\right)^{-2} = 0.0370$

$$\mathrm{h_i} = \mathrm{Nu_D k/D_i} = 18.8(0.091\mathrm{W/mK})/0.055\mathrm{m} = 31.1\mathrm{W/m^2 \cdot K}$$

For the external flow over the tube bank, we need to determine the maximum velocity of the flow and then $Re_{D,max}$. We note that we have to distinguish between the staggered and aligned tube arrangement. In this case we have aligned arrangement and we calculate:

$$\mathrm{V_{max}} = \left[\mathrm{S_T}/\left(\mathrm{S_T} - \mathrm{D_O}\right)\right]\mathrm{V} = [0.12\mathrm{m}/(0.12 - 0.08)\mathrm{m}]1\mathrm{m/s} = 3\mathrm{m/s}$$
$$\mathrm{Re_{D,max}} = \frac{\mathrm{V_{max} D_0}}{\nu} = \frac{3\mathrm{m/s}(0.08\mathrm{m})}{15.89 \cdot 10^{-6}\mathrm{m^2/s}} = 15,100$$

We then use the Zukauskas correlation for a tube bank, as the $Re_{D,max}$ is within its validity range:

$$\overline{\mathrm{Nu}}_\mathrm{D} = C(Re_{D,max})^m (Pr)^{0.36}(Pr/Pr_s)^{1/4}$$

For $Re_{D,max} = 10^3 - 2 \cdot 10^5$ and aligned tube banks we have: $C = 0.27$ and $m = 0.63$. Therefore:

$$\overline{\mathrm{Nu}}_{\mathrm{D}} = 0.27(15,100)^{0.63}(0.707)^{0.36}(0.707/0.709)^{1/4} = 102.3$$
$$\overline{\mathrm{h}}_{\mathrm{o}} = \overline{\mathrm{Nu}}\mathrm{D}\,(\mathrm{k}/\mathrm{D}_{\mathrm{o}}) = 102.3(0.0263\mathrm{W/mK})/0.08\mathrm{m} = 33.6\mathrm{W/m}^2\cdot\mathrm{K}$$

Hence, based on the inner surface, the *overall coefficient* is

$$\frac{1}{\mathrm{U_i}} = \frac{1}{\mathrm{h_i}} + \mathrm{R}''_{\mathrm{f,i}} + \frac{\mathrm{D_i}\ln\left(\mathrm{D_O}/\mathrm{D_i}\right)}{2\mathrm{k}} + \frac{\mathrm{D_i}}{\mathrm{D_0 h_o}}$$
$$\frac{1}{\mathrm{U_i}} = \left(0.0322 + 0.0002 + \frac{0.055\ln(0.08/0.055)}{40} + \frac{0.055}{0.08\times 33.6}\right)\mathrm{m}^2\cdot\mathrm{K/W}$$
$$\mathrm{U_i} = \left[0.0322 + 0.0002 + 0.001 + 0.0205\mathrm{m}^2\cdot\mathrm{K/W}\right]^{-1} = 18.6\mathrm{W/m}^2\cdot\mathrm{K}$$

Exercise 10.4 FOR REVISION

Saturated water at $1atm$ and velocity $2m/s$ flows over a cylindrical heating element of diameter $5mm$.

a) What is the maximum heating rate $[W/m]$ for nucleate boiling?

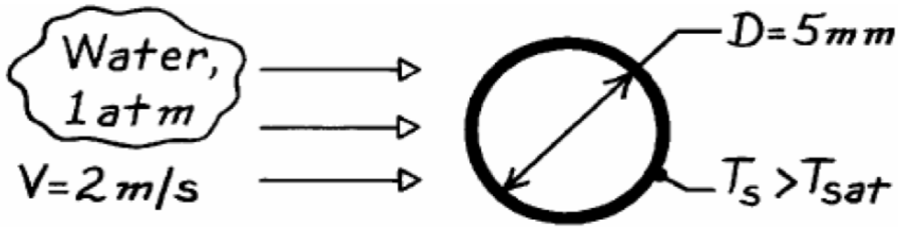
b) How much is the maximum heat flux $[W/m^2]$ enhanced compared to just pool boiling? (Hint: Assume it can be treated as a large horizontal cylinder).

Solutions

a) $q^{'}_{max} = 68kW/m$

b) $q^{"}_{max} = 1.107MW/m^2$

Solution



Assumptions: Nucleate boiling in the presence of external forced convection.

Properties: *Table A-6*, Water (1 atm): $T_{sat} = 100°C, \rho_\ell = 957.9 kg/m^3, \rho_v = 0.5955 kg/m^3, H = 2257 kJ/kg, \sigma = 58.9 \cdot 10^{-3} N/m$

Analysis: To solve this problem we have to refer to forced convection boiling. We see that in such a case to calculate the maximum heat flux there are two options, low velocity and high velocity case. We start by assuming high velocity and we need to later verify whether the obtained heat flux value satisfies the condition for the validity of this correlation. Thus we have:

$$q''_{max} = \frac{\rho_v h_{fg} V}{\pi}\left[\frac{1}{169}\left(\frac{\rho_\ell}{\rho_v}\right)^{3/4} + \frac{1}{19.2}\left(\frac{\rho_\ell}{\rho_v}\right)^{1/2}\left(\frac{\sigma}{\rho_v V^2 D}\right)^{1/3}\right]$$

Substituting numerical values, find

$$q''_{max} = \frac{1}{\pi}0.5955 kg/m^3 \times 2257 \cdot 10^3 J/kg \times 2 m/s \left[\frac{1}{169}\left(\frac{957.9}{0.5955}\right)^{3/4} + \frac{1}{19.2}\left(\frac{957.9}{0.5955}\right)^{1/2}\left(\frac{58.9\times10^{-3} N/m}{0.5955 kg/m^3 (2m/s)^2 0.005m}\right)^{1/3}\right]$$

$$q''_{max} = 4.331 MW/m^2$$

The high-velocity assumption is satisfied if

$$\frac{q''_{max}}{\rho_v h_{fg} V} < \frac{0.275}{\pi}\left(\frac{\rho_\ell}{\rho_v}\right)^{1/2} + 1$$

$$\frac{4.331 \times 10^6 W/m^2}{0.5955 kg/m^3 \times 2257 \cdot 10^3 J/kg \times 2 m/s} = 1.61 < \frac{0.275}{\pi}\left(\frac{957.9}{0.5955}\right)^{1/2} + 1 = 4.51$$

The inequality is satisfied. Using the q''_{max} estimate, the maximum heating rate is

$$q'_{max} = q''_{max} \cdot \pi D = 4.331 MW/m^2 \times \pi(0.005m) = 68.0 kW/m$$

For nucleate pool boiling we have to use:

$$q''_{max} = C h_{fg} \rho_v \left[\frac{\sigma g (\rho_l - \rho_v)}{\rho_v^2}\right]^{1/4}$$

where for large horizontal cylinders

$$C = \pi/24 = 0.131$$

.

Substituting we obtain:

$$q''_{max} = 1.107\text{MW/m}^2$$

and therefore the effect of the forced convection is to increase the critical heat flux by 4.331/1.107=3.91 over the pool boiling case.