Exercise 1(Points: 7.5)

The air inside a hot chamber must be kept at $T_{\infty,i} = 50C$. To achieve this, the air is heated convectively with $h_i = 20W/m^2K$ by a wall (thickness $L = 200mm$) having a uniform heat generation of $1000W/m^3$ and a thermal conductivity of $4W/mK$.

Initially, a very thick insulation layer is applied on the outside surface of the wall to prevent any heat generated within the wall from being lost to the outside.

a) Draw the temperature profile in the system (wall+chamber) as accurately as you can based on the information provided. You can use few words to clarify your choices.

b) Draw the thermal circuit labelling clearly (i) the temperatures of each node (ii) arrows for all heat fluxes involved (iii) all thermal resistances involved.

c) Write the general expression of the temperature profile in the wall and define the mathematical expressions for the boundary conditions at the outer (i.e. $T(x = 0)$) and inner (i.e. $T(x = L)$) surfaces of the wall.

d) Calculate the temperature on the outer and inner surfaces of the wall.

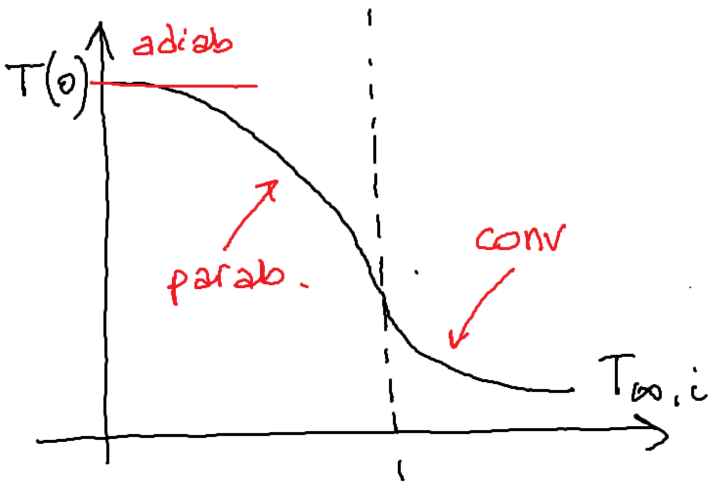
e) An engineer suggests to replace the bulky insulation layer on the outer surface of the wall with a very thin electrical strip heater, providing a uniform heat flux $q_o^{"}$. On the outside, air at $T_{\infty,o} = 25C$ can now convectively cool the wall with $h_o = 5W/m^2K$. Determine the value of $q_o^{"}$ that must be supplied by the strip heater so that all heat generated within the wall is still transferred to the inside of the chamber.

f) Because of processing needs, the heat generation in the wall is switched off while the heat flux to the strip heater remains constant. Draw the new thermal circuit representing the problem. Label clearly (i) the temperatures of each node (ii) arrows for all heat fluxes involved (iii) all thermal resistances involved.

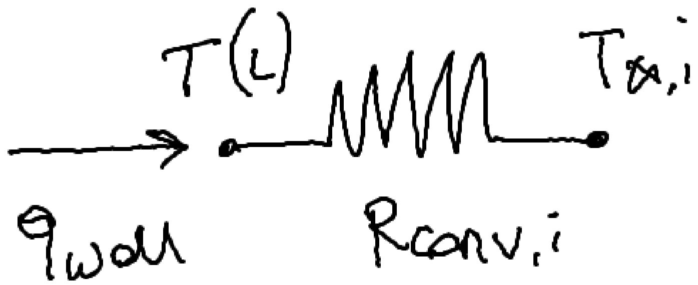
g) For this last situation, assuming that the temperature of the hot chamber remains at $T_{\infty,i} = 50C$, calculate the new steady state temperature of the outer wall surface, $T(x = 0)$.

Solution

a) **1.25 pts** : 0.5 adiabatic, 0.5 parabolic, 0.25 convection



b) **0.75 pts** : 0.25 convection resistance, 0.25 input heat flux from layer with heat sources, 0.25 for temperatures.



c) **1.25 pts** : 0.25 for the general expression (from formula sheet); 0.5 for outer wall BC (0.25 for left hand side and 0.25 for right hand side);0.5 for inner wall BC (0.25 for left hand side and 0.25 for right hand side - convection);

$$T(x) = -\frac{\dot{q}}{2k}x^2 + C_1 x + C_2$$

For the outer wall the heat flux is zero and the derivative of temperature is zero:

$$\frac{dT}{dx}_{x=0} = 0$$

For the inner wall we have a heat flux boundary condition given by the convective heat flux:

$$-k\frac{dT}{dx}_{x=L} = h(T(L) - T_{\infty,i})$$

d) **1.25 pts** : 0.25 for C_1; 0.25 for $T(L)$ expression, 0.25 for $T(L)$ value; 0.25 for C_2 expression; 0.25 for value of $T(0) = C_2$

From the first boundary condition we get:

$$\frac{dT}{dx}_{x=0} = C_1 = 0$$

From the second boundary condition we get:

$$-k(-\frac{\dot{q}}{k}x + C_1)_{x=L} = h(T(L) - T_{\infty,i})$$
$$\dot{q}L - kC_1 = h(T(L) - T_{\infty,i})$$
$$T(L) = \frac{\dot{q}L}{h} + T_{\infty,i} = 60C$$

From the general expression of the temperature profile we can also write:

$$T(L) = -\frac{\dot{q}L^2}{2k} + C_2$$

and thus equating with the expression above:

$$C_2 = \frac{\dot{q}L}{h} + T_{\infty,i} + \frac{\dot{q}L^2}{2k}$$

and therefore:

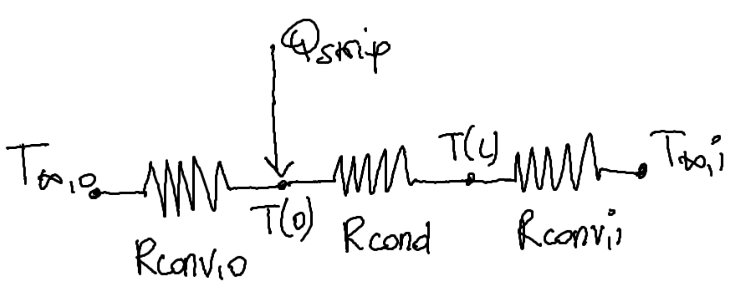
$$T(0) = C_2 = T(L) + \frac{\dot{q}L^2}{2k} = 65C$$

e) **0.75 pts** : 0.25 for the reasoning ; 0.25 for the expression and 0.25 for the value.

The heater strip must compensate exactly the heat lost by convection to the outside air. Therefore:

$$q_o^{"} = h_o(T(0) - T_{\infty,o}) = 200W/m^2$$

f) **1 pts** : 0.25x3 for the thermal resistances ; 0.25 for the input heat flux on the heater.



g) **1.25 pts** : 0.25 for writing the heat flux balance; 0.25 for the $q_{conv,o}^{"}$ expression; 0.5 for the $q_{cond+conv,i}$ expression (0.25 for conduction resistance, 0.25 for series); 0.25 for the value.

$$q_o^{"} = q_{conv,o}^{"} + q_{cond+conv,i}$$

$$q_o^{"} = \frac{T(0) - T_{\infty,o}}{1/h_o} + \frac{T(0) - T_{\infty,i}}{L/k + 1/h_i}$$

$$T(0) = 55C$$

Exercise 2(Points: 2.5)

Plasma spray coating processes are often used to provide surface protection for materials exposed to hostile environments. Ceramic coatings are commonly used for this purpose. By injecting ceramic powder through the nozzle of a plasma torch, the ceramic particles are accelerated and heated within the plasma jet. During their time-in-flight the ceramic particle must be heated to their melting point and experience complete conversion to the liquid state.

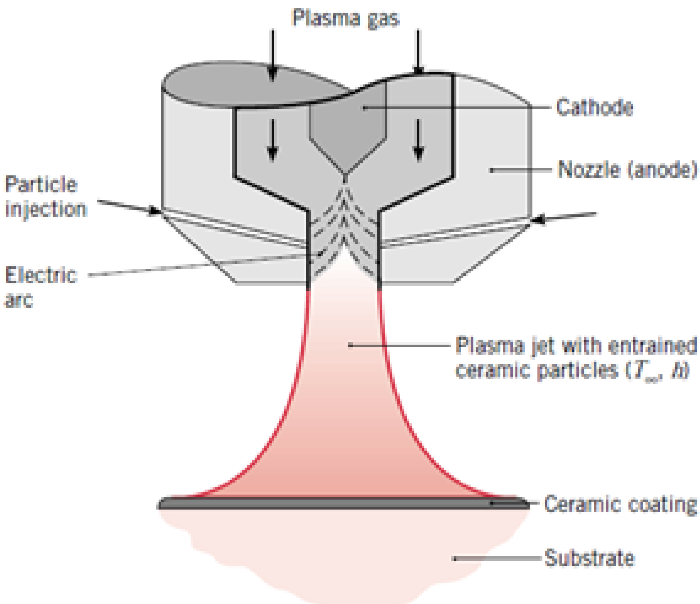
Consider the case of spherical alumina (Al_2O_3) particles of diameter $D = 50\mu m$, density $\rho = 3970kg/m^3$, thermal conductivity $k = 10.5W/mK$, specific heat $c_p = 1560J/kgK$, melting point $T_m = 2318K$ and latent heat of fusion $h_f = 3577kJ/kgK$.

The particles, initially at $T_i = 300K$, are injected into a plasma at $T_\infty = 10000K$ that provides a convection coefficient $h = 30000W/m^2K$. It is assumed that radiative heat transfer is negligible throughout the process.

a) Identify the correct model for this transient heating problem.

b) Calculate the time t_1 needed to heat up the nanoparticles up to their melting temperature.

c) Calculate the time t_2 required to achieve complete melting of the nanoparticle.



Solution

a) **0.75 pts**: 0.25 for the choice of Bi; 0.25 for value; 0.25 for Lumped capacitance model

We calculate the Biot number for this problem:

$$Bi = \frac{hV}{kA} = \frac{hR}{3k} = \frac{30000 \cdot 25 \cdot 10^{-6}}{3 \cdot 10.5} = 0.0238 << 0.1$$

Therefore we can use the lumped capacitance model.

b) **0.75 pts** : 0.25 for expression; 0.25 for correct T values; 0.25 for t_1 value

$$t_1 = \frac{\rho c_p V}{hA} \ln \frac{T_i - T_\infty}{T_m - T_\infty} = \frac{3970 \cdot 1560 \cdot 25 \cdot 10^{-6}}{3 \cdot 30000} ln \frac{300 - 10000}{2318 - 10000} = 0.0004s$$

c) **1 pts**: 0.5 for correct energy balance (0.25 left and 0.25 right hand side); 0.25 t_2 expression; 0.25 t_2 value

From an energy balance during the melting process we have:

$$\Delta E = \rho V h_f = \int_{t_1}^{t_1+t_2} q_{conv} dt = \int_{t_1}^{t_1+t_2} hA(T_\infty - T_m) dt = hA(T_\infty - T_m)t_2$$

$$t_2 = \frac{\rho R h_f}{3h} \frac{1}{(T_\infty - T_m)} = 0.0005s$$

Exercise 3(Points: 9.5)

One method to grow micro/nanowires is to deposit a small droplet of a liquid catalyst onto a flat surface. The surface and the catalyst are heated and simultaneously exposed to high-temperature air that contains a mixture of species from which the wire is to be formed. The catalytic liquid slowly absorbs the species from the gas and converts them to a solid material that is deposited onto the underlying solid/liquid interface. This results in the growth of a micro/nanowire. As this grows, the liquid remains suspended at its tip and moves further away from the flat surface (see Figure).

Consider the growth of a $5\mu m$ diameter silicon carbide microwire onto a silicon carbide surface. The surface is maintained at a temperature $T_s = 2000K$ and the thermal conductivity of the silicon carbide is $30W/mK$.

The hot air has a temperature of $T_\infty = 3000K$ and it is blown with a speed of $v = 30m/s$ through the chamber to continuously heat the catalyst and the microwire.

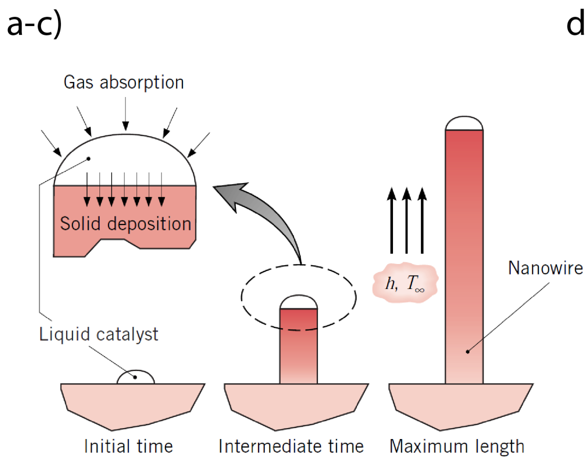
a) Motivating your choice of the convection correlation, calculate the convection coefficient between the nanowire and the hot air. Use $T_f = \frac{T_s+T_\infty}{2}$ for estimating the physical properties.

b) Sketch a qualitative temperature profile along the nanowire. Clearly indicate the boundary conditions at the base and tip of the nanowire.

c) The nanowire grows until the liquid catalyst reaches the temperature of $2500K$. For higher temperatures the growth stops. Under these conditions, is it possible to grow a nanowire with length $L = 50\mu m$?

d) The microwire growth technique is used to realize a micro-heat sink for an electronic package. This consists of a $100\mu m$ x $100\mu m$ silicon electronic device encapsulated by two silicon carbide sheets ($d_{sheet} = 100nm$, $k = 490W/mK$ see Figure). Heat is generated in the inner silicon layer. The chip is cooled blowing a dielectric coolant at $T_\infty = 20C$ both on top and bottom of the chip, achieving a convection coefficient $h = 10^5 W/m^2K$. The **finned** system presents two arrays of silicon carbide microwires (at the top and at the bottom), each consisting of 10 x 10 elements with $D = 5\mu m$ and length $L = 50\mu m$. Draw the thermal circuit for the finned electronic chip, labelling clearly (i) the temperatures of each node (ii) arrows for all heat fluxes involved (iii) all thermal resistances involved.

e) For the finned chip, calculate the allowable heat rate that can be generated in the chip if the silicon temperature should not exceed $T_max = 85C$. Use the Fin Efficiency Table provided below.



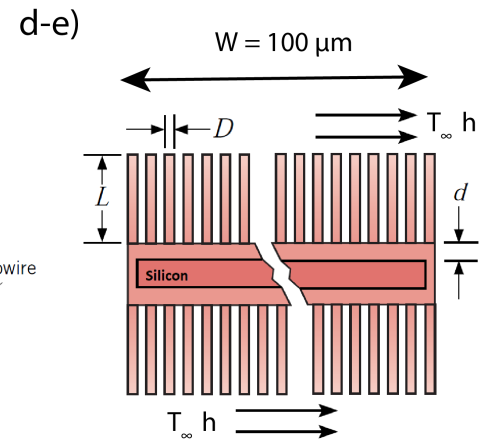
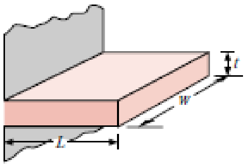
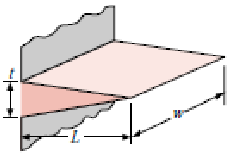
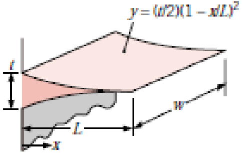
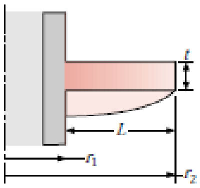
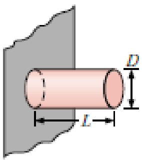
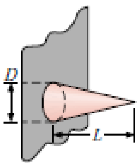


Table 3.5 Efficiency of common fin shapes

Straight Fins			
Rectangular[a] $A_f = 2wL_c$ $L_c = L + (t/2)$ $A_p = tL$		$\eta_f = \frac{\tanh mL_c}{mL_c}$	(3.89)
Triangular[a] $A_f = 2w[L^2 + (t/2)^2]^{1/2}$ $A_p = (t/2)L$		$\eta_f = \frac{1}{mL}\frac{I_1(2mL)}{I_0(2mL)}$	(3.93)
Parabolic[a] $A_f = w[C_1L + (L^2/t)\ln(t/L + C_1)]$ $C_1 = [1 + (t/L)^2]^{1/2}$ $A_p = (t/3)L$		$\eta_f = \frac{2}{[4(mL)^2 + 1]^{1/2} + 1}$	(3.94)
Circular Fin			
Rectangular[a] $A_f = 2\pi(r_{2c}^2 - r_1^2)$ $r_{2c} = r_2 + (t/2)$ $V = \pi(r_2^2 - r_1^2)t$		$\eta_f = C_2\frac{K_1(mr_1)I_1(mr_{2c}) - I_1(mr_1)K_1(mr_{2c})}{I_0(mr_1)K_1(mr_{2c}) + K_0(mr_1)I_1(mr_{2c})}$ $C_2 = \frac{(2r_1/m)}{(r_{2c}^2 - r_1^2)}$	(3.91)
Pin Fins			
Rectangular[b] $A_f = \pi DL_c$ $L_c = L + (D/4)$ $V = (\pi D^2/4)L$		$\eta_f = \frac{\tanh mL_c}{mL_c}$	(3.95)
Triangular[b] $A_f = \frac{\pi D}{2}[L^2 + (D/2)^2]^{1/2}$ $V = (\pi/12)D^2L$		$\eta_f = \frac{2}{mL}\frac{I_2(2mL)}{I_1(2mL)}$	(3.96)

Solution

a) **1.5 pts** : 0.25 for forced external convection; 0.25 for cylinder geometry and Churchill Bernstein relation; 0.25 for the physical properties from Table A4 (ρ, μ, k, Pr); 0.25 for Re; 0.25 for Nu ; 0.25 for convection coeff .

External forced convection around a cylinder (Churchill-Bernstein correlation):

$$Nu_D = 0.3 + \frac{0.62 Re_D^{1/2} Pr^{1/3}}{[1 + (0.4/Pr)^{2/3}]^{1/4}} \left[1 + \left(\frac{Re_D}{282000}\right)^{5/8}\right]^{4/5}$$

From Table $A.4$ for air at $T_f = 2500K$ we get:

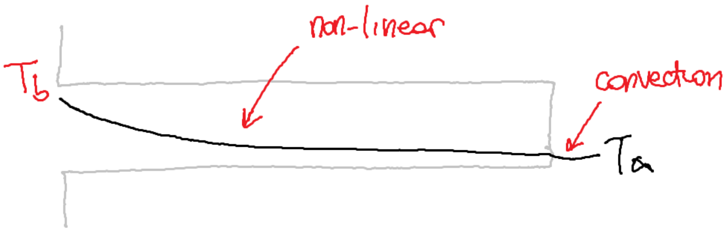
$$\rho(2500K) = 0.1389 kg/m^3$$
$$\mu(2500K) = 818 \cdot 10^{-7} Ns/m^2$$
$$k(2500K) = 0.222 W/mK$$
$$Pr(2500K) = 0.613$$

We thus get:

$$Re = \frac{\rho v D}{\mu} = \frac{0.1389 \cdot 30 \cdot 5 \cdot 10^{-6}}{818 \cdot 10^{-7}} = 0.255$$

$$Nu_D = 0.531$$
$$h = \frac{Nu_D k}{D} = \frac{0.531 \cdot 0.222}{5 \cdot 10^{-6}} = 23576.4 W/m^2K$$

b) **0.75 pts** : 0.25 non linear temperature profile (fin-like); 0.25 temperature base; 0.25 convection at the tip.



c) **2.25 pts** : 0.5 for recognizing the use of fins; 0.25 for the right expression (convection BC); 0.25 for $x = L$; 0.25 for correct A_c; 0.25 for correct P ; 0.25 for m value; 0.25 for $T(L)$ value; 0.25 for the correct answer (YES);

To know whether a $50\mu m$ long wire can be grown under these conditions, we need to calculate the temperature at the tip. If it is below $2500K$ then the wire can grow, if it is above it will not be able to grow.

We treat the microwire as a fin with convection at the tip.

$$\frac{T(L) - T_\infty}{T_s - T_\infty} = \frac{cosh(0) + (h/mk) sinh(0)}{cosh(mL) + (h/mk) sinh(mL)}$$

$$\frac{T(L) - 3000}{2000 - 3000} = \frac{1}{cosh(mL) + (h/mk) sinh(mL)}$$

where we have:

$$m = \sqrt{\frac{hP}{kA_c}} = \sqrt{\frac{4h}{kD}} = \sqrt{\frac{4 \cdot 23576.4}{30 \cdot 5 \cdot 10^{-6}}} = 25074m^{-1}$$

Therefore:

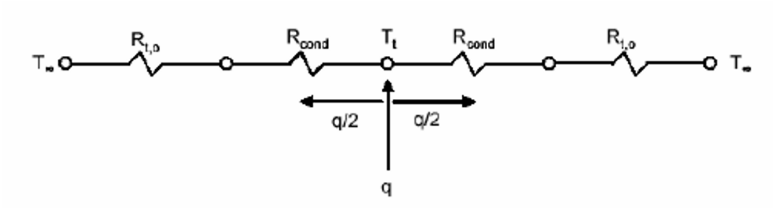
$$\frac{h}{mk} = \frac{23576.4}{25074 \cdot 30} = 0.0313$$
$$mL = 25074 \cdot 50 \cdot 10^{-6} = 1.2537$$

and we get:

$$T(L) = 3000 - 1000\frac{1}{cosh(1.2537) + 0.0313sinh(1.2537)} = 3000 - 1000\frac{1}{1.894 + 0.0313 \cdot 1.61} = 2485.7K$$

As the temperature is below the threshold value for growth, we can have a microwire of this length.

d) **2 pts**: 1 (0.25 x 4) for thermal resistances; 0.5 (0.25 x 2) for heat fluxes; 0.25 for T_c ; 0.25 for T_∞.



e) **3 pts**: 0.25 Rcond expression; 0.25 Rcond value; 0.25 Af; 0.25 Ab; 0.25 At; 0.25 etaf; 0.25 etat; 0.25 Rt expression and 0.25 Rt value; 0.25 serie resistance of Rcond and Rt; 0.25 expression of Q; 0.25 value of Q.

$$R_{cond} = \frac{d_{sheet}}{kA} = \frac{100 \cdot 10^{-9}}{490(100 \cdot 10^{-6})^2} = 0.0204K/W$$

$$R_t = \frac{1}{\eta_t h A_{tot}}$$

where:

$$\eta_t = 1 - \frac{NA_f}{A_{tot}(1 - \eta_f}$$
$$A_{tot} = NA_f + A_b$$

We have in this case:

$$A_f = \pi DL + \pi D^2/4 = \pi 5 \cdot 10^{-6} \cdot 50 \cdot 10^{-6} + \pi(5 \cdot 10^{-6})^2/4 = 8 \cdot 10^{-10}m^2$$

$$A_b = W^2 - N\pi D^2/4 = (100 \cdot 10^{-6})^2 - 200^2\pi(5 \cdot 10^{-6})^2/4 = 9.875 \cdot 10^{-9}m^2$$
$$A_t = 100 \cdot 8 \cdot 10^{-10} + 9.875 \cdot 10^{-9} = 8.9875 \cdot 10^{-8}m^2$$

We then have (we can accept also the use of only the length of the fin):

$$mL = \sqrt{\frac{4h}{kD}}L = \sqrt{\frac{4 \cdot 10^5}{490 \cdot 5 \cdot 10^{-6}}}(\pi DL + \pi D^2/4) = 0.655$$

From the table of efficiencies:

$$\eta_f = \frac{tanh(mL)}{mL} = \frac{0.575}{0.655} = 0.878$$

$$\eta_t = 1 - \frac{100 \cdot 8 \cdot 10^{-10}}{8.9875 \cdot 10^{-8}}(1 - 0.878) = 0.891$$

$$R_t = \frac{1}{0.891 \cdot 10^5 \cdot 8.9875 \cdot 10^{-8}} = 124.9K/W$$

Finally, using the thermal circuit we can calculate the heat rate that can be transferred in the presence of the fins is the temperature of the silicon is $T_c = 85C$:

$$\frac{Q}{2} = \frac{T_c - T_\infty}{R_{cond} + R_t}$$

$$Q = 2\frac{85 - 20}{0.0204 + 124.9} = 1.04W$$

Exercise 4(Points: 10)

Waste heat from the exhaust gas of an industrial furnace is recovered by mounting a bank of unfinned tubes in the furnace stack. Pressurized water makes a single pass through the tubes with a flow rate in **EACH** of the tubes of $0.075kg/s$. Instead, $2.25kg/s$ of exhaust gas, which has an upstream velocity of $15m/s$, move in cross flow over the tubes.

The tube bank consists of a square array of 100 thin-walled tubes (10x10), each $15mm$ in diameter and $4m$ long. The tubes are aligned with a transverse pitch of $50mm$.

The inlet temperatures of the water and the exhaust gas are $300K$ and $800K$, respectively. The water flow is fully developed and the gas properties my be assumed to be those of air at atmospheric pressure.

a) Sketch the temperature profile for the heat exchanger. Label clearly inlet and outlet temperatures and indicate the total mass flow rates for the two working fluids.

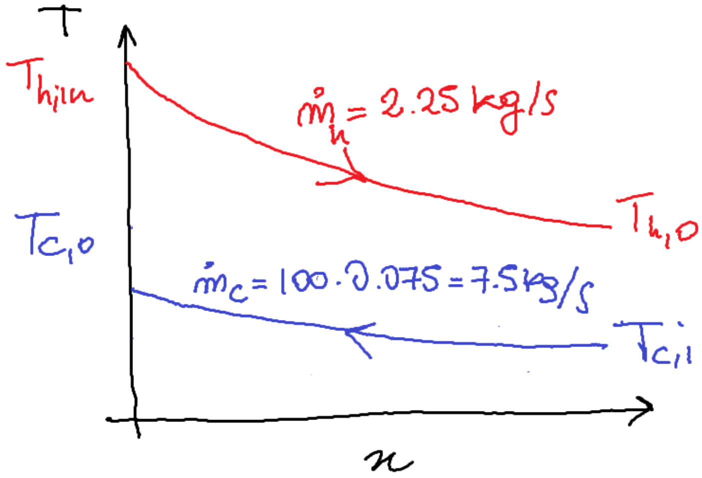
b) Calculate the overall heat transfer coefficient for this heat exchanger. Use $T_{m,w} = 325K$ to estimate the water properties and $T_{m,g} = 600K$ to estimate the exhaust gas properties and $T_{s,m} = 470K$ as the tube average surface temperature.

c) Calculate the water and exhaust gas outlet temperatures.

d) After some months of operation it is observed that the thin walled tubes are easily damaged, causing leakages. Therefore, it is necessary to replace them with copper tubes ($k = 15W/mK$) of inner diameter $D_i = 15mm$ and wall thickness of $10mm$. Assuming the convection coefficient are not affected by this change, calculate the new overall heat transfer coefficient with respect to the inner area.

Solution

a) **0.5 pts**: 0.25 for non-linear lines; 0.25 for total mass flow rate of water multiplied by the number of tubes.



b) **4.5 pts**: 0.25 for $UA = 1/Rtot$; 0.25 for series resistance; 0.25 for Rconv,in expression; 0.25 for Rconv,o expression; INNER Coeff: 0.25 for mu, Pr, k (values); 0.25 for ReD (value); 0.25 for right correlation; 0.25 for right exponent; 0.25 for NuD value; 0.25 for hi value; OUTER Coeff: 0.25 for physical properties at 600K (rho, mu, Pr, k); 0.25 for Prs; 0.25 for correct correlation; 0.25 for ReDmax; 0.25 for correct coeffients (C2, C, m); 0.25 for Nud value; 0.25 for ho value; 0.25 for U value. The overall heat transfer coefficient is:

$$UA = \frac{1}{R_{tot}} = \frac{1}{1/h_i A_i + 1/h_o A_o}$$

As the tubes are thin walled $A_i \approx A_o \approx A$ thus:

$$U = \frac{1}{1/h_i + 1/h_o}$$

For h_i we need to consider forced internal convection in circular tubes. Thus we have to compute Reynolds:

$$Re_D = \frac{4 \cdot \dot{m}}{\pi D \mu}$$

where: $\mu = 528 \cdot 10^{-6} Ns/m^2$, $k = 0.645 W/mK$ and $Pr = 3.42$ thus:

$$Re_D = \frac{4 \cdot 0.075}{\pi 0.015 \cdot 528 \cdot 10^{-6}} = 12063.3$$

As the water is heating we use:

$$Nu_D = 0.023 Re_D^{4/5} Pr^{0.4} = 0.023 \cdot 12063.3^{4/5} \cdot 3.42^{0.4} = 69.3$$

$$h_i = \frac{k_w}{D} Nu_D = \frac{0.645}{0.015} 69.3 = 2979.9 W/m^2K$$

For h_O we need to consider forced external convection on a bank of tubes. The tubes are aligned and we have 10 rows. Thus we calculate:

$$V_{max} = \frac{S_T}{S_T - D}v = \frac{0.05}{0.05 - 0.015} \cdot 15 = 21.4m/s$$

$$Nu_D = C_2 C Re_{D,max}^m Pr^{0.36} (\frac{Pr}{Pr_S})^{1/4}$$

where:

(a) $\rho = 0.5804kg/m^3$; $\mu = 305.8 \cdot 10^{-7} Ns/m^2$; $Pr = 0.685$; $Pr_S = 0.6852$; $k = 0.0469W/mK$

(b) $Re_{D,max} = \frac{\rho V_{max} D}{\mu} = 6092.5$

(c) $C = 0.27$; $m = 0.63$

(d) $C_2 = 0.97$

(e) $Pr_S = 0.6852$

hence:

$$Nu_D = 0.97 \cdot 0.27 \cdot 6092.5^{0.63} 0.685^{0.36} (\frac{0.685}{0.6852})^{1/4} = 55.4$$

$$h_o = \frac{k_g}{D} Nu_D = \frac{0.0469}{0.015} \cdot 55.4 = 173.2W/m^2K$$

$$U = \frac{1}{1/2979.9 + 1/173.2} = 163.7W/m^2K$$

c) **4 pts**: 0.25 for cp values; 0.25 for C values; 0.25 for right Cmin; 0.25 for NTU expression; 0.25 for NTU expression; 0.25 for NTU value; 0.25 for Cr; 0.25 for choosing right expression; 0.25 for epsilon; 0.25 for Qmax expression; 0.25 for Qmax value; 0.25 for Q expression; 0.25 for Q value; 0.25 for equation Tout; 0.25 vor Tho; 0.25 for Tco.

To solve for the outlet temperatures we need to use the ϵ-NTU method. As we know the ovreall heat transfer coefficient,we can obtain the NTU and then calculate ϵ to find the total heat transfer and the outlet temperatures.

$$NTU = \frac{UN\pi DL}{C_{min}}$$

where:$c_{p,w} = 4188J/kgK$ and $c_{p,g} = 1051J/kgK$ thus:

$$C_c = C_w = \dot{m}_w c_{p,w} = 0.075 \cdot 100 \cdot 4188 = 31410W/K = C_{unmixed}$$

$$C_h = C_g = \dot{m}_g c_{p,g} = 2.25 \cdot 1051 = 2364.75W/K = C_{mixed} = C_{min}$$

$$NTU = \frac{163.7(100\pi 0.015 \cdot 4)}{2364.75} = 1.3$$

$$C_r = \frac{C_{min}}{C_{max}} = \frac{C_{mixed}}{C_{unmixed}} = 0.0753$$

and from Table 11.3 we use:

$$\epsilon = 1 - exp(-\frac{1}{C_r}(1 - exp(-C_r \cdot NTU)))$$
$$\epsilon = 1 - exp(-\frac{1}{0.0753}(1 - exp(-0.0753 \cdot 1.3))) = 0.71$$

We also have that:

$$Q_{max} = C_{min}(T_{h,i} - T_{c,o}) = 2364.75(800 - 300) = 1.18MW$$

ans

$$Q = \epsilon Q_{max} = 837800W$$

Finally:

$$T_{h,o} = T_{h,i} - \frac{Q}{C_h} = 445.7K$$
$$T_{c,o} = T_{c,i} + \frac{Q}{C_c} = 326.7K$$

d) **1 pts**: 0.25 Rcondcyl; 0.25 multiplied by Ai; 0.25 value of conduction term; 0.25 value of U

If the tubes are substituted with thick wall tubes we need to account for the conduction resistance of the tube in the overall heat transfer coefficent/

$$A_i R_{cond} = \frac{ln(r_o/r_i)}{2\pi Lk} \cdot (2\pi r_i L) = \frac{r_i ln(r_o/r_i)}{k} = \frac{0.0075 ln(0.025/0.015)}{15} = 2.55 \cdot 10^{-4}$$

$$U = \frac{1}{1/h_i + A_i R_{cond} + 1/h_o} = \frac{1}{1/2979.9 + 2.55 \cdot 10^{-4} + 1/173.2} = 157.1W/m^2K$$

Exercise 5(Points: 5)

A counter-flow concentric tube heat exchanger (thin-walled tube) used for engine cooling has been in service for an extended period of time. The heat transfer surface area of the exchanger is $5m^2$ and the design value of the overall heat transfer coefficient is $U_{design} = 40W/m^2$.

During a test run, engine oil flowing at $0.1kg/s$ is cooled from $110C$ to $64C$ by water supplied at a temperature of $25C$ and a flow rate of $0.2kg/s$.

a) Sketch the temperature profile for the heat exchanger. Label clearly inlet and outlet temperatures and indicate the total mass flow rates for the two working fluids.

b) Determine whether fouling has occurred during the service period. Use $T_{m,w} = 305K$ to estimate the thermophysical properties of water.

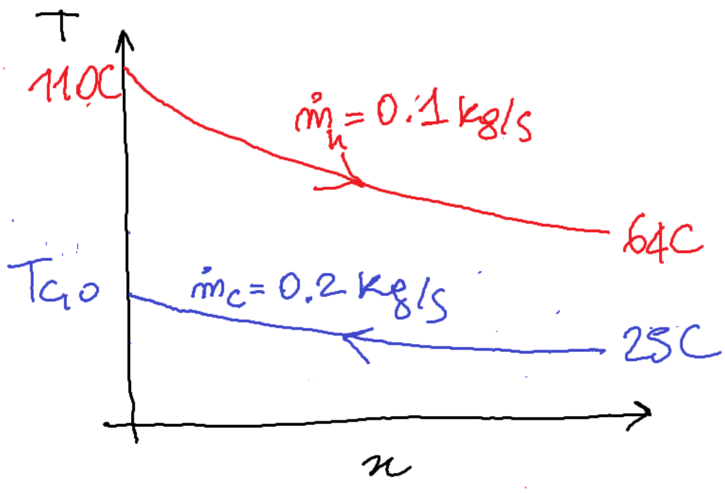
c) Calculate the fouling factor $R_f^{"}[m^2K/W]$.

d) To reduce water consumption, the heat exchanger is modified to use an evaporating refrigerant to cool the exhaust gas between the same temperatures. Sketch the temperature profile for the heat exchanger under the new operating conditions.

e) Calculate the necessary mass flow rate of refrigerant required in the closed-loop circuit.

Solution

a) **0.75 pts**: 0.5 for non-linear temperatures; 0.25 for temperature values and mass flow rates.



b) **2.25 pts**: 0.25 for physical properties (both together); 0.25 for energy balance and 0.25 for value of Q; 0.25 for energy balance cold side and 0.25 for value of Tco; 0.25 for concentricHE heat transfer expression; 0.25 for DTln value; 0.25 for value of U and 0.25 for answer to question (fouling occurred).

For the engine oil and hot water we get:

(a) $c_{p,h} = 2161 J/kgK$

(b) $c_{p,c} = 4178 J/kgK$

From an energy balance on the hot fluid we have:

$$Q = \dot{m}_h c_{p,h}(T_{h,i} - T_{h,o}) = 0.1 \cdot 2161 \cdot (110 - 64) = 9940.6W$$

Thus:

$$T_{c,o} = T_{c,i} + \frac{Q}{\dot{m}_c c_{p,c}} = 25 + \frac{9940.6}{0.2 \cdot 4178} = 36.9K$$

For the concentric heat exchangers we have:

$$Q = UA\Delta T_{ln} = UA\frac{(T_{h,i} - T_{c,o}) - (T_{h,o} - T_{c,i})}{ln((T_{h,i} - T_{c,o})/(T_{h,o} - T_{c,o}))}$$

Thus:

$$\Delta T_{ln} = \frac{(110 - 36.9) - (64 - 25)}{ln((110 - 36.9)/(64 - 25))} = 54.3K$$

$$U = \frac{Q}{A\Delta T_{ln}} = \frac{9940.6}{5 \cdot 54.3} = 36.6W/m^2K \neq U_{design}$$

Therefore, fouling has occurred.

c) **1 pts**: 0.25 for Rtot design; 0.25 for Rtot real; 0.25 for relation btw design and operation; 0.25 for value of Rf.

The overall heat transfer coefficient can be written as:

$$U = \frac{1}{AR_{tot}}$$

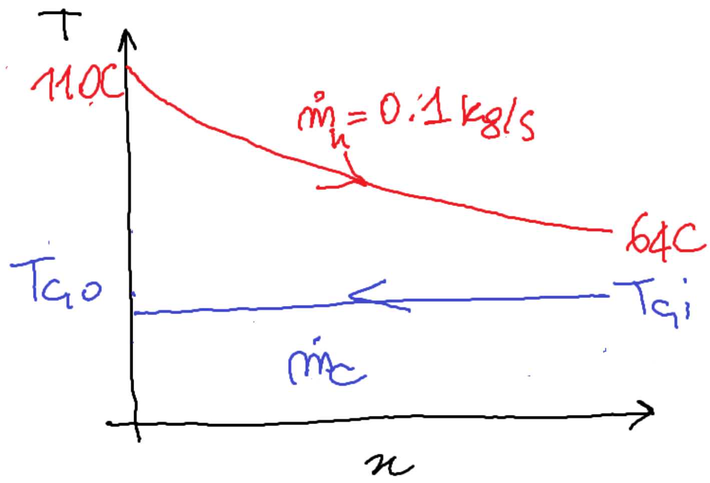
where :

$$AR_{tot} = \frac{1}{h_i} + R_f^{"} + \frac{1}{h_o} = AR_{tot,design} + R_f^{"} = \frac{1}{U_{design} + R_f^{"}}$$

thus:

$$R_f^{"} = AR_{tot} - AR_{tot,design} = \frac{1}{U} - \frac{1}{U_{design}} = 0.0023 m^2K/W$$

d) **0.5 pts**: 0.25 for non-linear temperature on the hot side; 0.25 for a flat line on the cold side.



e) **0.5 pts**: 0.25 for latent heat; 0.25 for the mass flow rate

From Table A.5 p 948, the latent heat of evaporation for the refrigerant is: $h_{fg} = 217000 J/kg$.

The required mass flow rate is equal to:

$$\dot{m} = \frac{Q}{h_{fg}} = \frac{9940.6}{217000} = 0.0458 kg/s$$